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# *PPTs and PDF materials*

# ***PPTs by Teachers***

# KLE's G. I. Bagewadi College Nipani Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: **C**alculus–II and 3–D Geometry

I online Class

Introduction of Partial differentiation

By

Dr. M. M. Shankrikopp

HOD of Mathematics

Date: 30.4.2021

# SYLLABUS

## UNIT-I

- Polar coordinates of a point and polar curve. Angle between the radius vector and the tangent at a point on the curve. Angle of intersection of two curves. Polar and pedal equation of the curves. Polar sub-tangent and polar sub - normal. **12 hours**

## UNIT-II

- Derivative of arc length, Curvature, Radius of curvature in Cartesian, Parametric, polar and pedal forms. Centre of curvature. Evolutes and Involutes. **12 hours**

# SYLLABUS

## UNIT-III

- Limits, continuity of functions of two variables. Partial derivatives, higher order partial derivatives, total derivatives and total differentials, Homogeneous functions, Euler's theorem on homogeneous functions. **12hours**

## UNIT-IV

- Reduction formulae for integration of $\sin^n x$, $\mathrm{Cos}^n x$, $\tan^n x$, $\cot^n x$, $\sec^n x$, $\mathrm{cosec}^n x$, $\sin^n x \cos^n x$, $x^n e^{ax}$ and $x^m(\log x)^n$. **12 hours**

# SYLLABUS

## UNIT-V

- **Sphere**: Equation of a sphere, section of a sphere by a plane, Equation of a sphere through a circle, Equation of a sphere through two given points as ends of a diameter. Equation to a tangent plane of a sphere, Condition for tangency, Orthogonality of two spheres.
- **Cone**: Equation of a cone, enveloping cone of a sphere, Right circular cone.
- **Cylinder:** Equation of cylinder, enveloping cylinder of a sphere, Right circular cylinder.
- **12 hours**

# BOOKS FOR REFERENCE

- Integral Calculus : Santinarayan and Dr. P.K. Mittal
- Differential Calculus and integral Calculus : N.P. Bali
- Text Book of B.Sc Mathematics: G. K. Ranganath
- Text book of Mathematics III: S.S. Bhoosnurmath and others

# COURSE OUTCOMES OF THE PAPER

- Able to find differentiation of polar curves, angle between two polar curves and pedal equation for all type of curves.
- Able to determine formula for curvature, evaluate and application of curvature in construction of lenses. Concavity and convexity and points of inflexion.
- Able to find limits, continuity of functions of two variables. Partial derivatives, higher order partial derivatives, total derivatives and total differentials, Homogeneous functions, Euler's theorem on homogeneous functions.
- Recognize to determine given equation represents sphere, Cone and Cylinder and gain the knowledge properties of these 3-D figures.

# DISTRIBUTUION OF SYLLABUS

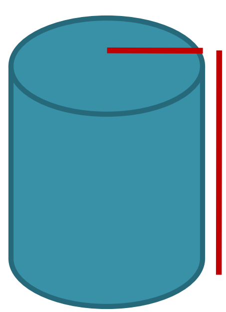
| Name | Units |
|---|---|
| 1. Dr. M. M. Shankrikopp | III: Partial Differentiation |
| | V: 3-D Geometry |
| 2. Dr. Ashok Rathod | I: Polar Coordinates |
| | II: Curvature and Evolute |
| 3. Miss G.L.Karaguppi | III: Reduction Formulae |

# UNIT III: PARTIAL DIFFERENTIATION

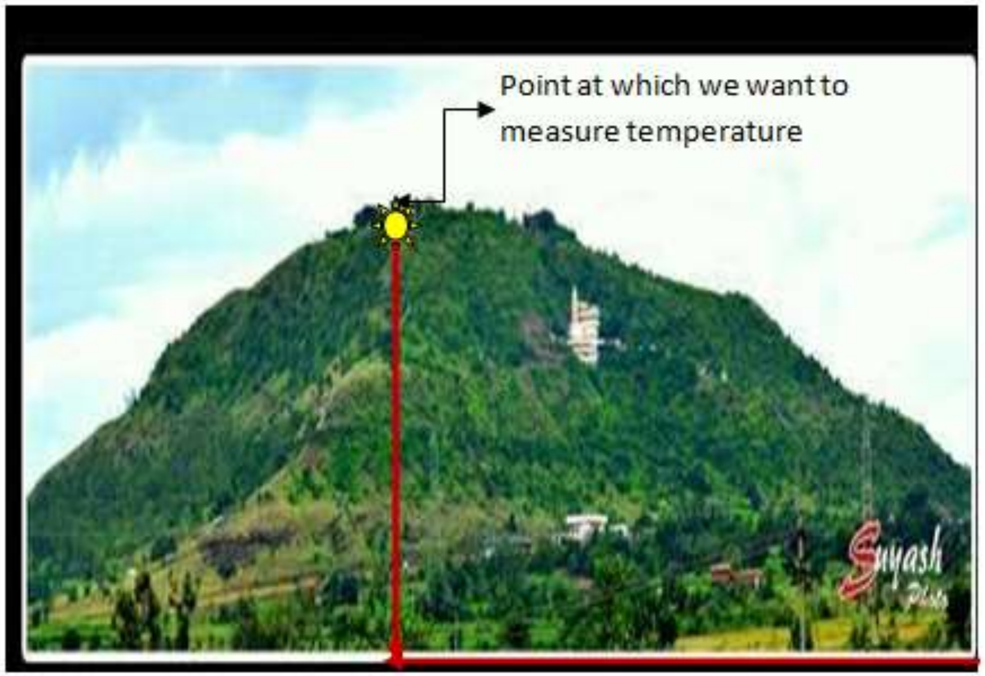
**Introduction:**

In PUC and B.Sc. I sem.we studied continuity and differentiability of function f(x) of one independent variable x. Sometimes the function or quantity depends on more than one variable.

**For example**: We know that the formula $V = \pi r^2 h$ gives the volume of cylinder. Here if volume V is the function depends on both radius and height of the cylinder which we can choose independently, ie if there is change in radius or height automatically there will be change in volume. So we conclude that V is function of two independent variables r and h. Symbolically , we write it as V =f(r, h).



Similarly, **one more example**: Let us choose a location 'Adi Hill'. If we want measure the temperature at particular point on the top of the hill (surface of the earth) at particular time, it depends on longitude and latitude i.e x and y coordinates and time t. So we say temperature T is a function of three variables x, y, and time t.




i.e T = u(x,y,t), so as a result T is a function of three independent variables, x,y and t.

Similarly, current in an electric circuit is a function of four variables; the electromotive force, the resistance, the capacitance and the induction.

So in this way one quantity is a function of more than one independent variables called function of several variables in which continuity and differentiability exists, called calculus of function of two or more variables. This chapter includes calculus of function of only two variables i.e, continuity, differentiability(called partial differentiation, chain rule and how to write total differentiability in terms of partial differentiation.

Historically speaking, Mathematicians like Euler, Lagrange and Jacobi among others, are the main contributors to the calculus of several variables.

## Applications:

This calculus has potential applications in Engineering and Physics. In physics, we come across partial differential equation which we can solve by using mathematics. Apart from this, vector calculus(studying in IV sem.), another branch of Mathematics, is depending on partial differentiation.

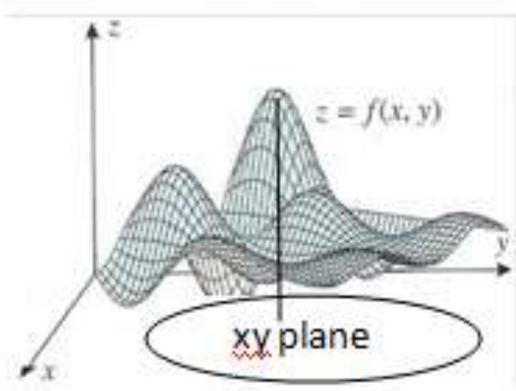
# Function of two variables

Let D be a subset of plane $R^2$, then a function of two variables is a rule f that assigns to each ordered pair (x,y) in a set D. i.e we get a unique no. z corresponding to function of x and y. i.e $f(x,y) = z$

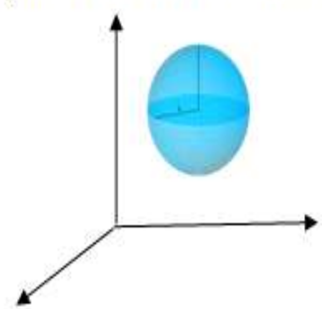
Symbolically, $f: R^2 \to R$ defined by $f(x, y) = z$.

**For example**: $f(x,y) = 1 - x^2 - y^2$ is a function of two variables.

**Geometrically**, the function $z = f(x, y)$ or $f(x,y,z)=0$ represents surface in $R^3$ (space)

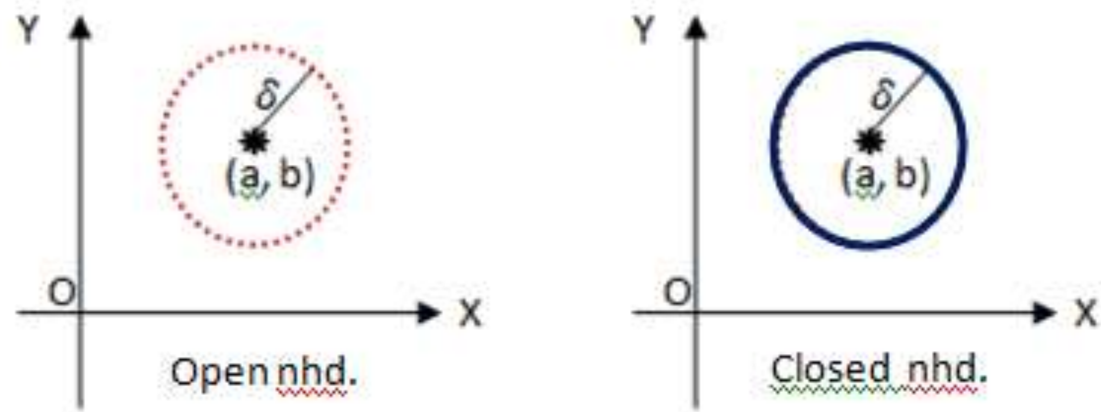



**For example**: Sphere is a surface wnose equation is $x^2+y^2+z^2+2ux+2vy+2wz+d=0$



# Basic Definitions

- **Neighbourhood of a point (a, b):**




Let (a,b) be any point in the plane, then collection or set of all the points (x,y) which are very close to (a, b) is called neighbourhood of the point (a,b). If I take the small distance as $\delta$

Then collection of all points (x,y) whose distance from (a,b) is less than $\delta$ is called nhd. of (a,b) or open disk. i.e nothing but collection of all points inside the circle with centre at(a,b) and radius $\delta$.

And collection of all points points (x,y) whose distance from (a,b) is less than or equal to $\delta$ is called nhd. of (a,b) or open disk.

- **For example**: If Niapni is the centre place i.e like (a, b) and be distance 5 kms. Then set of all villages which are within 5 kms is called nhd. of Nipani i.e they are neighbours of Nipani. i.e Lakanapur, Yamagarni, Chikli, Stavanidhi etc. are in nhd. of Nipani where as Galataga is not.

i.e collection of all neighbouring villeges of Nipani is callled Nhd. of Nipani.

Then what are the neighbouring cities of Nipani?

Answer for this?

BE  SAFE AT HOME

WEAR MASK IF YOU GO OUT FOR NECESSERY

SANITIZE HANDS OFTEN

## WE ALL PRAY TO GOD TO SAVE ALL PEOPLE FROM COVID

ಓಂ ನಮೋ ಭಗವತೆ ಮಹಾಸುದರ್ಶನ
ವಾಸುದೇವಾಯ
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ಸಕಲ ಭಯ ವಿನಾಶಾಯ
ಸರ್ವರೋಗ ನಿವಾರಣಾಯ
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ತ್ರಿಲೋಕ ನಿಧಯೆ
ಓಂ ಶ್ರೀ ಮಹಾವಿಷ್ಣುಸ್ವರೂಪ
ಶ್ರೀ ಧನ್ವಂತರಿ ಸ್ವರೂಪ
ಓಂ ಶ್ರೀ ಶ್ರೀ ಔಷಧಚಕ್ರ ನಾರಾಯಣಾಯ ನಮಃ ||
🙏🙏🙏

According to Veda, a very Powerful Mantra that keeps us Healthy..ಇದೊಂದು ಚಿಕ್ಕ ಪ್ರಾರ್ಥನೆ ಬೇರೆಯವರಿಗೆ ಕಳುಹಿಸಿ 11:08 AM

THANKYOU

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: Calculus–II and 3–D Geometry

II online Class

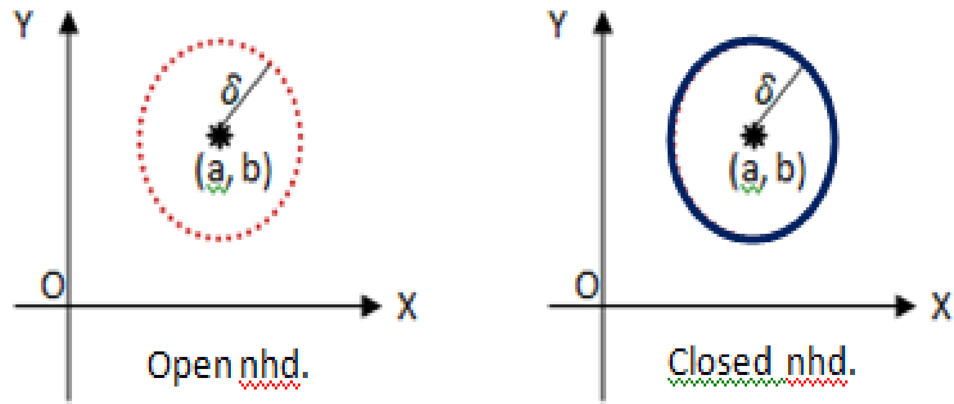
**Basic definitions and existence of limit for two variables**

By
Dr. M. M. Shankrikopp
HOD of Mathematics

Date: 4.5.2021

# Basic Definitions

**Neighbourhood of a point (a, b):**

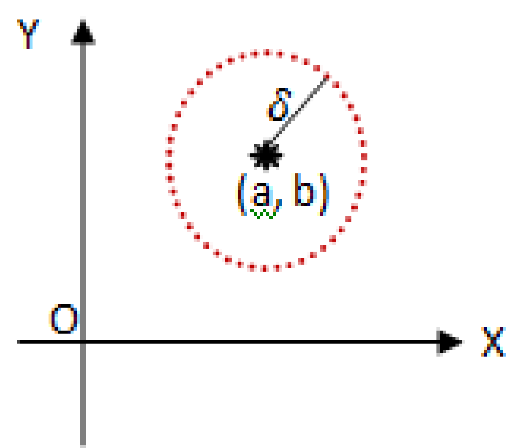



Let (a,b) be any point in the plane, then collection or set of all the points (x,y) which are very close to (a, b) is called neighbourhood of the point (a,b). If I take the small distance as $\delta$

Then collection of all points (x,y) whose distance from (a,b) is less than $\delta$ is called nhd. of (a,b) or open disk. i.e nothing but collection of all points inside the circle with centre at(a,b) and radius $\delta$.

And collection of all points points (x,y) whose distance from (a,b) is less than or equal to $\delta$ is called closed disk.

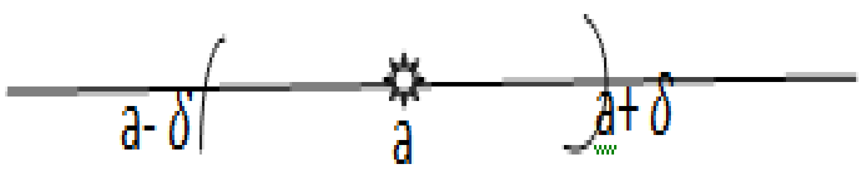
**For example:** If Niapni is the centre place i.e like (a, b) and $\delta$ be distance 5 kms. Then set of all villages which are within 5 kms is called nhd. of Nipani i.e they are neighbours of Nipani. i.e Lakanapur, Yamagarni, Chikli, Stavanidhi etc. are in nhd. of Nipani where as Galataga is not.



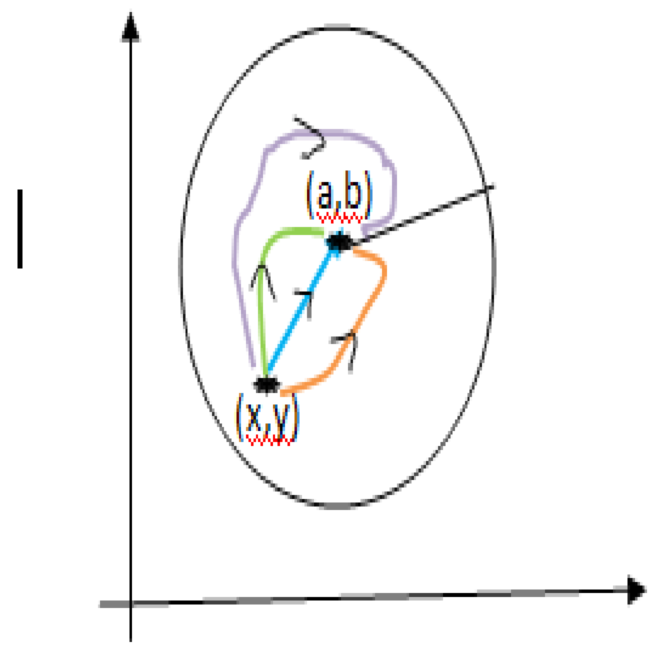
# Limit of function of two variables

**Definition:** A function u= f(x,y) is said to have the limit l as (x,y) tends to (a,b) if for given $\varepsilon > 0$,there exists $\delta>0$ such that $|f(x,y) - l| < \varepsilon$, for all points (x,y) lies in the nhd. of (a, b) along any path we choose.

Recall that incase of real function f(x), $\lim_{x \to a} f(x) = l$, if lest hand limit = right hand limit, and here path from x to a is always along x axis.



But in case of two variables point (x,y) moves towards (a,b) along any path as it is plane




In this fig. point (x, y) moves towards (a, b) ( which is in the nhd. of (a,b) only) along so many paths blue, brown, green or violet but all along these paths limit l should be same then only will say limit exists, otherwise no. i.e along one path limit is $l_1$ , another path if it is $l_2$ and so on then we say limit does exists.

If all are equal then we say that limit exists.

You come to know in examples.

And is denoted by $\lim_{(x,y)\to(a,b)} f(x,y) = l$,

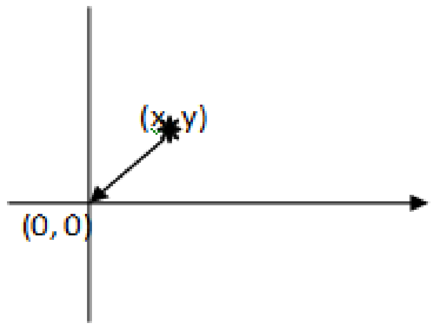
Meaning $\lim_{(x,y)\to(a,b)}$ means x→ $a$ and y→b

# EXAMPLES

**1. Show that for a function f(x, y) = $\frac{xy}{x^2+y^2}$ $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.**

Soln.: Now **f(x, y) = $\frac{xy}{x^2+y^2}$ , actually (0, 0) is not defined.**

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.




Therefore $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

Along y =mx means, substitute y =mx keep x as it is, and convert for single variable x.

i.e $\lim_{x\to 0} \frac{x(mx)}{x^2+(mx)^2} = \lim_{x\to 0} \frac{mx^2}{x^2+m^2x^2} = \lim_{x\to 0} \frac{mx^2}{x^2\,(1+m^2)} = \lim_{x\to 0} \frac{m}{(1+m^2)}$

$= \frac{m}{(1+m^2)}$ which depends on m,

hence limit is not unique.

i.e for different values of m we get different limits and hence limit does not exist.

**Note: For all the examples, first path is straight is st.line y =mx, along this path if limit is in terms of m, we say limit does not exist, suppose it is zero or constant we choose another path.**

**2. Evaluate the $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6}$ if it exists.**

Soln.: Now $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6}$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.

Then $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6} = \lim_{x\to 0} \frac{xm^3x^3}{x^4+m^6x^6} = \lim_{x\to 0} \frac{xm^3x^3}{x^4+m^6x^6}$

$= \lim_{x\to 0} \frac{m^3x^4}{x^4(1+m^6x^2)} = m^3$, depends on m and so limit depends on m, hence limit is not unique.

$\therefore \lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6}$ does not exist.

## 3. Show that $\lim_{(x,y)\to)(0,0)} \frac{xy^2}{x^2+y^2} = 0$

**Soln.:** Now $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.

Then $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2} = \lim_{x\to 0} \frac{xm^2x^2}{x^2+m^2x^2} = \lim_{x\to 0} \frac{m^2x^3}{x^2(1+m^2)}$

$= \lim_{x\to 0} \frac{m^2x}{(1+m^2)} = 0$ not depends on m , i.e for any value for m limit is 0 only.

And along any other path also it is 0

$\therefore \lim_{(x,y)\to)(0,0)} \frac{xy^2}{x^2+y^2} = 0$

**4. Evaluate** $\lim_{(x,y)\to(0,0)} \frac{x-y^2}{x^2+y^2}$ **, if exists.**

**Soln.:** Now $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{x-y^2}{x^2+y^2}$

Let us choose the path along the curve $y^2 = mx$, i.e point (x,y) moves towards (0,0) along the curve $y^2 = mx$.

Then $\lim_{(x,y)\to(0,0)} \frac{x-y^2}{x^2+y^2} = \lim_{x\to 0} \frac{x-mx}{x^2+mx} = \lim_{x\to 0} \frac{x(1-m)}{x(x+m)}$

$= \lim_{x\to 0} \frac{1-m}{x+m} = \frac{1-m}{m}$ , depends on m, does not exist.

I

## 5. Evaluate $\lim_{(x,y)\to)(0,0)} \frac{xy^3}{x^2+y^6}$ , if exists.

**Soln.:** Now $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,( along the line y =mx.

Then $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6} = \lim_{x\to 0} \frac{xm^3x^3}{x^2+m^6x^6} = \lim_{x\to 0} \frac{xm^3x^3}{x^2+m^6x^6}$

$$= \lim_{x\to 0} \frac{m^3x^4}{x^2(1+m^6x^4)} = 0$$

$\therefore \lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6}$ exists along the line y = mx

Let us choose another curve $y^3 = mx$, we get

$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6} = \lim_{x\to 0} \frac{xmx}{x^2+m^2x^2} = \lim_{x\to 0} \frac{mx^2}{x^2(1+m^2)}$

$$= \lim_{x\to 0} \frac{m}{(1+m^2)}$$

$= \frac{m}{(1+m^2)}$, depends on m, so limit does not exist.

## HOME WORK EXAMPLES

I

6. Evaluate $\lim_{(x,y)\to)(0,0)} \dfrac{x^2y^2}{x^4+y^4}$, if exists.

7. Evaluate $\lim_{(x,y)\to)(0,0)} \dfrac{x^4y^4}{(x^2+y^4)^3}$, if exists.

8. Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y}{x^2+y^2}$, if exists.

. A student scored 32% marks in science subjects out of 300. How much should he score in language papers out of 200 if he is to get overall 46% marks ?

- 72%
- 67%
- 66%
- 60%
- None of these

THANK YOU

BE SAFE AT HOME

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: Calculus–II and 3–D Geometry

III online Class

**Continuity of function of two variables**

By
Dr. M. M. Shankrikopp
HOD of Mathematics

Date: 5.5.2021

# CONTINUITY OF FUNCTION OF TWO VARIABLES

We define continuity of function of two variables in similar way as function of one variable.

In case of one variable we have f(x) is said to be continuous at x=a

if $\lim_{x\to a} f(x)$= f(a).  i.e LHL = RHL= f(a)

in the same way we define continuity of f(x,y) at (a, b).

**Definition:** A function f(x, y) is said to be continuous at the point (a,b) if

$\lim_{(x,y)\to(a,b)} f(x,y)$= f(a, b)

i.e in other words

A function f(x, y) is said to be continuous at the point (a,b) if

i) f(a,b) is defined.
ii) $\lim_{(x,y)\to(a,b)} f(x,y)$ exists
iii) $\lim_{(x,y)\to(a,b)} f(x,y)$= f(a, b)

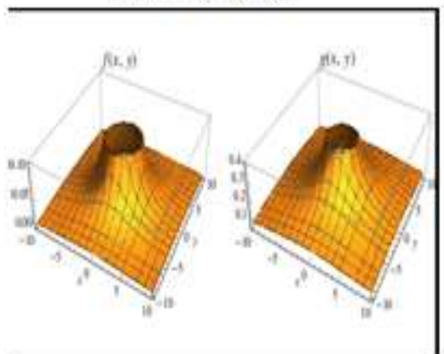
If any one of the above condition is not satisfied, it is discontinuous.

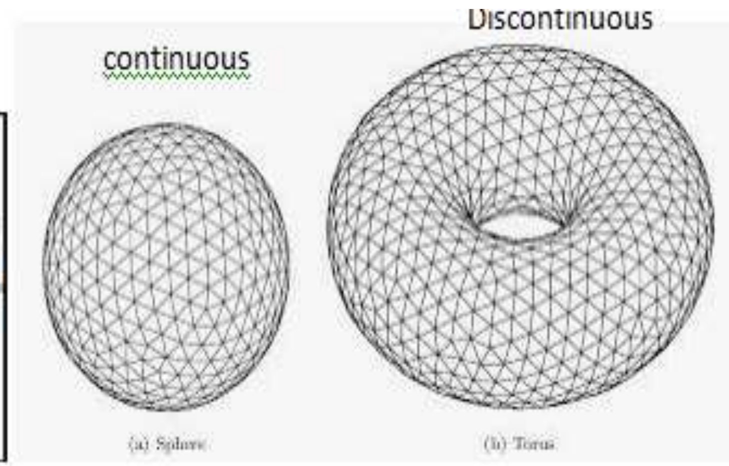
## Geometrical meaning of continuity of f(x,y)

Now $z = f(x,y)$ is surface and it is continuous mans it has no hole on its surface.

For example:

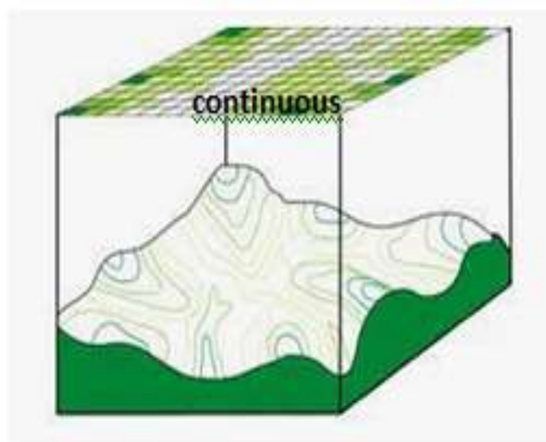
Discontinuous






(a) Sphere (b) Torus

Discontinuous




continuous

# Examples on continuity

**1. Show that the function**

$$f(x,y) = \begin{cases} \frac{2xy}{\sqrt{x^2+y^2}} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$$

is continuous at (0,0).

**Soln.:** Now given function is

$$f(x,y) = \begin{cases} \frac{2xy}{\sqrt{x^2+y^2}} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases}$$

At $(x, y) \neq (0,0)$,

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2+y^2}}$$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.

$$\text{Then } \lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2+y^2}} = \lim_{x\to 0} \frac{2xmx}{\sqrt{x^2+m^2x^2}} = \lim_{x\to 0} \frac{2mx^2}{x\sqrt{1+m^2}}$$

$$= \lim_{x\to 0} \frac{2mx}{\sqrt{1+m^2}} = 0$$

And all along other paths also limit is 0 = f(0,0)

Thus $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$

**2. If the function** $f(x,y) = \begin{cases} \frac{xy^2}{x^3+y^3}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0,0) \end{cases}$

Is the function f(x, y) continuous at (0, 0)?

**Soln.:** Now function $f(x,y) = \begin{cases} \frac{xy^2}{x^3+y^3}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0,0) \end{cases}$

Here, $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^3+y^3}$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.

$\lim_{(x,y)\to(0,0)} \frac{xm^2x^2}{x^3+m^3x^3} = \lim_{(x,y)\to(0,0)} \frac{m^2}{1+m^3} = \frac{m^2}{1+m^3}$ depends on m,

limit does not exist

So function is discontinuous at (0,0).

## Home work.

3. If the function 
$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^4}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0,0) \end{cases}$$

Is the function f(x, y) continuous at (0, 0)?

Thankyou

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: **C**alculus–II and 3–D Geometry

5th online Class

**Examples on Partial Derivatives**

By
Dr. M. M. Shankrikopp
HOD of Mathematics

Date: 7.5.2021

# EXAMPLES

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: **C**alculus–II and 3–D Geometry

6th online Class

**Higher Order Partial Derivatives**

By

Dr. M. M. Shankrikopp
HOD of Mathematics

Date: 11.5.2021

# Higher order Partial Derivatives

Before defining higher Order partial derivatives we know the following points

NOTE: (i) If u = f(x,y) then (i) the derivative of u w.r.t. x treating y as constant is called as partial derivative of u w.r.t.x and denoted by $\frac{\partial u}{\partial x}$ or $u_x$ and (ii) the derivative of u w.r.t. y treating x as constant is called as partial derivative of u w.r.t.y and denoted by $\frac{\partial u}{\partial y}$ or $u_y$ where symbol '$\partial$' is read as **'dov' or 'del'.**

(ii) The product rule, quotient rule and rule for composite function also same as function of one variable.

**We observe following comparisons:**

| Ordinary derivatives | | Partial derivatives | |
|---|---|---|---|
| 1. | $y = 3x^2 + 6x + 7$<br>$\frac{dy}{dx} = 6x+6+0 = 6x+6$ | 1. | $u = 3x^2y + 6xy^2 + 7$<br>$\frac{\partial u}{\partial x} = (6x)y + 6 .1.y^2 + 0 = 6xy + 6y^2$<br>(y treating as constant)<br>$\frac{\partial u}{\partial y} = 3x^2 + 6x(2y) + 0 = 3x^2 + 12xy$<br>(x treating as constant) |
| 2. | $y = e^{4x+3}$<br>$\frac{dy}{dx} = e^{4x+3}\ 4 = 4\ e^{4x+3}$ | 2. | $u = e^{4x+3y}$<br>$\frac{\partial u}{\partial x} = e^{4x+3y} \frac{\partial}{\partial x}(4x+3y)$<br>$= e^{4x+3y}\ 4 = 4\ e^{4x+3y}$<br>$\frac{\partial u}{\partial y} = e^{4x+3y} \frac{\partial}{\partial y}(4x+3y)$<br>$= e^{4x+3y}\ 3 = 3 e^{4x+3y}$ |
| 3. | $y = \sin 5x$<br>$\frac{dy}{dx} = 5 \cos 5x$ | 3. | $u = \sin(xy)$<br>$\frac{\partial u}{\partial x} = \cos(xy) \frac{\partial}{\partial x}(xy)$<br>$= \cos(xy) .y = y \cos(xy)$<br>$\frac{\partial u}{\partial y} = \cos(xy) \frac{\partial}{\partial y}(xy)$<br>$= \cos(xy) .x = x \cos(xy)$ |
| 4. | In general we conclude that<br>If $y = g(v)$ and $v = f(x)$ then<br>$\frac{dy}{dx} = g^{l}(v) \frac{dv}{dx}$ | 4. | If $u = f(r)$ and r is a function of x & y<br>Then $\frac{\partial u}{\partial x} = f^{l}(r) \frac{\partial r}{\partial x}$<br>And $\frac{\partial u}{\partial y} = f^{l}(r) \frac{\partial r}{\partial y}$ |

# Higher Order Partial Derivatives

- If the partial derivative of the function is again continuous function we can go for next derivative. So it is possible to take the partial derivative of a partial derivative. This is just like getting second derivative of f(x).
- The higher order partial derivatives for a function of two variables are defined as follows.

If z =f(x,y) then,

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}(f_x) = f_{xx} \text{ or } f_{x^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}(f_y) = f_{yy} \text{ or } f_{y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(f_y) = (f_y)_x \text{ or } f_{yx}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}(f_x) = (f_x)_y \text{ or } f_{xy}$$

The last two: Mixed partial derivatives

All four: Second order partial derivatives

Similarly

$$\frac{\partial^3 f}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x^2}\right) = f_{xxx}$$

$$\frac{\partial^3 f}{\partial y^3} = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right) = f_{yyy}$$

, $f_{xyy}$, $f_{xxy}$, $f_{yyx}$, $f_{yxx}$ are 3rd order partial order derivatives.

**For example**: If $u = x^3 - 3xy^2 + x + e^x \cos y + 1$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

**Soln.:** Now $u = x^3 - 3xy^2 + x + e^x \cos y + 1$ -----------------(1)

$\therefore \frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 1 + e^x \cos y$

Again diff. above eqn. p.w.r.t x we get

$\frac{\partial^2 u}{\partial x^2} = 6x - 0 + 0 + e^x \cos y = 6x + e^x \cos y$ ------------------(2)

diff. eqn. (1) p.w.r.t y we get

$\frac{\partial u}{\partial y} = 0 - 3x(2y) + 0 + e^x(-\sin y) + 0 = -6xy - e^x \sin y$

Again diff.w.r.t . y we get,

$\frac{\partial^2 u}{\partial y^2} = -6x - e^x \cos y$ -----------------------------------(3)

Adding (2) and (3) we get,

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + e^x \cos y - 6x - e^x \cos y = 0$

# Symmetric functions and partial derivatives of such functions

A function f(x,y) is said to be symmetric if f(x, y) =f(y, x) and f(x,y,z) is symmetric if f(x,y,z) = f(y,z,x) = f(z,x,y)

i.e if we change x,y, z in cyclic order there is no change in function

**For example:**

(i) $x+y$, $x^2 + y^2$, $\frac{x^2+y^2}{x+y}$, $x^2 + xy + y^2$, etc. are symmetric functions of 2 variables.

(ii) $x^2 + y^2 + z^2$, , $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, $x^3 + y^3 + z^3 - 3xyz$ etc. are symmetric functions of 3 variables.

**It is very important to note** that if given function is symmetric, then it is very easy to get partial derivative.

i.e if we get $u_x$, $u_{xx}$, $u_{xy}$ then directly we write $u_y$, $u_{yy}$, $u_{yx}$ etc. just by changing variables in cyclic order.

# EXAMPLES

**1. If $u = e^{ax-by}\sin(ax+by)$, show that $b\frac{\partial u}{\partial x} - a\frac{\partial u}{\partial y} = 2abu$**

**Soln.** Now $u = e^{ax-by}\sin(ax+by)$

$$\therefore \frac{\partial u}{\partial x} = e^{ax-by}(a)\sin(ax+by) + e^{ax-by}\cos(ax+by)\,a$$

$$= ae^{ax-by}(\sin(ax+by) + \cos(ax+by))$$

$$\text{And } \frac{\partial u}{\partial y} = e^{ax-by}(-b)\sin(ax+by) + e^{ax-by}\cos(ax+by)\,b$$

$$= be^{ax-by}(-\sin(ax+by) + \cos(ax+by))$$

$$b\frac{\partial u}{\partial x} - a\frac{\partial u}{\partial y} = ab\,e^{ax-by}[\sin(ax+by) + \cos(ax+by)] - ab\,e^{ax-by}[-\sin(ax+by) + \cos(ax+by)]$$

$$= 2ab\,e^{ax-by}[\sin(ax+by)]$$

$$= 2abu \text{ RHS}$$

**2. If $z = \sinh^{-1}\left(\frac{x}{y}\right)$ show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0.$**

**Soln.:** Now $z = \sinh^{-1}\left(\frac{x}{y}\right)$

$$\therefore \frac{\partial z}{\partial x} = \frac{1}{\sqrt{1+\left(\frac{x}{y}\right)^2}}\frac{1}{y} = \frac{y}{\sqrt{y^2+x^2}}\frac{1}{y} = \frac{1}{\sqrt{y^2+x^2}}$$

And $$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{1+\left(\frac{x}{y}\right)^2}}\frac{x}{-y^2} = \frac{-y}{\sqrt{y^2+x^2}}\frac{x}{y^2} = \frac{-x}{y\sqrt{y^2+x^2}}$$

$$\therefore x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{x}{\sqrt{y^2+x^2}} + y\left(\frac{-x}{y\sqrt{y^2+x^2}}\right)$$

$$= \frac{x}{\sqrt{y^2+x^2}} - \frac{x}{\sqrt{y^2+x^2}}$$

$$= 0$$

$$\therefore x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$

# PUZZLE

8. There are 2 jugs with 4 litres and 5 litres of water respectively. The objective is to pour exactly 7 litres of water in a bucket. How can it be accomplished?

Water and jugs puzzle

THANKYOU

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: **C**alculus–II and 3–D Geometry

13th online Class

**Examples on second Order Partial Derivatives**

By

Dr. M. M. Shankrikopp
HOD of Mathematics

Date: 25.5.2021

# Continued Examples on Second order partial derivatives in case of composite function

1. If $z = f(x, y)$ and $x = u\cos\alpha - v\sin\alpha$ and $y = u\sin\alpha + v\cos\alpha$ then prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}, \quad \alpha \text{ being constant}$$

**Soln.:** Now $z = f(x, y)$ and $x = u\cos\alpha - v\sin\alpha$ and $y = u\sin\alpha + v\cos\alpha$

i.e z is a composite function of u and v.

$$\therefore \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = \frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha$$

And $\frac{\partial^2 z}{\partial u^2} = \frac{\partial}{\partial u}\left(\frac{\partial z}{\partial u}\right) = \frac{\partial}{\partial u}(r)$ where $r = \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha$

$$= \frac{\partial r}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial r}{\partial y}\frac{\partial y}{\partial u}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha\right)\frac{\partial x}{\partial u} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha\right)\frac{\partial y}{\partial u}$$

$$= \left(\cos\alpha\frac{\partial^2 z}{\partial x^2} + \sin\alpha\frac{\partial^2 z}{\partial x \partial y}\right)\cos\alpha + \left(\cos\alpha\frac{\partial^2 z}{\partial y \partial x} + \sin\alpha\frac{\partial^2 z}{\partial y^2}\right)\sin\alpha$$

$$\frac{\partial^2 z}{\partial u^2} = \cos^2\alpha\frac{\partial^2 z}{\partial x^2} + 2\sin\alpha\cos\alpha\frac{\partial^2 z}{\partial x \partial y} + \sin^2\alpha\frac{\partial^2 z}{\partial y^2} \text{ -----------(1)}$$

Next $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = \frac{\partial z}{\partial x}(-\sin\alpha) + \frac{\partial z}{\partial y}\cos\alpha$

And $\frac{\partial^2 z}{\partial v^2} = \frac{\partial}{\partial v}\left(\frac{\partial z}{\partial v}\right) = \frac{\partial}{\partial v}(s)$ where $s = \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}(-\sin\alpha) + \frac{\partial z}{\partial y}\cos\alpha$

i.e $\frac{\partial^2 z}{\partial v^2} = \frac{\partial s}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial s}{\partial y}\frac{\partial y}{\partial v}$

$$= \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}(-\sin\alpha) + \frac{\partial z}{\partial y}\cos\alpha\right)\frac{\partial x}{\partial v} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}(-\sin\alpha) + \frac{\partial z}{\partial y}\cos\alpha\right)\frac{\partial y}{\partial v}$$

$$= \left(-\sin\alpha\frac{\partial^2 z}{\partial x^2} + \cos\alpha\frac{\partial^2 z}{\partial x\partial y}\right)(-\sin\alpha) + \left(-\sin\alpha\frac{\partial^2 z}{\partial y\partial x} + \cos\alpha\frac{\partial^2 z}{\partial y^2}\right)\cos\alpha$$

$$\frac{\partial^2 z}{\partial v^2} = \sin^2\alpha\frac{\partial^2 z}{\partial x^2} - 2\sin\alpha\cos\alpha\frac{\partial^2 z}{\partial x\partial y} + \cos^2\alpha\frac{\partial^2 z}{\partial y^2} \text{------------}(2)$$

Adding (1) and (2) we get, $\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = [\cos^2\alpha \frac{\partial^2 z}{\partial x^2} + 2 \sin\alpha \cos\alpha \frac{\partial^2 z}{\partial x \partial y} + \sin^2\alpha \frac{\partial^2 z}{\partial y^2}]$

$$+\sin^2\alpha \frac{\partial^2 z}{\partial x^2} - 2 \sin\alpha \cos\alpha \frac{\partial^2 z}{\partial x \partial y} + \cos^2\alpha \frac{\partial^2 z}{\partial y^2}$$

$$= \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$$

**Thus we have,** $\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$

2. If u = f(x, y) where x = r $\cos\theta$, $y$ = r $\sin\theta$ then show that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

***Soln***.: Now u = f(x, y) where x = r $\cos\theta$, $y$ = r $\sin\theta$

*i.e* u is composite function of r and $\theta$

$$\therefore \frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial r}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial r}=\frac{\partial u}{\partial x}\cos\theta+\frac{\partial u}{\partial y}\sin\theta$$

i.e $\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cos\theta+\frac{\partial u}{\partial y}\sin\theta$--------------(1)

Again differentiate (1) p.w.r.t r again ,

$$\frac{\partial^2 u}{\partial r^2}=\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}\right)=\frac{\partial s}{\partial r} \quad \text{where } s=\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\cos\theta+\frac{\partial u}{\partial y}\sin\theta$$

$$=\frac{\partial s}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial s}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\sin\theta\right)\frac{\partial x}{\partial r} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\sin\theta\right)\frac{\partial y}{\partial r}$$

i.e $\frac{\partial^2 u}{\partial r^2} = \left(\cos\theta\,\frac{\partial^2 u}{\partial x^2} + \sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y}\right)\cos\theta + \left(\cos\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \sin\theta\,\frac{\partial^2 u}{\partial y^2}\right)\sin\theta$

i.e $\frac{\partial^2 u}{\partial r^2} = \cos^2\theta\,\frac{\partial^2 u}{\partial x^2} + 2\sin\theta\cos\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \sin^2\theta\,\frac{\partial^2 u}{\partial y^2}$ ----------------(P)

$Next\ \frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} = \frac{\partial u}{\partial x}(-r\sin\theta) + \frac{\partial u}{\partial y}(r\cos\theta)$

i.e $\frac{\partial u}{\partial \theta} = \mathrm{r}\left(-\frac{\partial u}{\partial x}\sin\theta + \frac{\partial u}{\partial y}\cos\theta\right)$ ----------------(2)

Again differentiate (2) p.w.r.t $\theta$ again ,

$\therefore\ \frac{\partial^2 u}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(\frac{\partial u}{\partial \theta}\right) = \frac{\partial}{\partial \theta}(t)$ where $t = \frac{\partial u}{\partial \theta}$

$$= \frac{\partial t}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial t}{\partial y}\frac{\partial y}{\partial \theta} \text{ by chain rule}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial \theta}\right)\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial \theta}\right)\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial}{\partial x}\left(\mathrm{r}\left(-\frac{\partial u}{\partial x}\sin\theta + \frac{\partial u}{\partial y}\cos\theta\right)\right)\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}\left(\mathrm{r}\left(-\frac{\partial u}{\partial x}\sin\theta + \frac{\partial u}{\partial y}\cos\theta\right)\right)\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial}{\partial x}(\mathbf{r}\,(-\frac{\partial u}{\partial x}\ sin\theta + \frac{\partial u}{\partial y}\ cos\theta)\,)\,\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}(\mathbf{r}\,(-\frac{\partial u}{\partial x}\ sin\theta + \frac{\partial u}{\partial y}\ cos\theta)\,)\,\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial}{\partial x}(-\frac{\partial u}{\partial x}\ \boldsymbol{r}\ sin\theta + \frac{\partial u}{\partial y}\ \boldsymbol{r}\ cos\theta)\,)\,\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}(-\frac{\partial u}{\partial x}\ \boldsymbol{r}\ sin\theta + \frac{\partial u}{\partial y}\ \boldsymbol{r}\ cos\theta)\,)\,\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial}{\partial x}(-\frac{\partial u}{\partial x}\ y + \frac{\partial u}{\partial y}\ x)\,)\,\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}(-\frac{\partial u}{\partial x}\ y + \frac{\partial u}{\partial y}\ x)\,)\,\frac{\partial y}{\partial \theta}$$

$$=[\ \text{- y}\frac{\partial^2 u}{\partial x^2} + (-\frac{\partial u}{\partial x})\ 0 + x\,\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial u}{\partial y}\ 1]\ (\text{-r}\ \sin\theta) +$$

$$[\ \text{-y}\ \frac{\partial^2 u}{\partial x\,\partial y} + \left(-\frac{\partial u}{\partial x}\right) 1 + \ x\ \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\ (0)\ )\,]\ (\text{r}\ \cos\theta)$$

$$= [\ \text{- r}\ \sin\theta\ \frac{\partial^2 u}{\partial x^2} + \text{r}\ \cos\theta\ \frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial u}{\partial y}\,]\ (\text{-r}\ \sin\theta)$$

$$+ [\ \text{- r}\ \sin\theta\ \frac{\partial^2 u}{\partial x\,\partial y} + \left(-\frac{\partial u}{\partial x}\right) 1 + \text{r}\ \cos\theta\ \frac{\partial^2 u}{\partial y^2}\ )\,]\ (\text{r}\ \cos\theta)$$

$$= [\,-\text{r}\sin\theta\,\frac{\partial^2 u}{\partial x^2} + \text{r}\cos\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial u}{\partial y}\,]\,(-\text{r}\sin\theta)$$

$$+ [\,-\text{r}\sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \left(-\frac{\partial u}{\partial x}\right) 1 + \text{r}\cos\theta\,\frac{\partial^2 u}{\partial y^2}\,)\,]\,(\text{r}\cos\theta)$$

$$= r^2\,sin^2\theta\,\frac{\partial^2 u}{\partial x^2} - r^2 cos\theta\,sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} - \text{r}\sin\theta\,\frac{\partial u}{\partial y}$$

$$- r^2 cos\theta\,sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + r^2\,cos^2\theta\,\frac{\partial^2 u}{\partial y^2} - \text{r}\,cos\theta\,\frac{\partial u}{\partial x}$$

$$= r^2\,sin^2\theta\,\frac{\partial^2 u}{\partial x^2} + r^2\,cos^2\theta\,\frac{\partial^2 u}{\partial y^2} - 2\,r^2 cos\theta\,sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} - \text{r}\sin\theta\,\frac{\partial u}{\partial y} - \text{r}\,cos\theta\,\frac{\partial u}{\partial x}$$

$$\therefore \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = sin^2\theta\,\frac{\partial^2 u}{\partial x^2} + cos^2\theta\,\frac{\partial^2 u}{\partial y^2} - 2\,cos\theta\,sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} - \frac{1}{r}\sin\theta\,\frac{\partial u}{\partial y} - \frac{1}{r}\,cos\theta\,\frac{\partial u}{\partial x}$$

-----------------------------(Q)

And $\frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{r}\,cos\theta\,\frac{\partial u}{\partial x} + \frac{1}{r}\,sin\theta\,\frac{\partial u}{\partial y}$ -----------------------------------(R)

Adding (P), (Q) and (R )we get

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = [\cos^2\theta \frac{\partial^2 u}{\partial x^2} + \sin^2\theta \frac{\partial^2 u}{\partial y^2} + 2\sin\theta\cos\theta \frac{\partial^2 u}{\partial x\,\partial y}\ ]$$

$$+ [\sin^2\theta \frac{\partial^2 u}{\partial x^2} + \cos^2\theta \frac{\partial^2 u}{\partial y^2} - 2\cos\theta\sin\theta \frac{\partial^2 u}{\partial x\,\partial y} - \frac{1}{r}\sin\theta \frac{\partial u}{\partial y} - \frac{1}{r}\cos\theta \frac{\partial u}{\partial x}]$$

$$+ \frac{1}{r}\cos\theta \frac{\partial u}{\partial x} + \frac{1}{r}\sin\theta \frac{\partial u}{\partial y}$$

$$= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

**Thus we got** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$

# BRANCH ACCOUNTS

## Meaning:

**When producers or manufacturers open different shops in different places or streets, then those shops are called Branches.**

**Manufacturers office or producers office is called Head Office or Main Office.**

**Types Of Branches:**

1) **Foreign Branch**
2) **Inland Branch**

**1) Foreign Branch:**

**When the Head Office and Branches are in different countries then it is called Foreign Branch.**

## 2) Inland or Home Branch:

**When the Head Office and Branches are in the same country then it is called Inland or Home Branch.**

## Types of Inland or Home Branch:

**i) Dependent Branch**

**ii) Independent Branch**

**Dependent Branch:**

It is a branch which is mainly depending upon the head office for each & everything. It has to receive the goods from head office, cash for expenses etc. it has no power to purchase the goods from outsiders. Even it has no power to retain the cash.

# Futures of Dependent Branch:

1. Head Office maintains record of all transactions in respect of each branch.
2. All the requirements of branches are met by Head Office and branches are not allowed to make outside purchase.
3. Goods may by supplied by Head Office either at cost or invoice price.
4. All the expenses of branch are directly paid by Head Office.
5. All the petty expenses are paid by branch manager out of advance petty expenses sanctioned by H.O.
6. Generally branch may instructed to sell goods for cash or sometimes on credit basis also.
7. Branch should remit daily collection of the cash immediately to H.O.

**Journal entries in the books of Head Office:**

**1) When the goods sent to Branch.**

**Branch A/c---------------------Dr**

**To Goods sent to Branch A/c.**

**2) When the goods returned by Branch.**

**Goods sent to Branch A/c------------------Dr**

**To Branch A/c**

**3) When Branch Expenses paid.**

**Branch A/c--------------------------Dr**

**To Cash A/c or Bank A/c**

**4) When Branch Sells goods for cash.**

**Cash A/c-----------------------------Dr**

**To Branch A/c**

**5) Branch Sells goods on credit.**

**No entry**

- **Note:**

  **The transaction in between Branch & Branch Debtors are not appearing in the books of head office.**

  **Credit sales, Goods returned by customers, Bad debts, Discount allowed to customers etc. are the transactions in between branch and branch debtors. Hence these items are not appearing in the books of head office.**

6) When cash collected from Debtors.

Cash A/c---------------------------Dr

To Branch A/c

7) When there is profit

Branch A/c------------------------Dr

To General P&L A/c

8) When there is loss.

General P&L A/c-----------------Dr

To Branch A/c.

# Dr Branch Account Cr

| PARTICULARS | RS | PARTICULARS | RS |
|---|---|---|---|
| To Opening stock | XXX | By opening creditors | xxx |
| To Opening Debtors | XXX | By goods sent to Branch (Return) | xxx |
| To Opening Petty cash | XXX | By cash collected( Cash sale) | xxx |
| To Opening value of Assets | XXX | By cash collected ( From debtors) | xxx |
| TO Goods sent to Branch | XXX | By closing value of Assets | xxx |
| To cash/Bank A/c<br>Salaries xxx<br>Rent xxx<br>Stationery xxx | XXX | By General P&L A/c | xxx |

| | | | |
|---|---|---|---|
| To cash<br>( cash sent for petty cash) | xxx | | |
| To closing creditors | xxx | | |
| To General P&L A/c<br>(Profit) | xxx | | |
| | _XXX__ | | XXX |

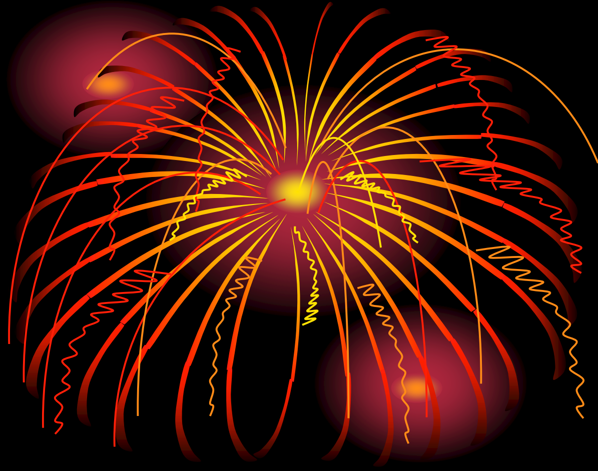
**Note:**

The following items are not appearing in Branch Account:

1) Credit sales, Bad debts, goods returned by customers & discount allowed to customers.

2) Petty expenses.

3) Depreciation on Asset.



# THANK YOU

# Business Law or Mercantile Law

## Introduction:

Man is a social being. He lives in society with his fellow beings. When living so, he has to observe a code of conduct or a set of rules for peaceful living and welfare of the whole society.

**These rules of conduct, when recognised by the State and enforced by it on people are termed as Law.**

Such law is not static. It changes when circumstances and conditions in the society change. Law is therefore dynamic.

# Need for the Knowledge of Law:

- Ignorance of law is not an excuse. Though it is not possible for a common man to learn every branch of law, yet he must know at least the general principles of the law of his country.
- Knowledge of Mercantile Law or Business Law is essential to people engaged in various economic and commercial activities i.e. business activities.
- The general knowledge of mercantile law will certainly help businessmen to solve their business problems and avoid conflicts with others.

# Meaning and Scope:

- The term ‘**Mercantile or Business Law**’ may be defined as that branch of law which deals with the rights and obligations arising out of mercantile or business transactions between businessmen.
- It consists of those rules that govern and regulate trade, commerce and industry. It is one of the important branches of Civil Law. Mercantile law is also known as Business Law.

- The scope of mercantile law is very wide and varied. It includes law relating to contracts, partnership, companies, sale of goods, negotiable instruments, carriage of goods, insolvency and arbitration and applies not only to businessmen but also to bankers and other professional men as well as to common people. Hence it is also known as Business Law.

# Law of Contract

## Introduction:

- Law of Contract is one of the most important branches of mercantile law. It is the foundation of modern business.
- In business, promises are made at one time and are performed at another time. To see that the promises made are duly performed by the parties to the Contract and to carry on the business smoothly, the law of Contract came into force.
- The law of Contract lays down the rules relating to promise, their formation, their performance and their enforceability.

# Indian Contract Act 1872

- It determines the circumstances in which promise made by the parties to a contract shall be legally binding on them.
- All of us enter into a number of contracts everyday knowingly or unknowingly.
- Each contract creates some right and duties upon the contracting parties. Indian contract deals with the enforcement of these rights and duties upon the parties.

# Meaning of Contract:

- According to Section 2(h) of the Indian Contract Act 1872, “A contract is an agreement enforceable by law”.
- According to Fredrick Pollock, “Every agreement and promise enforceable by law is a contract”.
- A contract is legally binding agreement between two or more persons.
- A contract is an agreement between two or more parties which is enforceable at law.

From the above definitions, a contract essentially consists of two elements:

1. an agreement and
2. its legal enforceability i.e. legal obligation.

## 1. Agreement:

- An agreement is defined as "every promise and every set of promises, forming consideration for each other." [(Sec 2.(e)]. Thus it is clear from this definition that a promise is an agreement. Sec. 2(b) defines. "A proposal, when accepted, becomes a promise."
- Thus, an agreement means an accepted proposal.

- Therefore, to form an agreement, there must be a proposal or an offer by one party and its acceptance by the other party. An agreement has two main characteristics.

a. **Plurality of Persons:** There must be two or more than two persons to make an agreement, because one person cannot enter into an agreement with himself.

b. **Consensus-ad-idam:** Both the parties to an agreement must agree upon the subject-matter of the agreement in the same sense and at the same time.

# 2. Legal Obligation:

- An agreement which creates a legal obligation between the parties becomes contract. If an agreement cannot create a legal obligation i.e., a duty enforceable by law, it is not a contract.
- Therefore, agreements of moral, religious or social nature are not contracts because they do not create any legal obligation.

- **Examples:** A promise to attend Pooja ceremony or dinner at your friend’s house is not a contract and failure to attend will not create any legal obligation.
- An agreement to buy a horse at Rs. 500 is a contract because it gives rise to legal obligation.
- Thus, there are various kinds of agreements but all these agreements are not contracts. “Only those agreements which are legally enforceable by law ad which therefore create legal obligation on the parties concerned constitute contracts.

# INDEXATION:

Indexation means converting purchase cost of the asset in to its present cost by using index number for inflation.

Note: Index is not applicable for debentures, bonds and depreciable assets, whose written down value is given in the problem.

# Indexed cost of acquisition:

Indexed cost of acquisition means the cost of acquisition which bears to CII for the year of sale, which bears to CII for the year of purchase.

Indexed cost of acquisition =

$$\frac{\text{Cost of acquisition} \times \text{CII for the year of sale}}{\text{CII for the year of Purchase or CII for 2001-02}}$$

# Index cost of improvement:

Index cost of improvement means an amount or cost of improvement bears to CII for the year sale which bears to CII for year of Improvement.

Index cost of Improvement =

$$\text{cost of improvement} \times \frac{\text{CII for the year of sale}}{\text{CII for the year of Improvement}}$$

Note:

CII= cost Inflation Index

# Note:

1. If any assets before 1-4-2001, the cost of acquisition or fair market value whichever is higher is taken as cost of acquisition.
2. Improvement cost before 1-4-2001 is ignored.
3. Cost Inflation Index is not applicable for debenture, bonds and depreciable assets whose written down value is given in the problem.
4. Index is not applicable to short-term capital gains.

# Conversion of capital asset in to stock in trade: Sec. 45(2)

Calculation of business income and capital gain:

| Particulars | Amount |
|---|---|
| Sale proceeds of converted capital asset | *XXX* |
| Less: Fair market value of converted capital asset on the date of conversion | *XXX* |
| **Business Income** | ***XXX*** |
| Fair market value of converted capital asset | *XXX* |
| Less: Cost of acquisition for STCG or Index cost of acquisition for LTCG | *XXX* |
| **CAPITAL GAINS** | ***XXX*** |

# Exemptions:

1. **Long-term capital gains on residential house invested in the residential house u/s 54:**

   **Any long term capital gain arising from the transfer of a residential house and invested in another residential house is exempt from tax subject to the following conditions:**

A. The income from such house property is taxable under the head income from house property.

B. The house property should be held for at least three years before its transfer

C. The assessee should purchase a new residential house within a period of one year before or two years after the date of transfer or construct a new residential house within a period of three years after the date of transfer.

D. If the assessee has not purchased or constructed a new residential house, he must deposit the amount of capital gain in a specified scheme called “Capital Gain Account Scheme” (CGAS)

## ***Amount of exemption*** u/s 54:

Cost of new residential house: xxx

Amount deposited in CGAS: xxx

xxx

OR

Long term capital gain xxx

Whichever is less xxx

# 2. Capital gain on agricultural land invested in other agricultural land: sec 54B:

Any capital gain arising from the transfer of agricultural land is exempt from tax subject to the following conditions:

a. The land must have been used for agricultural purpose for at least two years before the date of transfer either by assessee or his parents.

b. Assessee must purchase a new agricultural land within two years from the date of transfer.

***Amount of exemption:***

Cost of new agricultural land (CGAS) or capital gain whichever is less.

# 3. Capital gains on compulsory acquisition of land and building invested in other land and building: sec54D

**Any capital gain arising from the compulsory acquisition land and building exempt from tax subject to the following conditions:**

a. The property acquired is land and building forming part of industrial undertaking.

b. The asset must have been used for the purpose of business for at least two years immediately preceding the date of transfer.

c. Assessee should purchase new land and building for establishing another industrial undertaking, shifting or re-establishing the existing undertaking within a period of three years from the date of transfer.

***Amount of exemption:***

Cost of new land and building (CGAS) or capital gain whichever is less.

# 4. Long-term capital gains invested in certain bonds: sec 54EC:

**Any long –term capital gain is exempt from tax subject to the following conditions.**

A. The amount should be invested in long-term specified bonds within 6 months from the date of transfer.

B. Specified bonds redeemable after three years issued by:

1. National Highways Authority of India (NHAI)
2. Rural Electrification Corporation Ltd (REC)

*Amount of exemption:*

Amount invested in bonds (CGAS) or capital gain or maximum Rs.50,00,000 whichever is less.

## *5. Long-term capital gains other than residential house invested in residential house: sec 54F:*

**Any long-term capital gain other than residential house being invested in residential house is exempt from tax subject to the following conditions:**

1. The long-term capital gain should be the gain other than residential house.
2. Assessee should purchased a new residential house within a period of one year before or two year after the date of transfer or construct a new residential house within a period of three years after the date of transfer

3. The assessee should not own more than one residential house property on the date of transfer.
4. The assessee should not purchase within a period of one year or construct within a period of three years any residential house other than the new house.
5. The new residential house should not be sold within a period of three years from date of acquisition.

# *Amount of exemption: Sec. 54F*

**Long term capital gain X** $\dfrac{\textbf{cost of new house (CGAS)}}{\textbf{Net consideration}}$

### *6. Capital gains on shifting of industrial undertaking invested in new industrial undertaking in other than urban area: sec 54G:*

**Any capital gain arising from the transfer of machinery, plant, land and building is exempt from tax subject to the following conditions:**

1. Transfer of industrial undertaking should be any area other than an urban area.

2. The assessee within a period of one year before or three yearly after the date of transfer should purchase plant and machinery , land and building or construct a building and complete shifting to the new area.

***Amount of exemption:***

Cost of new plant, machinery, land and building (CGAS) or capital gain whichever is less.

### *7. Capital gain on shifting of industrial undertaking invested in new industrial undertaking in Special Economic Zone (SEZ) Sec:54GA:*

**Any capital gain arising from the transfer or shifting to SEZ is exempt from tax subject to the following conditions:**

1. The transfer of industrial undertaking should be to any SEZ
2. The assessee within a period of one year before or three years after the date of transfer should purchase plant and machinery and building or construct a building in SEZ

*Amount of exemption:*

Cost of new asset (CGAS) or capital gain whichever is less.

***8. Extension of time limit for acquiring new asset: sec 54H***

In case of compulsory acquisition of any asset under any law, the date of transfer is not considered for calculation of capital gain. The date of transfer is the date on which the assessee receives the compensation. In the view of this provision the period of making investment or acquiring the asset commences only from the date of compensation and not from the date of transfer of original asset.

# Retention money:

How do you treat advance money received and retained by the assessee?

If advance money received is forfeited on or after 01-04-2014 it will be taxable as “ Income from other sources”. But for advance money received is forfeited before 01-04-2014, it shall be deducted from cost of acquisition.

?

THANK YOU

# CONSIGNMENT ACCOUNTS

# *Introduction:*

When the owner of a business opens different shops in different cities or in different streets, in that time to look after those shops he will appoint some persons, who are called agents. Thus there will be an agreement between the owner and agent, that agreement is called consignment.

# ***Meaning***:

Consignment means “One person sends the goods to other persons to be sold on commission basis, on behalf of and at the risk of the sender of the goods”.

The agreement between consignor and consignee is called consignment.

The main object of such transactions is to increase sales.

# ***Consignor:***

The consignor is a person who sends the goods to be sold on his behalf and at his risk on commission basis.

He is the owner of the business. The goods so sent are called ‘consignment outwards’.

# ***Consignee:***

The consignee is a person who receives the goods to be sold on behalf of and at the risk of the sender on commission basis . The goods so received are called ‘consignment inwards’. He is the agent of the consignor.

# *Differences between consignment and sale:*

| *Point of distinction* | *Consignment* | *Sale* |
|---|---|---|
| 1) Transfer of ownership | Ownership of the goods is not transferred from the consignor to the consignee. | Ownership of goods is transferred from the seller to the buyer. |
| 2) Relationship between the parties | The relationship between the consignor and the consignee is that of the principal and agent | In case of credit sales the relationship between the seller and the buyer is that of creditor and debtor |
| 3) Physical transfer of goods | It is must | It is not compulsory |
| 4) Transfer of risk | The risk relating to the goods lies with the consignor until they are sold by the consignee | The risk relating to the goods passes to the purchaser |
| 5) Bearing the expenses | The expenses incurred on goods are to be borne by the consignor | The expenses incurred on goods are borne by the purchaser |
| 6) Right to call the goods back | Consignor can call back the goods as he is the owner | Seller cannot call back the goods once sold to the buyer |

# ***Commission***:

Consignee is an agent of his principal (i.e. consignor) & is entitled to a commission as remuneration for selling the goods on behalf of the consignor. Such commission is calculated generally on the gross sale proceeds of the goods. Some times it may be agreed to allow the commission on the invoice price of the goods sold and also on the excess amount realized over and above the invoice price or on the net profit made by the consignor.

# *TYPES OF COMMISSION TO THE CONSIGNEE*:

A. ***Ordinary commission***: It is the usual commission given to the consignee for sales made by him. Unless otherwise agreed upon this commission is calculated on the gross sales made by the consignee.

B. ***Del credere commission***: It is an extra commission given to the consignee in addition to the usual commission for collection of dues from the customers or debtors.

***Treatment of Bad debts***:

**When del-credere commission is allowed to consignee, the consignor need not worry about the bad debts, which is transferred to the consignee. Therefore, when the consignee is getting del credere commission, if there is any loss on account of bad debts that loss must be suffered by the consignee himself. When the del-credere commission is not given to the consignee, then the loss on account of bad debts to be suffered by consignor himself.**

**This commission is also calculated of gross sale proceeds unless otherwise it is stated in the problem.**

## C. ***Overriding commission***:

It is also an extra commission given to the consignee in addition to the usual commission and calculated of gross sale proceeds. The purpose of giving such commission is to induce the consignee to sell the new products in the market.

- ***Performa invoice***:

  It is a statement prepared by the consignor and sent to the consignee. It contains the details of quantity and quality of goods sent, the price of goods, expenses incurred by the consignor etc.

- ***Account sales***:

  It is a statement prepared by the consignee and sent to the consignor. It contains the details of the quantity and quality of goods sold (cash and credit), price of sale; less expenses incurred by consignee, his commission, any advance sent to consignor and the balance due to consignor.

***VALUATION OF CLOSING STOCK OR CONSIGNMENT STOCK***:

**Usually the consignee will not in a position to sell all the goods that he has received from the consignor. Thus, some stock will be left with the consignee, which is called closing stock.**

**The closing stock can be valued as follows:**

| | |
|---|---|
| **Cost price or market price of closing stock whichever is less** | **XXX** |
| **Add:** **Proportionate non- recurring expenses** | **<u>XXX</u>** |
| **Value of closing stock** | **XXX** |

**Proportionate non-recurring expenses =**

$$\text{Total NRE} \times \frac{\text{Closing Stock units}}{\text{Total units}}$$

# Consignment expenses:

Consignment expenses may be classified into two groups:

a. Non-recurring expenses and

b. Recurring expenses

a. Non-recurring expenses: Non-recurring expenses are those which are paid on the whole consignment only once.

   These are expenses which are paid from the time the goods are being sent by the consignor till the time the consignee brings them to his place.

   Non-recurring expenses are those expenses that are paid from the place of the consignor to the place of the consignee and not further.

**All the expenses paid by the consignor and consignee outside the godown of consignee are called non-recurring expenses.**

**Examples of NRE:**

**I. Non-recurring expenses paid by the consignor:**

1. **Packing**
2. **Carriage or cartage**
3. **Dock dues or import duty**
4. **Landing charges**
5. **Freight**
6. **Insurance for transporting goods**
7. **Octroi and loading charges**

**II. Non-recurring expenses paid by the consignee:**

1. Unloading charges
2. Dock dues
3. Octroi
4. Carriage charges to his place
5. Customs duty or import duty

# b. Recurring expenses:

Expenses incurred after the goods have reached the place of the consignee or godown of the consignee are called recurring expenses.

I. Recurring expenses paid by the consignor:

1. Bank charges for discounting bills, drafts, and cheques received from the consignee
2. Expenses incurred on damaged goods received

II. Recurring expenses incurred by the consignee:

1. Warehouse charges
2. Insurance on godown
3. Brokerage
4. Auction room expenses
5. Adverting
6. Salary and commission to salesman
7. Bad debts and discount
8. Expenses on goods returned
9. Expenses on damaged goods
10. Commission to consignee
11. Selling expenses
12. Rent on godown etc.

Note: Consignee does not buy the goods, but he undertakes to sell them on behalf of consignor. Whatever the consignee does is on behalf of the consignor and therefore the consignor must reimburse the consignee for his expenses unless otherwise agreed upon.

# *CONSIGNMENT LOSSES:*

When the consignor sends goods to the consignee sometimes a portion of goods is lost either in transit or after they reach the consignee.

There are two types of consignment losses:

1. Normal Loss &
2. Abnormal Loss

***1. <u>Normal Loss</u>***:

When the consignor sends goods to the consignee or when the consignee stores the goods in the godown, in that time there may be leakage, evaporation, driage etc. Due to that there will be some loss which is called normal loss.

The normal loss is allowed. The normal loss is suffered by good units. It means, the cost of the total goods will be treated as the cost of the goods available for sale. The normal loss will not come in the consignment account. But while valuating, the closing stock only the units of normal loss are to be taken in to consideration.

Normal loss is a loss which is unavoidable loss due to the inherent nature of the goods.

## 2. ***Abnormal Loss***:

Accidental loss or loss due to negligence is called abnormal loss. The abnormal loss is unexpected loss and which goes beyond the control of the businessman.

Abnormal loss is caused accidentally due to reasons such as fire, accident, theft, breakage, flood, earthquake, war etc.

Abnormal loss does not depend upon the nature of the goods and it can be avoided. This is also called accidental loss arising from carelessness and improper handling of the goods.

**This loss can be calculated as follows:**

**Calculation of Abnormal Loss:**

| | |
|---|---|
| **Cost price of abnormal loss units** | **XXX** |
| **Add: Proportionate Non-recurring expenses up to the point of loss** | **XXX** |
| **Abnormal Loss** | **XXX** |

$$\text{Proportionate NRE} = \text{Total NRE} \times \frac{\text{Abnormal loss units}}{\text{Total units}}$$

# *Journal entries for abnormal loss:*

**a. If the goods are not insured:**

**1. When there is abnormal loss**

**Abnormal Loss A/C……………………Dr**

**To Consignment A/C**

**2. When the abnormal loss is transferred to profit and loss account**

**Profit and Loss A/C ……………………Dr**

**To Abnormal Loss A/C**

**b. If the goods are insured:**

**1. When there is abnormal loss**

**Abnormal Loss A/C……………………Dr**

**To Consignment A/C**

**2. When the cash is received from the insurance company and loss transferred to profit and loss account**

**Cash A/C or Bank A/C …………………Dr**

**Profit and Loss A/C…………………….Dr**

**To Abnormal Loss A/C**

# Consignment Account:

**Consignment account is an account which is mainly prepared by the consignor in order to find out the profit or loss on the consignment.**

**This account is debited with the cost of goods sent, expenses incurred by both consignor and consignee, and commission payable to the consignee and credited with the gross sale proceeds of the goods, value of goods returned and the value of stock of unsold goods. The difference between the two sides is called profit or loss, which is transferred to profit and loss account.**

**How do you close the consignment account?**

**Consignment account can be closed by transferring the difference to profit and loss account.**

THANK YOU

# ELEMENTS OF COSTING

## Unit – 1: Introduction to Cost Accounting:

Limitations of financial accounting - meaning of cost, costing, cost accounting and cost Accountancy – Objectives and functions of cost Accounting – Advantages and limitations of Cost Accounting – Financial Accounting V/s Cost Accounting – Steps for installation of a costing system- Meaning of cost center, cost unit and cost Audit.

# Limitations of Financial Accounting:

1. **Collective Information:** Financial accounting supplies the information regarding profit and loss to the management collectively. Where in the whole industry is treated as one unit.

   But it is a difficult task for controlling or reducing the cost. Because if the factory is under loss in any year or profit is decreased, then the management is to locate the reasons, and also the person responsible.

But in financial accounts no one is responsible because every person escapes from his responsibility stating that his department is not responsible.

But in cost accounting the records are maintained units wise, process-wise or job wise and the responsibility can be fixed. It helps for controlling and reducing the cost.

2. **Historical in Nature:** Financial accounts are prepared at the end of the year. It means, for the whole year expenditure is incurred, then after closing the accounts or at the end of the accounting year calculation of how much expenditure is incurred and what will be the costs per unit is made. There is not proper system of control and calculation of the day to day cost.

   Financial accounting is like a thermometer which can tell us the temperature of the body only but cannot diagnose the disease.

3. **Materials and Supplies are not properly controlled:** There is no proper method for controlling of materials and supplies, which leads to loss, wastages, deterioration, excessive scraps and misappropriation of the materials.

4. **Expenses are not Classified:** Expenses are not classified, such as Controllable and Uncontrollable, Direct and Indirect, Fixed and Variable. This classification helps in controlling and reducing the cost.

5. **Fails to Help in Cost Reduction:** It is not possible to maximize the profits, since cost reduction is not possible under the financial accounting system. In the modern era, profits can be maximized only by increasing the volume of sales and sales can be increased if the prices charged are competitive and comparatively low.

6. **Fails to Provide Cost Information in Price Fixation:** It is very difficult to fix the prices of the product or services, or tenders because the financial accounts do not provide detailed cost information.

7. **Labour Cost is not Recorded Job-wise:** In financial accounting, labor costs are not recorded process-wise or job wise. It is very difficult to find out the cost per unit or process or job. There is no system of incentives which may be easily used to compensate the workers for their above standard performance.

**8. Fails to Provide Information for Appraisal and Comparison:** Financial accounting does not provide information to the management for appraisal and comparison of the profits under alternative methods. Not only this, even it fails to provide useful information to management for taking vital decisions such as replacement of labour, introduction of new techniques or make or buy the products etc.

## Introduction:

The term ‘COST’ plays vital role in success or failure of every business. The statement “conquer your costs before they conquer you,” signifies how dangerous the cost is.

In the LPG environment it has become still more dangerous and considered as serious issue.

The firm which neglects cost factor has no place to survive in the highly competitive global market.

It is worthwhile to state here that a firm which tries to raise its selling price, to realize desired profit, without making efforts at cost control will be soon itself out of the market.

Therefore, now it is indispensable for the firm to conquer their costs for gaining competitive edge.

Cost consciousness and cost control are the most urgent need of the hour.

It is the cost accounting which helps the firms in managing and controlling cost efficiently.

**Meaning and Definitions:**

**Cost:** Cost is defined as an amount of expenditure incurred on a product or service.

According to Gorden Shilling Law: "*Cost represents the resources that have been or must be sacrificed to attain a particular objective.*"

According to **ICMA (London):** "*Cost is the amount of expenditure incurred or attributable to given thing.*"

**Costing:** The term costing has been defined by ICMA (London) as "*the technique and process of ascertaining costs.*"

Thus, costing is routine process of ascertaining cost of a product or service or job. It consists of principles and rules which govern the process of ascertaining costs.

**Cost Accounting:** Cost accounting is a formal system of accounting for cost by means of which cost of products or services are ascertained and controlled.

a. According to **Erich Koher:** *"Cost accounting is the branch of accounting dealing with the classification, recording, allocation, summarization and reporting of current and prospective costs."*

b. According to **Van Sickle:** *"Cost accounting is the science of recording and presenting business transactions pertaining to the production of goods and services whereby these records become a method of measurement and a means of control."*

## Cost Accountancy:

Cost accountancy is a wider term. It refers to the principles, conventions, techniques and systems which are employed in a business to plan and control the utilisation of its resources.

Thus, cost accountancy includes costing, cost accounting, cost control and cost audit.

## Objectives of Cost Accounting:

1. **Ascertainment of Cost:** Cost accounting aims at ascertainment of cost which serves as basis for fixing selling price and making managerial decisions. It ascertains cost of each product, job, process etc., with the help of its methods like output costing, operating costing, job costing etc.

2. **Determination of Selling Price:** Cost is the basis for fixing selling price. Cost accounting provides necessary cost data and facilitates fixation of selling price. Even during deflation, it guides the management in deciding the extent to which selling price is reduced with the help of its marginal costing technique.

3. **Cost Control:** Cost control is most essential for improving efficiency and profitability of a firm. Cost accounting controls cost by comparing actual cost with predetermined cost by applying its standard costing and budgetary control techniques.

4. **Guiding Management in Business Policies and Decisions:** Cost accounting guides management in formulation of policies and decisions by providing relevant cost data. The management has to formulate suitable policies and decisions for carrying business operations efficiently. Cost accounting guides management in formulation of such policies and decisions by providing necessary cost data.

## Functions of Cost Accounting:

1. **Recording:** Recording of relevant transaction is the primary function of cost accounting. Various accounts are maintained according to the principles of Cost Accounting.
2. **Cost Analysis:** It means, costs are classified into different groups, Viz, direct and indirect cost, normal or abnormal cost etc.
3. **Cost Control:** Cost accounting establishes ideal standards and thereby controls the cost.

4. **Cost Comparison:** This is the function of comparing of cost for ascertainment of profitabilities, project proposals, plans and actions. This comparison helps to take the right decisions at crucial points of time.
5. **Quotation:** It estimates the cost of job or work order more scientifically to quote the price.
6. **Cost Planning:** Each element of cost should be properly planned and expenses incurred accordingly. The overall cost planning helps the management in order to activate the objectives.

7. **Cost Budgeting:** This function helps to formulate the cost budgets. The budget means fixing the overall limit of expenses and the cost information guides to be within the set framework.

8. **Providing Cost information for Decision Making:** Cost accounting provides useful information which is helpful to the management in making strategic decisions.

9. **Reporting:** Revealing and reporting inefficiencies of various elements of cost units, cost centers, products etc., to the management.

# THANK YOU

# DEPARTMENTAL ACCOUNTS

# Meaning:

A big business concerns dealing in different kinds of goods or services is usually divided into a number of departments.

A business having a number of departments each specializing in a particular line of activity is called departmental undertaking.

Under one management and under one roof the different goods and services are rendered is called departmental undertaking.

The accounts relating to different goods or services are called departmental accounting

# NEED FOR DEPARTMETNAL UNDERTAKING (OBJECTIVES OR ADVANTAGES):

1. To ascertain the result of each department
2. To compare the trading result of one department with the another department
3. To take necessary steps either to improve the department which is under loss or to close down all together the department which is under loss.

4. To evaluate the performance of each department

5. To reward the departmental managerial staff on the basis of trading results.

6. To have effective managerial control over the working of each department

# APPORTIONMENT OR ALLOCATION OF COMMON EXPENSES:

Apportionment of expenses means allocation of common expenses among the different departments on suitable basis

# APPORTIONMENT OR ALLOCATION OF COMMON EXPENSES:

| Sl. No | BASIS OF ALLOCATION | COMMON EXPENSES |
|---|---|---|
| 01 | Net purchase ratio ( Total purchases minus Purchase returns or returns outwards) | Carriage inwards, fright, octroi, duty etc |
| 02 | Net sales ratio (Total sales minus sales returns or returns inwards) | Commission on sales, discount allowed, carriage outwards, bad debts, RDD, RFDD, advertisement, sales tax etc |
| 03 | Staff appointed ratio (No of employees) | Salary, wages, labour welfare, canteen expenses etc. |

| | | |
|---|---|---|
| **04** | **Space or area occupied** | **Rent, rates, taxies, insurance on building, repairs of building, depreciation on building etc.** |
| 05 | Closing stock ratio | Insurance on goods, godown rent etc |
| 06 | Time spent or time devoted ratio | Salary of works manager |
| 07 | Value of assets ratio | Depreciation, repairs, maintenance of assets |
| 08 | Units consumed ratio | Lighting and heating, power, motive power, electricity etc. |

# IMPORTANT NOTES:

1. If the basis for allocation in not given in the problem, then those expenses should be apportioned on the basis of sales ratio ( turnover ratio)
2. However, there are certain common indirect expenses which cannot be apportioned on any one of the above basis. Such expenses should be directly recorded in the General Profit and Loss Account, which is meant for the entire organization. Such expenses are bank interest, accountancy charges, audit fees, income tax, and insurance on comprehensive policy.

## APPORTIONMENT OR ALLOCATION OF COMMON Incomes:

| Sl. No | BASIS OF ALLOCATION | COMMON INCOMES |
|---|---|---|
| 01 | Net purchase ratio | Discount received, commission earned |
| 02 | According to given ratio or allocated equally | Interest from bank, interest on investments etc. |

# INTERDEPARTMENTAL TRANSFER OR TRANSACTIONS:

Transfer of goods and services from one department to another department is called interdepartmental transactions.

# Treatment:

1. Interdepartmental transfer of goods from one department to another department is appearing in the trading account. The entry is:

   Receiving Department A/c ……………………………Dr

   To Giving Department A/c

2. Interdepartmental transfer of service from one department to another department is appearing in the profit and loss account. The entry is:

   Receiving Department A/c ……………………………Dr

   To Giving Department A/c

PURCHASES BOOK: It is a book meant for recording only credit purchases of goods.

Note: 1. Cash purchases of goods are appearing in the cash book.

2. Credit purchases of the things other than the goods are appearing in the journal proper.

SALES BOOK: It is a book meant for recording the credit sales of goods only.

Note: 1. Cash sales of goods are appearing in the cash book.

2. Credit sales of the things other than the goods are appearing in the journal proper.

# THANK YOU

# ENTREPRENEURSHIP DEVELOPMENT

## The Foundations of Entrepreneurship

Chapter #1

# What is an Entrepreneur?

An Entrepreneur is one who

- creates a new business in the face of risk and uncertainty
- for the purpose of achieving profit and growth
- by identifying opportunities &
- assembling the necessary resources to capitalize on them.

An entrepreneur is a

- Risk bearer
- Organizer
- Innovator

# Entrepreneurship

- Entrepreneurship is the attempt
  - to create value through recognition of business opportunity,
  - the management of risk-taking appropriate to the opportunity, and
  - through the communicative and management skills to mobilize human, financial and material resources necessary to bring a project to completion

| Difference between Entrepreneur and Entrepreneurship | |
|---|---|
| Person | Process |
| Organizer | Organization |
| Innovator | Innovation |
| Risk-bearer | Risk-bearing |
| Motivator | Motivation |
| Creator | Creation |
| Visualiser | Vision |
| Leader | Leadership |

# Characteristics of Entrepreneurs

- Desire for Responsibility-
  - prefer to be in control of their resources and use the resources to achieve self determined goal.
- Preference for Moderate Risk
  - They are calculated risk taker, rarely gamble
- Confidence in their ability to succeed
  - They are optimistic. This high level of optimism may explain why some of the most successful entrepreneur have failed in business before finally succeeding.
- Desire for Immediate Feedback
- High Level of Energy

# Characteristics of Entrepreneurs

- Future Orientation
  - Have a well-defined sense of searching opportunities. Entrepreneurs are less concerned about yesterday they are more focused on future.
- Skilled at Organizing
  - Effectively combining people and jobs.
- Value Achievement Over Money
  - Achievement seems to be the primary motivating source; money is symbol of achievement.
- High Degree of Commitment
  - Launching a company successfully needs total commitment from an entrepreneur .

# FARM ACCOUNTING

## Introduction

- In India Farming sector is dominated by small farmers.
- Average farmers cannot afford to appoint any accounts clerk.
- Recording of transaction may feel a troublesome .
- Hence the account maintained would be crude and unreliable due to some special features arising.
- Unawareness of the advantages for effective farm management.
- The well-organized and large scale farm can maintain the accounts adopting the principle of double-entry so that it will be possible to prepare the final accounts to ascertain the financial state of affairs of their farm activities.
- Profit and loss of each and every line of farm ascertained.
- Cost of production and yield obtainable by different activities can be ascertained.

# FRAM ACTIVITIES:

FRAM: an area of land or building attached to it used for growing crops or rearing animals

These include:

- Agriculture(food crops),
- Horticulture (fruits ,vegetables),
- Nurseries(growing plants under proper condition then transplanted to main field),
- Pisciculture (fish rearing),
- Sericulture(silk production),
- Dairy, Poultry, rearing of sheep pigs horse etc.

All these activities may be carried singly or in combination of two or more of them.

## **MEANING OF FARM ACCOUNTING**:

"Application of accounting principles and techniques to farming which constitutes the activities such as agricultural farming, dairy farming, poultry farming, etc."

**Kinds of farm Transaction**: (Four categories)

01. Cash transactions- involve receipts and payments of cash. Ex: wages to labourers.

02. Credit transactions- no receipts and payment of cash involved during transaction. But agreed for payment of cash in future. Ex: crop sold and delivered and payment received later.

03. Exchange transactions-exchange of goods or services between the parties. Ex: Exchange of workers between two agricultural farms

04. Notional transactions-transaction made between farm and household of same farm. Ex: Supply of food by farms to household.

## FEATURES OF FARM TRANSACTION

- Farming is like a family business
- Family members put their labour
- Payment of wages in kind
- Consumption of farm produce by household
- Farm produce from one section used by other section
- Farmer engaged in more than one activity
- Equipment etc. are used for more than one activity .
- Farming activities are prone to natural calamities (flood earthquake etc.)
- Valuation of inventory.

# SPECIAL FEATURES OF FARM ACCOUNTING

1. Problem of proper valuation
2. Value of products transferred on estimation
3. Allocation of common expenses
4. Farm products consumed by owners and workers
5. Family members assisting in farm work
6. Exchange of products on barter basis
7. Household property used in farm
8. Birth of death of calves, livestock, etc.
9. Allocation of depreciation in certain cases

# OBJECTIVES (PURPOSE) OF MAINTAINING FARM ACCONTING

1. To ascertain true cost and profit
2. To ascertain the overall profit of all activities
3. To take proper decision and control
4. To provide authentic (reliable or believable) information
5. To work out return on investment
6. To obtain agricultural credit or loan
7. To present the true financial position
8. To enable comparison of performance

# USE OF ACCOUNTING INFORMATION IN AGRICULTURAL FARM:

1. Analysis of performance
2. Detailed information of yield, etc.
3. Financial status of affairs of farm
4. Financial performance of the farm
5. Debt servicing ability of farm
6. Farm management survey
7. Assessment of agricultural income-tax
8. Data for decision making

# FARM ACCOUNTING IS NOT POPULAR IN INDIA

REASONS:

1. Farm sector is unorganized in India: activities are dominated by small-scale operations
2. Small farmers do not need accounting: due to their small landholdings
3. Lack of awareness in accounting in farms: the use of accounting is not clearly understood by many farmers
4. Cost of maintenance of accounts: farmers feel that cost is burdensome
5. Maintenance of cost is not compulsory: not made compulsory under any law
6. Data collections of farm operations: the data is usually collected through oral communication with farmers rather than any source information

# ASCERTAINMENT OF PROFIT OF EACH CROP SEPARATELY

- When two or more crops are produced by farmers, the profit of each crop should be ascertained separately.
- Direct cost of particular crop should be charged to that crop only.
- The common costs or expenses should be apportioned to each crop separately by following some generally accepted principles

Examples of common expenses :

1. Depreciation and repairs on common assets
2. Interest on fixed loan
3. Interest on working capital loan
4. Any other common expenses; they can be apportioned on the basis of generally accepted principles of accountancy and charged to respective crop account accordingly.

# REGISTERS MAINTAINED FOR ACCOUNTING DATA

The following six registers may be maintained by the farm to obtain data in accounting system:

a. Stock register: to record quantitative details of input stock (Ex: seeds), main products(crop) etc Analysis of stock
b. Debtors and creditors register: To record credit transactions :purchase and sales of goods
c. Fixed assets register : To record particulars of all fixed assets :land, shed, bore well, implements, tractors, etc. in respect to their location, description, cost, disposals or sales depreciation written off and their book value as on a given date
d. Loan register: To record particulars of borrowings made by the farm from govt., banks, cooperative societies or from any lending agencies
   Also shows interest paid and interest outstanding
e. Notional transaction register: To record transactions between farm and household.
f. Cost analysis register: To record crop wise apportionment and allocation of different common costs or expenses to compute crop profit of the farm.

Agriculture Activity

| Particular | | Amt | Particular | | Amt |
|---|---|---|---|---|---|
| To open stock | | XXX | By Sales | | |
| Corn | XXX | | Corn | XXX | |
| Growing crops | XXX | | Straw | XXX | XXX |
| Fertilizer | XXX | | By Drawings: | | |
| To Wages & Salaries | | XXX | Corn | XXX | |
| To Crop Expense | | XXX | Straw | XXX | XXX |
| To Live stock | | XXX | By Live stock | | |
| To Rent & Taxes | | XXX | Corn | XXX | |
| To Irrigation Expenses | | XXX | Straw | XXX | XXX |
| To Repairs | | XXX | By Closing Stock | | |
| To Other Expenses | | XXX | Corn | XXX | |
| To Depreciation | | XXX | Growing crops | XXX | |
| To General P & L account | | XXX | Fertiliser | XXX | XXX |
| | | XXX | | | XXX |

## CATTLE ACCOUNT FOR THE YEAR ENDED

| Particular | | Amt | Particular | | Amount |
|---|---|---|---|---|---|
| To open stock | XXX | XXX | By Sales | XXX | XXX |
| To Purchase | XXX | XXX | By Cattle Slaughter | XXX | XXX |
| To Clave born XXX | | | | | |
| Less Died XXX | XXX | | Sale Proceeds XXX | | |
| To Cattle food Consumed | | | Less Expenses: XXX | | |
| open stock XXX | | | | | |
| Add Purchase XXX | | | By Sales of Carcasses | XXX | XXX |
| XXX | | | By Closing Stock | XXX | XXX |
| Less closing Stock XXX | | | | | |
| To Wages & Salaries | | XXX | | | |
| To Veterinary Expense | | XXX | | | |
| To Crop Account | | XXX | | | |
| To Insurance | | XXX | | | |
| To Other Expenses | | XXX | | | |
| To Depreciation | | XXX | | | |
| To General P & L account | | XXX | | | |
| | XXX | XXX | | XXX | XXX |

**DAIRY ACTIVITY**

**Dairy ACCOUNT FOR THE YEAR ENDING**

| Particular | | Amt | Particular | | Amount |
|---|---|---|---|---|---|
| To open stock | | XXX | By Sales | | XXX |
| Clave Cattle | XXX | | Cattle | XXX | |
| Foodstuff | XXX | | Milk | XXX | |
| To Purchases | | | Butter | XXX | |
| Cattle | XXX | | By Drawing | | |
| Foodstuff | XXX | XXX | Milk | XXX | |
| To Wages & Salaries | | XXX | Butter | XXX | XXX |
| To Dairy Expense | | XXX | By Wages | | |
| To Insurance | | XXX | Milk | XXX | |
| To Other Expenses | | XXX | Butter | XXX | XXX |
| To Depreciation | | XXX | By Closing Stock | | |
| To General P & L account | | XXX | Cattle | XXX | |
| | | | Foodstuff | XXX | XXX |
| | | XXX | | | XXX |

# POULTRY ACCOUNT FOR THE YEAR ENDING

| Particular | | Amt | Particular | | Amount |
|---|---|---|---|---|---|
| To Open stock | | XXX | By Sales | | XXX |
| Poultry Birds | XXX | | Poultry Birds | XXX | |
| Poultry Eggs | XXX | | Poultry Eggs | XXX | |
| Poultry Food | XXX | | | | |
| To Purchases | | | By Drawing | | |
| Poultry birds | XXX | | Poultry Eggs | XXX | |
| Poultry Food | XXX | XXX | Poultry Birds | XXX | XXX |
| To Wages & Salaries | | XXX | By Wages | | |
| To Medicine Expense | | XXX | Poultry Eggs | XXX | |
| To Insurance | | XXX | Poultry Birds | XXX | XXX |
| To Other Expenses | | XXX | By Closing Stock | | |
| To Depreciation | | XXX | Poultry Eggs | XXX | |
| To General P & L account | | XXX | Poultry Birds | XXX | XXX |
| | | | Poultry Food | XXX | |
| | | XXX | | | XXX |

**FISH ACCOUNT FOR THE YEAR ENDING**

| Particular | | Amt | Particular | | Amount |
|---|---|---|---|---|---|
| To Open stock | | XXX | By Sales | | XXX |
| Fish | XXX | | By Drawing | | XXX |
| Foodstuff | XXX | | | | |
| To Purchases | | | By Wages | | XXX |
| Spawn | XXX | | | | |
| Foodstuff | XXX | XXX | By Closing Stock | | |
| To Wages & Salaries | | XXX | Fish | XXX | XXX |
| To Catching Expenses | | XXX | Foodstuff | XXX | |
| To Other Expenses | | XXX | | | |
| To Depreciation | | XXX | | | |
| To General P & L account | | XXX | | | |
| | | XXX | | | XXX |

## LIVE STOCK ACCOUNT FOR THE YEAR ENDING

| Particular | | Amt | Particular | | Amount |
|---|---|---|---|---|---|
| To Open stock | | XXX | By Sales | | XXX |
| Live Stock | XXX | | By Closing Stock | | |
| Feeding Materials | XXX | | Live Stock | XXX | |
| To Purchases | | | Feeding Materials | XXX | XXX |
| Live Stock | XXX | | | | |
| Feeding Materials | XXX | XXX | | | |
| To Wages & Salaries | | XXX | | | |
| To Insurance | | XXX | | | |
| To Crop account | | XXX | | | |
| To Other Expenses | | XXX | | | |
| To Depreciation | | XXX | | | |
| To General P & L account | | XXX | | | |
| | | XXX | | | XXX |

7. From the following information furnished by Mr. Krishik who keeps his books of account under single entry system ascertain his profit for the year ending 31-3-2012. Also prepare the revised statement affairs as on that date:

| Particulars | As on 1-4-2011 | As on 31-3-2012 |
|---|---|---|
| Land & Buildings | 1,00,000 | 1,00,000 |
| Farm Machinery | 30,000 | 30,000 |
| Implements | 16,000 | 16,000 |
| Stock of Crops | 45,000 | 68,000 |
| Sundry Debtors | 25,000 | 32,000 |
| Cash in Hand | 4,000 | 16,000 |
| Cash at Bank | 10,000 | 18,000 |
| Sundry Creditors | 10,000 | 12,000 |
| Loan from Bank | 20,000 | 8,000 |

His drawings during the year amounted to Rs. 30,000. The outstanding interest on bank loan was Rs. 2,000. Goods used by the proprietor for private purposes amounted to Rs.8,000. Depreciate land & buildings at 5%, machinery at 10% and the implements were revalued at Rs. 10,000 as at 31-3-2012.

# MR KRASHIKA'S STATEMENT OF AFFAIRS

| Liabilities | 1-4-2011 | 31-3-2012 | Assets | 1-4-2011 | 31-3-2012 |
|---|---|---|---|---|---|
| Sundry Creditors | 10000 | 12000 | Land & Buildings | 100000 | 100000 |
| Loan from Bank | 20000 | 8000 | Farm Machinery | 30000 | 30000 |
| | | | Implements | 16000 | 16000 |
| Capitals | 200000 | 260000 | Stock of Crops | 45000 | 68000 |
| | | | Sundry Debtors | 25000 | 32000 |
| | | | Cash in hand | 4000 | 16000 |
| | | | Cash at Bank | 10000 | 18000 |
| | 230000 | 280000 | | 230000 | 280000 |

**MR KRASHIKA'S**
**STATEMENT OF PROFIT OR LOSS**
**FOR THE YEAR ENDING 31ST MARCH 2012**

| Particular | Amt | Amount |
|---|---|---|
| Closing Capital | | 260000 |
| Add Drawing Cash | 30000 | |
| Goods | 8000 | 38000 |
| | | 298000 |
| Less Additional Capital | | 0000 |
| Adjusted Closing Capital | | 298000 |
| Less Opening Capital | | 200000 |
| Gross Profit | | 98000 |
| Add Incomes | | 0000 |
| | | 98000 |
| Less Expenses outstanding Interest | 2000 | |
| Depreciation on L & B 5% | 5000 | |
| on Implements | 6000 | |
| on Machinery 10% | 3000 | 16000 |
| Net Profit | | 82000 |

# MR KRASHIKA'S STATEMENT OF AFFAIRS AS ON 31$^{ST}$ MARCH 2012

| Liabilities | 1-4-2011 | 31-3-2012 | Assets | 1-4-2011 | 31-3-2012 |
|---|---|---|---|---|---|
| Opening Capitals | 200000 | 244000 | Land & Buildings | 100000 | 95000 |
| Add NP | 82000 | | Less Depreciation | 5000 | |
| | 282000 | | Farm Machinery | 30000 | 27000 |
| Less Drawings | 38000 | | Less Depreciation | 3000 | |
| | | | Implements | 16000 | 10000 |
| Sundry Creditors | | 12000 | Less Depreciation | 6000 | |
| Loan from Bank | 8000 | 10000 | Stock of Crops | | 68000 |
| Loan from Bank | 2000 | | Sundry Debtors | | 32000 |
| | | | Cash in hand | | 16000 |
| | | | Cash at Bank | | 18000 |
| | | 266000 | | | 266000 |

6. A farmer keeps his books of account under single entry system. You are required to ascertain his profit for the year ending 31-12-2010 and prepare revised statement affairs as on that date from the following information :

| Particulars | As on 1-1-2010 | As on 31-12-2010 |
|---|---|---|
| Sundry Creditors | 10,000 | 15,000 |
| Farm Equipments | 20,000 | 20,000 |
| Stock of Crops | 40,000 | 50,000 |
| Sundry Debtors | 15,000 | 25,000 |
| Cash in Hand | 5,000 | 10,000 |
| Cash at Bank | 30,000 | 40,000 |

His drawings during the year amounted to Rs. 16,000. He also introduced Rs. 6,000 as fresh capital during the year. He decided to depreciate farm equipments at 20% per annum.

8. A farmer keeps his books of account under single entry system. The following was his financial position as on 31-12-2011 :

| | | | |
|---|---|---|---|
| Capital | 1,50,000 | Tractor | 50,000 |
| Bank Loan | 35,000 | Stock of Farm Produce | 85,000 |
| Sundry Creditors | 45,000 | Sundry Debtors | 80,000 |
| | | Cash in Hand | 15,000 |
| | 2,30,000 | | 2,30,000 |

His assets and liabilities stood as under as at 31-12-2011 :

| | | | |
|---|---|---|---|
| Sundry Creditors | 75,000 | Tractor | 50,000 |
| Stock of Farm Produce | 80,000 | Sundry Debtors | 1,00,000 |
| Cash in Hand | 20,000 | Cash at Bank | 25,000 |

He consumed farm produce worth Rs. 20,000 for himself and his family during the year. Depreciate tractor by 10%.

Prepare the statement of profit for the year ending 31-12-2011 and revised statement of affairs as on that date.

# Zero rating:

Zero rating of goods and services means when the supply of certain goods and services are taxable at zero percentage and supplier can either claim the refund of the taxes paid on the input goods and services or gets exemptions from payment of taxes on such inputs.

Zero rating means getting exemption from payment of taxes on input goods and services supplied or claiming refund of the taxes paid on input goods and services for the supply of goods and services which are taxable at zero percentage.

**Zero Rated Supply:**

According to Section 16 of the IGST Act, zero rated supply means any of the following supplies of goods or services:

1. Export of goods or services or both;
2. Supply of goods or services or both to a Special Economic Zone developer
3. Supply of goods or services or both to a Special Economic Zone unit

# Features of Zero Rated Supply:

1. Zero rated supply includes supplies made to any country other than India (Exports) and supplies made to customers located in SEZ or SEZ developers.
2. Zero Rated Supply in IGST is being introduced through Chapter VIII.
3. The concepts of Zero Rated Supply are covered under Section 16 of the IGST Act.
4. Input Tax Credit is available in case of Zero rated supply.
5. Examples are Export of goods to USA, Supply of goods or services to SEZ or SEZ developers.

**List of Zero-Rated Supply:**

1. Agricultural products – paddy, fresh or chilled vegetables, certain provisionally preserved vegetables.
2. Essential foodstuff – oils, salt, flour, etc.
3. Livestock and livestock supplies or poultry – live animals and unprocessed meat.
4. Eggs.
5. Fish – live, fresh, frozen and dried.
6. First 200 units of electricity for domestic use.
7. Water for domestic users.
8. Exported goods.
9. Exported services – for example, architecture services in connection with land outside Malaysia.

# Benefits of Zero rated supply:

The following benefits available to zero rated supply:

1. Such supply is not eligible to any tax.
2. Credit of input tax may be availed for making zero rated supplies notwithstanding that such supply may be an exempt supply.
3. A registered person making zero rated supply shall be eligible to claim refund.
4. Zero rating expands supply chain system in export business
5. Cost of export and price of goods or services will become less
6. Overall countries business and earnings will rise
7. It maintains the country's economic growth.
8. It increases the employment opportunities.
9. It improves the balance of payment through exports.

# Consequences of zero rating:

1. The supplier must not collect any GST from the recipient of the good or service.
2. The supplier can claim input tax he paid to other GST registered persons in relation to the supply.

**Input Tax Credit (ITC) in zero rated supply:**

The input tax credit is available for zero rated supplies. This means that export without payment of duty and supply to SEZ will be considered as zero rated supply and credit will be available.

# Nil Rated supply:

**Meaning**: Nil rated supply is the supply of goods or services or both on which GST rate of zero percentage is applicable.

Nil rated supplies are listed in Schedule I in the GST rate schedule.

**Availability of input tax credit:** No ITC

**Examples:** Supply of jaggery, handloom, cereals, salt, accommodation in hotel with tariff below Rs. 1,000 per day.

# Non-taxable supplies:

- **Meaning:** Non-taxable supply is the supply of goods or services or both on which GST is not leviable.
- Non-taxable supplies are those goods or services or both kept out of the purview of GST.
- **Availability of input tax credit**: No ITC
- **Examples:** Supply of alcohol for human consumption, petroleum products, electricity

# Exempt supplies:

- **Meaning: Exempt supply under GST is a broad term which includes nil rate supplies, non-taxable supplies and specific supplies which are declared as exempt from tax by notification.**
- **Exempt supply is the supply of goods or services or both that does not attract GST.**
- **However, in GST returns, the Department requires a bifurcation between nil rates, non-taxable and exempt supplies.**
- **Availability of input tax credit: No ITC**
- **Examples: Bread, Fresh fruits, Fresh milk and curd etc.**

## Difference between Nil Rated, Zero Rated, Non-taxable supplies and Exempted supplies:

| Sl. No. | Nil Rated Supply | Zero Rated Supply | Non Taxable Supply | Exempted Supply |
|---|---|---|---|---|
| 01 | Nil rated supply is the supply of goods or services or both on which GST rate of zero percentage is applicable. | Zero rated supply includes supplies made to any country other than India (Exports) and supplies made to customers located in SEZ or SEZ developers. | Non-taxable supply is the supply of goods or services or both on which GST is not leviable. | Exempt supply is the supply of goods or services or both that does not attract GST. |
| 02 | Nil rated supplies are listed in Schedule I in the GST rate schedule. | The concepts of Zero Rated Supply are covered under Section 16 of the IGST Act. | Non-taxable supplies are those goods or services or both kept out of the purview of GST. | Exempt supply under GST is a broad term which includes nil rate supplies, non-taxable supplies and specific supplies which are declared as exempt from tax by notification. |
| 03 | No Input Tax Credit is available | Input Tax Credit is available | No Input Tax Credit is available | No Input Tax Credit is available |
| 04 | Example are Supply of jaggery, handloom, cereals, salt, accommodation in hotel with tariff below Rs. 1,000 per day. | Examples are Export of goods to USA, Supply of goods or services to SEZ or SEZ developers. | Examples are Supply of alcohol for human consumption, petroleum products, electricity | Examples are Bread, Fresh fruits, Fresh milk and curd etc. |

# Abatements (i.e. deductions):

- Abatement means reduction, discount, exemption and concession in the taxable value of supply.
- It is the partial exemption to the total amount of tax to be paid.
- Abatement is a reduction in or, an exemption of, the level of taxation faced by an individual or company.
- Examples are a decrease in tax, a reduction in penalties, reduction in heavy payment of taxes etc.

# Benefits of abatements:

1. It Encourages the specific activities such as investment in capital equipment.
2. It is one type of tax incentives.
3. It attracts some services at lower rate due to abatement.
4. It is the partial exemption to the total amount of tax to be paid.
5. It increases the job opportunities in the area.
6. It helps to invest in local infrastructure.

# Lower rate of tax for food and health:

Countries that use GST taxation route always want to make zero rated provisions on many types of food and health items like food, beverages, donation and charity to NGO’s and equipments in health like medical prescriptions, disabled wheelchairs, sewage services, medical book and any other useful daily use items are zero rated.

Government has given exemptions on lower rate of tax on food and health care education as they have importance for persons living below poverty line.

These exemptions in GST system of Organization for Economic Co-operative and Development (OECD) contain:

1. Hospital and medical care
2. Transport of sick injured persons
3. Human blood
4. Tissue organs
5. Dental care
6. Postal services
7. Education
8. Non-commercial activities of nonprofit making organizations
9. Cultural services excluding the following:

a) Radio
b) Television
c) Broadcasting
d) Insurance
e) Financial services etc.

# GST Council:

The Goods & Services Tax Council (GST Council) has been created in September 2016 under Article 279-A of the Constitution of India. The main objective of GST is to develop a harmonized national market of goods and services. It has its Secretariat office in New Delhi.

**Composition of GST Council:**

GST Council is a federal forum with both centre and states in India on board. It is made of:

**Chairman:**

The Union Finance Minister

**Members:**

The Union Minister of State in charge of Revenue or Finance, and

The Minister in charge of Finance or Taxation or any other Minister, nominated by each state government.

**Vice Chairman:**

The members of the goods and service council shall choose one amongst themselves to be the vice chairman of the council for such period as they may decide

# Functions or powers and responsibilities of GST Council:

As per Article 279A (4), the Council will make recommendations to the Union and the States on important issues related to GST, like

1. Taxes, cesses and surcharges to be subsumed under the GST
2. Goods and services which may be subject to, or exempt from GST
3. The threshold limit of turnover for application of GST;
4. Rates of GST
5. Model GST laws, principles of levy, apportionment of IGST and principles related to place of supply
6. Special provisions with respect to the states Arunachal Pradesh, Assam, Manipur, Meghalaya, Mizoram, Nagaland, Sikkim, Tripura, Himachal Pradesh, Jammu and Kashmir, and Uttara khand and
7. Other related matters

# GST Structure rates:

- GST Council will have statutory powers of recommendations on various issues relating to GST.
- GST Council specifies the various aspects like rate of GST, exemptions, which can be decided by Central Government on recommendations of GST council.
- GST Council suggests the GST rates. These rates are on the basis of classification of goods.
- The goods are classified on the basis of Harmonized System of Nomenclature (HSN) system

# Goods:

**HSN** code shall be used for classifying the goods under the GST Regime.

a) Tax payers whose turnover is above Rs. 1.5 crores but below Rs. 5 crores shall use 2 digit code.
b) Tax payers whose turnover is 5 crores and above shall use 4 digit code.
c) Tax payers whose turnover is below Rs. 1.5 crores are not required to maintain code in their invoices.
d) Those are engaging international business or export or import business shall mandatorily follow 8 digit code.

**Services:** Services will be classified as per Services Accounting Code (SAC).

# GST rate structure:

**On Goods:**

Schedule I : List of goods at nil rate

Schedule II : List of goods at 0.25% rate

Schedule III : List of goods at 3% rate

Schedule IV : List of goods at 5% rate

Schedule V : List of goods at 12% rate

Schedule VI : List of goods at 18% rate

Schedule VII : List of goods at 28% rate

**On Services:**

List of services at nil rate

List of services at 5% rate

List of services at 12% rate

List of services at 18% rate

List of services at 28% rate

**GST TAX RATES INDIA 2019: Latest Rates as per the GST Council in its meeting held on 24th January, 2019 :**

**Schedule I: List of Goods at Nil rate:**

1. Unpacked food grains
2. Fresh vegetables and fruits
3. Unbranded maida, besan, gur, milk, eggs, curd, lassi
4. Unpacked paneer
5. Unbranded natural honey
6. All types of salt
7. Fresh meat, fish, chicken
8. Butter milk
9. Cereal grains

10. Jute
11. Flour
12. Bread
13. Prasad
14. Bindi, sindoor
15. Stamps, judicial papers, printed books, newspapers
16. Bangles
17. Children's' picture, drawing or colouring books
18. Human hair.

Services:

1. All hotels and lodges in India with tariff below Rs.1,000 are exempted from paying taxes.

# 0% GST Items

**Goods**

- Unpacked foodgrains, fresh vegetables & fruits, fresh meat, fish, chicken
- milk, eggs, curd, lassi, unpacked paneer, unbranded natural honey
- printed books, newspapers, bangles, handloom

**Services**

- all hotels and lodges in India with tariff below Rs.1,000
- 0.25% tax for Rough precious and semi-precious stones

## Schedule II: List of Goods at 0.25% rate:

**1.** Rough industrial diamonds including unsorted rough diamonds

2. Rough precious and semi-precious stones

3. Admission to "protected monuments"

## Schedule III: List of Goods at 3% rate:

1. Pearls, natural or cultured, whether or not.
2. Diamonds, whether or not worked.
3. Precious stones (other than diamonds) and semi-precious stones, whether or not worked.
4. Synthetic or reconstructed precious or semi-precious stones, whether or not worked or graded.

5. Dust and powder of natural or synthetic precious or semi-precious stones.

6. Silver (including silver plated with gold or platinum), unwrought or in semi-manufactured forms, or in powder form.

7. Gold (including gold plated with platinum) unwrought or in semi-manufactured forms, or in powder form

8. Platinum, unwrought or in semi-manufactured forms, or in powder form

9. Base metals, silver or gold, clad with platinum, not further worked than semi-manufactured

10. Articles of jewellery and parts thereof, of precious metal or of metal clad with precious metal
11. Articles of goldsmiths’ or silversmiths’ wares and parts thereof, of precious metal or of metal clad with precious metal
12. Other articles of precious metal or of metal clad with precious metal
13. Articles of natural or cultured pearls, precious or semi-precious stones (natural, synthetic or reconstructed)
14. Imitation jewellery
15. Coin

**Schedule IV: List of Goods at 5% rate:**

1. Apparel below Rs 1000
2. Footwear below Rs 500
3. Sugar, tea, roasted coffee beans
4. Edible oils
5. Cream, skimmed milk powder
6. Fish fillet
7. Branded paneer
8. Frozen vegetables
9. Coffee
10. Spices

11. Pizza bread
12. Sabudana
13. Kerosene
14. Coal
15. Medicines
16. Stent
17. Lifeboats
18. Cashew nut, Cashew nut in shell
19. Ice and snow
20. Biogas
21. Insulin
22. Agarbatti

**Services** :
All restaurants of hotels with room tariff of less than Rs 7,500, Food parcels, Textile job work, Transport services (Railways, air transport)
Supply of e-waste

# 5% GST Items

**Goods**

- Apparel below Rs 1000, footwear below Rs 500, Sugar, tea
- medicines, Insulin, Biogas, Postage or revenue stamps
- kerosene, Cashew nut, frozen vegetables, coffee, spices

**Services**

- Transport services (Railways, air transport)
- Small restaurants

**Schedule V: List of Goods at 12% rate:**

1. Cell phones and Apparel above Rs 1000
2. Sewing machine
3. Umbrella
4. Ayurvedic medicines
5. Tooth powder
6. Butter
7. Ghee
8. Fruit juice
9. Packed coconut water
10. Preparations of vegetables, fruits, nuts or other parts of plants including pickle
11. Chutney, jam, jelly, fruit juices, frozen meat products, dry fruits in packaged form

12. Animal fat and sausage
13. Cheese
14. Colouring books  and picture books
15. Ketchup & Sauces
16. All diagnostic kits
17. Exercise books and note books
18. Spoons and forks
19. Ladles
20. Skimmers

21. Cake servers
22. Fish knives
23. Spectacles
24. Playing cards
25. Chess board, carom board and other board games.

**Services**:

State-run lotteries, Non-AC hotels, business class air ticket, fertilizers, Work contracts

# 12% GST Items

**Goods**

- Cell phones, Apparel above Rs 1000, umbrella, Spectacles
- Ayurvedic medicines, tooth powder, packed coconut water
- Playing cards, chess board, carom board, other board games like ludo

**Services**

- Non-AC hotels, business class air ticket
- State-run lotteries, fertilisers, Work Contracts

**Schedule VI: List of Goods at 18% rate:**

1.Footwear above Rs.500

2. Camera, speakers and monitors

3. Headgear and parts thereof

4. Trademarks and goodwill

5. Software

6. Bidi Patta

7. Biscuits - All categories

8. Flavored refined sugar

9. Pasta

10. Pastries and cakes

11. Preserved vegetables
12. Jams, sauces and soups
13. Ice cream
14. Instant food mixes
15. Mineral water
16. Envelopes,
17. Steel products
18. Printed circuits
19. Kajal pencil sticks
20. Aluminium foil

21. Weighing Machinery [other than electric or electronic weighing machine]
22. Printers [other than multifunction printers]
23. Electrical Transformer
24. CCTV
25. Optical Fiber
26. Bamboo furniture
27. Swimming pools and paddling pools
28. Curry paste
29. Mayonnaise and salad dressings
30. Mixed condiments and mixed seasonings.

**Services** :

1. Restaurants in hotel premises having room tariff up to Rs 7500
2. Telecom services
3. IT services
4. Branded garments
5. Financial services
6. Outdoor catering

# 18% GST Items

**Goods**

- Footwear above Rs.500, camera, speakers and monitors, Headgear
- Software, printed circuits, CCTV, Swimming pools, steel products
- All categories of Biscuits, flavoured refined sugar, ice cream, mineral water

**Services**

- AC hotels that serve liquor, Room tariffs between Rs.2,500 and Rs.7,500, Restaurants inside five-star hotels
- telecom services, IT services, branded garments and financial services

**Schedule VII: List of Goods at 28% rate:**

1. Automobiles
2. Motorcycles
3. ATM
4. Washing machine
5. Shavers
6. Hair clippers
7. Bidis
8. Chewing gum
9. Molasses
10. Chocolate not containing cocoa

11. Waffles and wafers coated with chocolate
12. Pan masala
13. Aerated water
14. Paint
15. Deodorants
16. Shaving creams
17. After shave
18. Hair shampoo
19. Dye
20. Sunscreen
21. Wallpaper

22. Ceramic tiles
23. Water heater
24. Dishwasher
25. Weighing machine
26. Vending machines
27. Vacuum cleaner
28. Aircraft for personal use.

**Services**:

1. Private-run lotteries authorized by the states
2. Race club betting
3. Cinema
4. 5-star hotels
5. Hotels with room tariffs above Rs 7,500

# 28% GST Items

## Goods

- Automobiles, Motorcycles, ATM, washing machine, shavers, water heater
- Bidis, chewing gum, molasses, chocolate not containing cocoa, pan masala
- vending machines, vacuum cleaner, aircraft for personal use, ceramic tiles, paint, deodorants, hair shampoo, dye, sunscreen

## Services

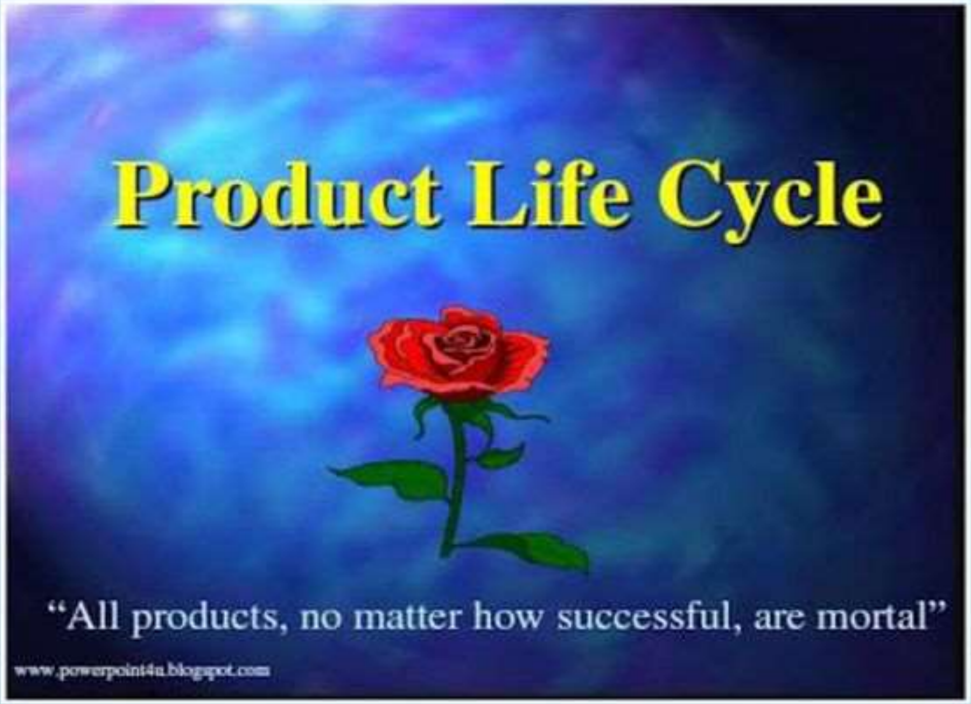
- Cinema, 5-star hotels, hotels with room tariffs above Rs 7,500
- Private-run lotteries authorised by the states, race club betting

# THANK YOU

# Product Life Cycle

# Product Life Cycle

- The stages of product development is called the **"Product Life Cycle".**
- The product life cycle has four stages introduction; growth; maturity and decline



# Product Life Cycle

**Introduction stage**

- High failure rates
- Little competition
- Frequent product modification
- Limited distribution
- High marketing and production costs
- Negative profits with slow sales increases
- Promotion focuses on awareness and information
- Communication challenge is to stimulate primary demand

# Product Life Cycle

Growth stage

- Increasing rate of sales
- Entrance of competitors
- Market consolidation
- Initial healthy profits
- Aggressive advertising of the differences between brands
- Wider distribution

# Payment Process in GST and Returns of GST

Every registered person is required to compute his tax liability on a monthly basis by setting off the Input Tax Credit (ITC) against the Outward Tax Liability. If there is any balance tax liability the same is required to be paid to the government.

**Electronic Ledgers (E-Ledgers):**

Electronic Ledgers are statements of cash and input tax credit in respect of each registered taxpayers.

Every tax payer required to be maintained 3 types of ledgers prescribed by the Government. They are:

1. Electronic Cash Ledger
2. Electronic Credit Ledger
3. Electronic Tax Liability Ledger

**1.Electronic Cash Ledger:** Electronic cash ledger is a ledger that is maintained by every taxpayer on common portal of GST and contains deposits that a taxpayer has made and any tax payment made through cash.

Electronic cash ledger is an account of the taxpayer maintained by GST system reflecting the cash deposits and payment of taxes and other dues made by taxpayer.

2. **Electronic Credit Ledger:** The taxes paid by the dealer on the inputs will be recorded in the electronic credit ledger. The credit for input tax credit is credited to the electronic credit ledger of the taxpayer.

Electronic credit ledger is maintained in GST portal. All the approved claims for input tax credit are credited to the electronic credit ledger under the appropriate head of CGST, SGST, IGST, UGST and GST cess.

**3. Electronic Tax Liability Ledger:** Electronic tax liability ledger shows the total tax liability of a registered dealer for a particular month. This total tax liability can be accessed on the GST portal. The total tax liability includes:

a. Amount of tax payable.
b. Interest, penalty and late fees.
c. Any other amount payable as determined in a proceeding by an assessing authority.
d. Tax Deducted at Source (TDS).
e. Tax Collected at Source (TCS).
f. Tax payable under reverse charge.

**Features of GST payment process:**

The following are the main features of GST payment process:

1. Electronically generated challan from GSTN Common Portal in all modes of payment and no use of manually prepared challan.
2. Facilitation for the taxpayer by providing hassle free, anytime, anywhere mode of payment of tax.
3. Convenience of making payment online.
4. Logical tax collection data in electronic format.
5. Faster remittance of tax revenue to the Government Account.
6. Paperless transactions.
7. Speedy Accounting and reporting.
8. Electronic reconciliation of all receipts.
9. Simplified procedure for banks.
10. Warehousing of Digital Challan.

**Methods or Modes of Payment under GST:**

1. **Internet banking through authorized banks (Debit card or Credit card).**
2. **Payment through NEFT (National Electronic Fund Transfer) or RTGS (Real Time Gross Settlement) from any bank.**
3. **Over The Counter Payment (OTC) through authorized banks for deposits up to ten thousand rupees per challan per tax period by cash, cheque or demand draft.**

**What are CPIN, CIN, BRN, E-FPB and UNI?**

1. **CPIN** stands for Common portal Identification Number. It is created for every Challan successfully generated by the taxpayer. It is a 14-digit unique number to identify the challan. CPIN remains valid for a period of 15 days.
2. **CIN** or Challan Identification Number is generated by the banks, once payment in lieu of a generated Challan is successful. It is a 17-digit number that is 14-digit CPIN plus 3-digit Bank Code. CIN is generated by the authorized banks/Reserve Bank of India (RBI) when payment is actually received by such authorized banks or RBI and credited in the relevant government account held with them. It is an indication that the payment has been realized and credited to the appropriate government account. CIN is communicated by the authorized bank to taxpayer as well as to GSTN.
3. **BRN** or Bank Reference Number is the transaction number given by the bank for a payment against a Challan
4. **E-FPB** stands for Electronic Focal Point Branch. These are branches of authorized banks which are authorized to collect payment of GST. Each authorized bank will nominate only one branch as its E-FPB for pan India transaction.
5. **UNI stands for Unique Identification Number**

**What is Common Portal?**

Common portal means the common goods and services tax electronic portal referred to in section 146 of the GST Act.

All returns are to be filled and payments to be made through this common portal

**What is Return?**

Return is a statement of information furnished by the taxpayer to tax administrators at regular intervals.

A return is a document containing details of income which a taxpayer is required to file with the tax administrative authorities. This is used by tax authorities to calculate tax liability.

Under GST, a registered dealer has to file GST returns that include:

a. Purchases
b. Sales
c. Output GST (On sales)
d. Input tax credit (GST paid on purchases)

**Different Types of Returns applicable under GST:**

| Return form | Who should file the return and what should be filed? | Due date for filing returns |
|---|---|---|
| GSTR-1 | Registered taxable supplier should file details of outward supplies of taxable goods and services as affected. | 10th of the subsequent month. |
| GSTR-2 | Registered taxable recipient should file details of inward supplies of taxable goods and services claiming input tax credit. | 15th of the subsequent month. |
| GSTR-3 | Registered taxable person should file monthly return on the basis of finalization of details of outward supplies and inward supplies plus the payment of amount of tax. | 20th of the subsequent month. |
| GSTR-4 | Composition supplier should file quarterly return. | 18th of the month succeeding quarter. |
| GSTR-5 | Return for non-resident taxable person. | 20th of the subsequent month. |
| GSTR-6 | Return for input service distributor. | 13th of the subsequent month. |

| | | |
|---|---|---|
| **GSTR-7** | **Return for authorities carrying out tax deduction at source.** | **10th of the subsequent month.** |
| GSTR-8 | E-commerce operator or tax collector should file details of supplies affected and the amount of tax collected. | 10th of the subsequent month. |
| GSTR-9 | Registered taxable person should file annual return. | 31 December of the next fiscal year. |
| GSTR-10 | Taxable person whose registration has been cancelled or surrendered should file final return. | Within 3 months of date of cancellation or date of cancellation order, whichever is later. |
| GSTR-11 | Person having UIN claiming refund should file details of inward supplies. | 28th of the month, following the month for which the statement was filed. |

**GSTR-1 Return for outward supplies (Section 37):**

GSTR-1 return form has to be filed by a registered taxable supplier with details of the outward supplies of goods and services. This form is filled by the supplier.

GSTR-1 is to be filled by all taxpayers including casual tax payer except the following persons:

I. Input Service Distributor (ISD)
II. Non-Resident taxable person
III. Composite Tax payer
IV. Person Deducting Tax at Source (TDS)
V. Person Collecting Tax at Source (TCS)
VI. A supplier of online Information and Database Access or Retrieval (OIDAR) Services

- While filing the GSTR-1 invoices has to be uploaded depending upon supply is made to registered person (B2B supply) or unregistered person (B2C supply).
- **B2B Supply:** B2B supply means business to business transactions, where the recipient is also a registered person, hence he can take ITC.
- **B2C Supply:** B2B supply means business to consumer transaction, where recipient is a consumer or unregistered person, hence he cannot take ITC.

**Contents of GSTR-1:**

1. GSTIN: Goods and Services Taxpayer Identification Number.
2. Name of the registered person.
3. Aggregate Turnover in the previous Year
4. Taxable outward supplies made to registered persons: All B2B supplies should be mentioned in this section.
5. Taxable outward inter-State supplies to un-registered persons (B2C) where the invoice value is more than Rs 2.5 lakh.
6. Zero-rated supplies and deemed exports.
7. Taxable outward supplies to consumer (B2C) where invoice value is less than 2.5 lakh.
8. Nil-rated, exempt and non-GST outward supplies.

9. Amendments to taxable outward supply details furnished in returns for earlier tax periods in points 4, 5 and 6.
10. Amendments to taxable outward supplies to unregistered persons furnished on returns for earlier tax periods.
11. Consolidated Statement of Advances Received or adjusted in the current tax period, plus amendments from earlier tax periods.
12. HSN-wise summary of outward supplies: This section requires a registered dealer to provide HSN wise summary of goods sold.
13. Documents issued during the tax period: This head will include details of all invoices issues in a tax period, any kind of revised invoice, debit notes, credit notes, etc.

**GSTR-1 has to be filed by 10th of the following month.**

## GSTR-2 Return for inward supplies (Section 38):

GSTR-2 return form has to be filed by a registered taxable recipient with details of the inward supplies of goods and services.

GSTR-2 is to be filled by all taxpayers including casual tax payer except the following persons:

I. Input Service Distributor (ISD)
II. Non-Resident taxable person
III. Composite Tax payer
IV. Person Deducting Tax at Source (TDS)
V. Person Collecting Tax at Source (TCS)
VI. A supplier of online Information and Database Access or Retrieval (OIDAR) Services

**Contents of GSTR-2:**

1. GSTIN
2. Name of the registered person
3. Inward supplies received from a registered person other than the supplies attracting reverse charge
4. **Inward supplies on which tax is to be paid on reverse charge**
5. **Inputs/Capital goods received from Overseas or from SEZ units on a Bill of Entry**
6. **Amendments to details of inward supplies furnished in returns for earlier tax periods in points 3, 4 and 5 including debit notes or credit notes issued.**
7. **Supplies received from composition taxable person and other exempt or nil rated or Non GST supplies received.**

**8. ISD credit received**

**9. TDS and TCS credit received**

**10. Consolidated Statement of Advances paid or Advance adjusted on account of receipt of supply**

**11. Input Tax Credit Reversal or Reclaim**

**12. Addition and reduction of amount in output tax for mismatch and other reasons**

**13. HSN summary of inward supplies**

**GSTR-2 has to be filed by 15th of the following month.**

**GSTR-3 Monthly returns (Section 39):**

GSTR-3 return form has to be filed by a registered taxpayer with details that are automatically populated by from GSTR-1 and GSTR-2 returns forms. The taxpayer has to verify and make modifications, if any. GSTR-3 return form will contain the following details:

1. Details about Input Tax Credit, liability, and cash ledger.
2. Details of tax paid under CGST, SGST, and IGST.
3. Claim a refund of excess payment or request to carry forward the credit.

**GSTR-3 has to be filed by 20th of the following month**.

**GSTR-4 Return for compounding taxable person:**

GSTR-4 return form has to be filed by taxpayers who have opted for the Composition Scheme. Taxpayers with small business or a turnover of up to Rs.75 lakh can opt for the Composition Scheme wherein he or she have to pay tax at a fixed rate based on the type of business. Taxpayers under this scheme will not have input tax credit facility. GSTR-4 quarterly return form will contain the following details:

1. The total value of consolidated supply made during the period of return.
2. Details of tax paid.
3. Invoice-level purchase information.

**GSTR-4 has to be filed by 18th of the following month.**

**GSTR-5 Return for Non-Resident foreign taxable person:**

GSTR-5 return form has to be filed by all registered non-resident taxpayers. This form will contain the following:

1. Name and address of the taxpayer, GSTIN, and period of return.
2. Details of outward supplies and inward supplies.
3. Details of goods imported, any amendments in goods imported during the previous tax periods.
4. Import of services, amendments in import of services
5. Details of credit or debit notes, closing stock of goods, and refund claimed from cash ledger.

**GSTR-5 has to be filed by 20th of the following month.**

**GSTR-6 Return for Input Service Distributor:**

GSTR-6 return form has to be filed by all taxpayers who are registered as an Input Service Distributor. This form will contain the following:

1. Name and address of the taxpayer, GSTIN, and period of return.
2. Details of input credit distributed.
3. Supplies received from registered persons.
4. The amount of input credit availed under the current tax period.
5. Details of inward supplies will be auto-populated from GSTR-1 and GSTR-5 return forms.
6. Details of the receiver of input credit corresponding to his or her GSTIN.
7. Details of credit or debit notes.
8. Input tax credit received, input tax credit reverted, and input tax credit distributed as SGST, CGST, and IGST.

**GSTR-6 has to be filed by 13th of the following month.**

**GSTR-7 Return for authorities deducting tax at source:**

GSTR-7 return form has to be filed by all registered taxpayers who are required to deduct tax at source under the GST rule. This form will contain the following:

a. Name and address of the taxpayer, GSTIN, and period of return.
b. TDS details and amendments in invoice amount, TDS amount or contract details.
c. TDS liability will be auto-populated. Details of fees for late filing of return and interest on delayed payment of TDS.
d. Refund received from Electronic Cash Ledger will be auto-populated.

**GSTR-7 has to be filed by 10th of the following month.**

**GSTR-8 Details of supplies effected through e-commerce operator and the amount of tax collected:**

GSTR-8 return form has to be filed by all e-Commerce operators who are required to collect tax at source under the GST rule. This form will contain details of supplies effected and the amount of tax collected under Sub-section (1) of Section 43C of Model GST Law. Other details include:

a. Name and address of the taxpayer, GSTIN, and period of return.
b. Details of supplies made to registered taxable person and amendments, if any.
c. Details of supplies made to unregistered persons.
d. Details of Tax Collected at Source.
e. TDS liability will be auto-populated. Details of fees for late filing of return and interest on delayed payment of TDS.

**GSTR-8 has to be filed by 10th of the following month.**

**GSTR-9 Annual Returns:**

GSTR-9 return form is filed by normal taxpayers with details of all income and expenditure for the year. This detail will be regrouped in accordance with the monthly returns. The taxpayer will have the opportunity to make modifications in the information provided if required.

**Contents of GSTR-9:**

a. GSTIN
b. Legal name of registered person
c. Date of statutory audit
d. Auditors
e. Details of expenditure
f. Details of income
g. Return reconciliation statement
h. Profit and Loss statement

**GSTR-9 has to be filed by 31st December of the following financial year along with the audited copies of the annual accounts.**

**GSTR-10 Final returns:**

GSTR-10 return form has to be filed by any taxpayer who opts for cancellation of GST registration. This form will contain the following:

a. Application Reference Number (ARN).
b. Date of cancellation of GST registration.
c. Unique ID of cancellation order.
d. Date of cancellation order.
e. Details of closing stock including amount of tax payable on closing stock.

**GSTR-10 final return form has to be filed within 3 months of the date of cancellation or date of cancellation order, whichever is later.**

**GSTR- 11 Details of Inward supplies:**

GSTR-11 return form has to be filed by everyone who has been issued a Unique Identity Number (UIN) and claims a refund of the taxes paid on inward supplies. This form will contain the following details:

a. Name of the government entity, UIN, and period of return.
b. All inward purchases from GST registered supplier will be auto-populated.

**Based on the above mentioned details, the tax refund will be made. GSTR-11 form has to be filed on 28th of the month, following the month for which supply was received.**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# Payment Process in GST and Returns of GST

# Introduction:

Every registered person is required to compute his tax liability on a monthly basis by setting off the Input Tax Credit (ITC) against the Outward Tax Liability. If there is any balance tax liability the same is required to be paid to the government.

# Features of GST payment process:

1. Electronically generated challan from GSTN Common Portal in all modes of payment and no use of manually prepared challan.
2. Facilitation for the taxpayer by providing hassle free, anytime, anywhere mode of payment of tax.
3. Convenience of making payment online.
4. Logical tax collection data in electronic format.
5. Faster remittance of tax revenue to the Government Account.
6. Paperless transactions.
7. Speedy Accounting and reporting.
8. Electronic reconciliation of all receipts.
9. Simplified procedure for banks.
10. Warehousing of Digital Challan.

# Methods or Modes of Payment under GST:

1. **Internet banking through authorized banks (Debit card or Credit card).**
2. **Payment through NEFT (National Electronic Fund Transfer) or RTGS (Real Time Gross Settlement) from any bank.**
3. **Over The Counter Payment (OTC) through authorized banks for deposits up to ten thousand rupees per challan per tax period by cash, cheque or demand draft.**

# What are CPIN, CIN, BRN and E-FPB?

**CPIN** stands for Common portal Identification Number. It is created for every Challan successfully generated by the taxpayer. It is a 14-digit unique number to identify the challan. CPIN remains valid for a period of 15 days.

**CIN** or Challan Identification Number is generated by the banks, once payment in lieu of a generated Challan is successful. It is a 17-digit number that is 14-digit CPIN plus 3-digit Bank Code. CIN is generated by the authorized banks/Reserve Bank of India (RBI) when payment is actually received by such authorized banks or RBI and credited in the relevant government account held with them. It is an indication that the payment has been realized and credited to the appropriate government account. CIN is communicated by the authorized bank to taxpayer as well as to GSTN.

**BRN** or Bank Reference Number is the transaction number given by the bank for a payment against a Challan

**E-FPB** stands for Electronic Focal Point Branch. These are branches of authorized banks which are authorized to collect payment of GST. Each authorized bank will nominate only one branch as its E-FPB for pan India transaction.

# What is Common Portal?

Common portal means the common goods and services tax electronic portal referred to in section 146 of the GST Act.

All returns are to be filled and payments to be made through this common portal.

**What is Return?**

A return is a document containing details of income which a taxpayer is required to file with the tax administrative authorities. This is used by tax authorities to calculate tax liability.

Under GST, a registered dealer has to file GST returns that include:

a. Purchases
b. Sales
c. Output GST (On sales)
d. Input tax credit (GST paid on purchases)

# Different Types of Returns applicable under GST:

GST return can be filed using different forms depending on the type of transaction and registration of the taxpayer. Return forms for normal taxpayers are:

**GSTR-1**

GSTR-1 return form has to be filed by a registered taxable supplier with details of the outward supplies of goods and services. This form is filled by the supplier. The buyer has to validate the auto-populated purchase information on the form and make modifications if required. The form will contain the following details:

a. Business name, period for which the return is filed, Goods and Services Taxpayer Identification Number (GSTIN).
b. Invoices issued in the previous month and the corresponding taxes collected.
c. Advances received against a supply order that has to be delivered in the future.
d. Revision in outward sales invoices from the previous tax periods.

GSTR-1 has to be filed by 10th of the following month.

**GSTR-2**

GSTR-2 return form has to be filed by a registered taxable recipient with details of the inward supplies of goods and services. The form will contain the following details:

a. Business name, period for which the return is filed, Goods and Services Tax Identification Number (GSTIN).
b. Invoices issued in the previous month and the corresponding taxes collected.
c. Advances received against a supply order that has to be delivered in the future.
d. Revision in outward sales invoices from the previous tax periods.

GSTR-2 has to be filed by 15th of the following month.

**GSTR-3**

GSTR-3 return form has to be filed by a registered taxpayer with details that are automatically populated by from GSTR-1 and GSTR-2 returns forms. The taxpayer has to verify and make modifications, if any. GSTR-3 return form will contain the following details:

a. Details about Input Tax Credit, liability, and cash ledger.
b. Details of tax paid under CGST, SGST, and IGST.
c. Claim a refund of excess payment or request to carry forward the credit.

GSTR-3 has to be filed by 20th of the following month.

**GSTR-4**

GSTR-4 return form has to be filed by taxpayers who have opted for the Composition Scheme. Taxpayers with small business or a turnover of up to Rs.75 lakh can opt for the Composition Scheme wherein he or she have to pay tax at a fixed rate based on the type of business. Taxpayers under this scheme will not have input tax credit facility. GSTR-4 quarterly return form will contain the following details:

a. The total value of consolidated supply made during the period of return.
b. Details of tax paid.
c. Invoice-level purchase information.

GSTR-4 has to be filed by 18th of the following month.

**GSTR-5**

GSTR-5 return form has to be filed by all registered non-resident taxpayers. This form will contain the following:

a. Name and address of the taxpayer, GSTIN, and period of return.
b. Details of outward supplies and inward supplies.
c. Details of goods imported, any amendments in goods imported during the previous tax periods.
d. Import of services, amendments in import of services
e. Details of credit or debit notes, closing stock of goods, and refund claimed from cash ledger.

GSTR-5 has to be filed by 20th of the following month.

**GSTR-6**

GSTR-6 return form has to be filed by all taxpayers who are registered as an Input Service Distributor. This form will contain the following:

a. Name and address of the taxpayer, GSTIN, and period of return.
b. Details of input credit distributed.
c. Supplies received from registered persons.
d. The amount of input credit availed under the current tax period.
e. Details of inward supplies will be auto-populated from GSTR-1 and GSTR-5 return forms.
f. Details of the receiver of input credit corresponding to his or her GSTIN.
g. Details of credit or debit notes.
h. Input tax credit received, input tax credit reverted, and input tax credit distributed as SGST, CGST, and IGST.

GSTR-6 has to be filed by 13th of the following month.

**GSTR-7**

GSTR-7 return form has to be filed by all registered taxpayers who are required to deduct tax at source under the GST rule. This form will contain the following:

a. Name and address of the taxpayer, GSTIN, and period of return.
b. TDS details and amendments in invoice amount, TDS amount or contract details.
c. TDS liability will be auto-populated. Details of fees for late filing of return and interest on delayed payment of TDS.
d. Refund received from Electronic Cash Ledger will be auto-populated.

GSTR-7 has to be filed by 10th of the following month.

**GSTR-8**

GSTR-8 return form has to be filed by all e-Commerce operators who are required to collect tax at source under the GST rule. This form will contain details of supplies effected and the amount of tax collected under Sub-section (1) of Section 43C of Model GST Law. Other details include:

a. Name and address of the taxpayer, GSTIN, and period of return.
b. Details of supplies made to registered taxable person and amendments, if any.
c. Details of supplies made to unregistered persons.
d. Details of Tax Collected at Source.
e. TDS liability will be auto-populated. Details of fees for late filing of return and interest on delayed payment of TDS.

GSTR-8 has to be filed by 10th of the following month.

## GSTR-9

GSTR-9 return form is filed by normal taxpayers with details of all income and expenditure for the year. This detail will be regrouped in accordance with the monthly returns. The taxpayer will have the opportunity to make modifications in the information provided if required. GSTR-9 has to be filed by 31st December of the following financial year along with the audited copies of the annual accounts.

**GSTR-10**

GSTR-10 return form has to be filed by any taxpayer who opts for cancellation of GST registration. This form will contain the following:

a. Application Reference Number (ARN).
b. Date of cancellation of GST registration.
c. Unique ID of cancellation order.
d. Date of cancellation order.
e. Details of closing stock including amount of tax payable on closing stock.

GSTR-10 final return form has to be filed within 3 months of the date of cancellation or date of cancellation order, whichever is later.

**GSTR- 11**

GSTR-11 return form has to be filed by everyone who has been issued a Unique Identity Number (UIN) and claims a refund of the taxes paid on inward supplies. This form will contain the following details:

a. Name of the government entity, UIN, and period of return.
b. All inward purchases from GST registered supplier will be auto-populated.
c. Based on the above mentioned details, the tax refund will be made.

GSTR-11 form has to be filed on 28th of the month, following the month for which supply was received.

# *PROCESS COSTING*

***Meaning and Definition***: "*Process costing is used to ascertain the cost of each stage of manufacturing where material is passed through various processes to obtain a final product to result, with by products in many cases at different stages*".

---------- By Lunt and Riply

Process costing is a method applied to industries where the material has to pass through two or more production processes for being converted into a finished product.

Where the production is continuous and the end product is the result of a sequence of process and where each plant is divided into separate process centers, it becomes necessary to ascertain the cost of production in each process.

Thus, process costing is a method of costing designed to ascertain the cost of production in each process.

In this method the finished product of one process will be transferred to the next process for the further production. Thus, when the finished product of one process is transferred to the next process it becomes input or raw material for the next process.

When the finished product or output is transferred to the next process the cost of one process are also transferred to the next process and it is only in the last process the total cost of production is ascertained.

***For which industries the process costing is applied?***

The process costing can applied to the following industries:

a. Chemical industries
b. Paper industries
c. Rubber industries
d. Oil refining industries
e. Textile industries
f. Soap making industries
g. Biscuit works etc.

## *Application of the process costing:*

1. The factory is divided into different processes and an account is kept for each process.
2. The costs of each process will be debited to the respective process account.
3. As the production moves from one process to another process, the output and the total cost of that process will be transferred to the next process. Thus, the output of one process becomes input of another next process.

4. The total cost of each process is divided by the units manufactured in that process to get the cost per unit of that process.
5. The record of each process is to be maintained in such a way that, the output, input, waste, scrap, loss and gain of each process can be easily ascertained.

## *PROCESS LOSSES:*

In all manufacturing industries some kind of loss is inevitable or unavoidable. Generally, this loss is classified into two groups namely:

I) Normal loss and

II) Abnormal Loss

I) ***Normal loss:*** It is such kind of loss which is a part of normal production. It is the loss which is unavoidable on account of inherent nature of materials. Since such loss is expected under normal conditions it is always estimated in terms of percentage well in advance on the basis of past experience.

***Treatment of normal loss in process account:***

The normal loss is calculated in terms of percentages on the input of the process. This normal loss is recorded only in terms of quantity. Thus, the normal loss will reduce the quantity of output. The units of normal loss therefore, recorded on the credit side of concerned process account. Where such normal loss possesses some scrap value such value is also credited to the concerned process account.

Therefore, when there is a normal loss in the process, then the normal production or normal out put is:

***Normal production or Normal output***

***= Input - % of normal loss***

***II. Abnormal Loss***:

A loss which is not common to the production or the occurrence of which is not generally expected in the ordinary course of production is called abnormal loss.

Where the loss is caused by unexpected or abnormal conditions and if it is beyond the limit and control is called abnormal loss.

***Treatment of abnormal loss in process account:***

The abnormal loss is not allowed to affect the normal cost of production. To find out the units of abnormal loss the actual output of the process and normal output of the process will be compared. If the actual production is less than the normal production then, there is abnormal loss.

Therefore,

***Units of abnormal loss***

***= Normal production (output) – Actual Production (output)***

The units of abnormal loss along with the value of abnormal loss are credited to the concerned process account. Then, the calculation of abnormal loss is as follows:

# Value of Abnormal loss =

$$\frac{Total\ Debit\ of\ \Pr ocess - Total\ Credit\ of\ \Pr ocess}{Normal\ \Pr oduction\ (Output)}\ X\ Abnormal\ Loss\ Units$$

Where:

1. Normal production = Input - % of normal loss
2. Units of abnormal loss= Normal production – Actual production.

***Abnormal Gain***:

When the actual production or actual output is more than the normal production (output) then there is abnormal gain. The difference between actual production and normal production is called abnormal gain units.

Abnormal gain units

= Actual production – Normal production

## ***Treatment of abnormal gain in process account***:

The units of abnormal gain along with the value of abnormal gain are debited to the concerned process account.

**Value of Abnormal gain =**

$$\frac{Total\ Debit\ of\ \Pr ocess - Total\ Credit\ of\ \Pr ocess}{Normal\ \Pr oduction\ (Output)} X\ Abnormal\ Gain\ Units$$

***Note:*** While transferring the units from one process to another process, only actual output is to be transferred irrespective of abnormal loss or abnormal gain.

***Steps to be followed to prepare the process account:***

a. Normal loss units to be calculated based on the percentage on input and recorded on credit side of process account along with scrap value.

b. Normal production or output

   = Input – Normal loss units

c. Actual output to be taken

d. Compare the actual output with normal output

e. The difference may be abnormal loss or abnormal gain

f. If actual output is less than normal output, then there is abnormal loss

g. If actual output is more than normal output, then there is abnormal gain

h. Units of abnormal loss =

Normal production (output) – Actual Production (output)

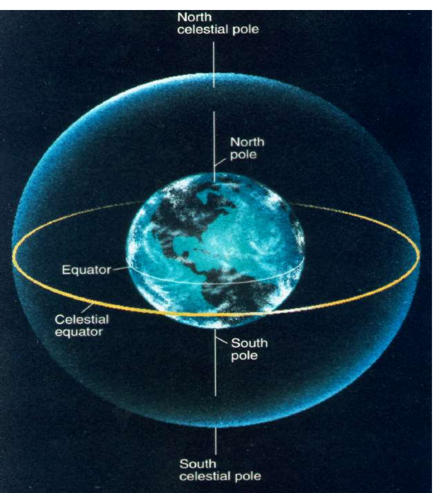
# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: Calculus–II and 3–D Geometry

22nd online Class

**Section of a sphere by the Plane**




By

Dr. M. M. Shankrikopp

HOD of Mathematics

Date: 8.6.2021

# Examples on finding equation of sphere passing through given points

1. **Find equation of sphere passing through origin and the points(0, 1, -1), (-1, 2, 0) and (-1,2, 3).**

Soln.: Let the required equation of the sphere be

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \text{----------}(1)$$

**It passes through the origin, we put x=0 , y =0, z=0 in (1)**

$0 + d = 0$

$\Rightarrow d = 0$ ---------------(2)

Next (1) passes through the point (0,1,-1), (-1,2,0) and (-1, 2, 3)

Therefore we have,

$0+1+1+2u(0) + 2v(1)+2w(-1)=0$

i.e $2+2v-2w=0$

i.e. **$1+u-w=0$------------(3)**

Next $1+4+0+2u(-1)+2v(2)+2w(0)=0$

i.e **$5 -2u+4v=0$----------(4)**

Also

$1+4+9+2u(-1) +2v(2) + 2w(3)=0$

i.e **$14-2u+4v+6w=0$ -------(5)**

**Substitute $-2u +4v =-5$ from (4) in (5) we get**

$14 - 5 + 6w = 0 \Rightarrow 6w = -9$

$\Rightarrow$ **$w = -3/2$**

Put w in (3) we get, $1 + u + 3/2 = 0$

$\Rightarrow$ **$u = -5/2$**

From (4) we have

**$5 - 2u + 4v = 0$**

**i.e $5 - 2(-5/2) + 4v = 0$**

**i.e.** $10 + 4v = 0$

$\therefore v = -5/2$

Then from(1), req. eqn.of sphere is

**$x^2+y^2+z^2+2(-5/2)x+2(-5/2)y+2(-3/2)z+d=0$**

**$x^2+y^2+z^2 -5x-5y+2(-3/2)z+0=0$**

**i.e $x^2+y^2+z^2+ -5x-5y -3z = 0$**

**2. Find equation of sphere passing through origin and the points(-1, 1, 1), (1-1, 1) , (1,1,-1).**

Soln.: Let the required equation of the sphere be

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \text{----------} (1)$$

**It passes through the origin, we put x=0 , y =0, z=0, in (1)**

$0 + d = 0$

$\Rightarrow d = 0$ ---------------(2)

Next (1) passes through the point (-1,1,1), (1,-1,1) and (1, 1, -1)

Therefore we have,

$1+1+1+2u(1) + 2v(-1)+2w(1)=0$

i.e $3+2u-2v+2w=0$

i.e. **$u-v+w= -3/2$-----------------------(3)**

Similarly **we have $u+v-w= -3/2$-----(4)**

**And $-u+v+w= -3/2$---------------------(5)**

Adding (3) and (5) we get $2w = -6/2$

**$\therefore w = - 3/2$**

Similarly ,

From (4) and (5) we get $v = -3/2$

And from (3) and (4) we get $u = -3/2$

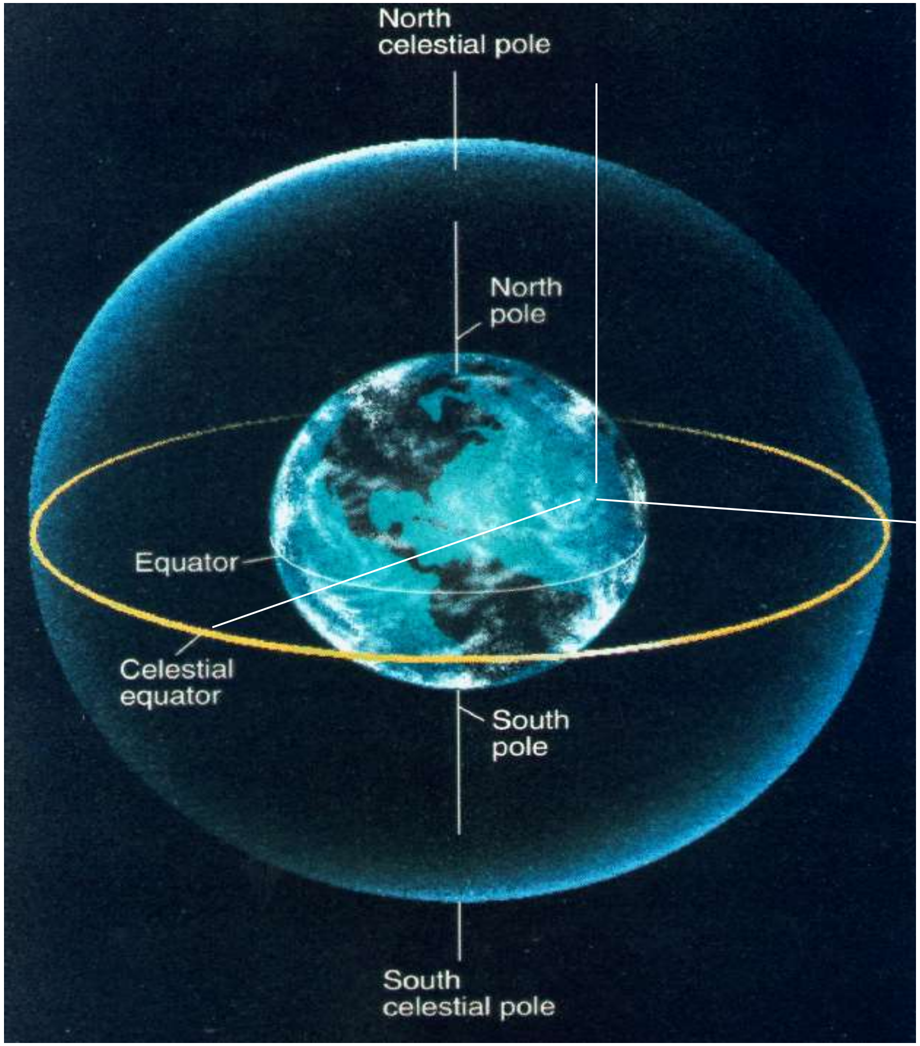
Then from(1), req. eqn.of sphere is

**$x^2+y^2+z^2 + 2(-3/2)x+ 2(-3/2)y+2(-3/2)z+0=0$**

**i.e $x^2+y^2+z^2 - (3)x- 3y- (3)z =0$**

**i.e $x^2+y^2+z^2 -3x-3y -3z =0$**

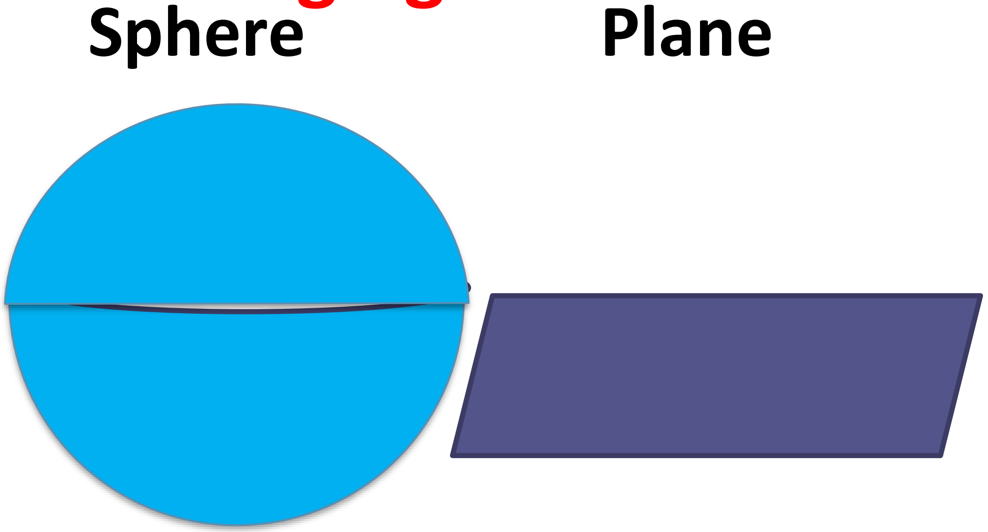
# Sphere with centre at origin



# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

We observe the following figure:



# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux +2vy+2wz+d=0$ then what is the section?

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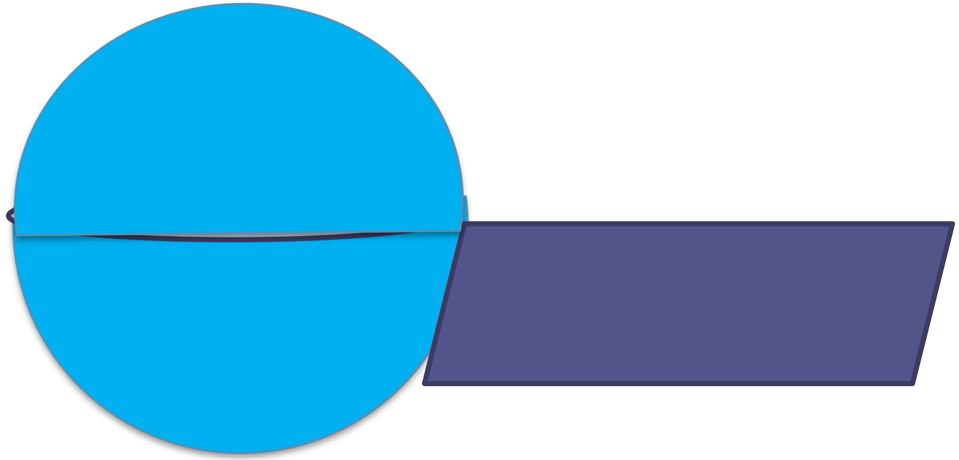

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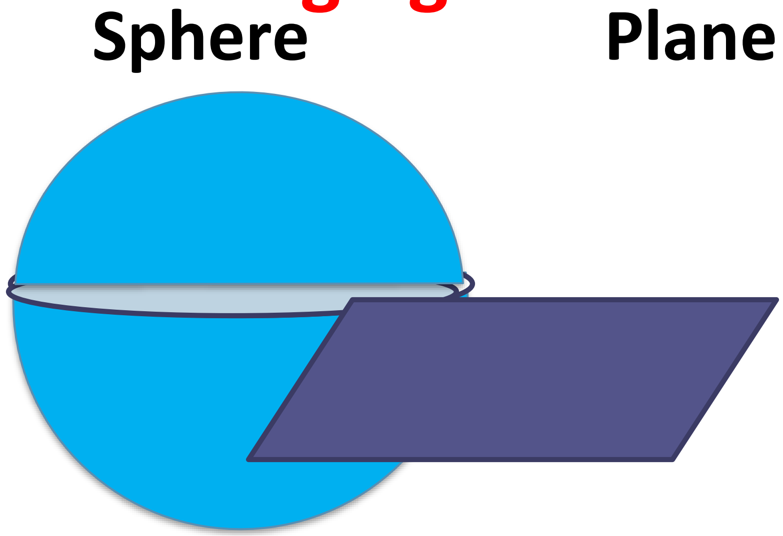

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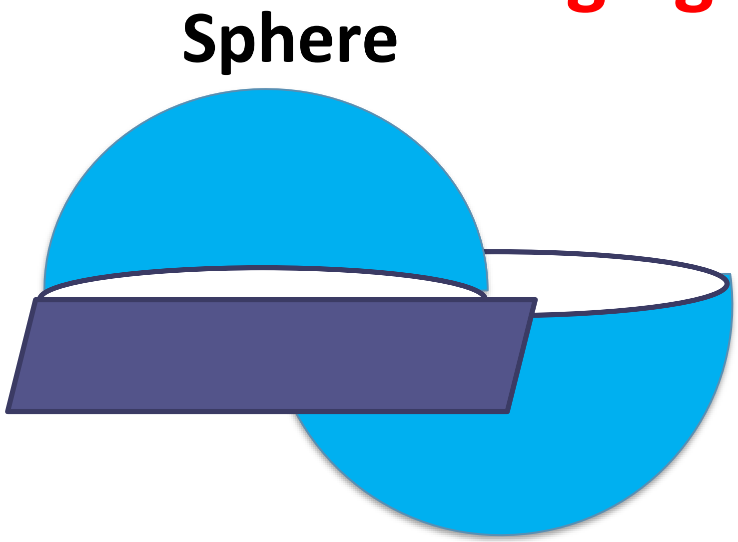

# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

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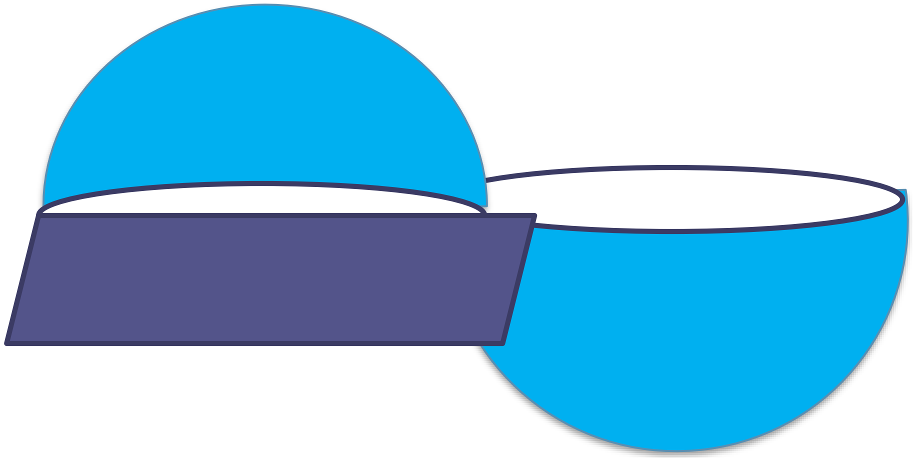
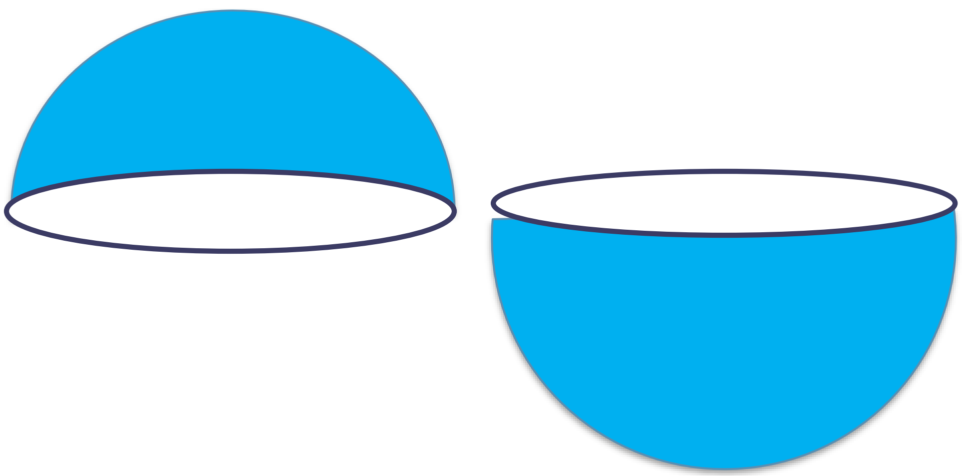


# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

We observe the following figure

See, we get two sections in both surface of section is circle



# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

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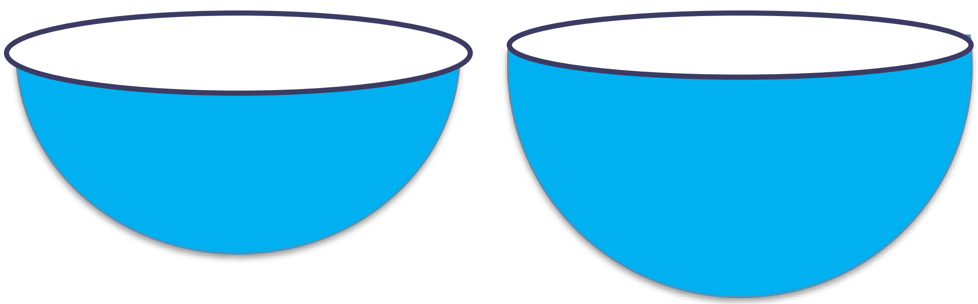

# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

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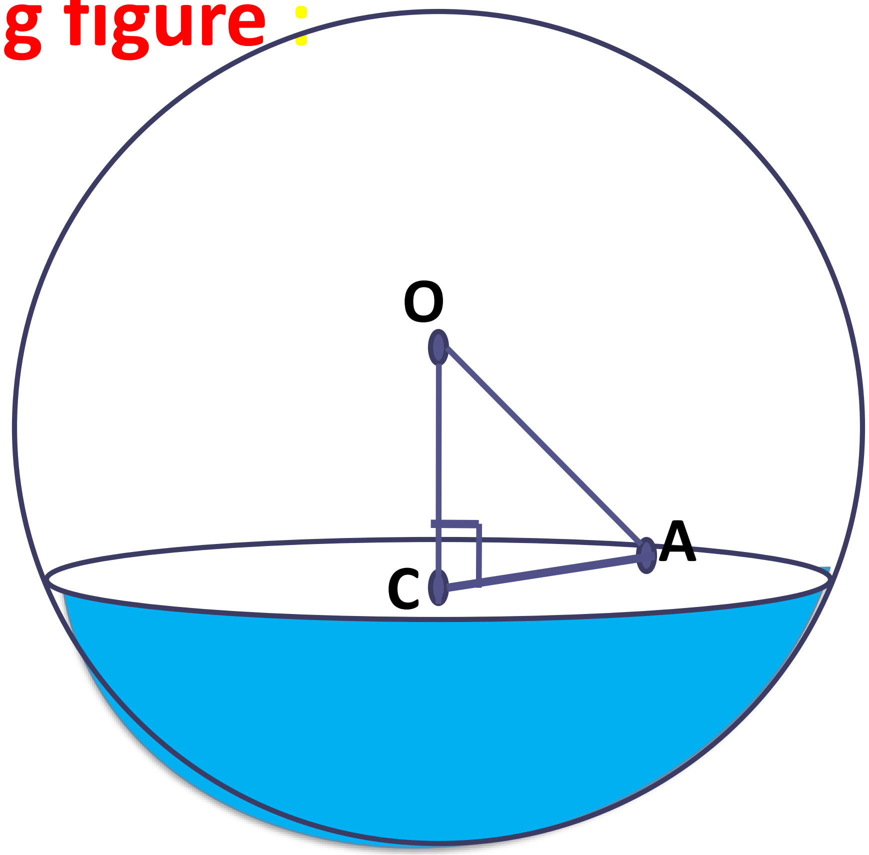
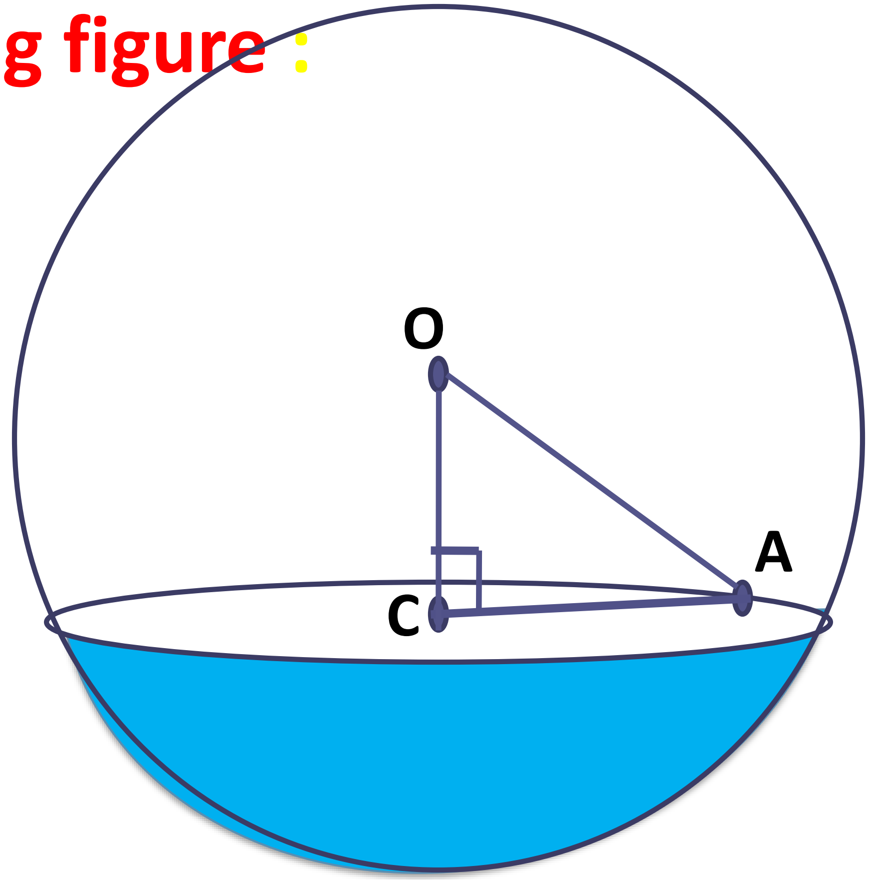


# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then **what is the section?**

**We observe the following figure :**



# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

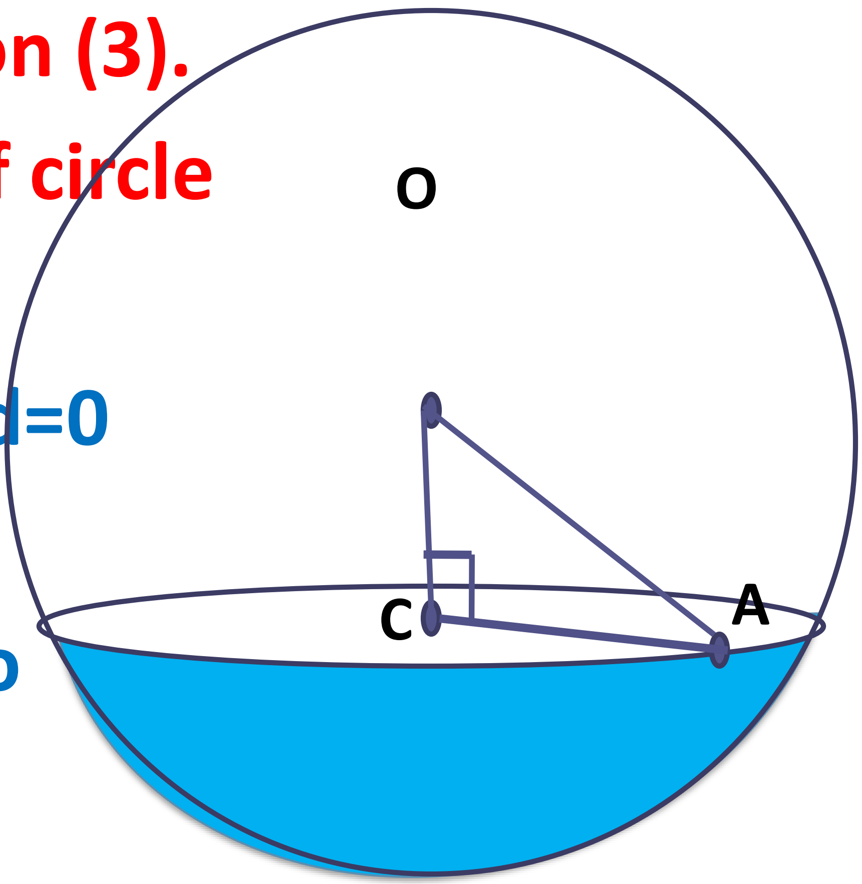
We obtain CA by equation (3).

Thus general equation of circle can be given by

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

and $ax+by+cz+d_1=0$

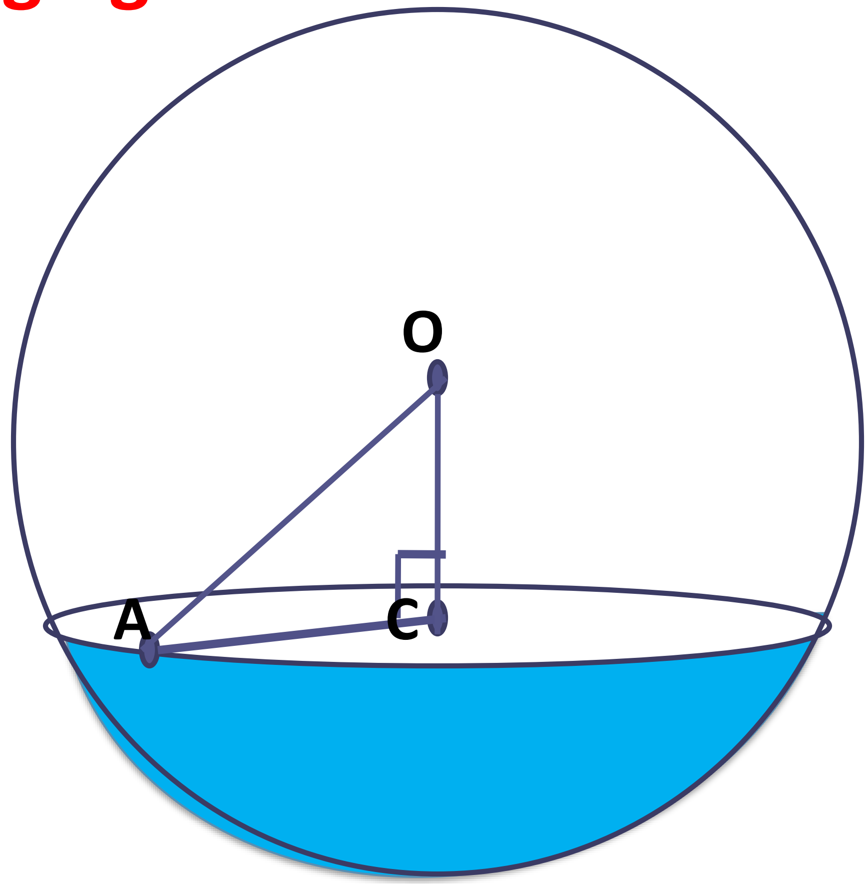
i.e combinedly these two gives circle.



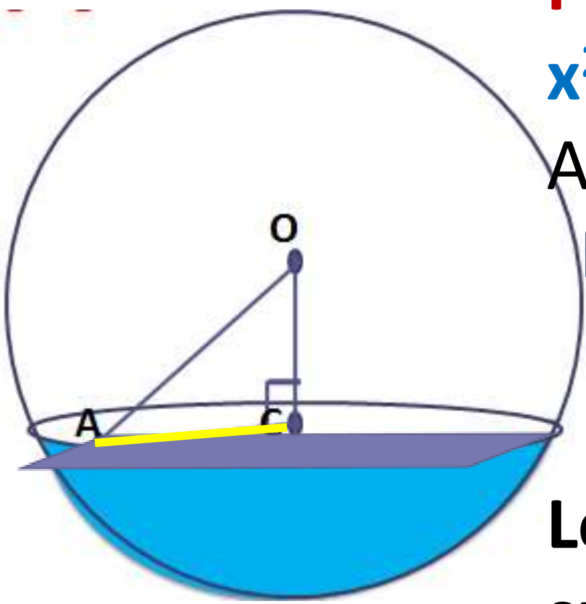
# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

We observe the following figure :



**Theorem**: Prove that the section of the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ by the plane $ax+by+cz+d=0$ **is a circle and hence find its centre and radius**




**Proof**: Given Sphere is

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ ---------(1)

And given plane $ax+by+cz+d=0$--------(2)

**Let O(-u, -v, -w) be centre of sphere (1)**

**Let OC be the perpendicular drawn from O to the plane (2)**

**Let A be any point on the section of a sphere (1) by plan (2).**

Let CA be any line through C, hence CA is perpendicular to OC, and **OA is radius of the sphere** (b'cz, O is the centre of the sphere and A be any point on the surface of the sphere)

Hence $OA = \sqrt{u^2 + v^2 + w^2 - d}$ = constant

And OC = Length of Perpendicular from O(-u,-v,-w) to the plane (2)

$$OC = \left|\frac{a(-u)+b(-v)+c(-w)+d}{\sqrt{a^2+b^2+c^2}}\right| = \text{constant}$$

From the right angled triangle OCA, we have

$OA^2 = OC^2 + CA^2$ or $CA^2 = OA^2 - OC^2$ = constant

as OA and OC are constant

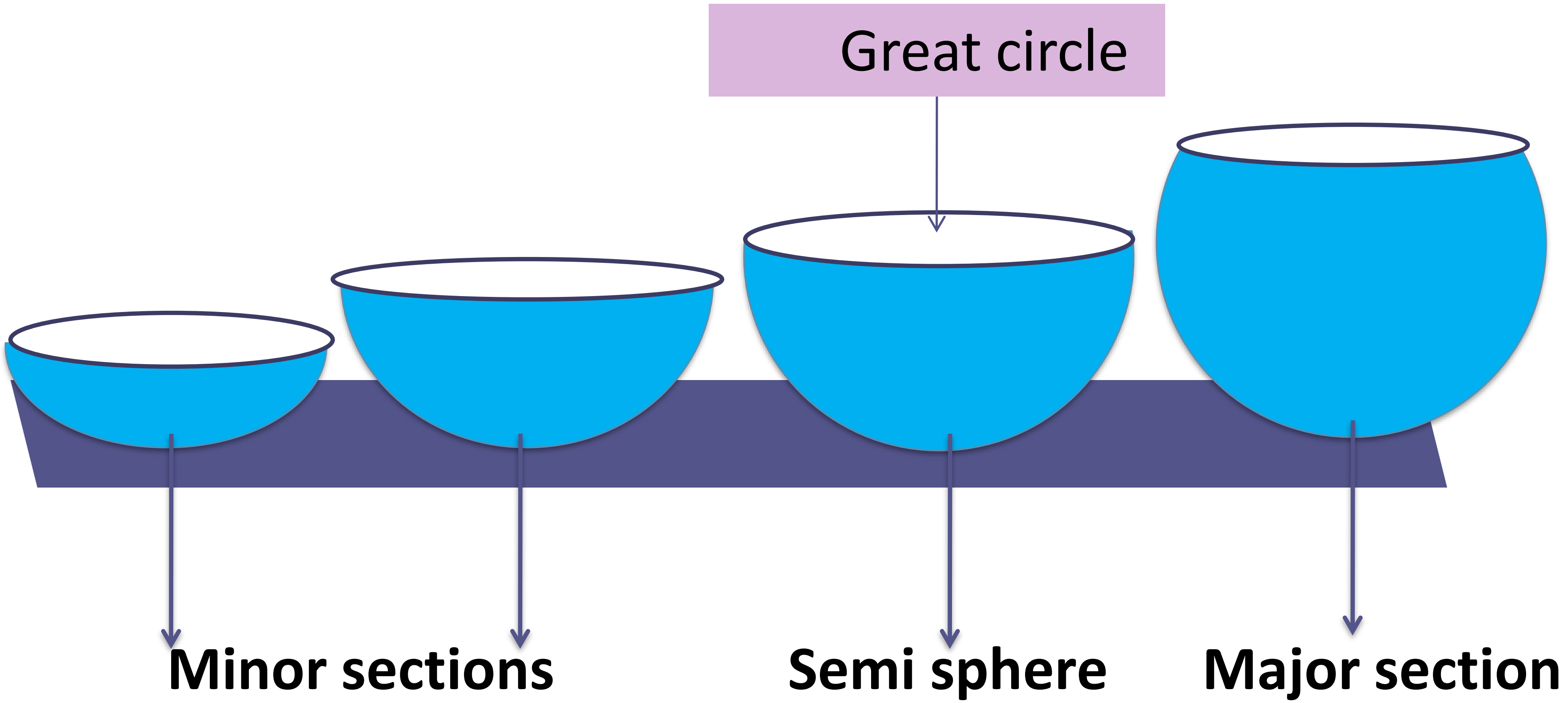
Hence distance of any point A from C is constant

This is true only when section is circle.

Thus the section of the sphere **$x^2+y^2+z^2+2ux+2vy+2wz+d=0$** by the plane **$ax+by+cz+d=0$ is a circle. And its radius is CA and centre is C.**

We have to find coordinates of C and length OC .

# Different Sections of sphere by the Plane




**For all these sections , surface is circle.**

**Note: (i) Hence equation of circle of section is given by**

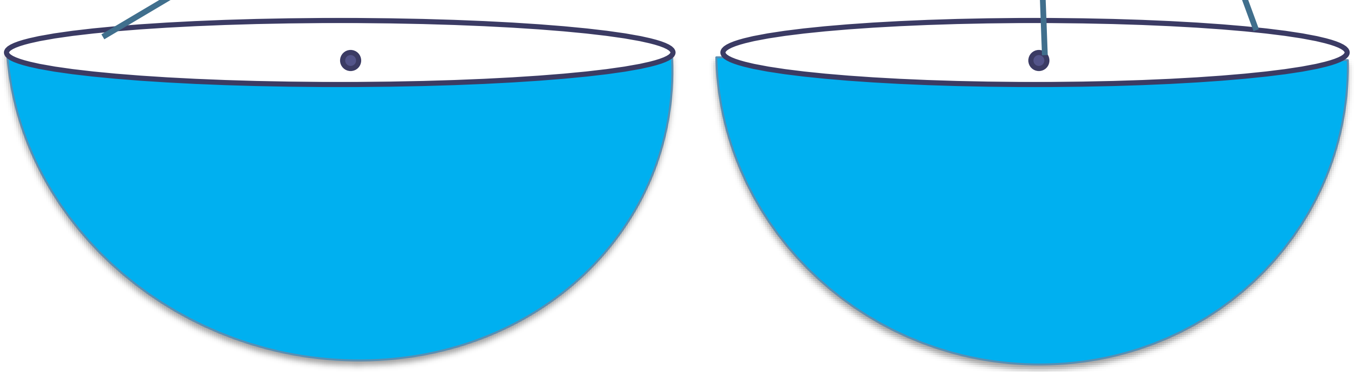
**S: $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ and Plane $ax+by+cz+p=0$**

# (ii) GREAT CIRCLE

- CIRCLE OF HEMISPHERE IS A GREAT CIRCLE

If the section of a sphere by plane passes through centre of sphere , then section is called as great circle .

CENTRE OF CIRCLE IS SAME AS CENTRE OF SPHERE.

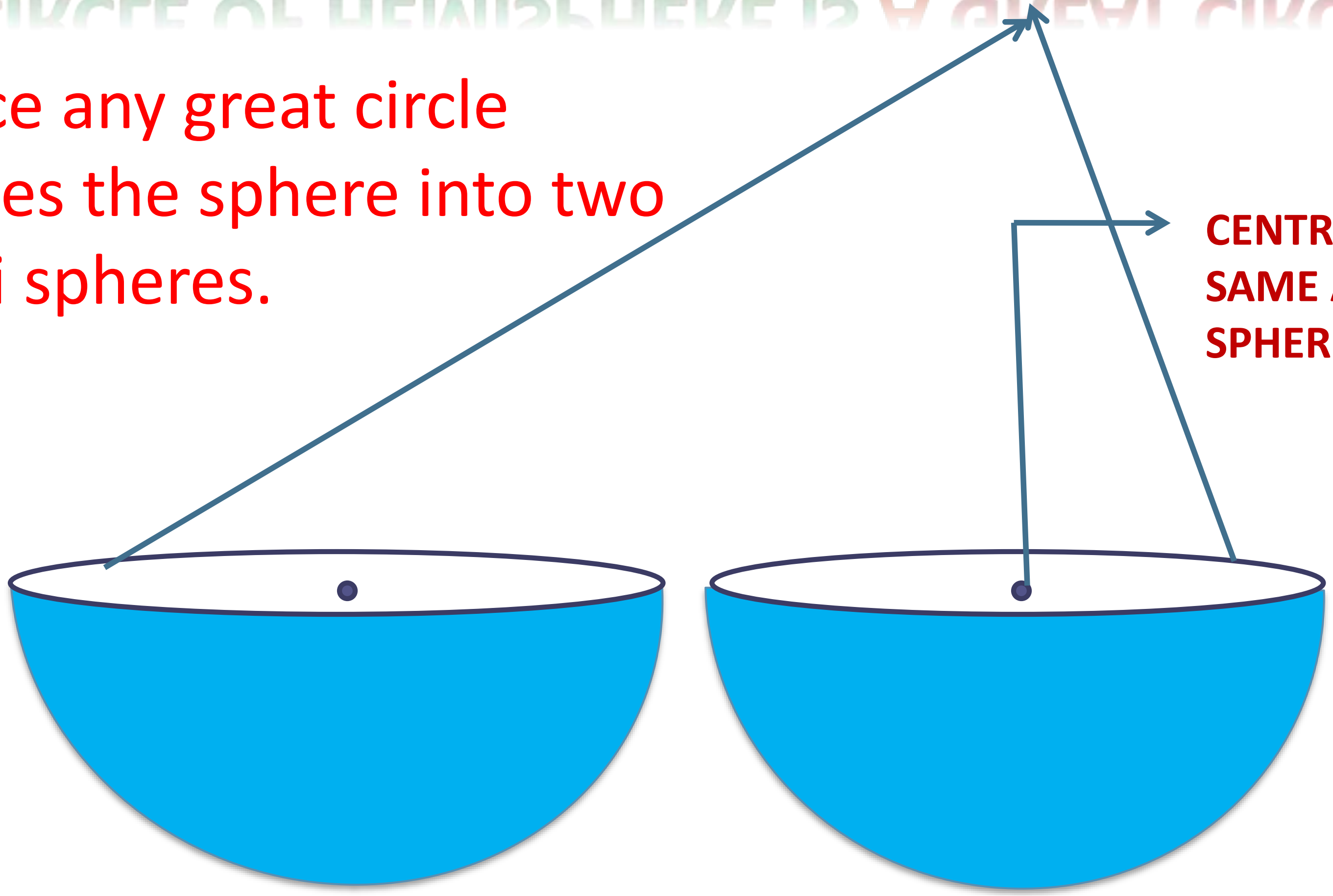


# (ii) GREAT CIRCLE

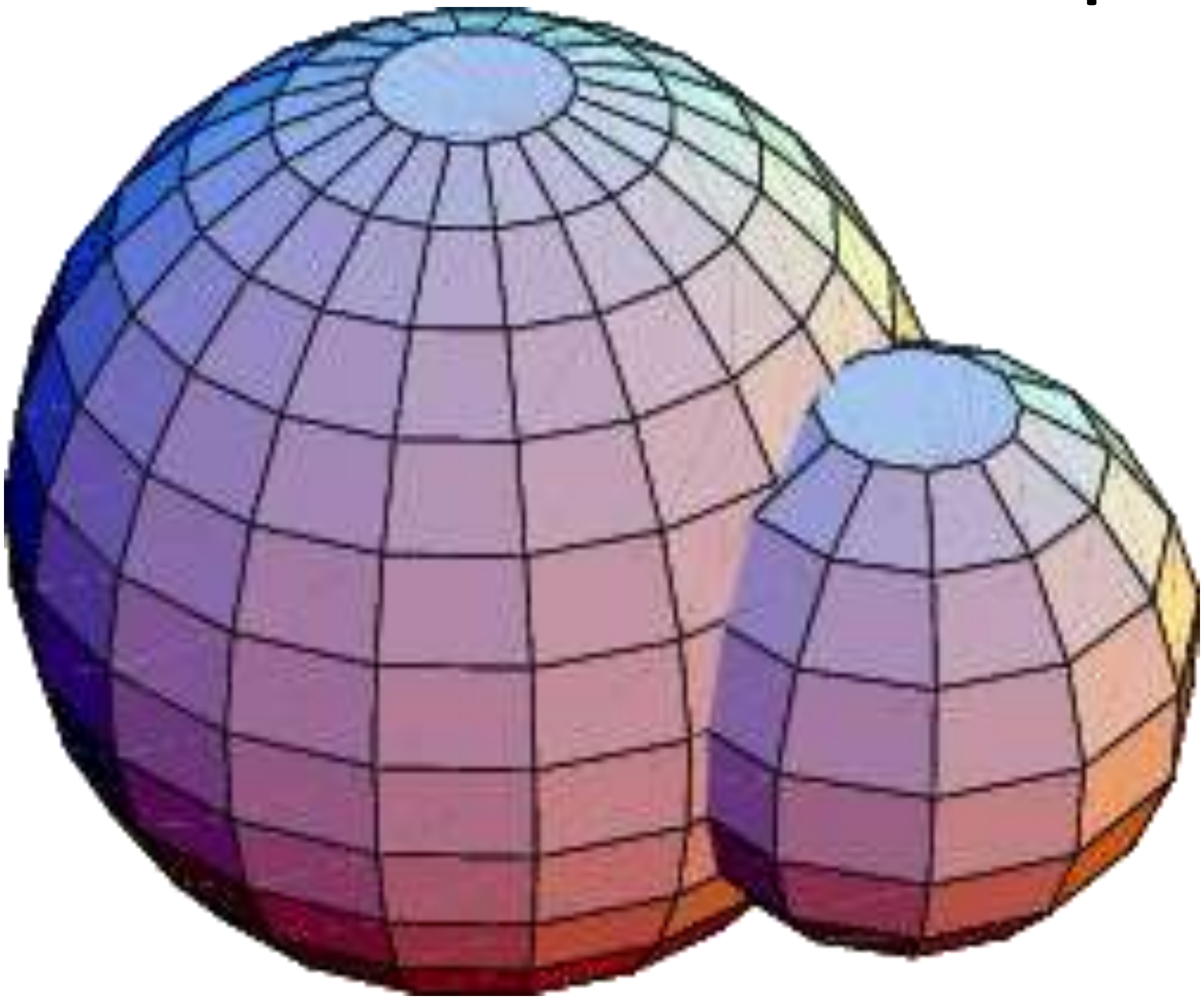
- CIRCLE OF HEMISPHERE IS A GREAT CIRCLE

Hence any great circle divides the sphere into two hemi spheres.

CENTRE OF CIRCLE IS SAME AS CENTRE OF SPHERE.



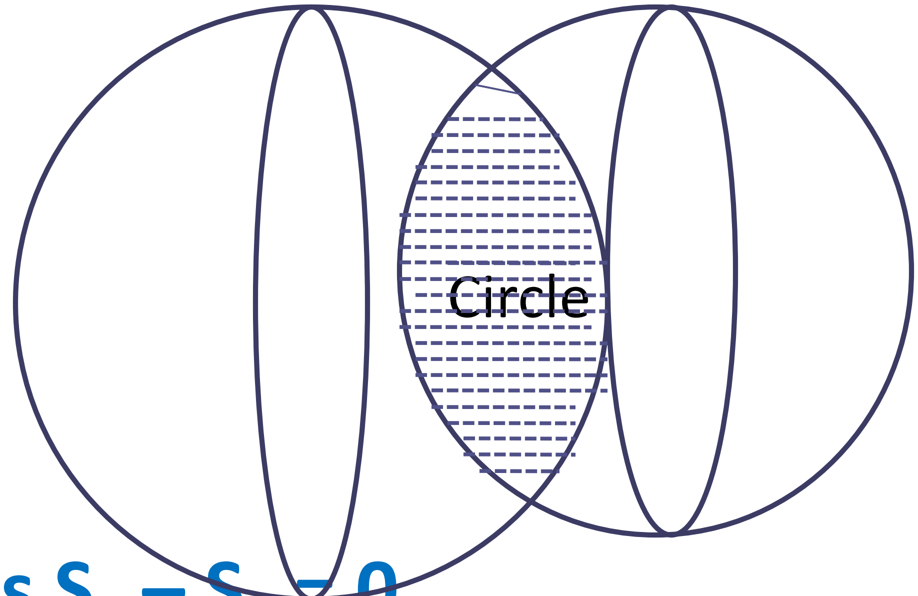
# Intersection of two spheres



## (iii). Intersection of two spheres whose equations are

$S_1 = x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ and

$S_2 = x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$



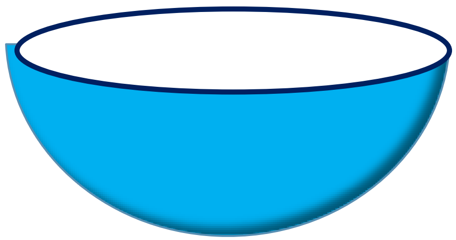

is taken as $S_1 - S_2 = 0$

i.e $2(u_1 - u_2)x + 2(v_1 - v_2)y + 2(w_1 - w_2)z + d_1 - d_2 = 0$

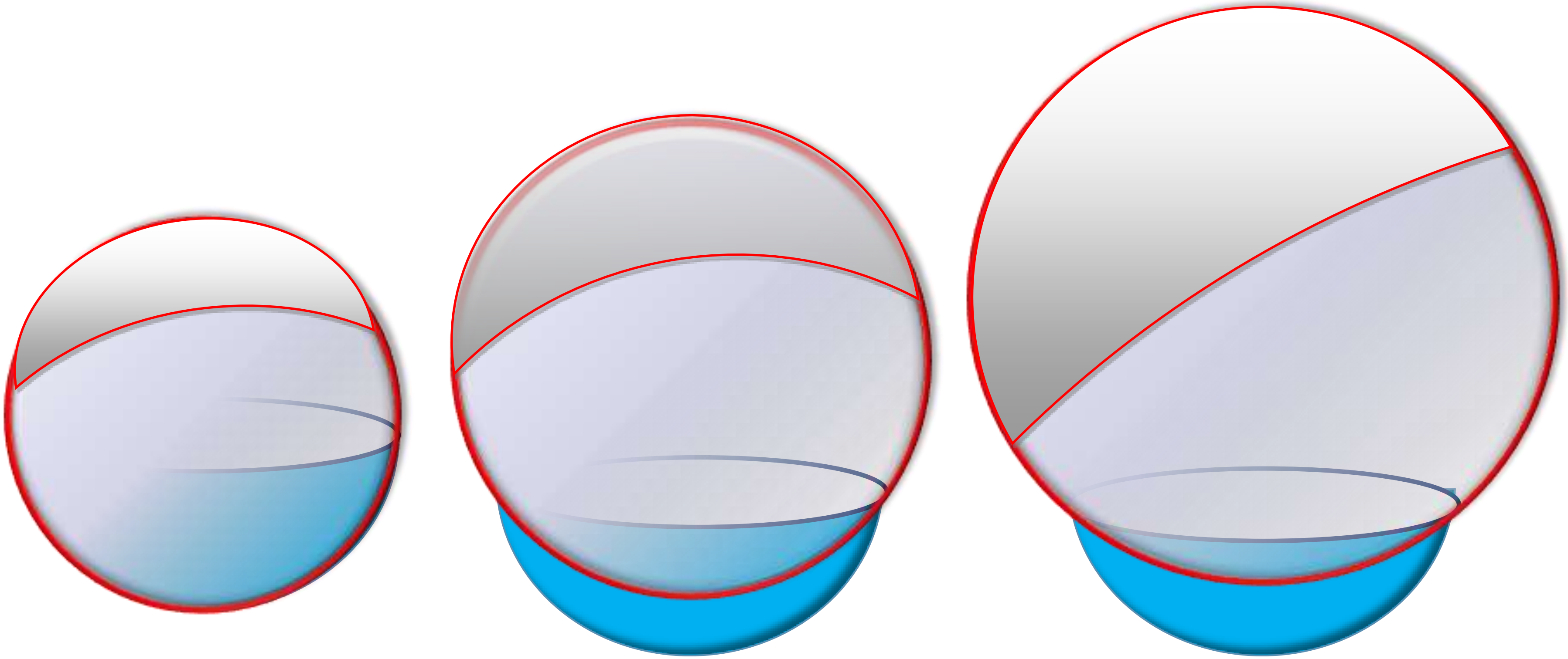
which is plane i.e circle.

4. **Spheres passing through the circle,** $x^2+y^2+z^2+2ux+2vy+2wz+d=0,$

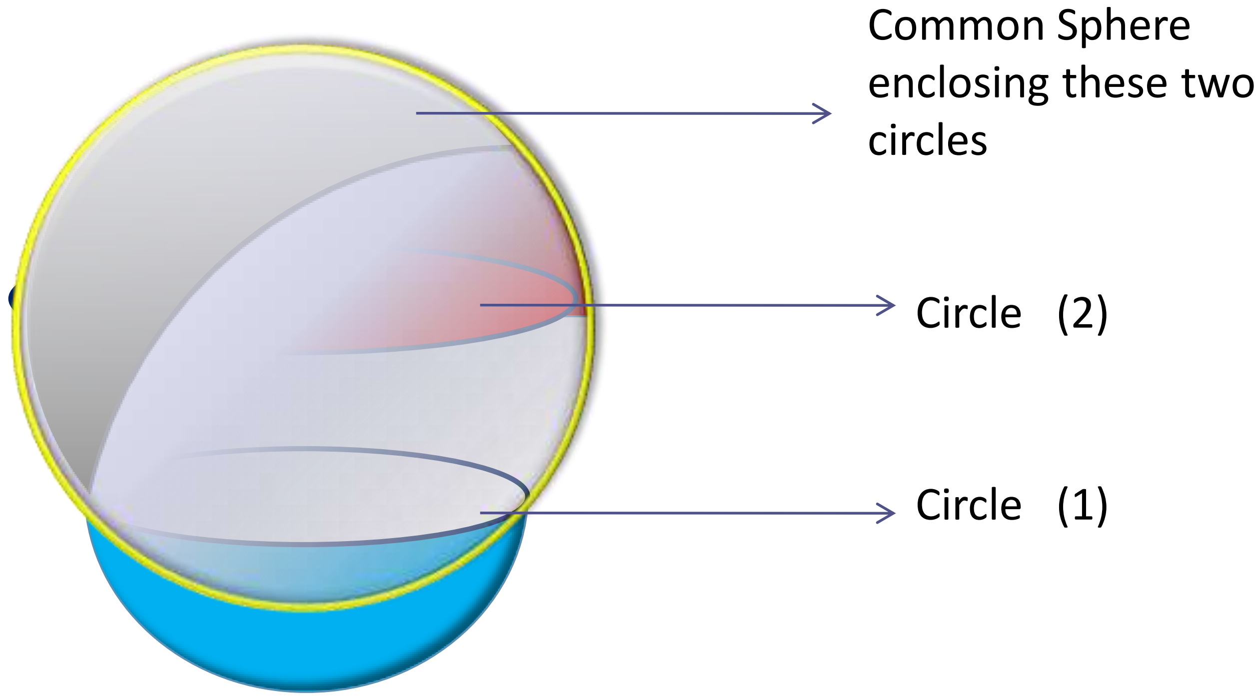
$$ax+by+cz+p=0$$



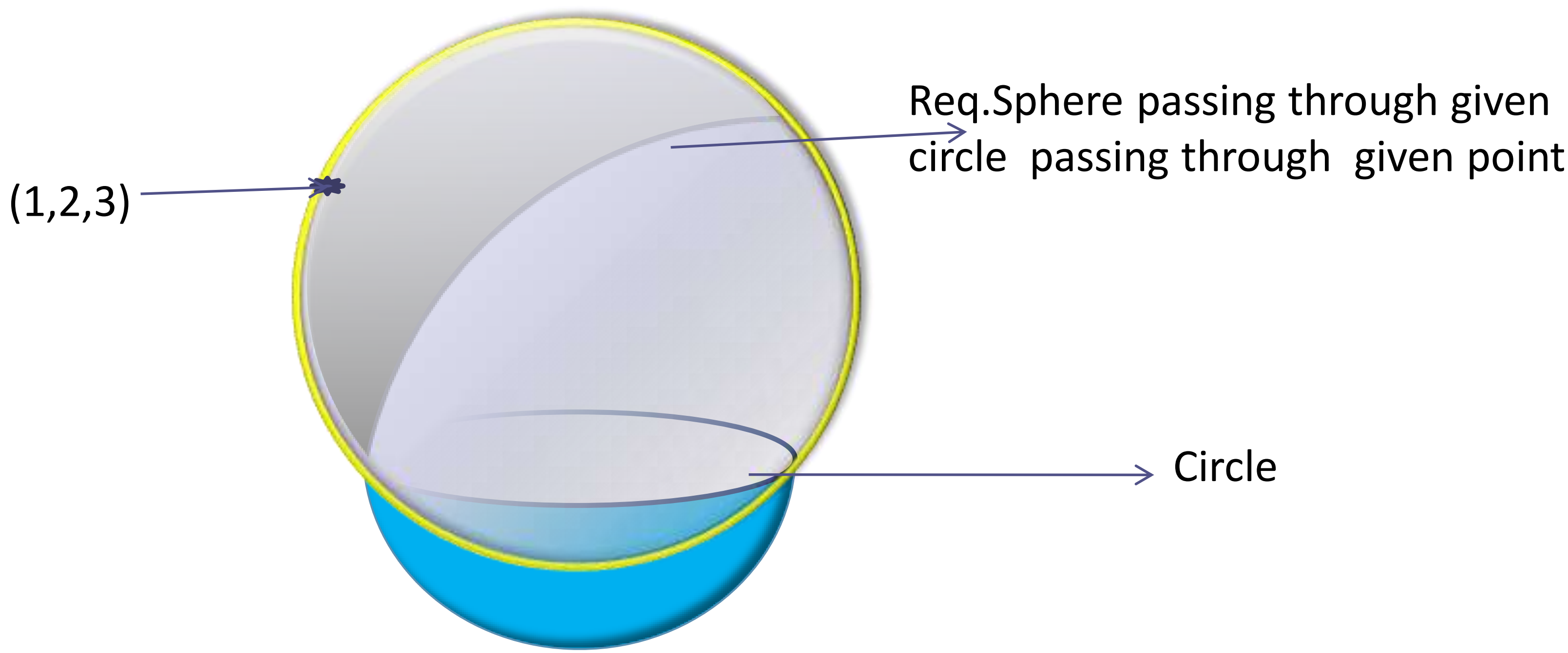
4. **Spheres passing through the circle,**
**S :$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ and**
**P: $ax+by+cz+p=0$ is $S+\lambda P=0$**



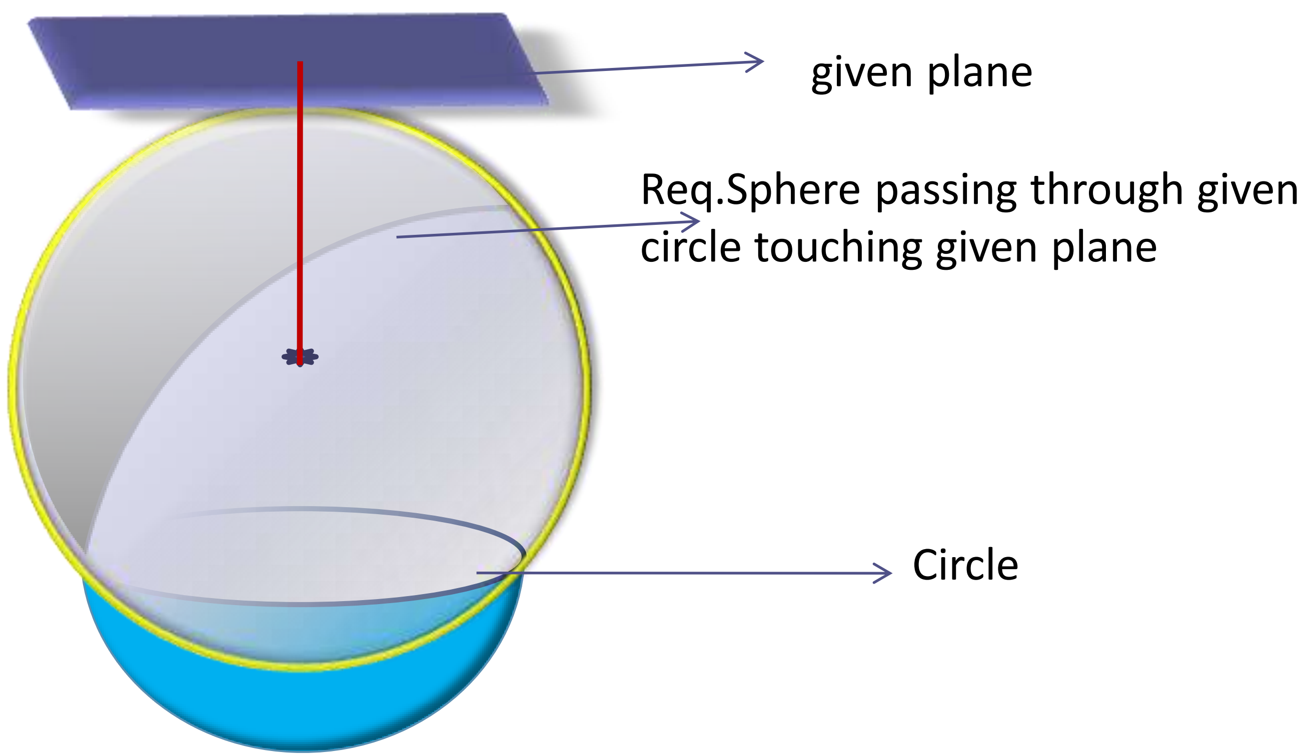
# 5. Sphere which is common for two circles



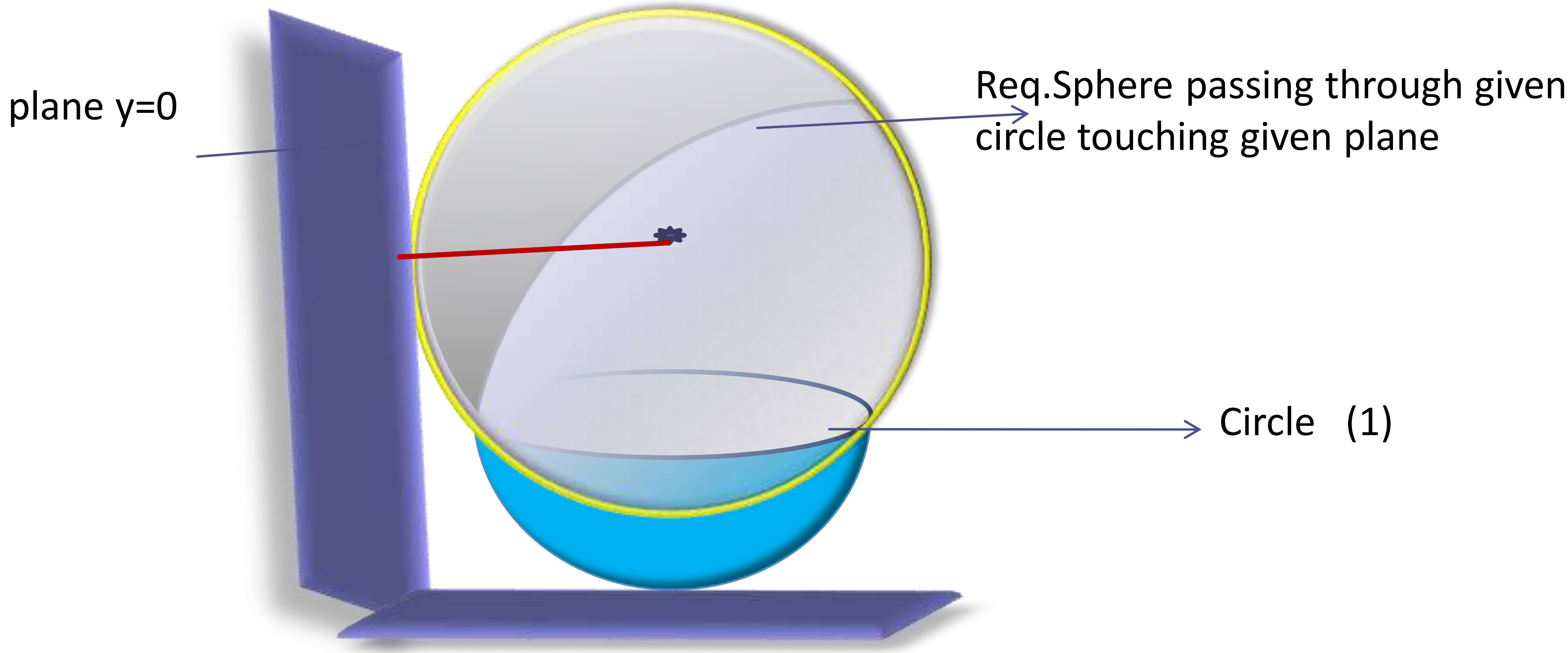
Eg . Find equation of **Sphere passing through circle $x^2+y^2+z^2=9$ and $2x+3y+4z=5$ and passing through the point (1,2,3)**



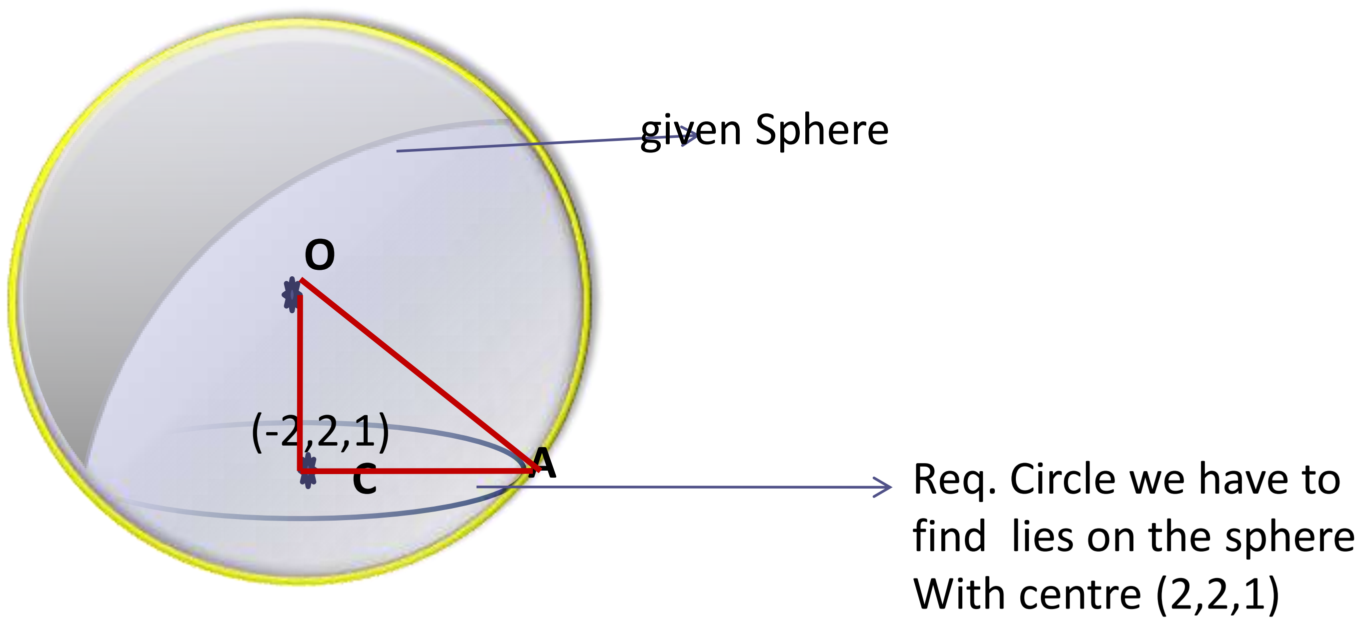
Eg . Find equation of **Sphere passing through circle $x^2+y^2+z^2-2x+2y+4z-3=0$ and $3x+4y=14$ and touching the plane $2x+y+z+4=0$**



Eg . Find equation of **Sphere passing through circle $x^2+y^2+z^2-2x+2z=2$ and $y+z=7$ and touching the plane $y=0$**



Eg . Find equation to the circle which has centre at (-2,2,1) lies on the sphere $x^2+y^2+z^2+5x-7y+2z-8=0$.



# Business Law or Mercantile Law

## Introduction:

Man is a social animal. He lives in society with his fellow beings. When living so, he has to observe a code of conduct or a set of rules for peaceful living and welfare of the whole society.

**These rules of conduct, when recognised by the State and enforced by it on people are termed as Law.**

Such law is not static. It changes when circumstances and conditions in the society change. Law is therefore dynamic.

## What is Law?

The word 'law' is a general term and has different connotations for different people, e.g.,

- A citizen may think of law as a set of rules which he must obey.
- A lawyer who practices law may think of law as a vocation.
- A legislator may look at law as something created by him.
- A judge may think of law as guiding principles to be applied in making decisions.

## Need for the Knowledge of Law:

- Ignorance of law is not an excuse. Though it is not possible for a common man to learn every branch of law, yet he must know at least the general principles of the law of his country.
- Knowledge of Mercantile Law or Business Law is essential to people engaged in various economic and commercial activities i.e. business activities.
- The general knowledge of mercantile law will certainly help businessmen to solve their business problems and avoid conflicts with others.

- **Meaning and Scope:**
  - The term '**Mercantile or Business Law**' may be defined as that branch of law which deals with the rights and obligations arising out of business transactions between businessmen.
  - It consists of those rules that govern and regulate trade, commerce and industry.
  - It is one of the important branches of Civil Law. Mercantile law is also known as Business Law.

## The scope of mercantile law is very wide and varied.

It includes law relating to contracts, partnership, companies, sale of goods, negotiable instruments, carriage of goods, insolvency and arbitration and applies not only to businessmen but also to bankers and other professional men as well as to common people. Hence it is also known as Business Law.

# Law of Contract

## Introduction:

Law of Contract is one of the most important branches of mercantile law. It is the foundation of modern business.

In business, promises are made at one time and are performed at another time.

To see that the promises made are duly performed by the parties to the Contract and to carry on the business smoothly, the law of Contract came into force.

The law of Contract lays down the rules relating to promise, their formation, their performance and their enforceability.

## Indian Contract Act 1872

It determines the circumstances in which promise made by the parties to a contract shall be legally binding on them.

All of us enter into a number of contracts everyday knowingly or unknowingly. Each contract creates some right and duties upon the contracting parties.

Indian contract deals with the enforcement of these rights and duties upon the parties.

**Meaning of Contract:**

According to Section 2(h) of the Indian Contract Act 1872, "A contract is an agreement enforceable by law".

According to Fredrick Pollock, "Every agreement and promise enforceable by law is a contract".

A contract is legally binding agreement between two or more persons.

A contract is an agreement between two or more parties which is enforceable at law.

From the above definitions, a contract essentially consists of two elements:

1. an agreement and
2. its legal enforceability i.e. legal obligation.

## 1. Agreement:

An agreement is defined as "every promise and every set of promises, forming consideration for each other." [(Sec 2.(e)].

Thus it is clear from this definition that a promise is an agreement.

Sec. 2(b) defines. "A proposal, when accepted, becomes a promise."

Thus, an agreement means an accepted proposal.

To sum up:

Agreement = Offer + Acceptance

Therefore, to form an agreement, there must be a proposal or an offer by one party and its acceptance by the other party. An agreement has two main characteristics.

a. **Plurality of Persons:** There must be two or more than two persons to make an agreement, because one person cannot enter into an agreement with himself.
b. **Consensus-ad-idam:** Both the parties to an agreement must agree upon the subject-matter of the agreement in the same sense and at the same time.

2. **Legal Obligation:** An agreement which creates a legal obligation between the parties becomes contract. If an agreement cannot create a legal obligation i.e., a duty enforceable by law, it is not a contract.

Therefore, agreements of moral, religious or social nature are not contracts because they do not create any legal obligation.

**Examples:**

A promise to attend Pooja ceremony or dinner at your friend's house is not a contract and failure to attend will not create any legal obligation.

An agreement to buy a horse at Rs. 500 is a contract because it gives rise to legal obligation.

Thus, there are various kinds of agreements but all these agreements are not contracts. "Only those agreements which are legally enforceable by law and which therefore create legal obligation on the parties concerned constitute contracts.

- **ESSENTIAL ELEMENTS OF A VALID CONTRACT**

According to Sec. 10, the following are the essential elements of a Contract.

1. **Offer and Acceptance:** There must be an agreement based on a lawful offer made by one person to another and lawful acceptance of that offer by the other person.
2. **Intention to Create Legal Relationship:** The intention of the parties entering into an agreement must be to create legal relationship between them. If there is no such intention on the part of the parties, there cannot be any contract between them.

3. **Lawful Consideration:** An agreement to form a valid contract should be supported by consideration. Consideration means "something in return" It can be cash, kind, an act or abstinence. It can be past, present or future. However, consideration should be real and lawful.

4. **Capacity of Parties:** In order to make a valid contract the parties to it must be competent to be contracted. According to section 11 of the Contract Act, a person is considered to be competent to contract if he satisfies the following criterion:
   a. The person has reached the age of maturity.
   b. The person is of sound mind.
   c. The person is not disqualified from contracting by any law

5. **Free Consent:** To constitute a valid contract there must be free and genuine consent of the parties to the contract. It should not be obtained by misrepresentation, fraud, coercion, undue influence or mistake.
6. **Lawful Object:** The object of the agreement must not be illegal or unlawful.
7. **Agreement not declared Void:** Agreements which have been expressly declared void or illegal by law are not enforceable at law; hence they do not constitute a valid contract.

8. **Certainty:** The agreement must be certain and not vague or indefinite. It must be possible to ascertain the meaning of the agreement.

**Example:** X agrees to sell one of his horses to Y at a reasonable price. Here the agreement is vague as to the horse and the price. Hence it is a void contract

9. **Possibility of Performance:** The agreement must be capable of being performed. "An agreement to an act impossible in itself is void." An agreement to do an impossible act either physically or legally cannot be enforced by law.
10. **Legal Formalities:** The agreement may be oral or in writing. Where it is to be in writing, it must comply with the necessary legal requirements as to writing, registration, attestation etc. If the agreement does not comply with these requirements, it cannot be enforced.

## Differences between an agreement and contract:

The terms agreement and contract are not one and the same. They differ from each other in some respects. The main differences are as follows.

1. Every promise or every set of promises, forming consideration for each other, is an agreement. On the other hand an agreement enforceable at law is a contract.

2. An agreement is the sum total of offer and its acceptance. But a contract is the sum total of agreement and its enforceability at law

   ( i.e. the sum total of offer, its acceptance and the enforceability of the obligation at law.)

2. An agreement may or may not create legal relationship, whereas a contract necessarily creates legal relationship.
3. An agreement is a wider concept and contract is a narrow contract.
4. All agreements are not contracts, where as all contracts are agreements.

## Classification of Contract:

Contract may be classified on the basis of their

(a) enforceability,

(b) formation or

(c) performance.

(a) **Classification on the basis of enforceability** they are classified as

(1) Valid
(2) Voidable
(3) Vide or
(4) Unenforceable or illegal.

1. **Valid Contract:** Agreements enforceable by law are valid contracts.
2. **Voidable:** Agreements which are enforceable at the option of one or more of the parties are voidable.
3. **Void Contract:** Agreements not enforceable by law are void contracts.
4. **Un-enforceable Contracts:** Agreements which are not enforceable because of some technical defects.
5. **Illegal Contracts:** Agreements which are contrary to the law, that means forbidden by law are illegal contracts.

## (b) Classification on the basis of Formation:

Contracts are classified as express, implied and constructive on the basis of formation.

1. **Express Contracts:** Where the terms and conditions are expressly agreed by words spoken or written.
2. **Implied Contracts:** Where the terms and conditions are inferred from the acts or conduct of the parties.
3. **Constructive:** Where the terms and conditions do not arise out of agreement but are created by law.

**(c) Classification the basis of performance:**

On the basis of performance contracts are classified as executed, or executory.

1. **Executed Contracts:** It means both the parties to the contract have performed their obligations.

2. Executory Contracts: Executory contract is one in which the parties to the contract have not yet performed their obligations.

## Offer and Acceptance:

Every contract begins with an offer. The first step towards the creation of a contract is that one party should make a proposal or an offer to the other party to do or not to do something. So, one can rightly say that an offer is an essential element of a contract.

**Meaning of Proposal or Offer:**

According to Sec. 2(a), "When one person signifies to another his willingness to do or to abstain from doing anything, with a view to obtaining the assent of that other to such an act or abstinence, he is said to make a proposal."

The person making the offer or proposal is called the offeror, proposer, or promisor and the person to whom it is made is called the oferee or promise.

## Essentials of a Valid Offer or Proposal:

1. **A valid offer must intend to create legal relations:** It must not be a casual statement. If the offer is not intended to create legal relationship, it is not an offer in the eyes of law.

   For example; Sunil invites Sridhar to a dinner party and Sridhar accepts the invitation. Sridhar does not turn up at the dinner party. Sunil cannot sue Sridhar for breach of contract as there was no intention to create legal obligation. Hence, an offer to perform social, religious or moral acts without any intention of creating legal relations will not be a valid offer.

2. **The terms of an offer must be definite, unambiguous and certain**: They must not be loose and vague. A promise to pay an extra Rs. 500 if a particular house proves lucky is too vague to be enforceable. for example; Sridhar says to Sunil "I will give you some money if you marry my daughter". This is not an offer which can be accepted because the amount of money to be paid is not certain.

3. **An offer may be made to a definite person or to the general public:** When offer is made to a definite person or to a special class of persons, it is called "specific offer".

When an offer is made to the world at large or public in general, it is called "general offer".

A specific offer can be accepted only by that person to whom it has been made and a general offer can be accepted by any person.

For example; Sunil promises to give Rs.1,000 to Sridhar, if he brings back his missing dog. This is a specific offer and can only be accepted by Sridhar.

Sunil issues a public advertisement to the effect that he would give Rs.1,000 to any one who brings back his missing dog. This is a general offer. Any member of the public can accept this offer by searching for and bringing back Sunil's missing dog.

4. **An offer to do or not to do must be made with a view to obtaining the assent of the other party:** Mere enquiry is not an offer.
5. **An offer should contain any term or condition:** The offeror may prescribe any mode of acceptance. But he cannot prescribe the form or time of refusal so as to fix a contract on the acceptor. He cannot say that if the acceptor does not communicate his acceptance within a specified time, he is deemed to have accepted the offer.

**6. The offeror is free to lay down any terms and conditions in his offer:** If the other party accepts it, then he has to abide by all the terms and conditions of the offer. It is immaterial whether the terms and conditions were harsh or ridiculous. The special terms or conditions in an offer must be brought to the notice of the offeree at the time of making a proposal.

7. **An offer is effective only when it is communicated to the offeree:** Communication is necessary whether the offer is general or specific. The offeror may communicate the offer by choosing any available means such as a word of mouth, mail, telegram, messenger, a written document, or even signs and gestures.

   Communication may also be implied by his conduct. A person can accept the offer only when he knows about it. If he does not know, he cannot accept it. An acceptance of an offer, in ignorance of the offer, is no acceptance at all.

**ACCEPTANCE:**

Section 2(b) of Indian Contract Act defines an acceptance as" When the person to whom the proposal is made signifies his assent thereto, the proposal is said to be accepted." A proposal when accepted becomes a contract.

Acceptance is the act of assenting by the offeree to the offer. An acceptance is the expression, by the offeree, of his assent, consent or willingness to the terms of the offer and establishment of legal relations.

**Acceptance may be express or implied.**

**Express acceptance:** When the acceptance is communicated by words spoken or written or by doing some required act, then it is called express acceptance.

**Implied acceptance:** When acceptance is to be gathered from the surrounding circumstances or the conduct of the parties, then it is called implied acceptance.

**Who can accept?**

**Acceptance of particular offer:** When an offer is made to a particular person, it can be accepted by him only. If it is accepted by any other person, there is no valid acceptance.

**Acceptance of general offer:** When an offer is made to world at large, any person to whom the offer is made can accept it.

## Essentials of a Valid Acceptance:

1. **Acceptance by the Offeree:** Offeree is the person to whom the proposal or offer is made. Acceptance must be made by the person to whom the proposal is made.

   If any other person accepts the proposal, it is not valid in law. He will not get any right in law unless it is a general offer.

   The offeror has made the offer to a specific person in order to get his assent. So it must be accepted by that specific person. Then only there will be an agreement and a resulting contract.

**Example:** 'A' advertises a reward of Rs 10,000/- to anyone who gives information of his lost son: B gives the information but is ignorant of the reward. After some time, B claims the reward. It was held that B is not entitled to the reward as he gave the information without being aware of the offer.

2. **Acceptance in time:** In case of specific offers, the one who makes the offer generally gives a certain time within which the offer should be accepted by the offeree.

This is done for giving certainty to the offer. So the acceptance must be within the time specified. But there are cases in which no time is specified. In such cases the acceptance must be within a reasonable time.

After the time is over, the offer is not valid and cannot be accepted in the eyes of law. (Ramsgate Victoria Hotel Co. Vs Montefore)

3. **Acceptance must be unconditional:** The acceptance must be for the whole offer and without any change in the terms of the offer.

A conditional or qualified acceptance is no acceptance in the eyes of the law. Even a slight deviation from the terms of offer would make the acceptance invalid.

In fact a conditional acceptance is a counter offer and not an acceptance. If A offers an article to B for Rs. 100/- the acceptance by B to buy the article for Rs. 90/- is no acceptance in the eyes of the law.

4. **Communication:** Very much like the offer the acceptance should also be communicated. Mere silence cannot be considered accepting the terms of the offer. So the acceptance must be made in some visible form.

**(Powell Vs Lee)**

5. **Particular method of acceptance:** If the terms of the offer specify it to be accepted in some particular form, it should be made so. The person who made the offer can rightly reject the acceptance if it is not made in the form prescribed.

**Example:** If the offeror prescribes 'acceptance by telegram' and the offeree sends his acceptance by ordinary post, there is no acceptance of the offer, if the offeror informs the offeree that his acceptance is not according to the prescribed mode.

But if the offeror fails to do so, it will be presumed that he has accepted the acceptance and a valid contract is created between them.

6. **Acceptance must succeed an offer:** Acceptance must be given only after receiving the offer and not before receiving it. If the acceptance precedes the offer, it is not a valid acceptance and does not result into a contract. (**Lalman Shukla Vs Gouri Dutt**).

In this case, Gouri Dutt's nephew was missing. So, Gouri Dutt sent his servant, Lalman Shukla to search for the boy. After Lalman Shukla left for searching the boy, Gouri Dutt announced a reward of Rs. 501/- to anybody who would trace the boy.

Lalman Shukla traced the boy in ignorance of Gouri Dutt's offer of reward. But, later on, when he came to know of the reward, he demanded the reward offered. Gouri Dutt refused to give the reward. At this, Lalman Shkla filed a suit against Gouri Dutt for the reward offered. It was held by the court that Lalman Shukla was not entitled to the reward of which he was ignorant.

## Communication of Offer, Acceptance and Revocation:

- **Section 2(a)** of Indian Contract Act 1972 says that when a person signifies his willingness to do or to abstain from doing something to another, with a view to obtaining the assent of that another, he is said to make a proposal.
- Further, **section 2(b)** says that when the person to whom the proposal is made signifies his assent, the proposal is said to be accepted. The important point to note here is that the party making the proposal or the party accepting the proposal must "signify" their willingness or assent to the other party.

- Thus, a promise cannot come into existence unless the willingness or assent is communicated to the other party. Further, even the revocation, if any, must be communicated to the other party for it to take effect. Therefore, communication is the most critical aspect in the making of a contract.

1. **Communication of an offer:** The communication of an offer is complete when it comes to the knowledge of the person to whom it is made i.e. when the letter containing the offer reaches the offeree and not when it is posted.

**Example:** For example, if A sends a proposal in the mail to B and if the mail is lost, it can be held that the communication of the proposal is not complete. In the case of Lalman vs Gauridatta 1913, it was held that the reward for the missing child cannot be claimed by a person who traced the child without any knowledge of the announcement. There was no contract between the two in the first place because the proposal never came to the knowledge of the person who found the child and thus he could never accept it.

**2. Communication of an acceptance:** Communication of the acceptance is complete, as against the promisor, when it is put in course of transmission to the promisor so as to be out of the power of the acceptor, as against the acceptor, when it comes to the knowledge of the promisor.

**Example:** As soon as B drops a letter of acceptance in mail back to A, A is bound by the promise. However, B is not bound by it unless A receives the acceptance letter.

In the case of Adams vs Lindsell 1818, it was held that a contract arose as soon as the acceptance was posted by the acceptor. In this case, the plaintiff received the offer to sell wool on 5th and they posted an acceptance, which was received on 9th by the defendants. The defendants, however, had already sold the wool on 8th. The court observed that the contract must arise as soon as the acceptance is posted and is gone out of the reach of acceptor otherwise this will result in an infinite loop.

3. **Communication of revocation:** Communication of a revocation is complete as against the party who makes it when it is put in course of transmission to the party to whom it is made, so as to be out of the power of the party who makes it; as against the party to whom it is made, when it comes to the knowledge of the party to whom it is made.

- For example, if A sends a letter revoking his proposal, it will be complete against A as soon as the letter is dropped in the mailbox and is out of his control. However, the revocation will be held complete against B only when B receives the letter.

- Further, if B revokes his acceptance by telegram, it will he deemed complete against B as soon as he dispatches the telegram. It will be held complete against A, when A receives the telegram.

## Consideration:

Consideration is the foundation of every contract. The law enforces only those promises which are made for consideration.

Where one party promises to do something, it must get something in return. This something in return is called consideration.

Consideration is the very life blood of every contract. In the absence of consideration a promise or undertaking is purely gratuitous. However, sacred and binding in honour, it creates no legal obligation.

**Definition.**

Consideration has been defined in many ways. According to Pollock, "Consideration is the price for which the promise of other is bought and the promise thus given for value is enforceable."

It is something which is some value in the eyes of law. It may be of some benefit to the plaintiff or some detriment to the defendant.

It is also used in the sense of quid pro quo i.e. something in return. A most commonly accepted definition of consideration is given in the famous English case Currie vs Misa as " some right , interest, profit or benefit accruing to one party or some forbearance, detriment, loss or responsibility, given, suffered or undertaken by the other".[(1875)10 Ex.162].

**Section 2 (d) of the Indian Contract Act defines consideration as-**

"When at the desire of the promisor, the promisee or any other person, has done or abstained from doing , or does or abstains from doing, or promises to do or abstain from doing something, such act or abstinence or promise is called consideration for the promise."

From the above definition it shows that the consideration has four components.

a. It may be an act or abstinence or a return promise done at the desire of the promisor.
b. It may be done by the promisor or any other person.
c. It must have been already executed or it is in the process of being done or it may be executed.
d. It must be something to which the law attaches a value.

Thus, the consideration need not necessarily be in cash or in kind.

It may be even an act or abstinence, forbearance or a promise to do or not do something.

It must be present in the sense or benefit to one party and a detriment or loss to the other party or detriment to both.

- **Examples:**
- A agrees to sell his car for Rs. 76,000 to B and who accepts the offer. Here B's promise to pay Rs. 76,000 is the consideration for A's promise to sell his car, and A's promise to sell his car is the consideration for B's promise to pay Rs. 76, 000
- X promises his debtor Y not file a suit against him for six months on Y's agreement to pay Rs. 200 more. The abstinence of X is the consideration for Y's promise to pay extra Rs. 200.
- A promise to act as a legal advisor to B and in return B promises to give tution to A's son. The promise of each party is the consideration for the promise of the other party.

# ESSENTIALS OF VALID CONSIDERATION

1. **Consideration must move at desire of the promisor.** An act or abstinence must have been done at the desire of the promisor only. Any act performed at the desire of the third party cannot be valid consideration. *(Durga Prasad Vs. Baldeo.)*

- *In this case Durgaprasad spent money and built a market at the request of the District collector. The stalls in the market were occupied by many shop-keepers, promising to pay commission on the articles sold in the market. One of the shop-keeper Baldeo failed to pay the promised commission. Durga Prasad sued Baldeo to recover the promised commission.*
- *It was held by the court that the promise of Baldev could not be enforced against him because the act of Durga Prasad (construction of the market) was not at the desire of Baldeo but at the desire of the third party (the collector of the district).*

2. **It may move from the promisee or any other person**. Under the English Law, consideration must move from the promisee. Under the Indian Law, consideration may move from the promisee or any other person, i.e., even a stranger. This means that as long as there is consideration for a promise it is immaterial who has furnished it. But the stranger to consideration will be able to sue only if he is a party to the contract. (Chinnaya Vs Ramayya).

- *In this case an old lady, by a deed of gift, made over certain property to her daughter D, under the direction that she should pay her aunt, P (sister of the old lady), a certain sum of money annually. The same day D entered into an agreement with P to pay her the agreed amount. Later, D refused to pay the amount on the plea that no consideration had moved from P to D.*
- *It was held that, P was entitled to maintain suit as consideration had moved from the old lady, sister of P, to the daughter D.*

## 3. Consideration may be past, present or future.

(a) **Past Consideration:-**A consideration for the act done in past is a past consideration. Past consideration valid in Indian Contract Act, but it is no consideration in English Law.

(b) **Present Consideration:-**When both the parties are ready to move consideration at the same time, it is a present consideration.

(c) **Future Consideration:-**When a party promises to or abstain from doing something in future, it is a future consideration.

**4. Consideration need not to be adequate:** The real meaning of consideration is something in return. This ‘something in return’ need not necessarily be equal in value to ‘something given’. Adequacy is for the parties to decide at the time of making the agreement. No contract can be refused on the ground of inadequacy of the consideration.

5. **Consideration must be real and not illusory:** Although consideration need not be adequate, it must be real, competent and of some value in the eyes of the law.

6. **Consideration must be lawful**: In valid contract it is necessary that the consideration should be lawful, otherwise it will become void and unenforceable.

- ***EXCEPTIONS:***
  No doubt without consideration agreement is void but it has also exceptions which are following:

1. **Case of Love :-**
   Consideration is not compulsory if an agreement made between the parties for natural love and affection.
2. **Case of An Agent:-**
   The contract of agency requires consideration, where the contract is a promise to appoint an agent.
3. **Case of Voluntary Services:-**
   In case of compensation for voluntary services there is a relaxation of consideration.

## 4. Case of Donation:-

Agreement made for donation is not enforceable for want of consideration. A promised amount can not be legally recovered where the promisee has done nothing on the basis of promise.

**Example:** - If Mr. Shah promised to donate one lakh rupees for the repair of college. College principal did nothing for repair. Mr. Shah refused to pay. On a suit by principal it was held the Mr. Shah is not liable because it did not result any loss to promisee.

**5. Case of Gift:-**

In case of gift there is no need of any consideration. According the law any gift which is actually delivered will be valid. It cannot be demanded back on the ground that there was no consideration for him.

**6. Extension in Time Limit:-**

There is no need of any consideration if agreement is made to extend time for the enforcement of the contract.

**Example :-** Mr. Chun agrees to construct the shop for Mr. Raju within one year against Rs. 20 lac. Later on the request to Mr. Raju to extend the time period for the completion of the shop. Mr. Raju accepts the request. It is a valid agreement without consideration.

**7. Case of Time Barred Debt :-**

If a debtor promises to pay a time barred debt, then there is no need of consideration. The promise must be in written and signed by the debtor or his agent.

**8. Contract Under Seal :-**

A contract without consideration is valid if it is made under seal.

**Example :-** Mr. Nehra and Mr. Adit enter into agreement by writing the partnership deed to form a partnership. This contract is valid

## FREE CONSENT

Free consent of all the parties to a contract is one of the essential elements of a valid contract as per requirement of section 10. The parties to a contract should have identity of minds. This is called consensus ad idem in English law.

Two or more persons are said to consent when they agree upon the same thing in the same sense. (Section 13).

Free consent is the consent which has been obtained by the free will of the parties out of their own accord.

According to Section 14, consent is said to be free when it is not caused by

1. Coercion – as per Sec. 15 or
2. Undue influence – as per Sec. 16 or
3. Fraud – as per Sec. 17 or
4. Misrepresentation – as per Sec. 18
5. Mistake, subject to the provisions of Sections 20, 21, and 22

When consent to an agreement is caused by coercion, undue influence, misrepresentation or fraud, the contract is voidable at the option of the party whose consent was so caused. But when the consent is caused by mistake, the agreement is void.

## 1. COERTION:

In simple words, coercion is threat or force used by one party against another for compelling him to enter in to an agreement,

Section 15 of the Indian Contract Act defines 'coercion as the committing or threatening to commit any act forbidden by the Indian Penal Code  or an unlawful detaining or threatening to detain, any property to the prejudice of any person with the intention of inducing any person to enter into an agreement.'

Coercion is said to have been employed when a person was forced o enter into a contract by use or under the threat of use of physical force by the other person committing or threatening to commit any act forbidden by Indian Penal Code.

- **Example:**
- X threatens to kill Y if he does not sell his house for Rs. 1, 00,000 to X. Y sells his house to X and receives the payments. Here, Y's consent has been obtained by coercion. Hence, this contract is voidable at the option of Y.
- If Y decides to avoid the contract, he will have to return Rs 1,00,000 which he had received from X. "Y" (aggrieved party) will return Rs. 1,00,000. "X" (defendant party) will return the house and any benefit from the goods.

- **When voidable contract cannot be canceled:** When the third party become interested into a voidable contract.
- **Example:** A obtains the car of B through coercion. Let, A sold it to "C" an innocent buyer, now B cannot get the contract canceled.
  When the aggrieved party ratify / confirm / affirm, then contract cannot be canceled.

## 2. UNDUE INFLUENCE:

**Meaning:** [section 16(1)]: The term 'undue influence' means dominating the will of the other person to obtain an unfair advantage over the other.

**According to section 16(1),** a contract is said to be induced by undue influence
"Where the relations subsisting between the parties are such that one of them is in a position to dominate the will of the other, and the dominant party uses that position to obtain an unfair advantage over the other."

- When two-partner are in relation, and one of them is dominant and other is in weaker position and dominant person takes undue-Advantage, then it is called "Undue- influence."

  There is no presumption of undue influence in the following relationships:

  Husband and wife, landlord and tenant, Creditor and debtor

- **Effect of undue influence [section 19A]:** when consent to an agreement is caused by undue influence, the agreement is a contract voidable at the option of the party whose consent was so caused.

**Comparison between coercion and undue influence:**
**Similarities:** **In case of both coercion and undue influence, the consent is not free and the contract is voidable at the option of the aggrieved party.**

## 3. FRAUD:

Meaning and essential elements of fraud [section 17] :

The term 'fraud' means a false representation of fact made willfully with a view to deceive the other party. Fraud includes following:

Wrong suggestion about a fact, knowing that it is not-true;

**Example:** X sells to Y locally manufactured goods as imported goods charging a higher price, it amounts to fraud. OR

A seller claimed that his projector is made in Singapore, and sold it for Rs. 100,000/- However the fact is that "Projector was made in south India".

**Active concealment (Hide) of defect in goods:**

**Example:** "A carpenter uses paint to hide the scratches over the old furniture and sold it claiming that is New". This is fraud. OR

X a furniture dealer, conceals the cracks in furniture sold by him by using some packing material and polishing it in such a way that the buyer even after reasonable examination cannot trace the defect, it would tent amount to fraud through active concealment.

**Promise made without intention to perform:**

**Example:**

"A man and a woman underwent a ceremony of marriage with the husband not regarding it as a real marriage. Held, the husband had no intention to perform the promise from the time he made it and hence the consent of the wife was obtained under fraud. OR

"A farmer agrees to supply 100 kg potato that will be produced by him out of his field, after three month". Two months has been lapsed, but the farmer neither implants seeds, nor does cultivation. This is case of fraud.

Any activity declared fraud as per other law; under companies act and insolvency acts, certain kinds of transfers have been declared to be fraudulent.

**Note:** In case of fraud, the seller is always liable even though buyer has an opportunity to check the fraud.

## Whether silence is fraud?

**General concept:** According to explanation to section 17, "Mere silence as to facts likely to affect the willingness of a person to enter into a contract is not fraud".

In other words, Silence is not fraud. It is buyer, who must check the goods & suitability.

**Example:** X purchased a used computer from Z thinking it as a computer imported from USA, Z failed to disclose the fact to X. On knowing the fact X wants to repudiate the contract. So, here X cannot repudiate/ cancel the contract.

**Exceptions to the general rule:**

The general rule that silence does not amount to fraud has the following exceptions.

**When silence is equivalent to speech:**

**Example:** "A student of BBA selects a Business law-book and asks the seller". If seller doesn’t stop me from buying this book, I will assume that "it is the best". The seller remained silent here the student will treat "silence" as speech. If the book was inferior, then it is a case of fraud.

**Disclosure of dangerous nature:**

**Example:** Shyam sold his horse to Ram a buyer for Rs. 11000/- Shyam knows that horse was "wicked" but fails to disclose it to buyer. Here seller has committed fraud by remaining silent.

## 4. Misrepresentation: (Section 18)

The term "misrepresentation" means a false representation of fact made innocently or non-disclosure of a material fact without any intention to deceive the other party.

“Misrepresentation does not involve deception but is only an assertion of something by a person which is not true, though he believes to be true. Misrepresentation could arise because of innocence of the person making it or because he lacks sufficient or reasonable ground to make it. A contract which is hit by misrepresentation can be avoided by the person who has been misled.

-

**For example:** A makes the statement on the information derived, not directly from C but from M. B applies for shares on the faith of the statement which turns out to be false. The statement amounts to misrepresentation, because the information received second-hand did not warrant A to make positive statements to B. (Section 18(1)).

## 5. Mistake:

An error committed innocently is called mistake. Mistake may be ;

A. **Mistake of Law**

i. Mistake of General Law of Country.

ii. Mistake of Foreign Law

iii. Mistake of Private Rights of a Party Relating to Property and Goods etc.

B. **Mistake of Fact**

i. Bilateral Mistake

ii. Unilateral Mistake

## A. Mistake of Law:

i. **Mistake of General Law:** Every citizen of the country is expected to know the law of his nation. The maxim *'ignorance of law is no excuse'* is applicable and the party cannot be allowed any relief on that ignorance.

No citizen can claim excuse on the grounds of ignorance of law. It does not give right to the parties to avoid the contract, stating the effect of mistake as to law. According to Sec. 21 'a contract is not voidable because it was caused by a mistake as to any law in force in India.

ii. **Mistake of Foreign Law:** Even though the ignorance of law is no excuse, the ignorance of foreign law is excused. Because everyone cannot be expected to know the law of all foreign nations. Therefore, mistake of foreign law is considered as void, if it is bilateral.

iii. **Mistake of Private Rights:** Mistake of rights relating to property or goods is treated as mistake of fact and hence void.

## B. Mistake of Fact:

i. **Bilateral Mistake:** Where both the parties to an agreement are under a mistake as to a matter of fact essential to an agreement it is called bilateral mistake and it is void.

ii. **Unilateral Mistake:** When only one party to the agreement is mistaken about the contents of the agreement it is called unilateral mistake. But when it is caused by fraud or misrepresentation on the part of other party, it can be avoided.

# LEGALITY OF OBJECT AND CONSIDERATION

## INTRODUCTION:

Section 10 (Indian Contracts Act) states that all agreements are contracts if made for lawful considerations and with lawful object. Considerations should be lawful, as otherwise, it would vitiate the whole contract and make it void.

**For example:** A promises to pay B Rs 500/- if he commits a theft in C's house. Such a promise will not be enforced by law even if B has committed a theft because the object of consideration of the promise is unlawful.

Section 23 also lays down that every agreement of which the object or consideration is unlawful is void. It, therefore follows that every contract, in order to be valid must be made for lawful consideration with a lawful object.

**Illustrations of Lawful Considerations**:

1. A agrees to sell his house to B for Rs 10,000. Here B's promise to pay the sum of Rs 10,000 is the consideration for A's promise to sell the house, and A's promise to sell the house is the consideration for B's promise to pay Rs 10,000. These are lawful considerations.

2. A promises to pay B Rs 10,000 at the end of six months, if C who owes that sum to B, fails to pay it. B promises to grant time to C accordingly. Here the promise of each party is the consideration for the promise of the other party and they are lawful considerations
3. A promise for a certain sum paid to him by B to make good to B the value of his ship if it is wrecked on a certain voyage. Here A's promises is the consideration for B's payment and B's payment is the consideration for A's promise and these are lawful considerations.

4. A promises to maintain B's child, and B promises to pay A Rs 1,000 yearly for the purpose. Here the promise of each party is the consideration for the promise of the other party. These are lawful considerations.

## Lawful considerations and lawful object distinguished:

Object of an agreement should be differentiated from consideration for an agreement. Object is different from consideration. Object means purposes or design. However, certain difficulties are faced in practice to distinguish between the two, particularly when considerations consist in a promise to do or not to do something.

**Illustrations:**

A promises to obtain or B an employment in the public services and B promises to pay A Rs 100/- The agreements is void as the consideration being A’s promise to procure an employment in the public services is opposed to public policy and hence unlawful.

## Illustrations of unlawful object:

A promise to drop a prosecution which he has instituted against B for robbery and B promises to restore the value of things taken. The agreement is void as its object to save a robber from punishment is unlawful.

A, B and C enter into an agreement for the division among them of the gains to be acquired by them by fraud. Because object of the agreements is to practice fraud on others, it is unlawful.

## What is unlawful consideration?

In the following cases, the consideration or object of an agreement is unlawful: if

1. it is forbidden by law; or
2. is of such a nature that, if permitted, it would defeat the provisions of any law; or
3. is fraudulent; or
4. involves or implies injury to the person or property of another; or
5. the court regards it as in moral or opposed to public policy.

An act promised to be done may be either unlawful to perform (illustrations (1) and (3) below); or the act may be lawful but law will not enforce it for reasons of public policy like wagering agreements. Law means the law for the time being in force in India and includes Hindu and Muslim laws.

## What is a Void Agreement?

Section 2 (g) of the Indian Contract Act, states "that a void agreement is one which is not enforceable by law. A void agreement does not create rights, obligations or duties. It does not give rise to any legal consequences. Such agreements are void. The courts can only enforce those agreements that according to Section 10 fulfill the conditions of the Indian Contract Act. It should not be declared void by any law in the country. There is a difference between void agreements and void contracts.

## Void Agreement:

- A void agreement is not valid.
- The agreement is not enforceable by law.
- It is void from the very beginning of the making of the agreement.
- Agreement by a minor or a person of unsound mind.[Sec(11)]
- Agreement of which the consideration or object is unlawful[Sec(23)]
- Agreement made under a bilateral mistake of fact material to the agreement[Sec(20)]
- Agreement of which the consideration or object is unlawful in part and the illegal part cannot be separated from the legal part [Sec(24)]

- Agreement made. without consideration.[Sec(25)]
- Agreement in restraint of marriage [Sec(26)]
- Agreement in restraint of trade [Sec(27)]
- Agreement in restraint of legal proceedings[Sec(28)]
- Agreement the meaning of which is uncertain [Sec(29)]
- Agreement by way of wager [Sec(30)]
- Agreement contingent on impossible events [Sec(36)]
- Agreement to do impossible acts [Sec(56)]

# MARGINAL COSTING

**For example:**

A company is producing 100 cell phones per month. The total fixed cost per month is Rs 10,000 and variable cost per phone is Rs. 500. The total cost per month is:

| | |
|---|---|
| Marginal Cost (Variable) of 100 phones = 100 X 500 | 50,000 |
| Fixed Cost | 10,000 |
| **Total Cost** | **60,000** |

**If the output is increased by one unit, the cost will be:**

| | |
|---|---|
| Marginal Cost (Variable) of 101 phones = 101 X 500 | 50,500 |
| Fixed Cost | 10,000 |
| **Total Cost** | **60,500** |

Thus, the additional cost of producing one additional unit is Rs. 500. It is known as Marginal Cost.

OR

Marginal costing is the change in total cost on account of adding/ subtracting one additional unit.

## Characteristics of Marginal Costing:

1. It is a technique of analysis and presentation of cost rather than an independent method of costing
2. Total costs are classified into fixed costs and variable costs.
3. It considers only variable costs in analysis.
4. It guides pricing and other managerial decisions on the basis of 'contribution'. Contribution is the difference between sales value and variable costs.
5. It valuates finished stock and work-in-progress at marginal cost only.
6. It charges the fixed costs against 'contribution'
7. It takes the difference between contribution and fixed cost as profit or loss.

## Assumptions:

1. All costs are divisible into fixed costs and variable costs.
2. Selling price and variable cost per unit will remain the same.
3. Total fixed costs will remain constant.
4. Volume is the only factor which influences the costs.

## Marginal Cost Equation:

The following equation is known as basic marginal cost equation:

- **If there is a Profit :**

Sales – Variable Cost = Fixed Cost + Profit

- **If there is a Loss:**

Sales – Variable Cost = Fixed Cost – Loss

# Determination of Profit under Marginal Costing

| Particulars | | Total (Rs.) | Per Unit (Rs.) |
|---|---|---|---|
| **Sales** | | xxx | xxx |
| **Less:** Variable costs | | | |
| Direct Materials | xxx | | |
| Direct Wages | xxx | | |
| Direct Expenses | xxx | | |
| Variable Overheads | xxx | xxx | xxx |
| **Contribution** | | **xxx** | **xxx** |
| **Less:** Fixed Cost | | xxx | xxx |
| **Profit/Loss** | | **xxx** | **xxx** |

## Contribution:

Contribution is the difference between sales and variable cost. In other words, contribution is the excess of sales over the variable cost. It is also known as gross margin or marginal income. It enables to meet fixed costs and contributes to profit.

Contribution = Sales – Variable Cost

Contribution = Fixed Cost + Profit

Contribution = Fixed Cost – Loss

Contribution = Sales x P/V Ratio

**Example: 1**

Fixed Cost Rs. 10,000, SP per unit Rs. 20, and Variable cost per unit Rs. 8

**Statement of Marginal Cost**

| Particulars | Per unit cost | 1,000 Units | 5,000 Units | 10,000 Units | 500 Units |
|---|---|---|---|---|---|
| Selling Price | 20-00 | 20,000 | 1,00,000 | 2,00,000 | 10,000 |
| **Less:** Variable Cost | 08-00 | 8,000 | 40,000 | 80,000 | 4,000 |
| **Contribution** | 12-00 | 12,000 | 60,000 | 1,20,000 | 6,000 |
| **Less:** Fixed Cost | -- | 10,000 | 10,000 | 10,000 | 10,000 |
| **Profit** | -- | 2,000 | 50,000 | 1,10,000 | - 4,000 |

**Example: 1**

Fixed Cost Rs. 10,000, SP per unit Rs. 20, and Variable cost per unit Rs. 8

## Statement of Marginal Cost

| Particulars | Per unit cost | 1,000 Units | 5,000 Units | 10,000 Units | 500 Units |
|---|---|---|---|---|---|
| Selling Price | 20-00 (100%) | 20,000 (100%) | 1,00,000 (100%) | 2,00,000 (100%) | 10,000 (100%) |
| **Less:** Variable Cost | **08-00 (40%)** | **8,000 (40%)** | **40,000 (40%)** | **80,000 (40%)** | **4,000 (40%)** |
| **Contribution** | 12-00 (60%) | 12,000 (60%) | 60,000 (60%) | 1,20,000 (60%) | 6,000 (60%) |
| **Less:** Fixed Cost | -- | **10,000** | **10,000** | **10,000** | **10,000** |
| **Profit** | -- | 2,000 | 50,000 | 1,10,000 | - 4,000 |
| **P/V Ratio** | 12/20 = **60%** | **60%** | **60%** | **60%** | **60%** |

## Profit Volume Ratio (P/V Ratio):

P/V Ratio is a ratio of contribution to sales.

It states the relationship between contribution and sales.

Therefore it is also called as contribution/sales ratio, or contribution ratio or marginal ratio.

It is calculated by using the following formula:

$$\text{P/V Ratio} = \frac{Sales - Variable\ Cost}{Sales}$$

$$\text{OR} = \frac{Fixed\ Cost + Profit}{Sales}$$

$$\text{OR} = \frac{Fixed\ Cost - Loss}{Sales}$$

$$\text{OR} = \frac{Contribution\ per\ unit}{Selling\ price\ per\ unit}$$

$$\text{OR} = \frac{Change\ in\ Profit}{Change\ in\ Sales}$$

$$\text{OR} = \frac{Change\ in\ Contribution}{Change\ in\ Sales}$$

Let us see how change in Contribution/Profit/Loss divided by Change in Sales will give same P/V Ratio

## Statement of Marginal Cost

| Particulars | Per unit cost | 1,000 Units | 5,000 Units | Change | 10,000 Units | 500 Units | Change |
|---|---|---|---|---|---|---|---|
| Sales | 20-00 | 20,000 | 1,00,000 | 80,000 | 2,00,000 | 10,000 | 1,90,000 |
| Less: VC | 08-00 | 8,000 | 40,000 | 32,000 | 80,000 | 4,000 | 76,000 |
| Contribution | 12-00 | 12,000 | 60,000 | 48,000 | 1,20,000 | 6,000 | 1,14,000 |
| Less: FiC | -- | 10,000 | 10,000 | -- | 10,000 | 10,000 | -- |
| Profit | -- | 2,000 | 50,000 | 48,000 | 1,10,000 | - 4,000 | 1,14,000 |

Problem No. 1

Calculate P/V ratio, from the following:

| Particulars | Years | | Change |
| --- | --- | --- | --- |
| | 2010 | 2011 | |
| Sales (Rs.) | 1,50,000 | 2,00,000 | 50,000 |
| Profit (Rs.) | 25,000 | 40,000 | 15,000 |

$$P/V\ Ratio = \frac{Change\ in\ Profit}{Change\ in\ Sales}$$

$$= \frac{15,000}{50,000}$$

= 3/10 or 30%

Problem 2:

From the following particulars, calculate P/V Ratio:

| Year | Sales(Rs.) | Profit/Loss (Rs.) |
|---|---|---|
| 2010 | 6,00,000 | 60,000 (loss) |
| 2011 | 12,00,000 | 90,000 (Profit) |
| **Change** | **6,00,000** | **1,50,000** |

$$P/V \text{ Ratio in } \% = \frac{Change\ in\ Profit}{Change\ in\ Sales} \times 100$$

$$= \frac{1,50,000}{6,00,000} \times 100 = 25\%$$

**P/V Ratio = 25%**

## Break Even Point (BEP):

Break-even point is a point at which the total costs are equal to sales.

It is a volume of sales at which there is neither profit nor loss. Hence, it is also called as no profit no loss point.

If the sale is increased beyond break-even-point level, profit will accrue and if sale is decreased below the BEP level, loss will occur.

1. BEP (in units) = $\frac{Fixed\ Cost}{Contribution\ \ per\ unit}$

2. BEP (in Rs.) = BEP units × Selling price per unit

3. BEP (in Rs.) = $\frac{Fixed\ Cost}{P/V\ \ Ratio}$

**Problem 1:** Fixed Cost Rs. 10,000, SP per unit Rs. 20, and Variable cost per unit Rs. 12.

## Statement of Marginal Cost

| Particulars | Per unit cost | 2,000 Units | 1,250 Units | 3,750 Units |
|---|---|---|---|---|
| Sales | 20-00 (100%) | 40,000 | 25,000 | 75,000 |
| Less: Variable Cost | 12-00 (60%) | 24,000 | 15,000 | 45,000 |
| Contribution | 8-00 (40%) | 16,000 | 10,000 | 30,000 |
| Less: Fixed Cost | -- | 10,000 | 10,000 | 10,000 |
| Profit | -- | 6,000 | 00,000 | 20,000 |

1. BEP(Rs.) = $\frac{Fixed\ Cost}{P/V\ Ratio}$ = $\frac{10,000}{0.40} = Rs.\ 25,000$

2. BEP (Units) = $\frac{Fixed\ Cost}{Contribution\ per\ unit}$ = $\frac{10,000}{8} = 1,250\ Units$

**Margin of Safety:**

Margin of safety is the excess of actual sales over sales at break-even-point. In other words, sales over and above the break-even point are known as margin of safety.

If the margin of safety is large, it is the sign of soundness of the business and if the margin of safety is small, it is a sign of weak position of business.

The margin of safety can be expressed in absolute sales amount or in terms of percentage to sales.

Margin of safety can be ascertained by;

1. Margin of Safety (Amount)

   = Actual sales – Sales at BEP

2. Margin of Safety (Units)

   = Actual Sales units – BEP Sales units

3. Margin of Safety (Amount)

$$= \frac{Profit}{P/V\ Ratio}$$

4. Margin of Safety (Units)

$$= \frac{Profit}{Contribution\ per\ unit}$$

**Estimated Sales or Profit:**

In order to calculate the estimated sales at a given profit or estimated profit at given volume of sales the following formulae are used.

1. Estimated Sales (units) = $\frac{Fixed\ Cost + Given\ Profit}{Contribution\ per\ unit}$
2. Estimated Sales (amt)

   = Estimated Sales (units) × Selling price per unit

1. Estimated Sales (amt) = $\frac{Fixed\ Cost + Given\ Profit}{P/V Ratio}$

**Problem 1:**

A company has a sales of Rs. 12,00,000 with fixed cost of Rs. 3,60,000 and Profit Rs. 2,40,000. If the company desires to earn a profit Rs. 3,00,000 in the next period, what would be its sales?

**Solution:**

**Contribution** = Fixed Cost + Profit

= Rs. 3,60,000 + Rs. 2,40,000

= **Rs. 6,00,000**

**P/V Ratio**

$$= \frac{Contribution}{Sales} \times 100$$

$$= \frac{6,00,000}{12,00,000} \times 100$$

$$= \mathbf{50\%}$$

**Sales in the next period if the company desires to earns a profit of Rs. 3,00,000.**

$$= \frac{Fixed\ Cost + Desired\ Profit}{P/V\ Ratio}$$

$$= \frac{3,60,000 + 3,00,000}{50\%}$$

$$= \frac{6,60,000}{0.50}$$

= **Rs. 13,20,000**

If the company wants to earn a profit of Rs. 3,00,000 it has to achieve the sales target of Rs. 13,20,000

**Problem 2:**

A company has a sales of Rs. 6,00,000 with fixed cost of Rs. 1,80,000 and Profit Rs. 1,20,000. Calculate sales volume if the company suffered a loss of Rs. 60,000, in the next period .

**Solution:**

**Contribution** = Fixed Cost + Profit

= Rs. 1,80,000 + Rs. 1,20,000

= **Rs. 3,00,000**

$$\textbf{P/V Ratio} = \frac{Contribution}{Sales} \times 100$$

$$= \frac{3,00,000}{6,00,000} \times 100$$

$$= \textbf{50\%}$$

**Sales in the next period if the company suffered a loss of Rs. 60,000.**

$$= \frac{Fixed\ Cost\ -\ Expected\ Loss}{P/V\ Ratio}$$

$$= \frac{1,80,000\ -\ 60,000}{50\%}$$

$$= \frac{1,20,000}{0.50}$$

= **Rs. 2,40,000**

If the company suffered a loss of Rs. 60,000 in the next period, its sales should be at Rs. 2,40,000

# Ascertainment of Variable Cost:

1. Variable Cost = Sales – Contribution
2. Variable Cost = Total Cost – Fixed Cost
3. Variable Cost ratio to sales = $\frac{Change\ in\ total\ cost}{Change\ in\ Sales}$

   Therefore,

   Variable Cost = Sales × Variable cost ratio
4. Variable Cost = Sales (1- P/V Ratio)

1. From the following information of Asha Co. Ltd. Calculate P/V Ratio and Margin of Safety.

| | | | |
|---|---|---|---|
| a. | Sales | -- | Rs. 10, 00,000 |
| b. | Variable Cost | -- | Rs. 4, 00,000 |
| c. | Profit | -- | Rs. 3, 00,000 |

**Solution:**

**Contribution** = Sales – Variable Cost

= Rs. 10,00,000 – Rs. 4,00,000

**= Rs. 6,00,000**

**Fixed Cost** = Sales – Variable Cost – Profit

= 10,00,000 – 4,00,000 – 3,00,000

= Rs. 10,00,000 – Rs. 7,00,000

**= Rs. 3,00,000**

**OR** Fixed Cost = Contribution - Profit

= Contribution – Profit

= Rs. 6,00,000 – Rs. 3,00,000

**= Rs. 3,00,000**

**P/V Ratio** = $\frac{Contribution}{Sales}$ = $\frac{6,00,000}{10,00,000} \times 100$

= **60%**

**BEP (Value)** = $\frac{Fixed\ Cost}{P/V\ Ratio}$ = $\frac{3,00,000}{0.6}$

= **Rs. 5, 00,000**

**Margin of Safety** = Sales – BEP Sales

= Rs. 10,00,000 – Rs. 5,00,000

= **Rs. 5,00,000**

# THANK YOU

# VALUATION OF GOODWILL

- Meaning and Definition
- FEATURES OF GOODWILL
- NEED FOR VALUATION OF GOODWILL
- FACTORS AFFECTING THE VALUE OF GOODWILL
- TYPES OF GOODWILL
- METHODS OF VALUATION OF GOODWILL

# Meaning and Definition

- Goodwill means good name or fame or reputation of the company which attracts more customers to earn more profits. Goodwill is one of the assets of a business. Goodwill contributes to the profit earning capacity of the company.

# Meaning and Definition

- According to the Institute of Chartered Accountants of India, “Goodwill is an intangible asset arising from sound business connections or trade name or reputation of an enterprise”.

# Meaning and Definition

- Goodwill may be defined as “the present value of firm’s anticipated excess earnings”.
- In the words of Spicer and Pegler “Goodwill may be said to be that element arising from the reputation, connection or other advantage possessed by a business which enables it to earn greater profits than the return, normally to be expected on the capital represented by the net tangible assets employed in the business”.

# Meaning and Definition

- According to Kohler "Goodwill is the current value of expected future income in excess of a normal return on investment in net tangible assets".
- Goodwill may be defined as "an extra value attached to and established business beyond the intrinsic value of the net assets arising from reputation, trade name, sound connections, high profit earning capacity of the business and the same profitability will be continued in future despite a change in the ownership of the business".

# FEATURES OF GOODWILL

- Goodwill is an intangible asset but not fictitious asset.
- Goodwill helps the business to increase the profit earning capacity of the business.
- It helps the business to earn more than the normal profits.
- Goodwill is not separate from other assets. So it cannot be sold separately. However, it can be sold with the whole business of an enterprise.
- The value of goodwill fluctuates from time to time depending upon the changing circumstances which may be either internal or external to the business.

# FEATURES OF GOODWILL

- Goodwill is created on the basis of different factors such as location of the business, management, functioning of the business, sound connections and so on.
- Indian Accounting Standard-10 states that only purchased goodwill should be recognized in the accounts of the enterprises.
- The value of goodwill does not depend upon the money invested and expenses incurred on building up of the same.
- Valuation of the goodwill is not objective but it is subjective and also differs from one valuer to another.

## NEED FOR VALUATION OF GOODWILL

- When the business of one company is sold to another company.
- When the business of a company is taken over by another company e.g., in case of amalgamation or absorption.
- When the company’s shares are not quoted on the stock exchange and their value is to be determined for the purposes of estate duty and wealth tax etc

# NEED FOR VALUATION OF GOODWILL

- When a person wants to purchase a large block of the company's shares with a view to acquire control over the management of the company.
- When the business of the company is being taken over by the Government.
- When the Management wants to write back goodwill which it wrote off earlier to reduce or eliminate the debit balance in the profit and loss account.

## FACTORS AFFECTING THE VALUE OF GOODWILL

- Efficient management team
- Effective adverting campaign
- Loyal, efficient and sincere employees
- Trade mark, patents etc, owned by the enterprise
- Regular research and developmental activities

## FACTORS AFFECTING THE VALUE OF GOODWILL

- Good relation with other industries, suppliers, etc
- Strategic location of business premises
- Sufficient number of outlets and service centers for the products
- Training programmes for employees at various levels
- Good name and reputation in the general public
- Proper planning in tax matters
- High quality and reliability of products

## FACTORS AFFECTING THE VALUE OF GOODWILL

- Favourable government rules and regulations
- Absence of competition
- Existence of full or partial monopoly
- Good and sound public image
- Earning capacity of the business

# TYPES OF GOODWILL

- **Purchased goodwill:** The purchased goodwill is one which arises when one enterprises purchase the business of another enterprises.
- In this case, if the purchase consideration payable is more than the value of net assets, such excess amount is known as goodwill.
- Cost of goodwill = Purchase price – Net assets purchased
- Net assets = Total assets – outside liabilities
- The purchased goodwill is recognized by the Indian Accounting Standard-10. Hence, it is shown in the balance sheet of a purchasing company.
- The factors such as reputation of the business, trade name, sound connections, financial advantages, etc will contribute to the value of purchased goodwill

# TYPES OF GOODWILL

- **Non-purchased goodwill**: Non-purchased goodwill is one which is generated by the business enterprise itself internally. Since this goodwill is self-generated no costs can be placed on it. This goodwill is not shown in the balance sheet of the enterprise. The factors such as efficient management, strategic location, high quality products, public image, etc will contribute to the value of goodwill. Non-purchased goodwill is also known as inherent or self-generated goodwill

## METHODS OF VALUATION OF GOODWILL

- Average profits method
  - Simple average profit method
  - Weighted average profit method
- Capitalization of average profit method
- Super profits method
  - Purchase of super profit method
  - Sliding-scale valuation of super profit method
  - Annuity method of super profit
  - Capitalization of super profit method

# Average profits method:

- **Simple average profit method:** According to this method goodwill is valued on the basis of a certain number of years' purchase of the average profits of the past given number of years.
- Average simple profit =

$$\frac{\text{Total profit of given number of years}}{\text{Total number of years}}$$

- Value of Goodwill = Average simple profit X No of years of purchase

# Average profits method:

- **Weighted average profit method:** According to this method the value of goodwill is the number of years purchase of the weighted average profit.
- Weighted average profit =

$$\text{Weighted average profit} = \frac{\text{Total product of weighted profit}}{\text{Total number of weights}}$$

- Value of goodwill = Weighted average profit X No of years of purchase

# Capitalization of average profit method

- According to this method the future maintainable profits of the business are capitalized applying the normal rate of return. Then the value of net tangible assets is to be calculated. The difference between the capitalized value of the business and the value of net tangible asset is the value of the goodwill.

# Capitalization of average profit method

- The normal rate of return is a rate which is equivalent to the average return from a similar business of an enterprise.
- Under this method the following steps are to be followed to find out the value of goodwill:
  - Find out the average maintainable profits
  - Find out the capitalized value of business

## Capitalization of average profit method

- Capitalized value of business =

$$\frac{\text{Average maintainable profits} \times 100}{\text{Normal rate of return}}$$

  - Calculate the value of net tangible assets of the business
- Net tangible asset = Total tangible assets – Outside liabilities
- **Value of Goodwill = Capitalized value of business – Net tangible assets**

## Super profit method

- According to this method the value of goodwill depends upon the super profits earned by an enterprise.

**Super profit:**

- If the average profit earned by a business is more than the normal profit, then the difference between these two profits is called as super profit.
- The difference between the average profit and normal profit is called as super profit.
- **Super Profit = Average profit – Normal profit**

# Super profit method

- **Normal Profit**
- Normal profit is a profit which is obtained by multiplying the average capital employed in the business with the normal rate of return and then divided by 100.
- **Normal profit = Average capital employed X Normal rate of return**

$$\text{Normal profit} = \frac{\text{Average capital employed} \times \text{Normal rate of return}}{100}$$

# Average capital employed

Average capital is to be calculated according to:

- Assets side approach or
- Liabilities side approach

# Assets side approach:

| Particulars | Rs. |
|---|---|
| Assets at market value | XXX |
| Less: Outside Liabilities | XXX |
| Capital employed | XXX |
| Less: Half of current year profit (after adjustment) | XXX |
| Average capital employed | XXX |

# Assets side approach:

Note: 1. Goodwill, Preliminary expenses, discount on issue of shares and debentures and losses should not be included

- 2. Debentures, Creditors, Bills payable, Provision for taxation and such other liabilities should only be included in outside liabilities
- 3.Non-trading assets such as investment in Government securities etc also should not be included in assets.
- 4.When market value of assets is not given in the problem, then the book value of assets is taken into account.

# Liabilities side approach: Average capital employed

| Particulars | Rs. |
|---|---|
| Equity and preference share capital | XXX |
| Less: Goodwill, losses etc | XXX |
| Capital employed | XXX |
| Less: Half of current year profit (after adjustment) | XXX |
| Average capital employed | XXX |

# Liabilities side approach:

Note:

- All reserves, profits, profit on revaluation of assets and liabilities should be added to share capital.
- All losses, loss on revaluation, preliminary expenses, fictitious assets etc should be deducted from the share capital.

# Different methods of super profit:

- **Purchase of super profit method**: According to this method the value of goodwill is certain number of years' of super profit.
- **The value of goodwill = Super profit X No. of years' of purchase**

# Sliding scale valuation of super profit method

- This method of valuation is a slight variation of the purchase of super profit method. It is developed by A.E. Cutforth. According to first method the amount of super profit remains same for a fixed number of years and after that period super profit disappears

# Sliding scale valuation of super profit method

- According to the sliding scale valuation method, the amount of super profit goes on decreasing year by year and after a fixed period of time the super profit disappears. Hence, this method of valuation of goodwill is considered more realistic than the purchase of super profit method.

# Sliding scale valuation of super profit method

- The present method is based on the general theory that higher the amount of super profit earned by a business it will be rather difficult for it to maintain the same in future. The reason is that if there is higher profit in a type of business more traders will be attracted in the same type of business with a result that the ability to earn more profit will be reduced.

# Sliding scale valuation of super profit method

- . Therefore, instead of multiplying the whole amount of super profit by a given number of years, a grading scale of valuation of goodwill will be adopted as below:

| | | | |
|---|---|---|---|
| Year | Amount of Division of super profit | No of years' of purchase | Product(Amount X No of years' of purchase) |
| | | | |

# Annuity method of valuation of super profit

- This method makes use of annuity value for valuation of goodwill of the business. In this method of valuation it takes into account the time gap between the actual payment of goodwill amount and the actual earning of the super profit in future. Hence, the present value of super profit is calculated to find out the value of goodwill. The present value of goodwill depends upon the annuity table.

- **Value of goodwill = Super profit X Annuity value**
- **The value of goodwill calculated according to this method is considered as the most realistic one.**

# Capitalization of super profit method

- According to this method the value of goodwill of a business is determined by capitalizing the amount of super profit at the normal rate of return. This method tries to find out the amount of capital needed for earning the super profit.

$$\text{Value of goodwill} = \frac{\text{Super profit} \times 100}{\text{Normal rate of return}}$$

# *PPTs by Students*

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: **C**alculus–II and 3–D Geometry

I online Class

Introduction of Partial differentiation

By


Dr. M. M. Shankrikopp

HOD of Mathematics


Date: 30.4.2021

# SYLLABUS

## UNIT-I

- Polar coordinates of a point and polar curve. Angle between the radius vector and the tangent at a point on the curve. Angle of intersection of two curves. Polar and pedal equation of the curves. Polar sub-tangent and polar sub - normal. **12 hours**

## UNIT-II

- Derivative of arc length, Curvature, Radius of curvature in Cartesian, Parametric, polar and pedal forms. Centre of curvature. Evolutes and Involutes. **12 hours**

# SYLLABUS

## UNIT-III

- Limits, continuity of functions of two variables. Partial derivatives, higher order partial derivatives, total derivatives and total differentials, Homogeneous functions, Euler's theorem on homogeneous functions. **12hours**

## UNIT-IV

- Reduction formulae for integration of $\sin^n x$, $\cos^n x$, $\tan^n x$, $\cot^n x$, $\sec^n x$, $\mathrm{cosec}^n x$, $\sin^n x \cos^n x$, $x^n e^{ax}$ and $x^m (\log x)^n$. **12 hours**

# SYLLABUS

## UNIT-V

- **Sphere**: Equation of a sphere, section of a sphere by a plane, Equation of a sphere through a circle, Equation of a sphere through two given points as ends of a diameter. Equation to a tangent plane of a sphere, Condition for tangency, Orthogonality of two spheres.
- **Cone**: Equation of a cone, enveloping cone of a sphere, Right circular cone.
- **Cylinder:** Equation of cylinder, enveloping cylinder of a sphere, Right circular cylinder.
- **12 hours**

# BOOKS FOR REFERENCE

- Integral Calculus : Santinarayan and Dr. P.K. Mittal
- Differential Calculus and integral Calculus : N.P. Bali
- Text Book of B.Sc Mathematics: G. K. Ranganath
- Text book of Mathematics III: S.S. Bhoosnurmath and others

# COURSE OUTCOMES OF THE PAPER

- Able to find differentiation of polar curves, angle between two polar curves and pedal equation for all type of curves.

- Able to determine formula for curvature, evaluate and application of curvature in construction of lenses. Concavity and convexity and points of inflexion.

- Able to find limits, continuity of functions of two variables. Partial derivatives, higher order partial derivatives, total derivatives and total differentials, Homogeneous functions, Euler's theorem on homogeneous functions.

- Recognize to determine given equation represents sphere, Cone and Cylinder and gain the knowledge properties of these 3-D figures.

# DISTRIBUTUION OF SYLLABUS

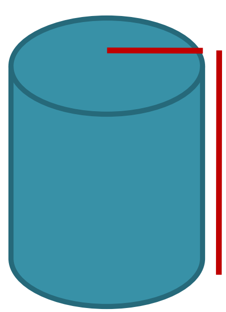
| Name | Units |
|---|---|
| 1. Dr. M. M. Shankrikopp | III: Partial Differentiation |
| | V: 3-D Geometry |
| 2. Dr. Ashok Rathod | I: Polar Coordinates |
| | II: Curvature and Evolute |
| 3. Miss G.L.Karaguppi | III: Reduction Formulae |

# UNIT III: PARTIAL DIFFERENTIATION

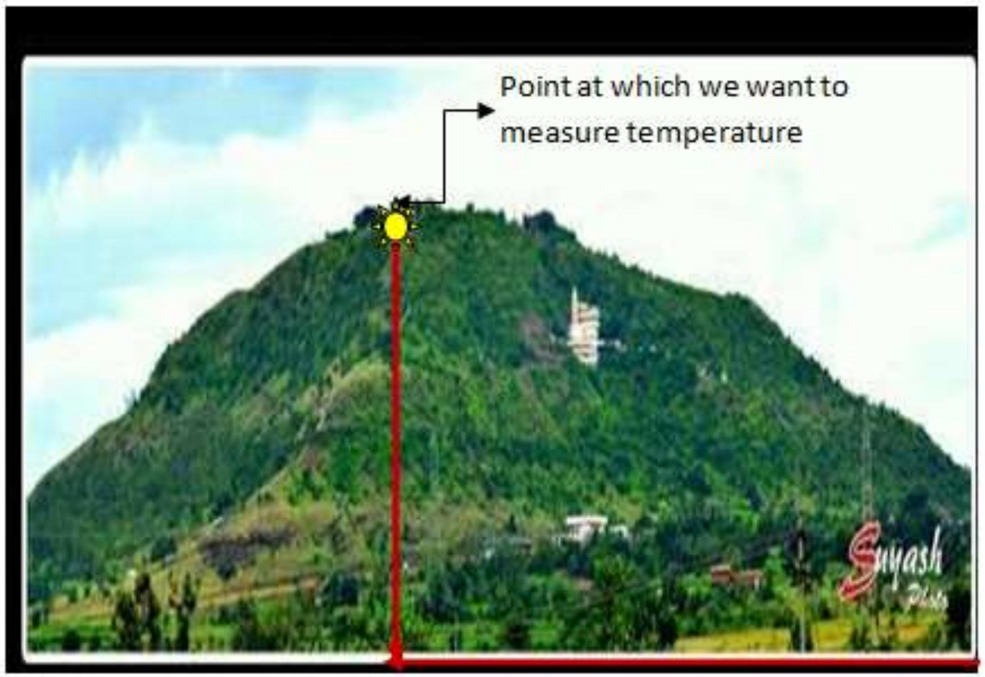
**Introduction:**

In PUC and B.Sc. I sem.we studied continuity and differentiability of function f(x) of one independent variable x. Sometimes the function or quantity depends on more than one variable.

**For example**: We know that the formula $V = \pi r^2 h$ gives the volume of cylinder. Here if volume V is the function depends on both radius and height of the cylinder which we can choose independently, ie if there is change in radius or height automatically there will be change in volume. So we conclude that V is function of two independent variables r and h. Symbolically , we write it as V =f(r, h).



Similarly, **one more example**: Let us choose a location 'Adi Hill'. If we want measure the temperature at particular point on the top of the hill (surface of the earth) at particular time, it depends on longitude and latitude i.e x and y coordinates and time t. So we say temperature T is a function of three variables x, y, and time t.




i.e $T = u(x,y,t)$, so as a result T is a function of three independent variables, x,y and t.

Similarly, current in an electric circuit is a function of four variables; the electromotive force, the resistance, the capacitance and the induction.

So in this way one quantity is a function of more than one independent variables called function of several variables in which continuity and differentiability exists, called calculus of function of two or more variables. This chapter includes calculus of function of only two variables i.e, continuity, differentiability(called partial differentiation, chain rule and how to write total differentiability in terms of partial differentiation.

Historically speaking, Mathematicians like Euler, Lagrange and Jacobi among others, are the main contributors to the calculus of several variables.

## Applications:

This calculus has potential applications in Engineering and Physics. In physics, we come across partial differential equation which we can solve by using mathematics. Apart from this, vector calculus(studying in IV sem.), another branch of Mathematics, is depending on partial differentiation.

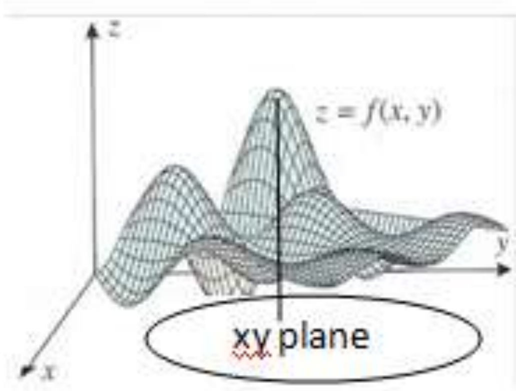
# Function of two variables

Let D be a subset of plane $R^2$, then a function of two variables is a rule f that assigns to each ordered pair (x,y) in a set D. i.e we get a unique no. z corresponding to function of x and y. i.e $f(x,y) = z$

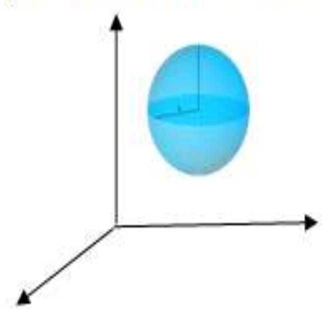
Symbolically, $f: R^2 \to R$ defined by $f(x, y) = z$.

**For example**: $f(x,y) = 1 - x^2 - y^2$ is a function of two variables.

**Geometrically**, the function $z = f(x, y)$ or $f(x,y,z)=0$ represents surface in $R^3$ (space)

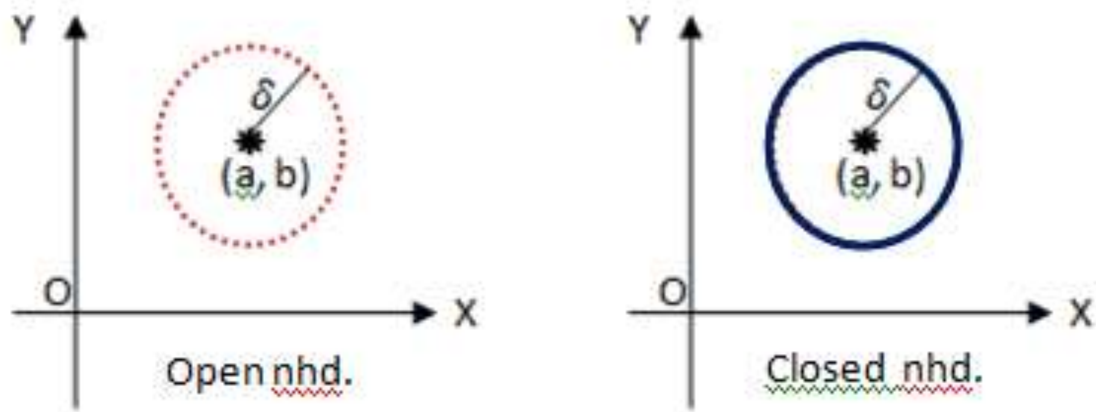



**For example**: Sphere is a surface wnose equation is $x^2+y^2+z^2+2ux+2vy+2wz+d=0$



# Basic Definitions

- **Neighbourhood of a point (a, b):**




Let (a,b) be any point in the plane, then collection or set of all the points (x,y) which are very close to (a, b) is called neighbourhood of the point (a,b). If I take the small distance as $\delta$

Then collection of all points (x,y) whose distance from (a,b) is less than $\delta$ is called nhd. of (a,b) or open disk. i.e nothing but collection of all points inside the circle with centre at(a,b) and radius $\delta$.

And collection of all points points (x,y) whose distance from (a,b) is less than or equal to $\delta$ is called nhd. of (a,b) or open disk.

- **For example**: If Niapni is the centre place i.e like (a, b) and be distance 5 kms. Then set of all villages which are within 5 kms is called nhd. of Nipani i.e they are neighbours of Nipani. i.e Lakanapur, Yamagarni, Chikli, Stavanidhi etc. are in nhd. of Nipani where as Galataga is not.

i.e collection of all neighbouring villeges of Nipani is callled Nhd. of Nipani.

Then what are the neighbouring cities of Nipani?

Answer for this?

BE SAFE AT HOME

WEAR MASK IF YOU GO OUT FOR NECESSERY

SANITIZE HANDS OFTEN

# WE ALL PRAY TO GOD TO SAVE ALL PEOPLE FROM COVID

ಓಂ ನಮೋ ಭಗವತೆ ಮಹಾಸುದರ್ಶನ
ವಾಸುದೇವಾಯ
ಧನ್ವಂತರಾಯ
ಅಮೃತಕಳಶ ಹಸ್ತಾಯ
ಸಕಲ ಭಯ ವಿನಾಶಾಯ
ಸರ್ವರೋಗ ನಿವಾರಣಾಯ
ತ್ರಿಲೋಕ ಪತಯೆ
ತ್ರಿಲೋಕ ನಿಧಯೆ
ಓಂ ಶ್ರೀ ಮಹಾವಿಷ್ಣುಸ್ವರೂಪ
ಶ್ರೀ ಧನ್ವಂತರಿ ಸ್ವರೂಪ
ಓಂ ಶ್ರೀ ಶ್ರೀ ಔಷಧಚಕ್ರ ನಾರಾಯಣಾಯ ನಮಃ ||
🙏🙏🙏

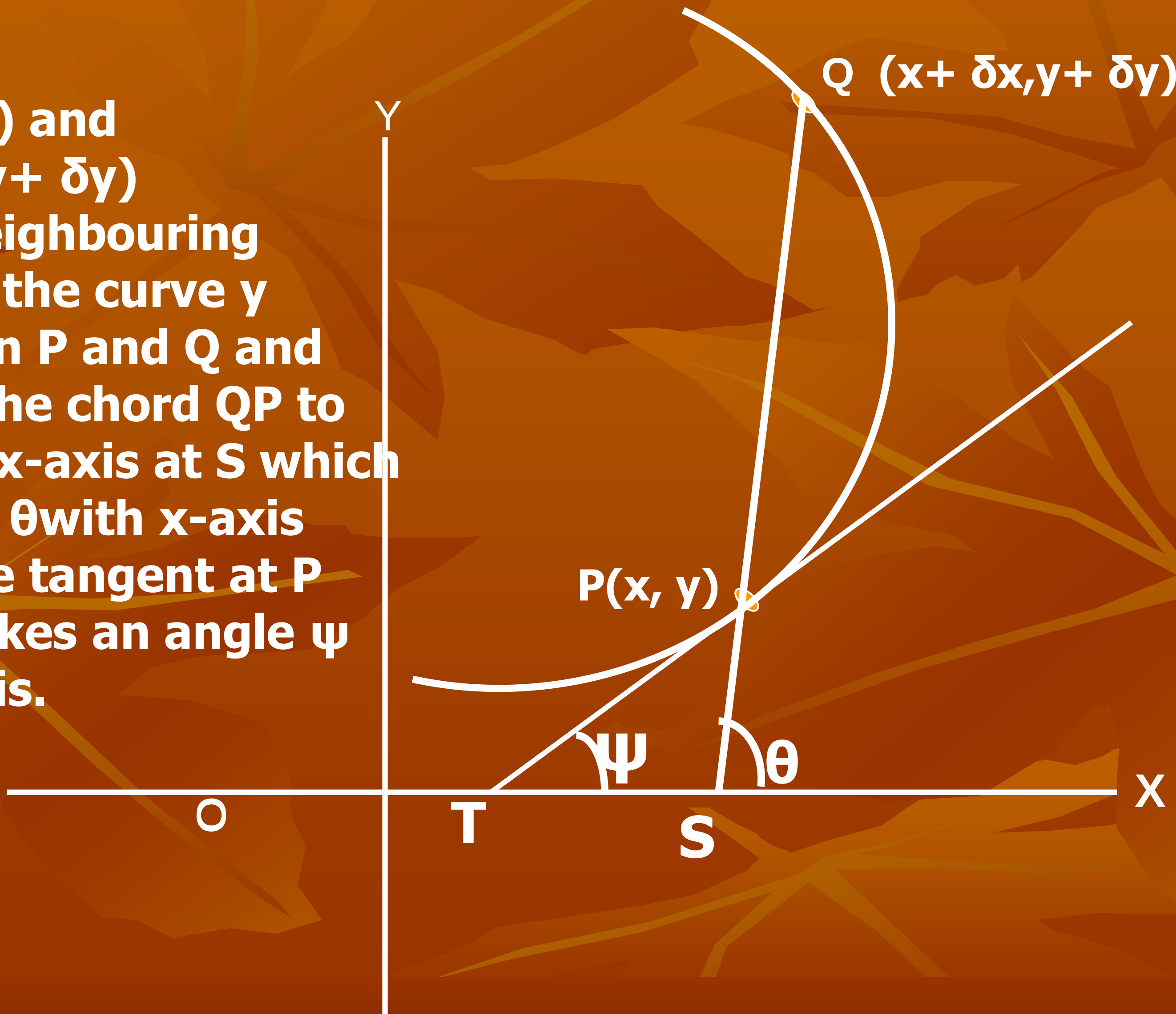
According to Veda, a very Powerful Mantra that keeps us Healthy..ಇದೊಂದು ಚಿಕ್ಕ ಪ್ರಾರ್ಥನೆ ಬೇರೆಯವರಿಗೆ ಕಳುಹಿಸಿ 11:08 AM

Dr. M.M. Shankrikopp

THANKYOU

# Geometrical Representation Of Differentiation

**Let P(x, y) and Q(x+ δx,y+ δy) be two neighbouring points on the curve y =f(x). Join P and Q and produce the chord QP to meet the x-axis at S which makes an θwith x-axis .Draw  the tangent at P which makes an angle ψ with x-axis.**



# Geometrical Representation Of Differentiation

**Draw PM and QN ⊥ʳ to x-axis and PR ⊥r to PN. From the figure $\angle PQR = \theta$ and PR = MN**

**= δx and QR = δy**

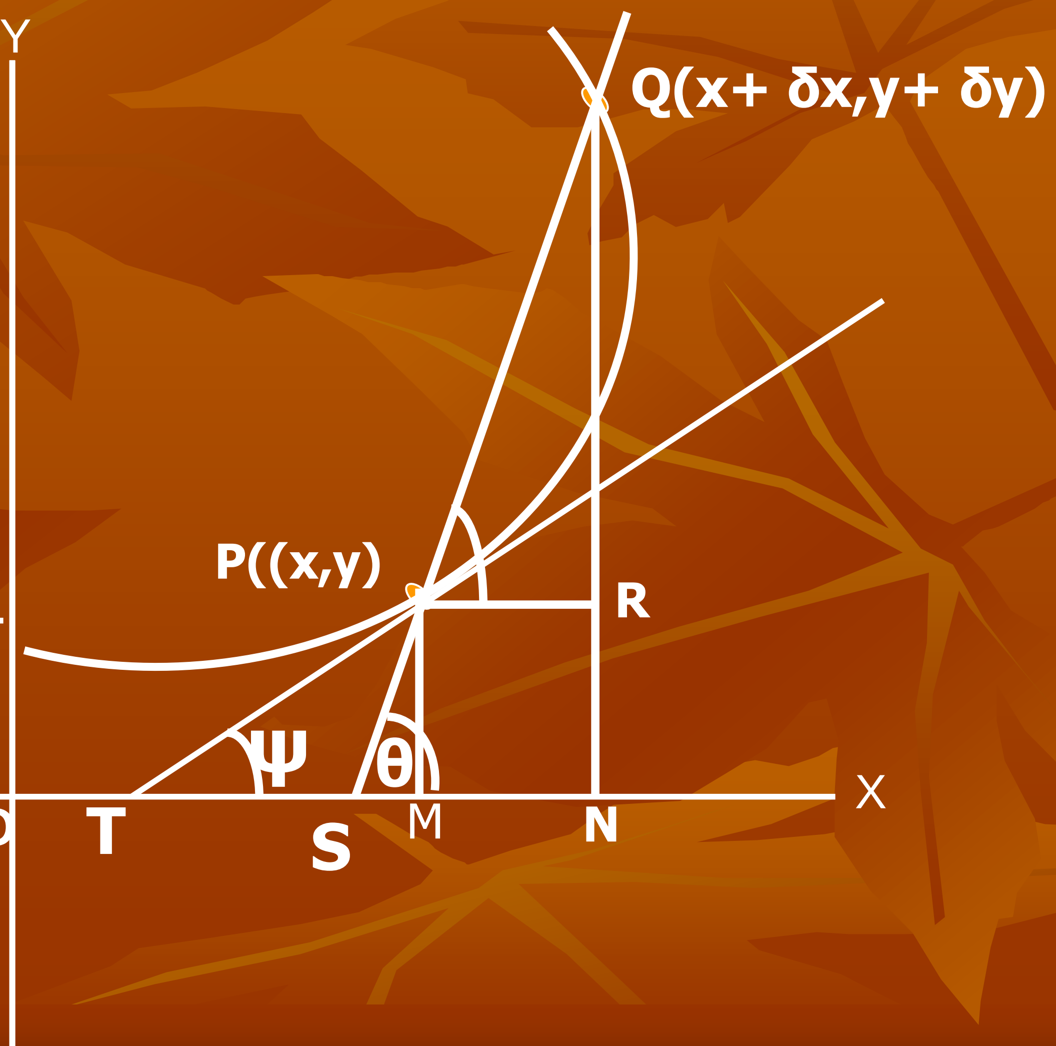
**From the rt. angled triangle PQR**

**$\tan(\angle PQR) = \tan\theta = \dfrac{QR}{PR}$**

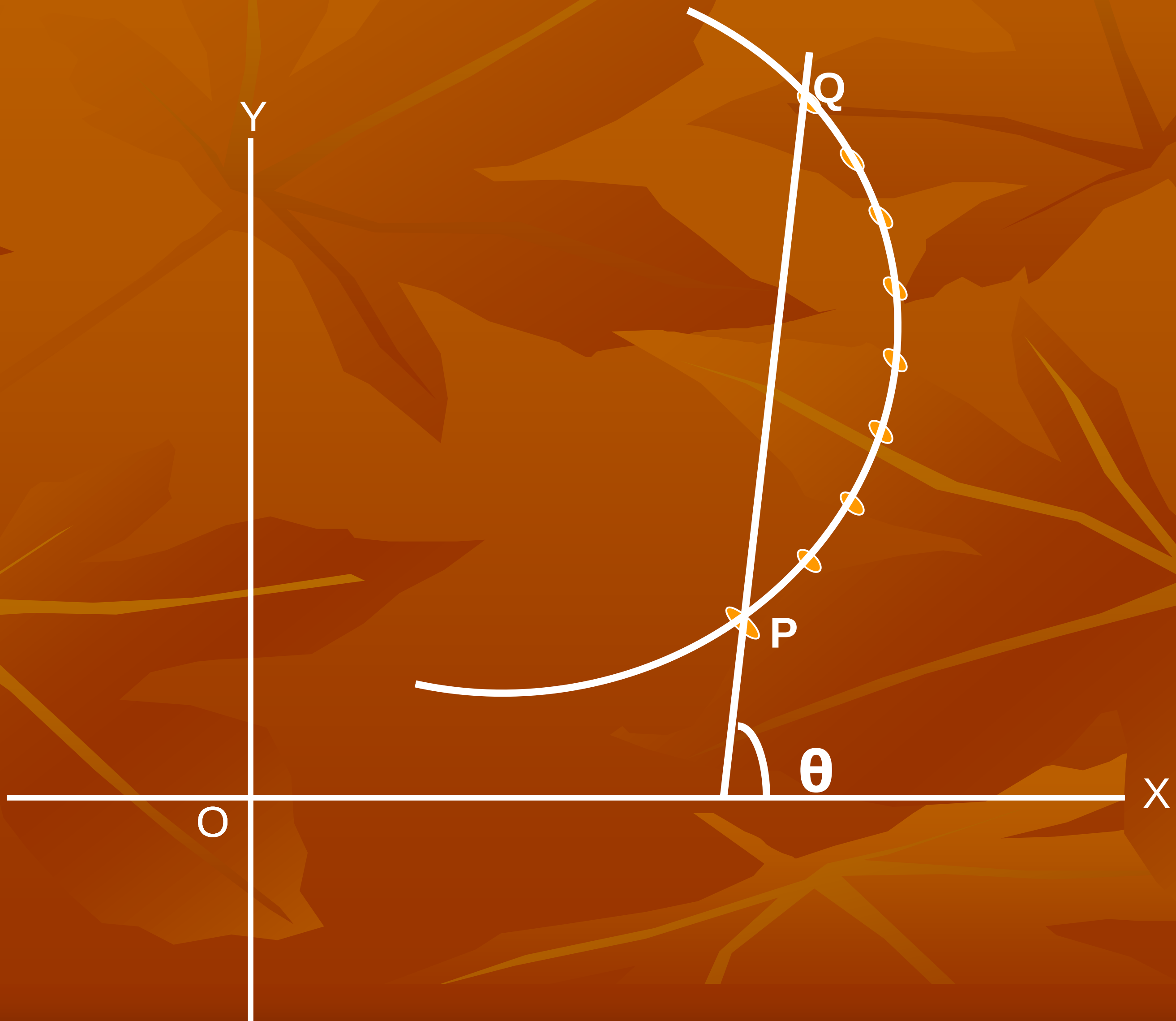
$\tan\theta = \delta y / \delta x$ i.e

slope of PQ = $\boldsymbol{\delta y/\delta x}$

**As Q moves towards P**



# Geometrical Representation Of Differentiation



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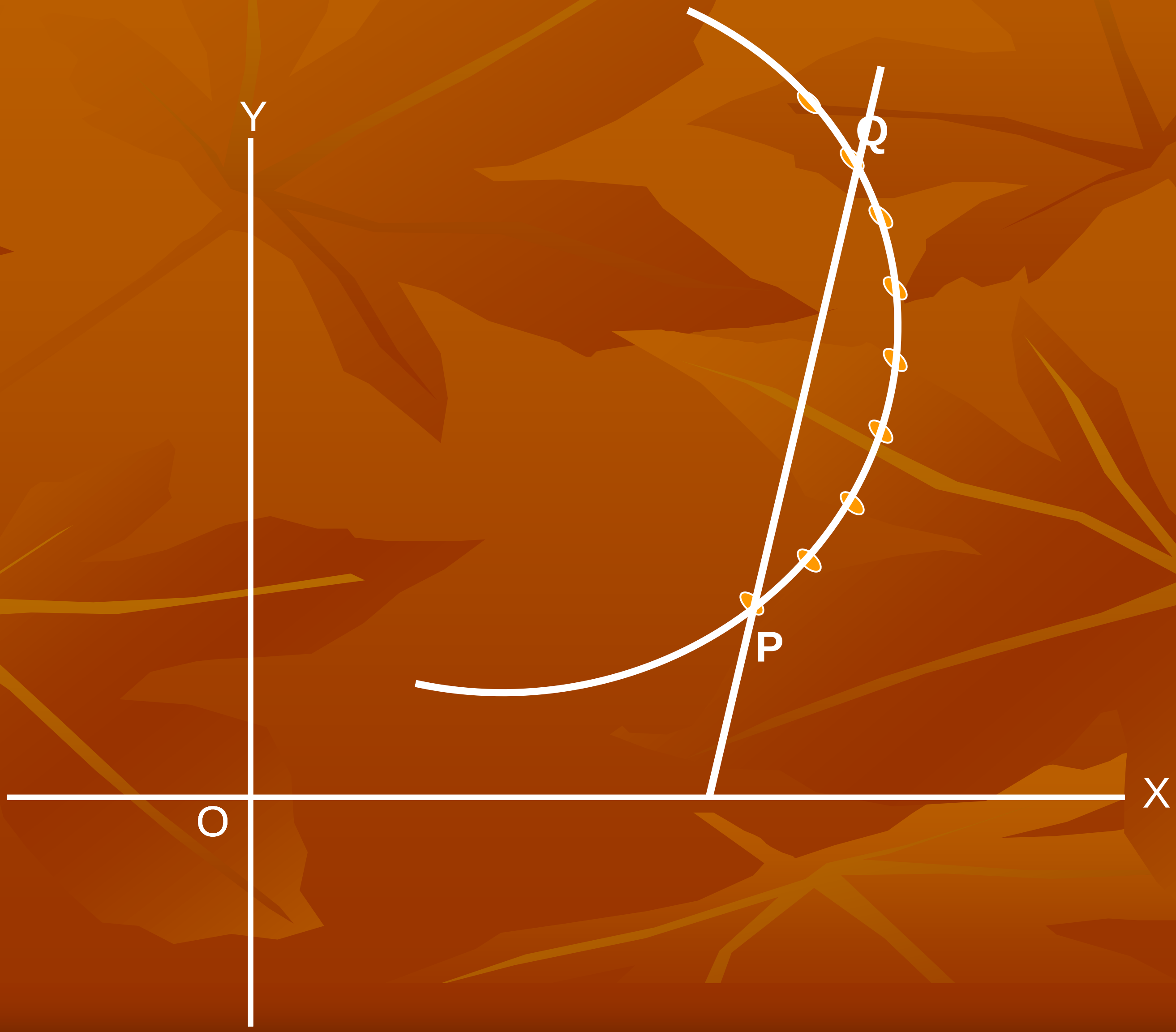

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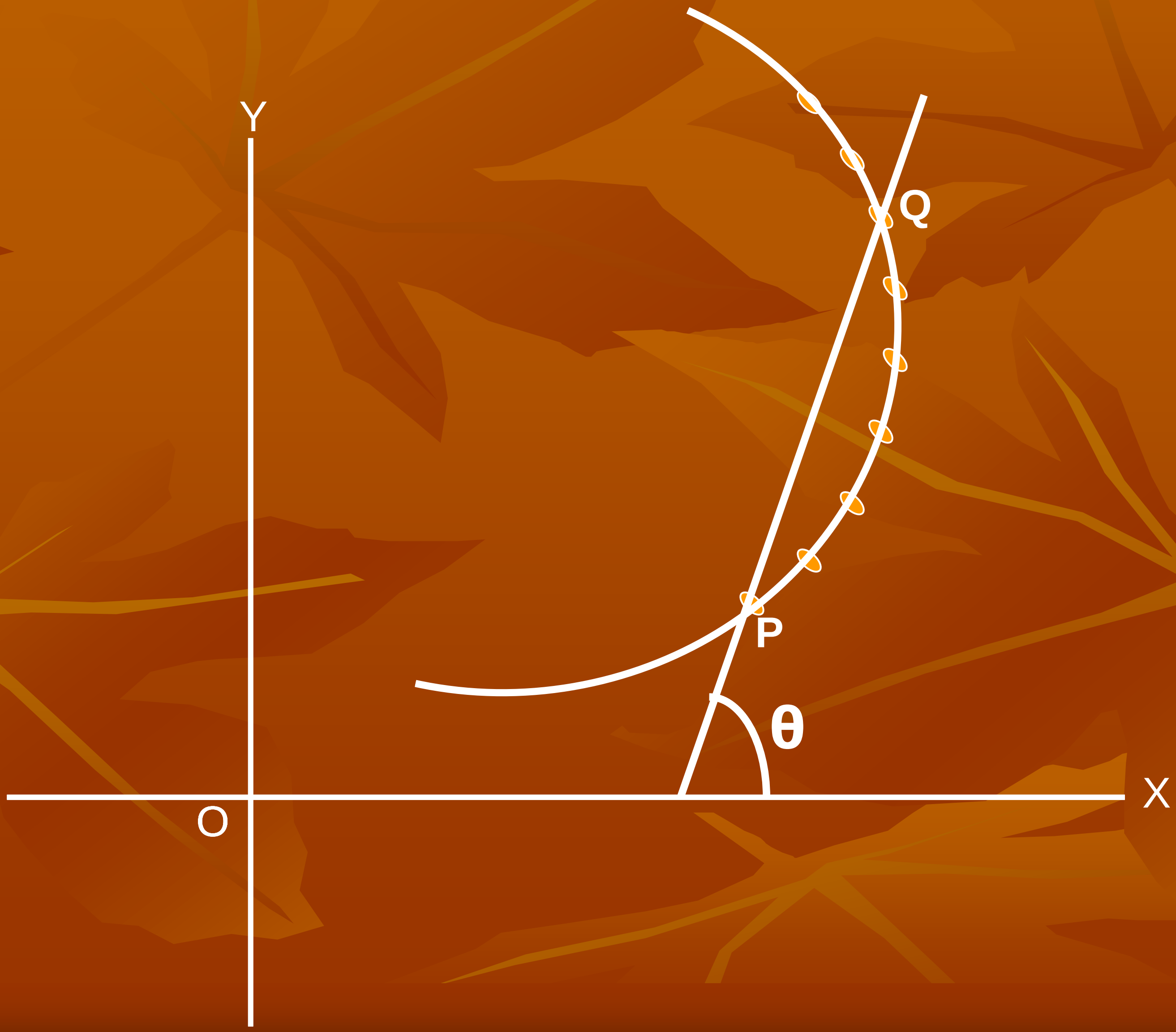

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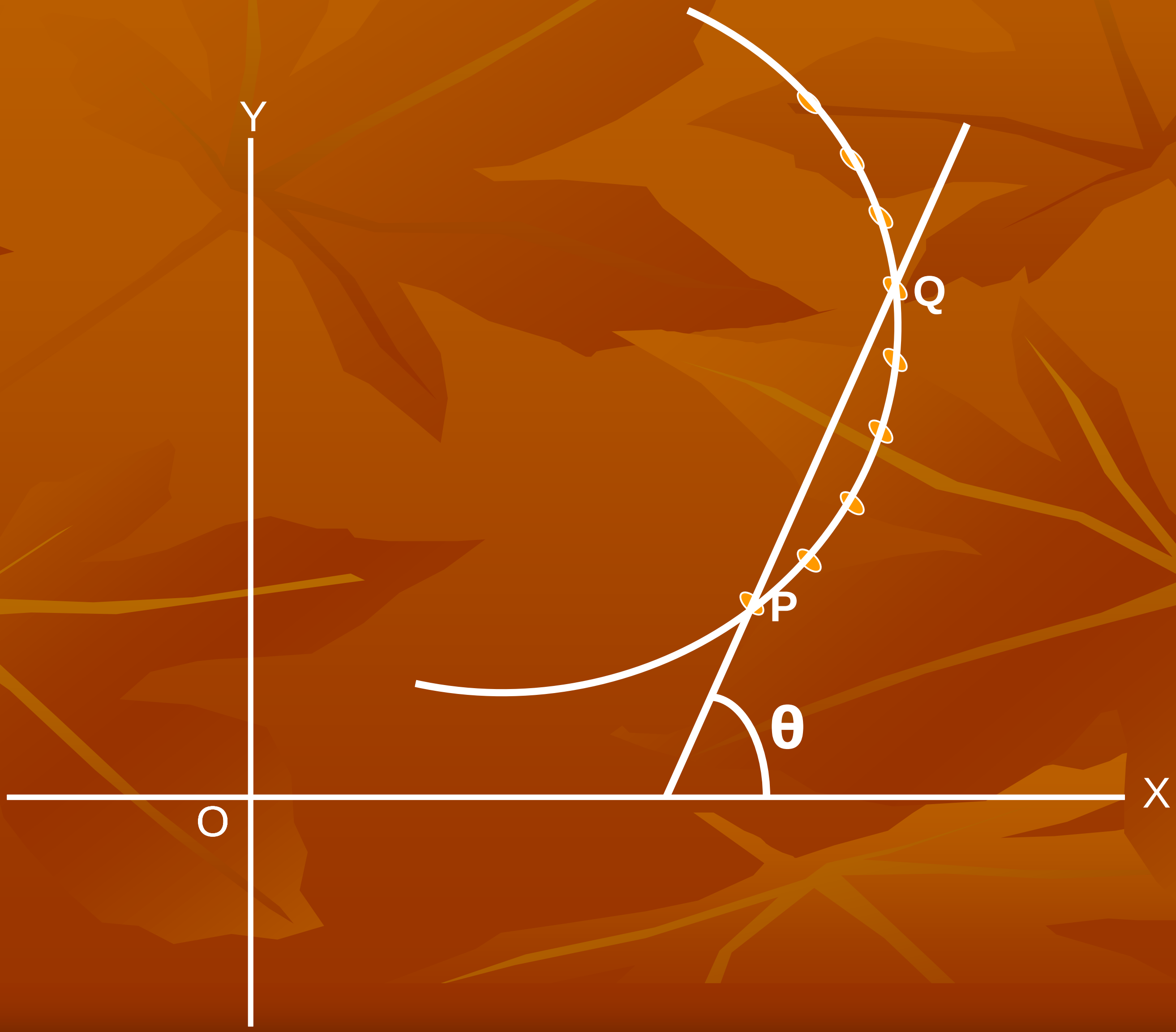

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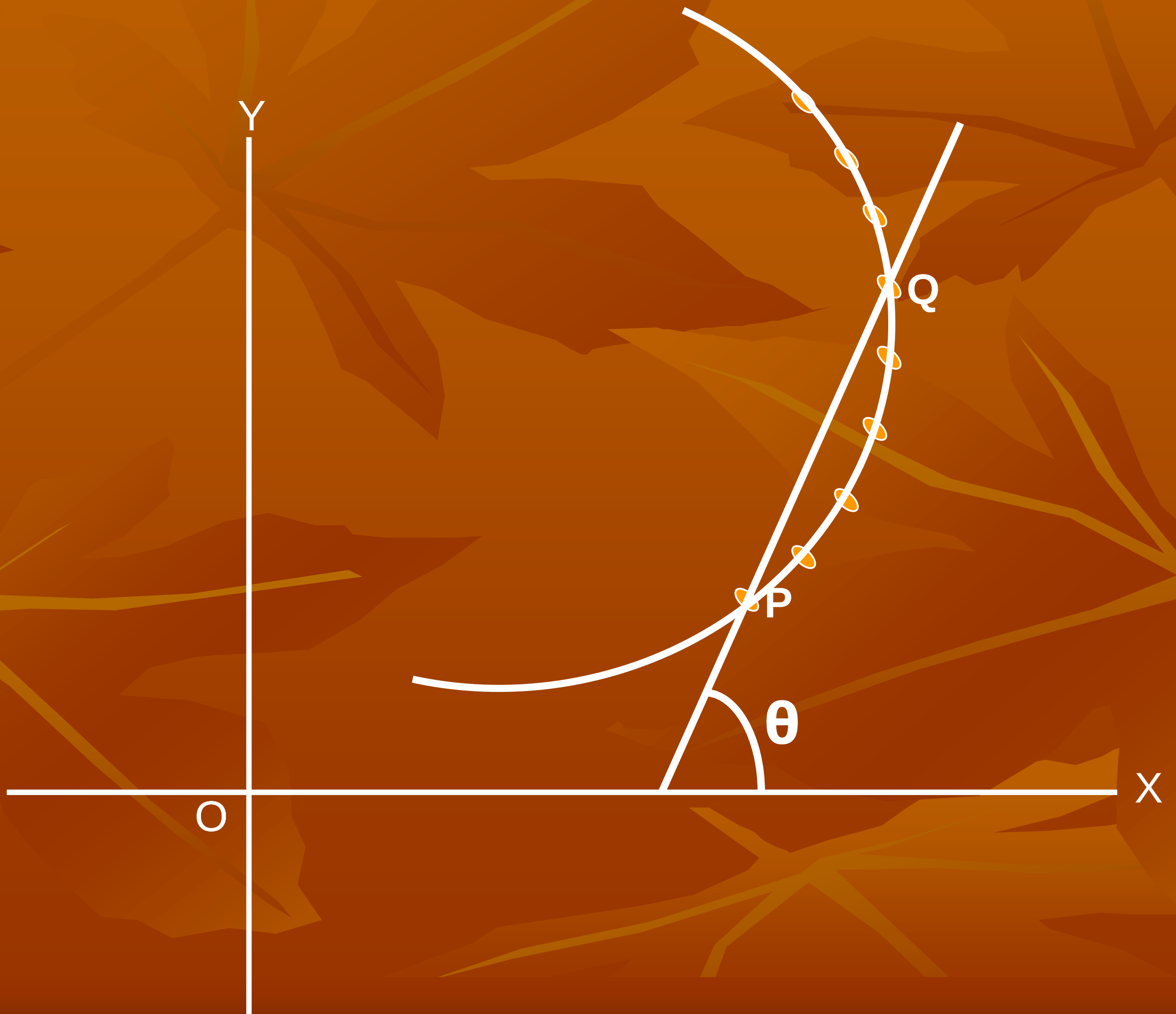

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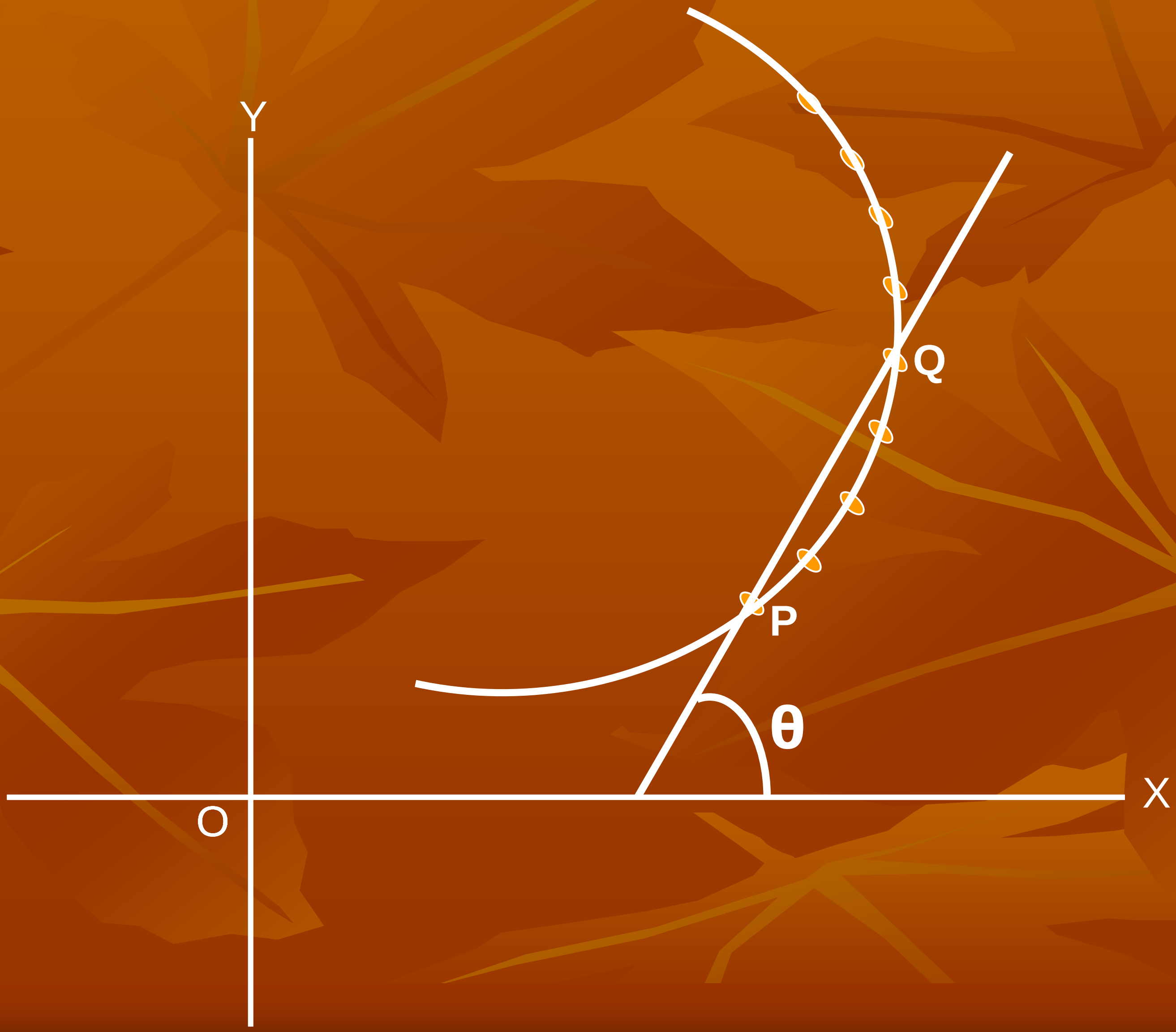
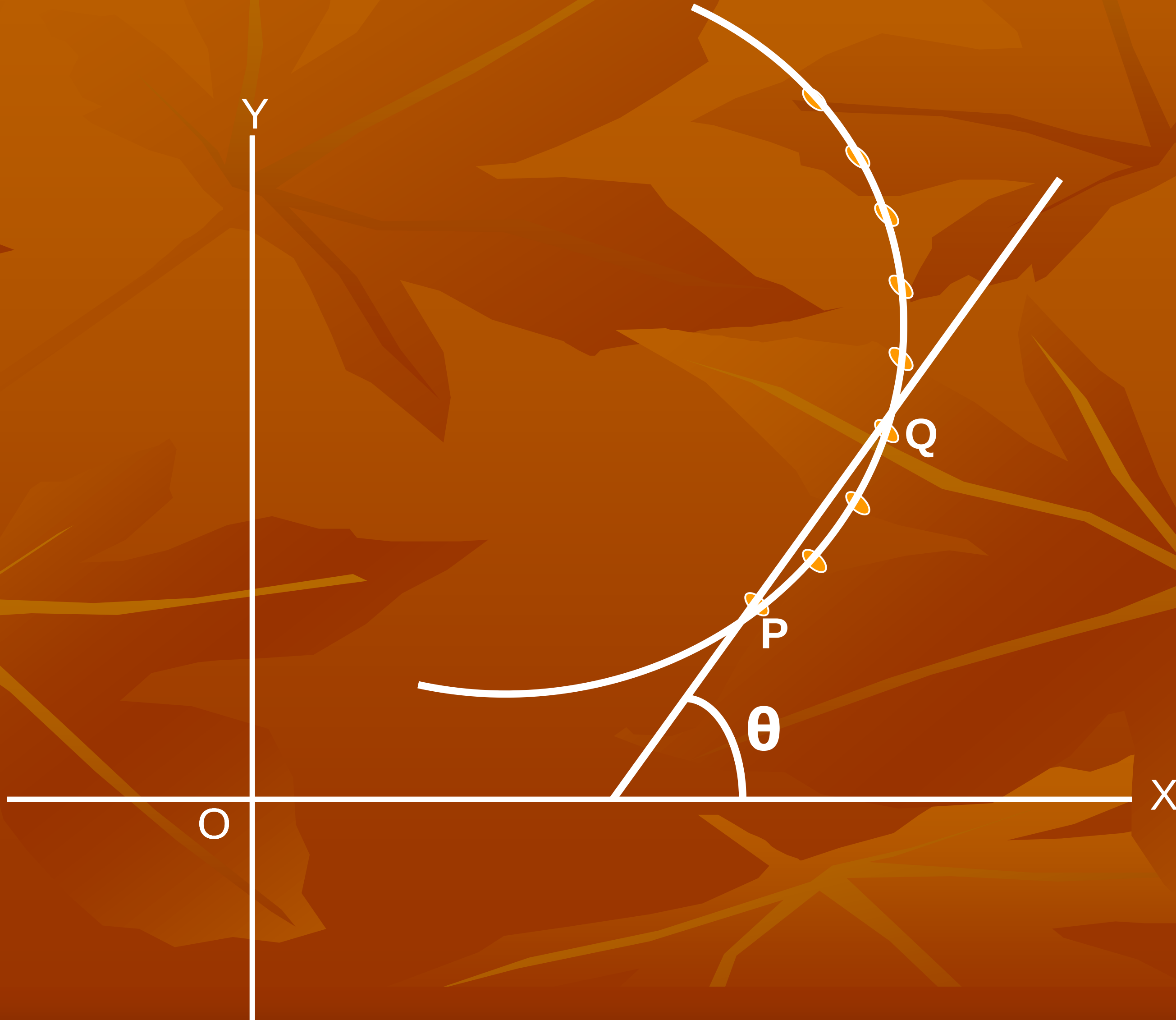


# Geometrical Representation Of Differentiation



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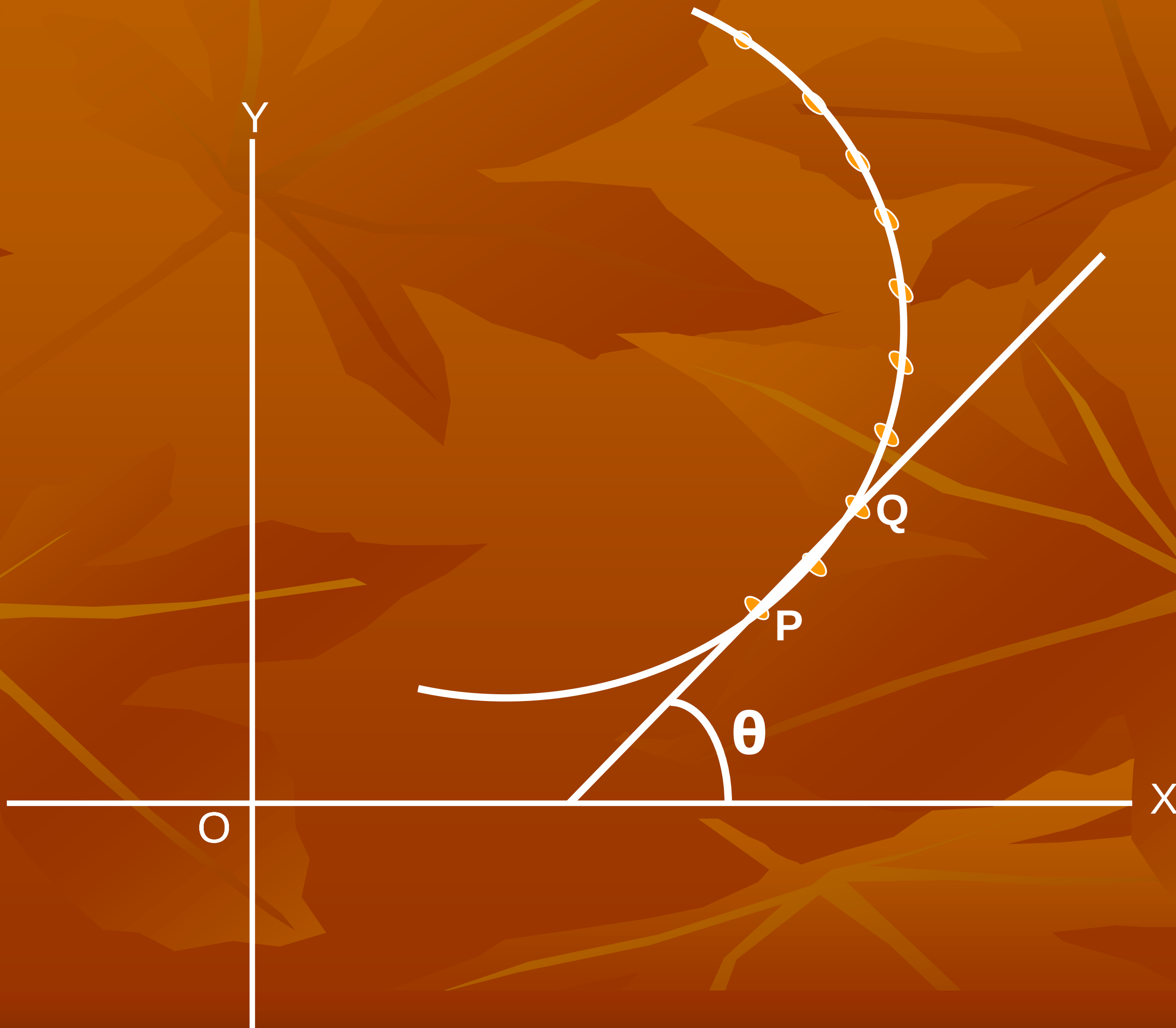

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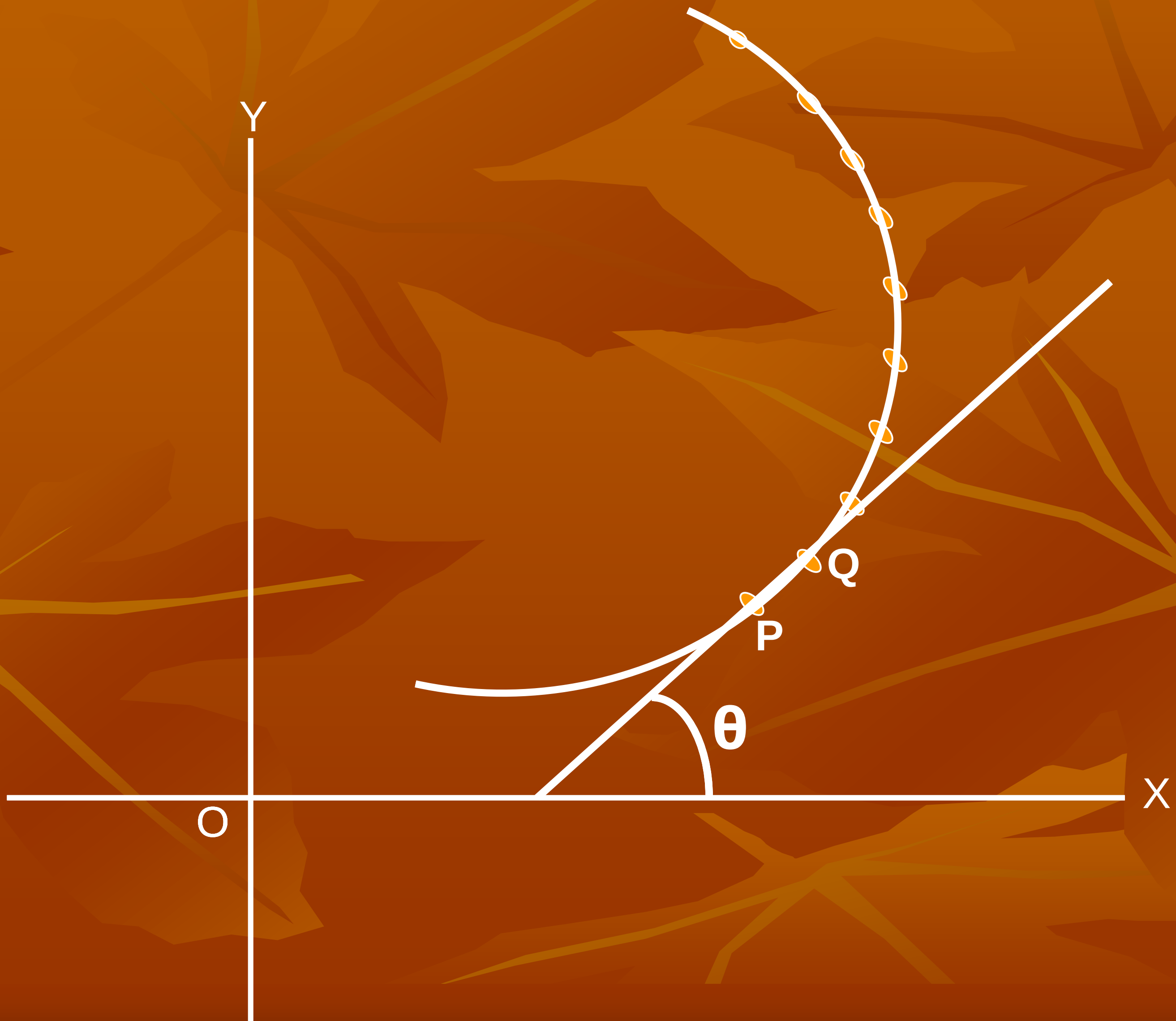
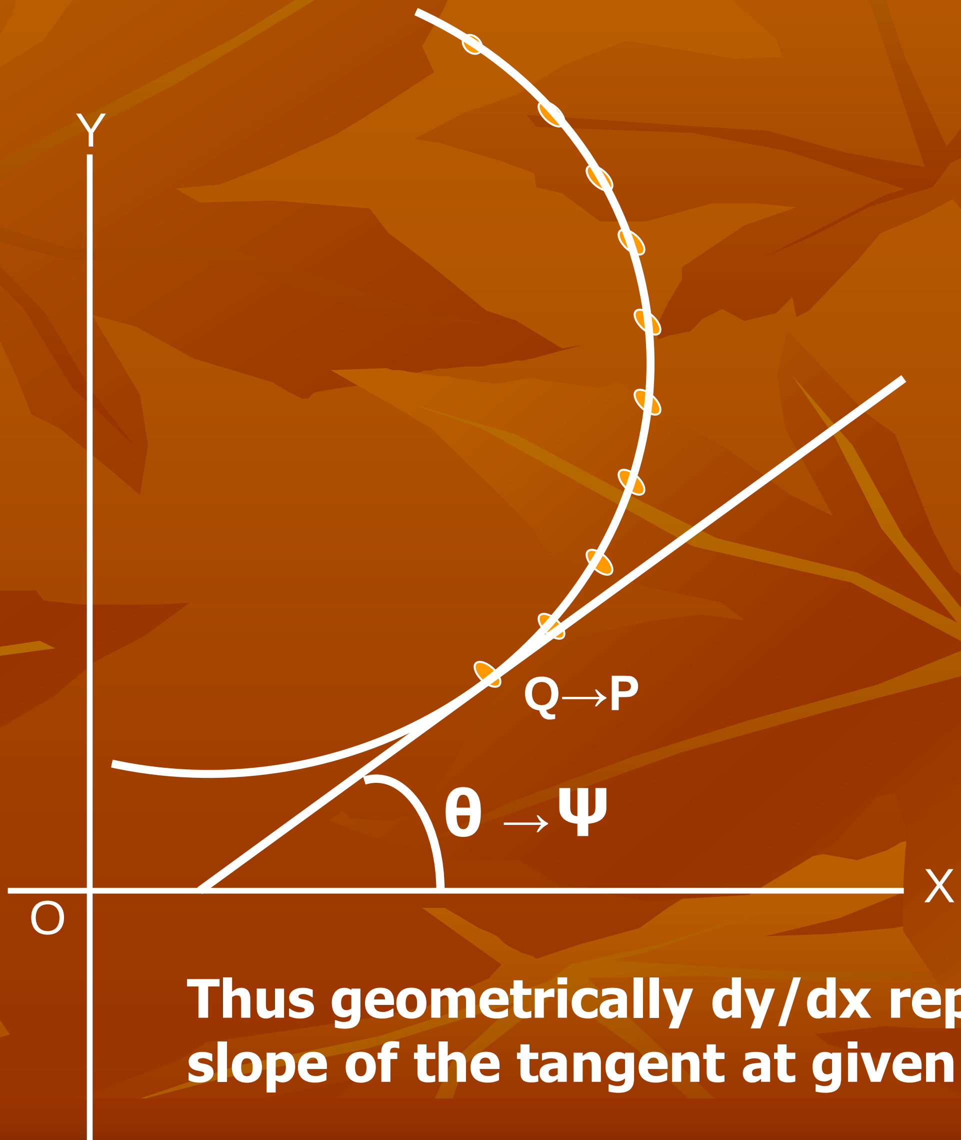
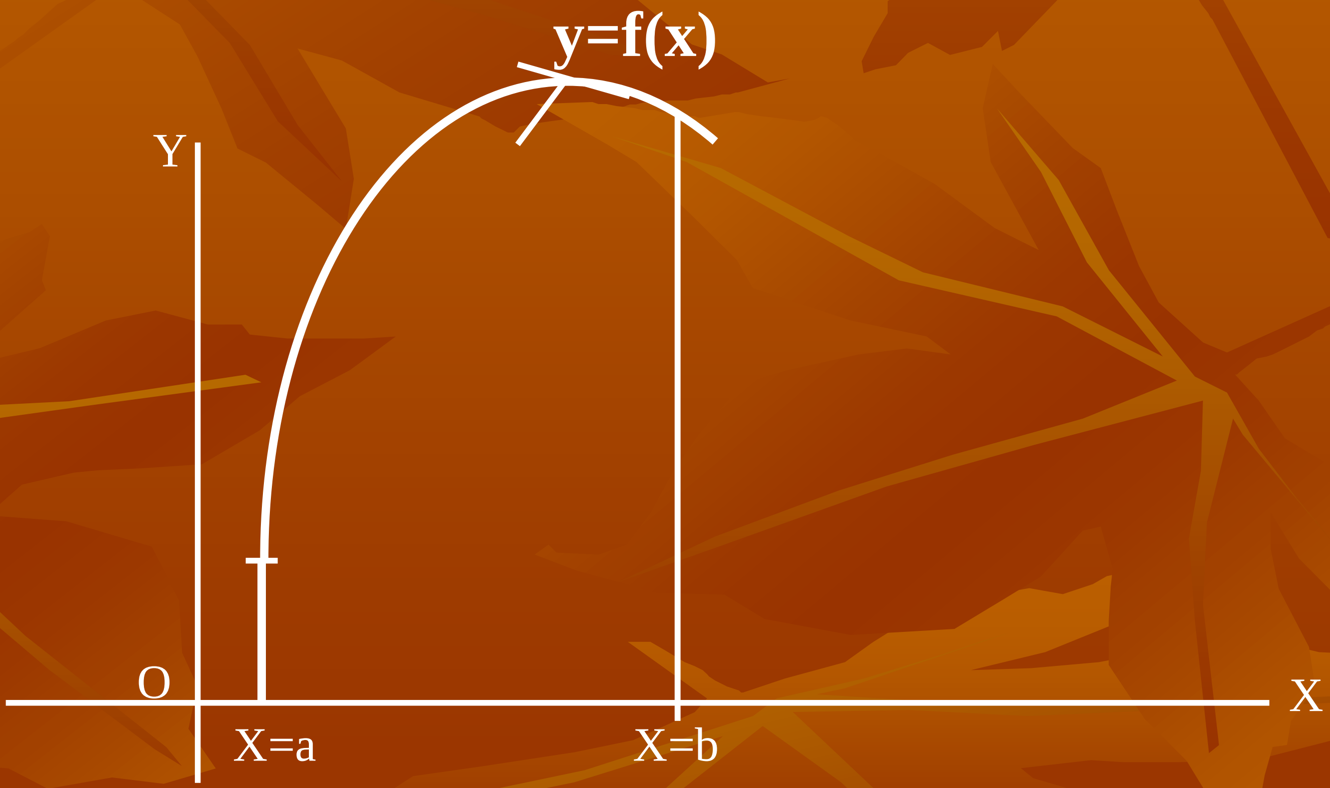


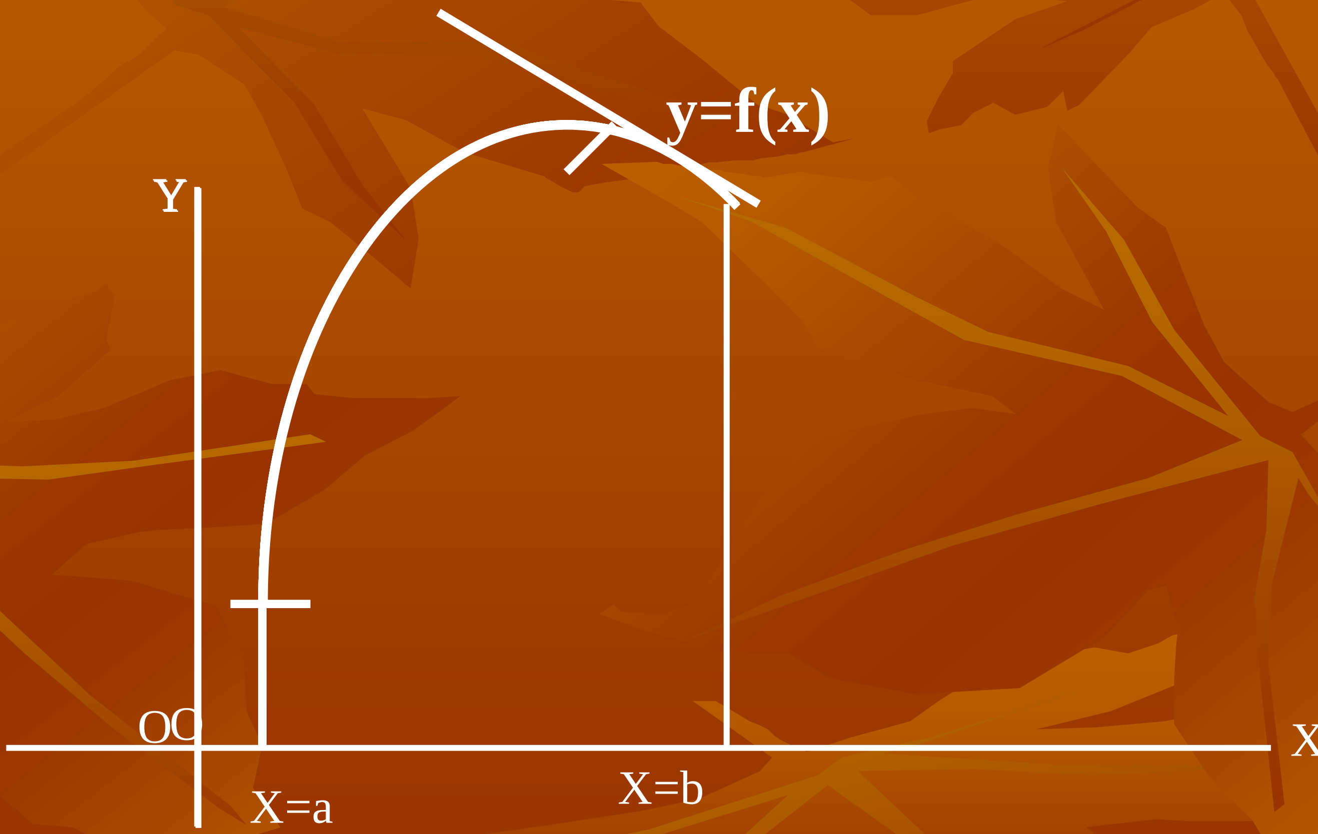
# Geometrical Representation Of Differentiation



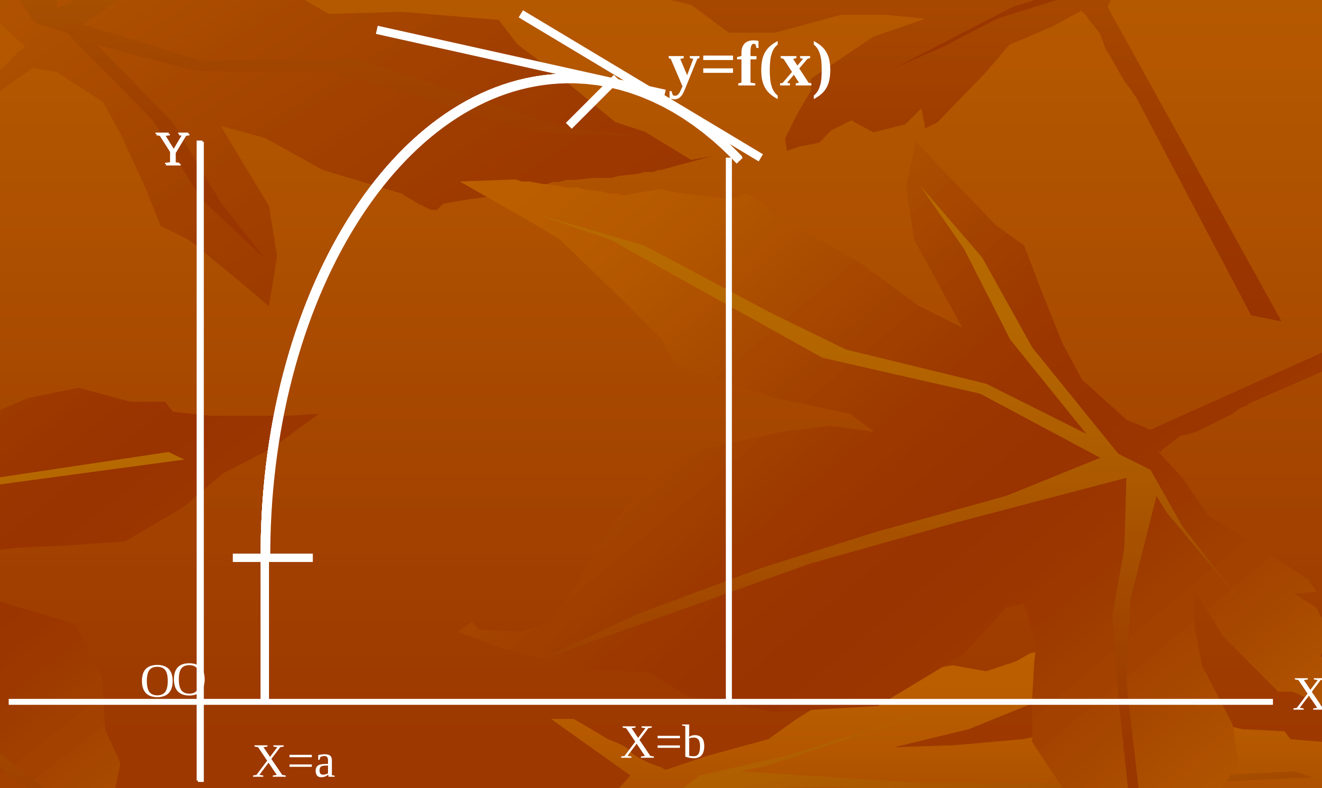

**Thus geometrically dy/dx represents slope of the tangent at given point**

# Geometrical meaning of the function y =f(x) is differentiable in the interval (a, b)





y=f(x)
Y
OO
X
X=a
X=b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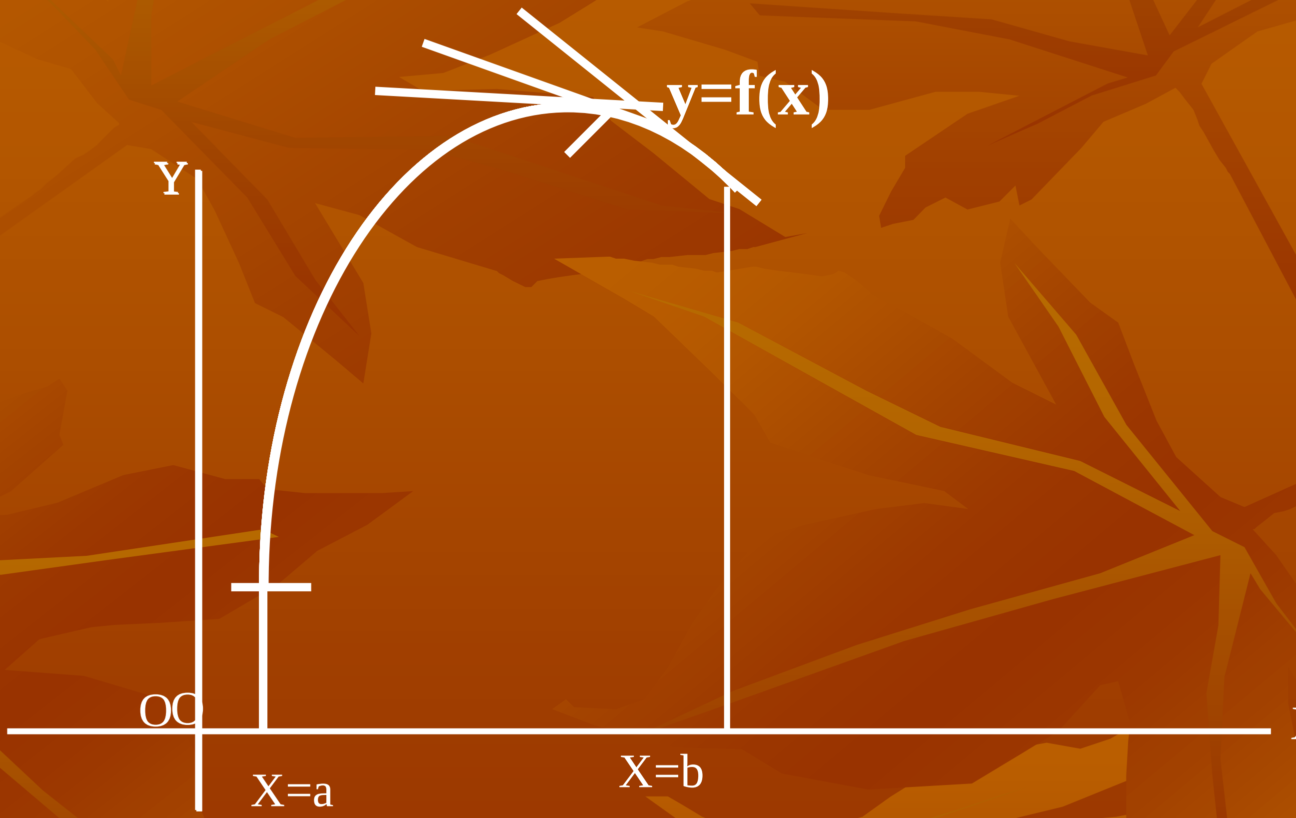
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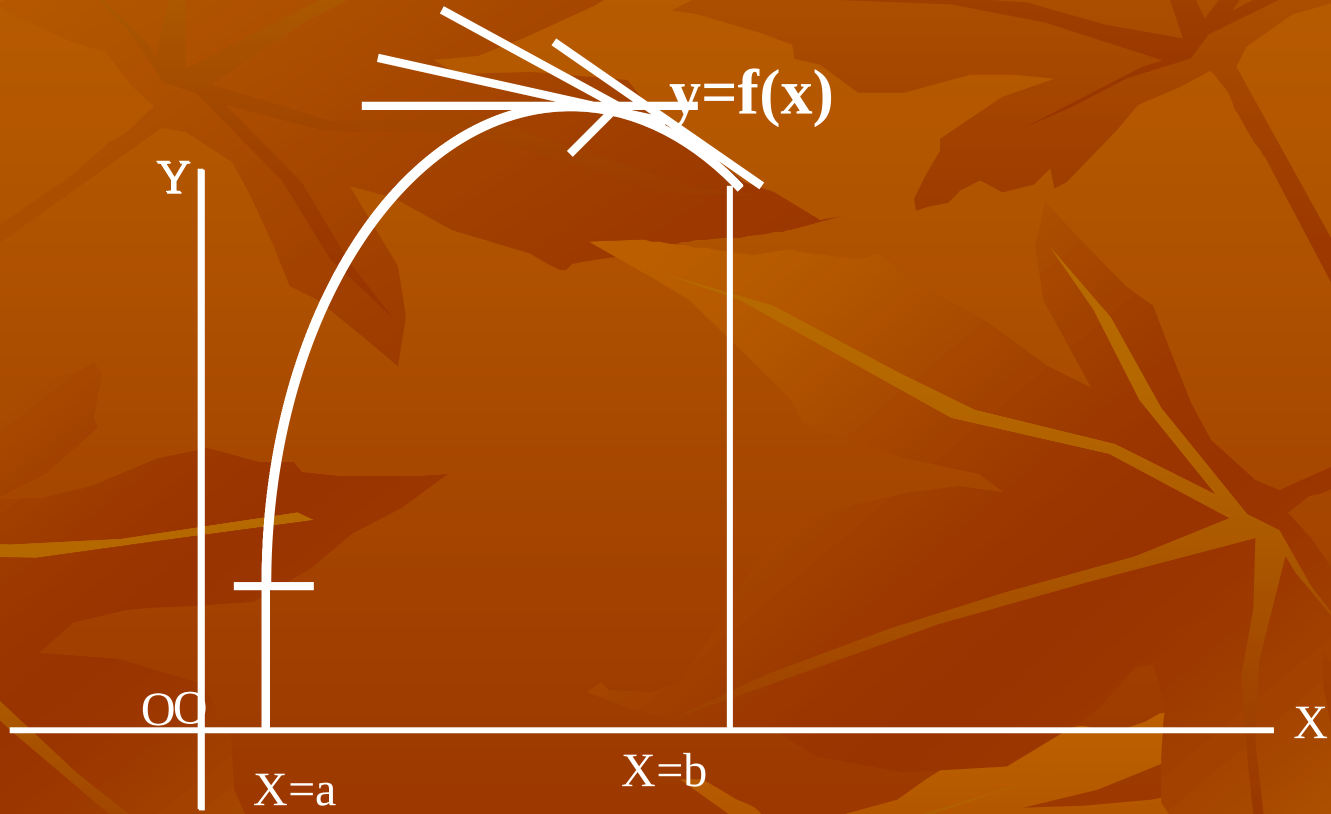
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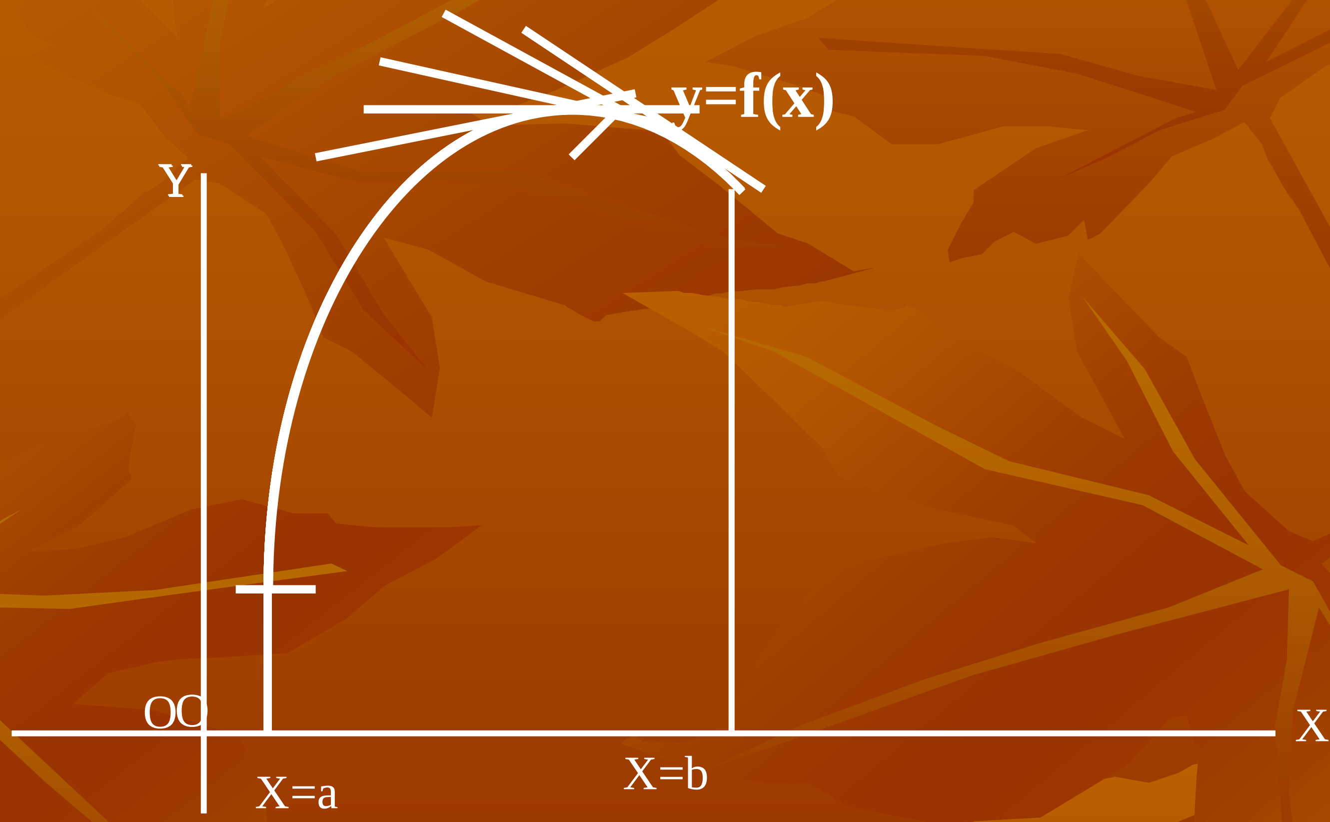

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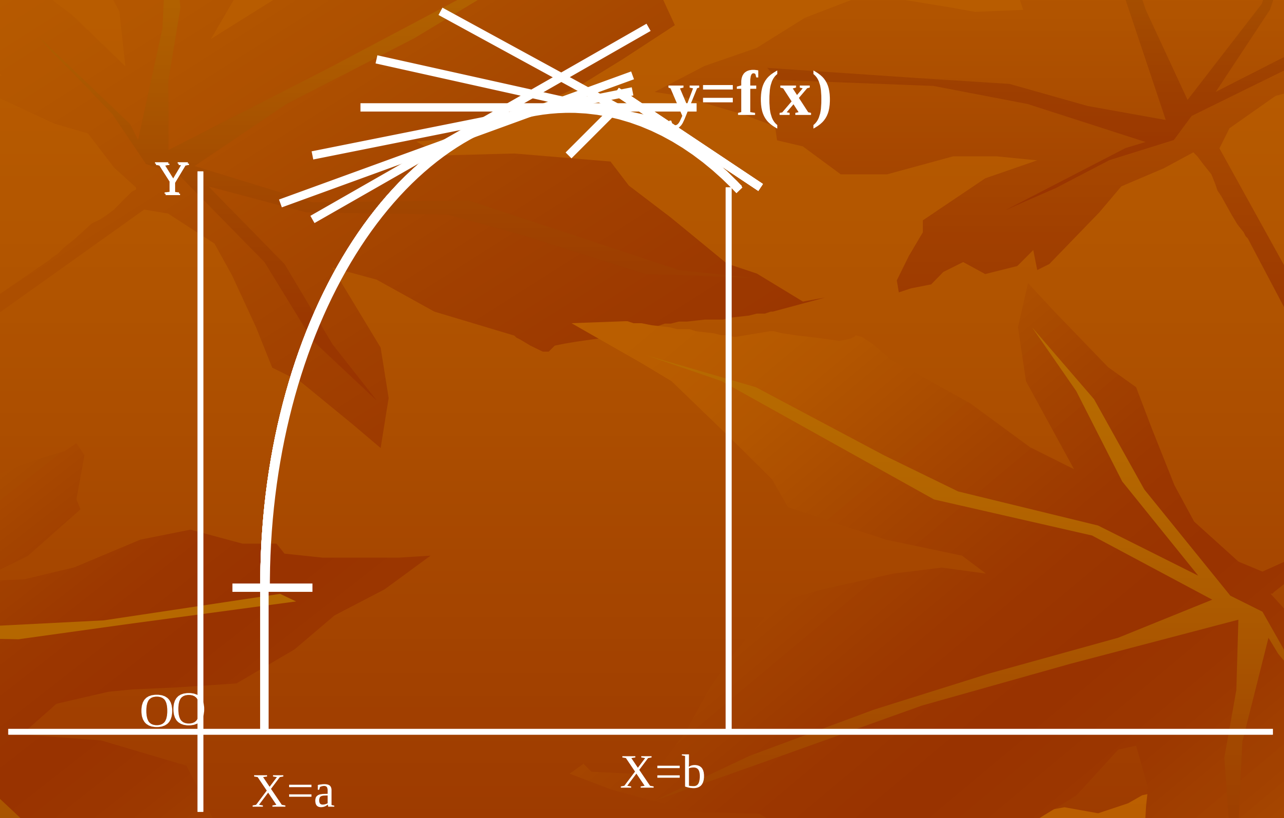

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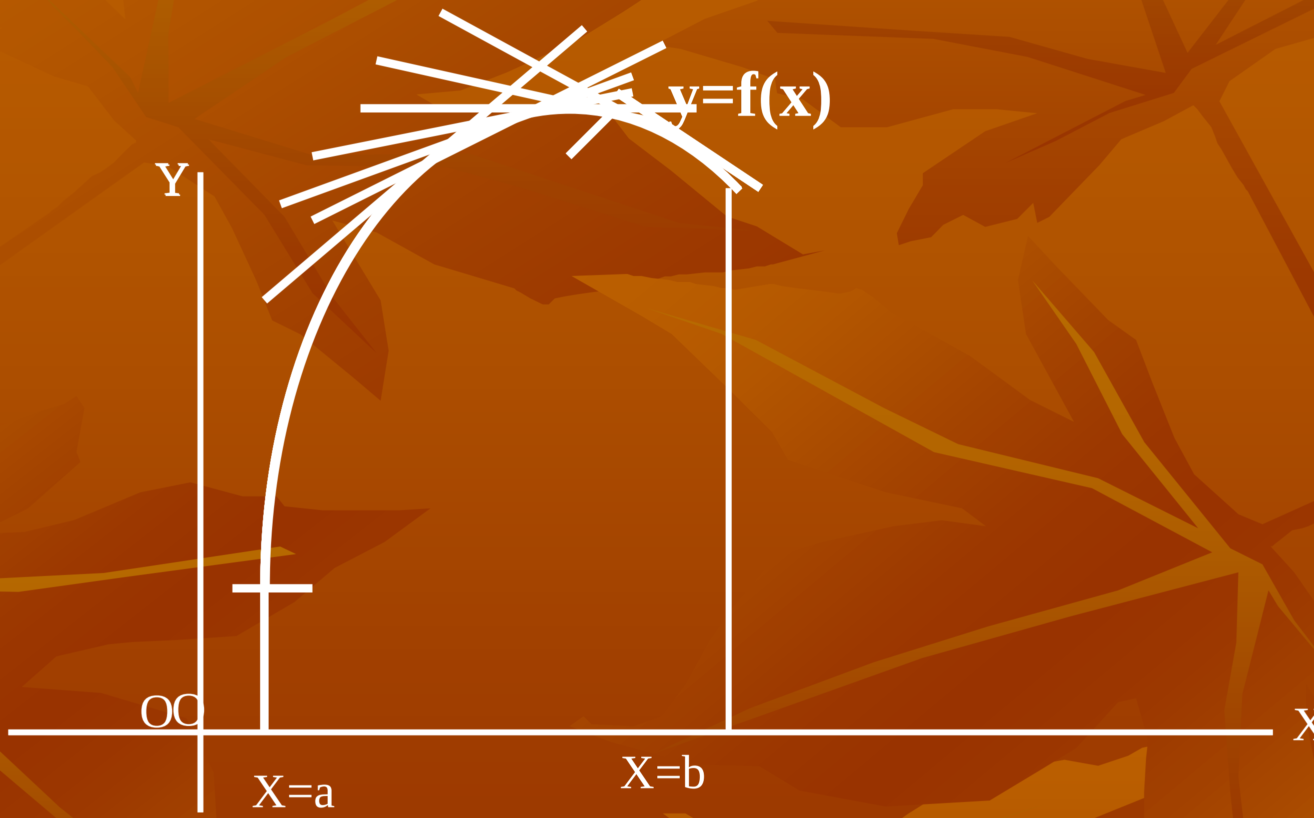
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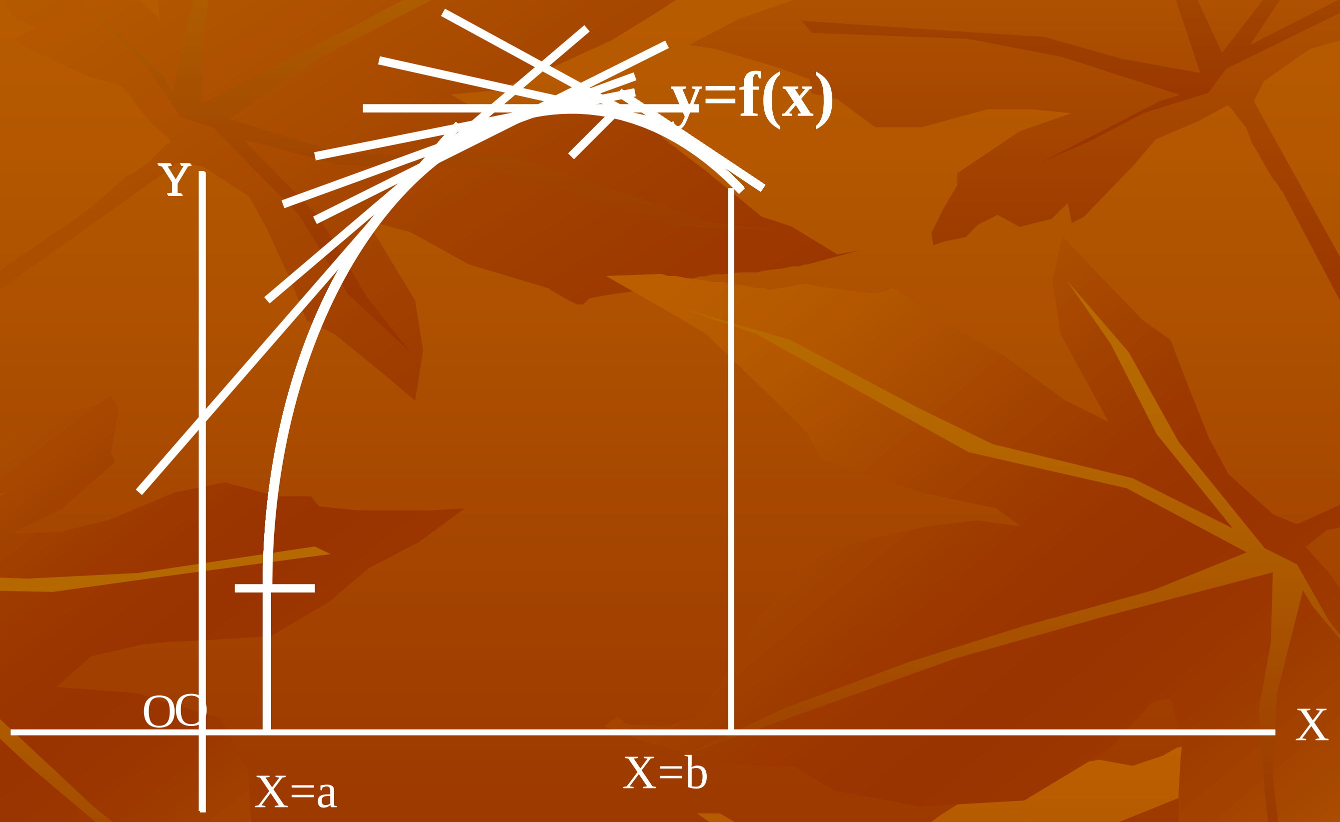

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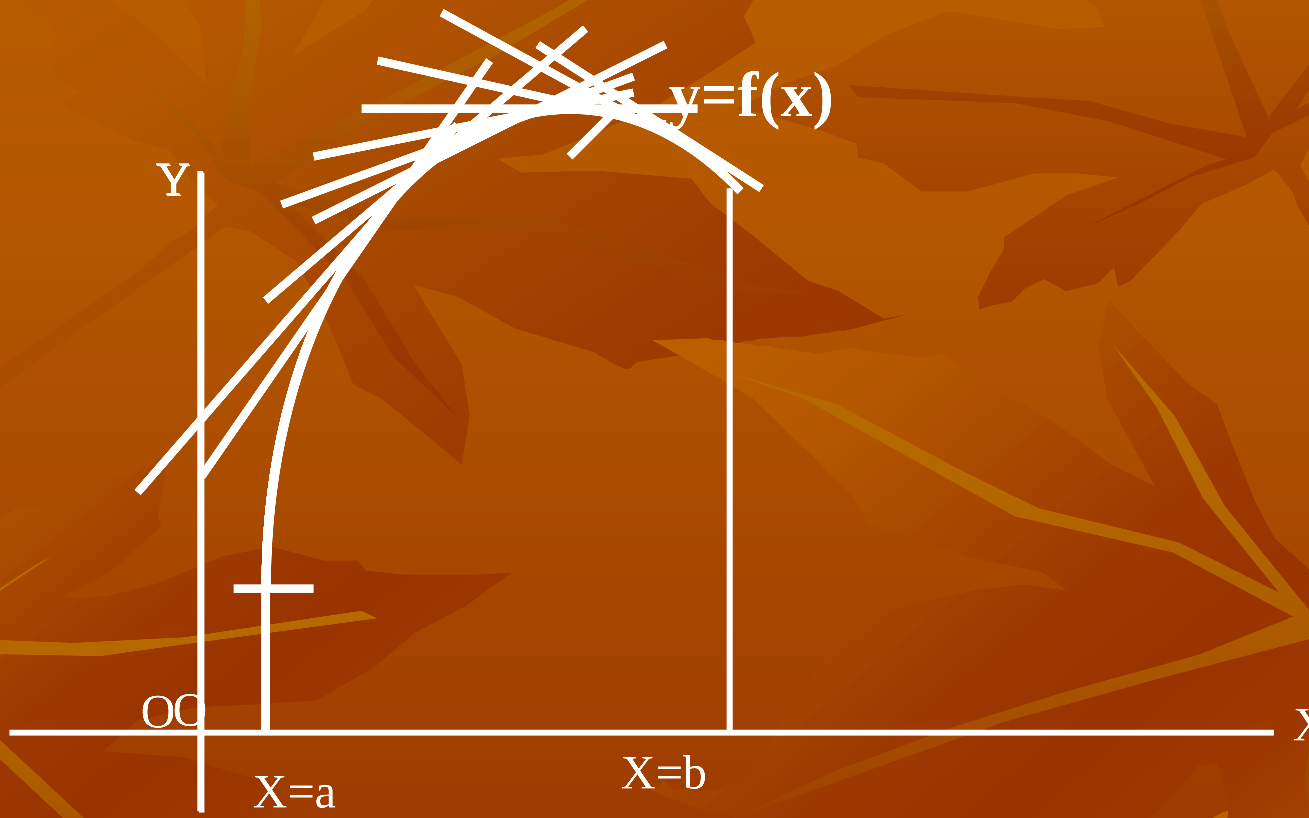
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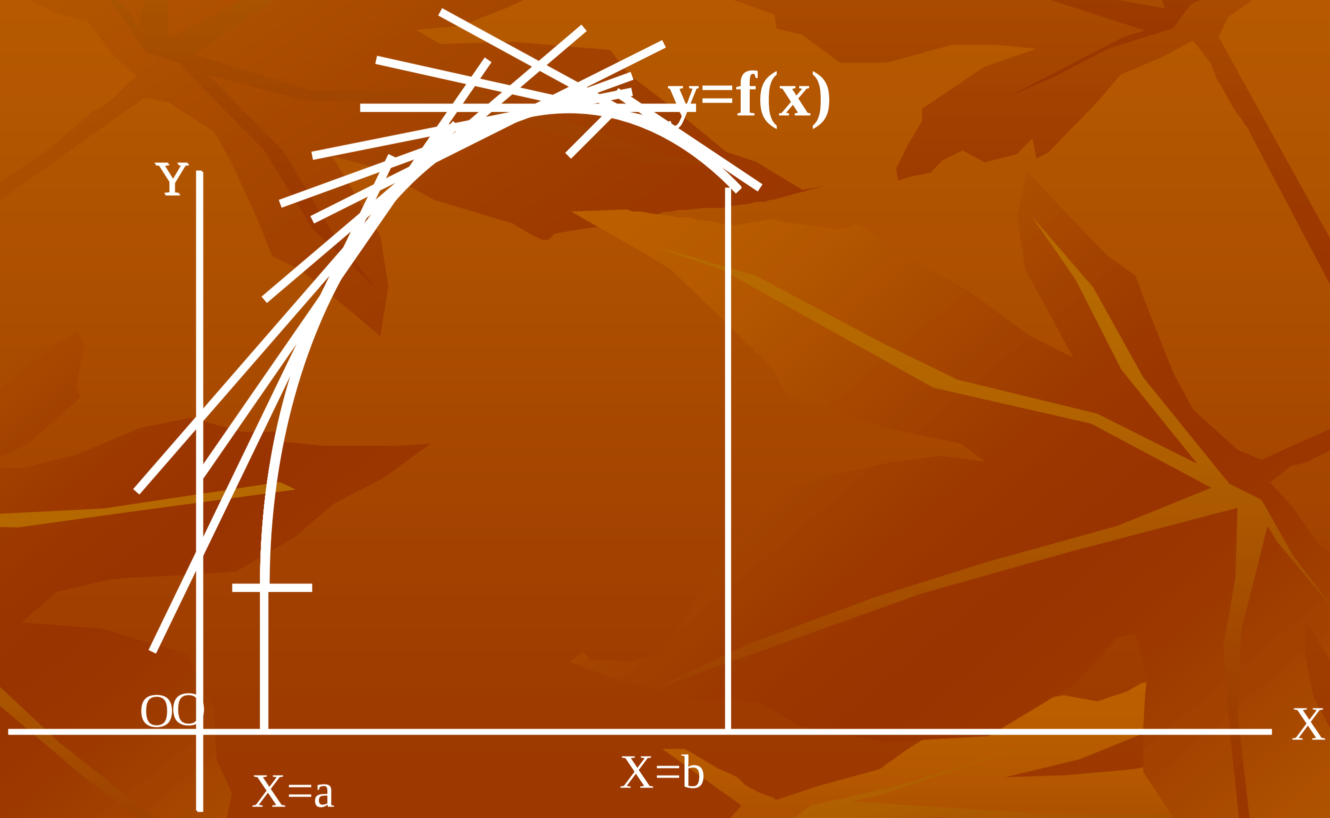

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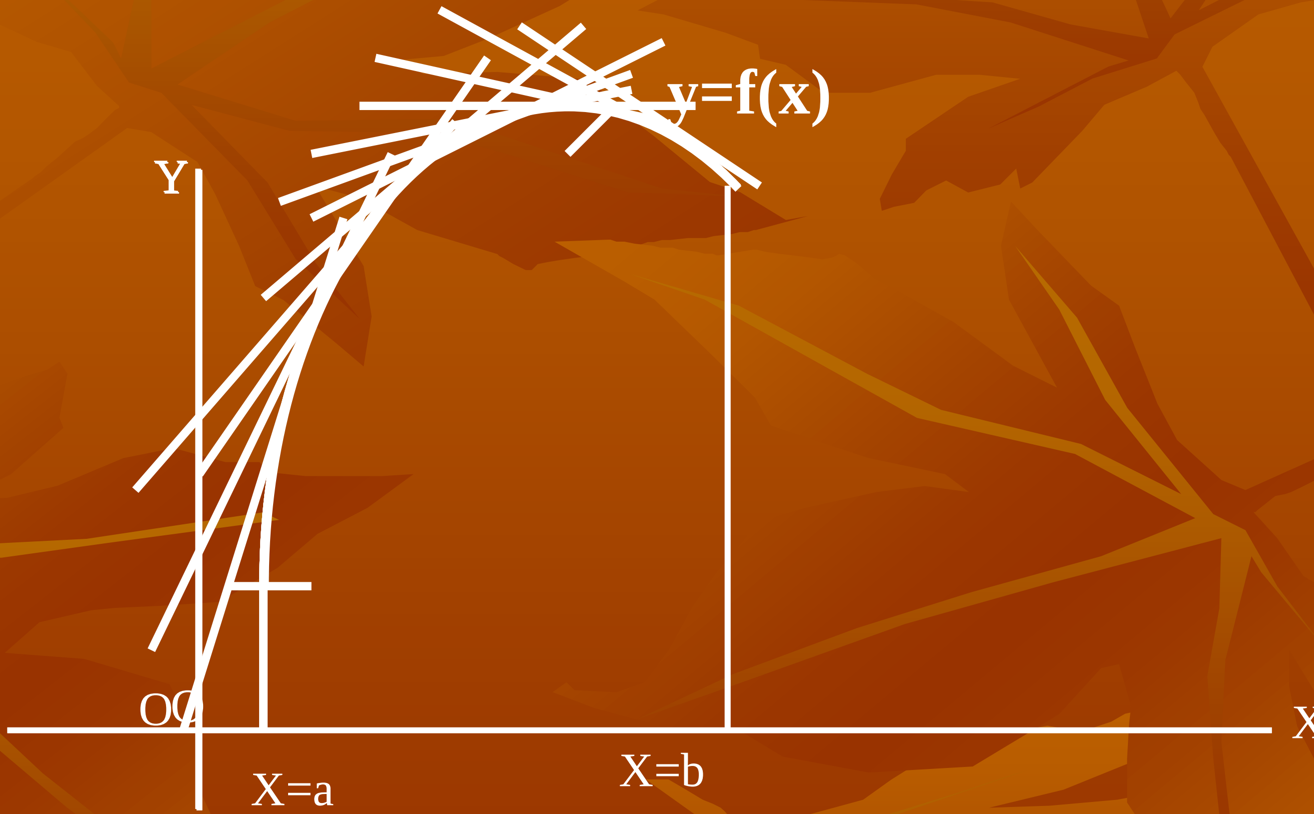
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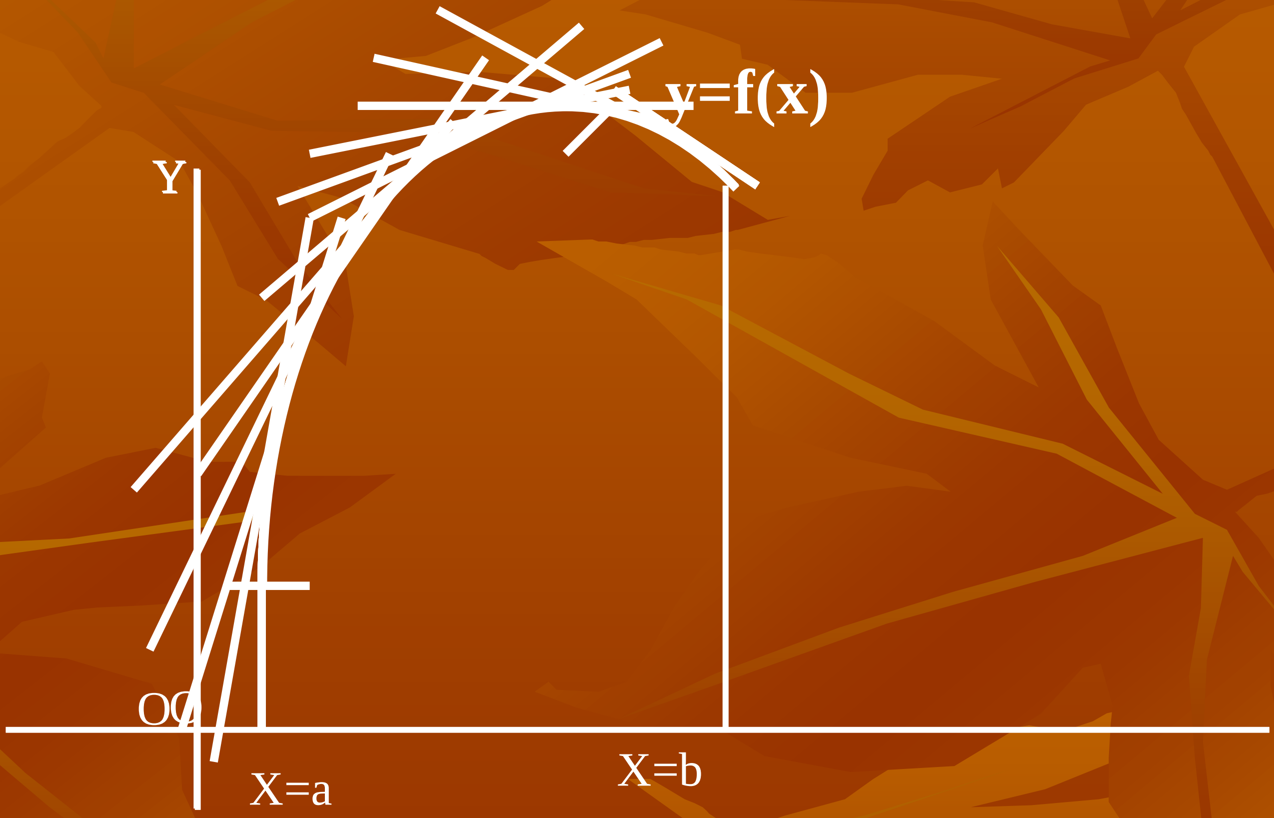
y=f(x)
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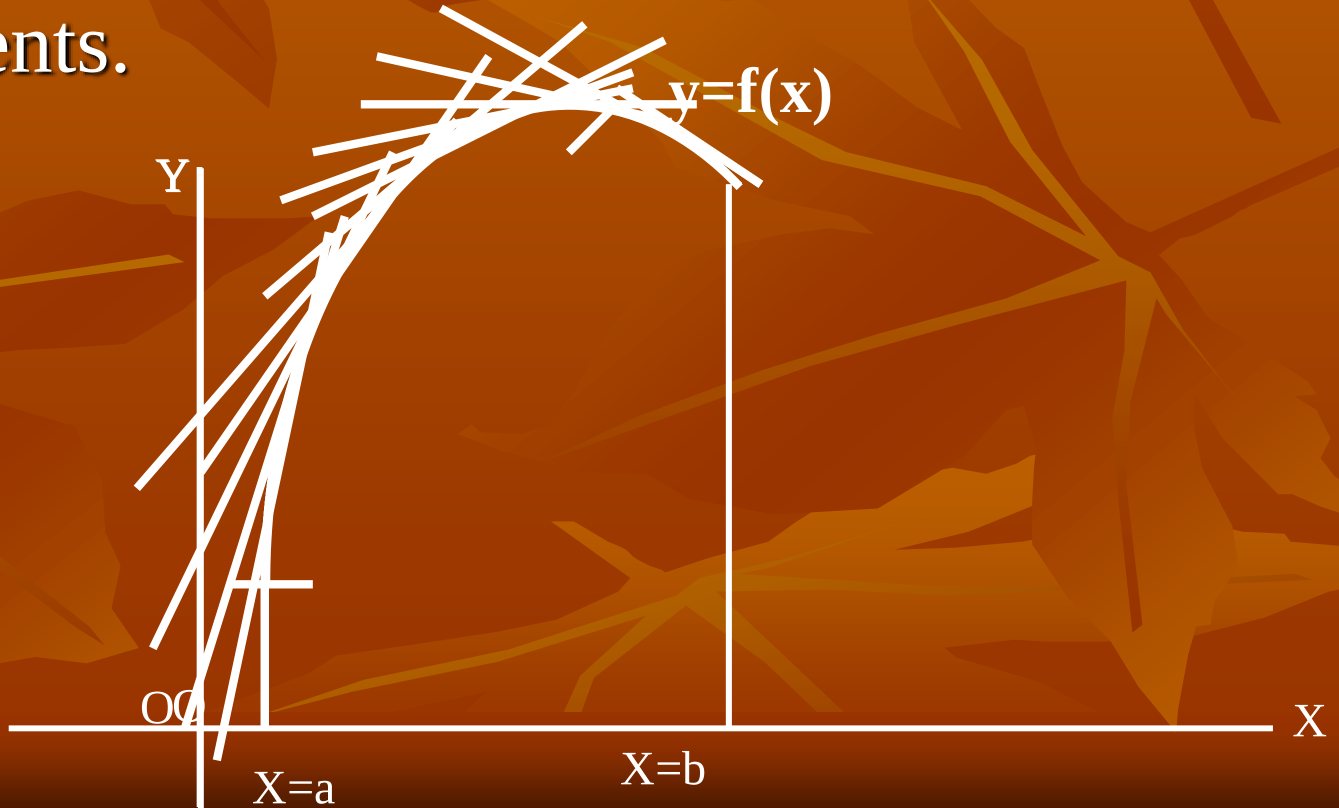
X
X=a
X=b

# Geometrical meaning of the function y =f(x) is differentiable in the interval (a, b)

- Now y = f(x) means it is a curve, and is differentiable in the interval (a,b) means at each point on the curve between x=a and x=b curve is having tangents.



# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: Calculus–II and 3–D Geometry

II online Class

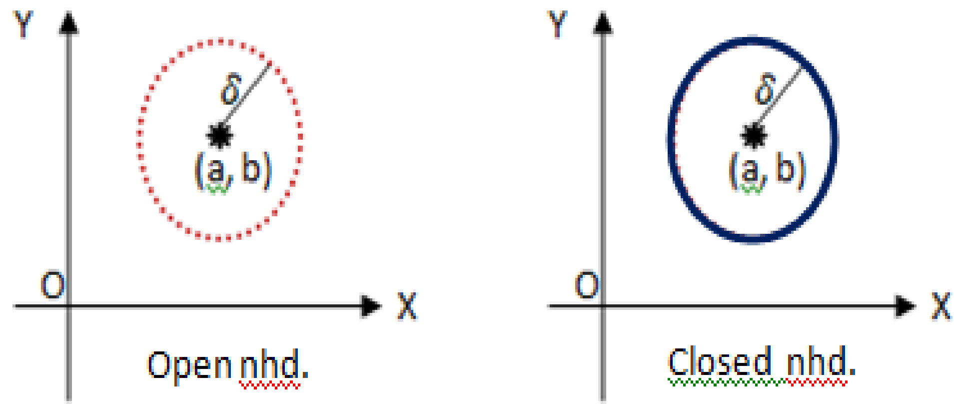
**Basic definitions and existence of limit for two variables**

By
Dr. M. M. Shankrikopp
HOD of Mathematics

Date: 4.5.2021

# Basic Definitions

**Neighbourhood of a point (a, b):**

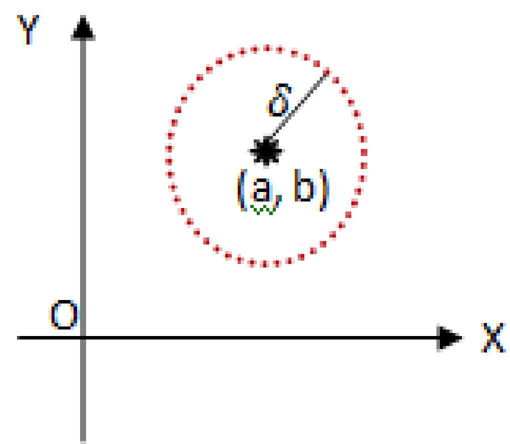



Let (a,b) be any point in the plane, then collection or set of all the points (x,y) which are very close to (a, b) is called neighbourhood of the point (a,b). If I take the small distance as $\delta$

Then collection of all points (x,y) whose distance from (a,b) is less than $\delta$ is called nhd. of (a,b) or open disk. i.e nothing but collection of all points inside the circle with centre at(a,b) and radius $\delta$.

And collection of all points points (x,y) whose distance from (a,b) is less than or equal to $\delta$ is called closed disk.

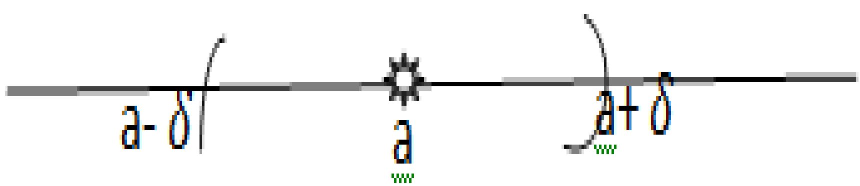
**For example:** If Niapni is the centre place i.e like (a, b) and $\delta$ be distance 5 kms. Then set of all villages which are within 5 kms is called nhd. of Nipani i.e they are neighbours of Nipani. i.e Lakanapur, Yamagarni, Chikli, Stavanidhi etc. are in nhd. of Nipani where as Galataga is not.



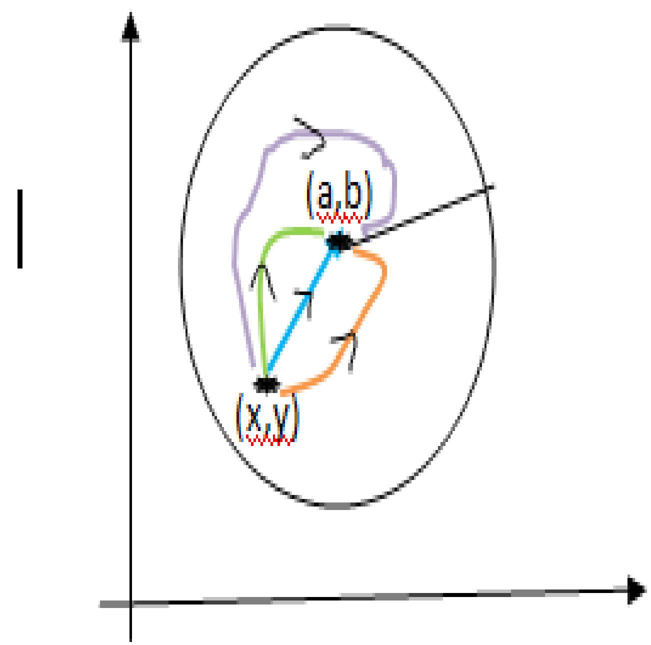
# Limit of function of two variables

**Definition:** A function u= f(x,y) is said to have the limit l as (x,y) tends to (a,b) if for given $\varepsilon > 0$, there exists $\delta > 0$ such that $|f(x,y) - l| < \varepsilon$, for all points (x,y) lies in the nhd. of (a, b) along any path we choose.

Recall that incase of real function f(x), $\lim_{x \to a} f(x) = l$, if lest hand limit = right hand limit, and here path from x to a is always along x axis.



But in case of two variables point (x,y) moves towards (a,b) along any path as it is plane




In this fig. point (x, y) moves towards (a, b) ( which is in the nhd. of (a,b) only) along so many paths blue, brown, green or violet but all along these paths limit l should be same then only will say limit exists, otherwise no. i.e along one path limit is $l_1$ , another path if it is $l_2$ and so on then we say limit does exists.

If all are equal then we say that limit exists.

You come to know in examples.

And is denoted by $\lim_{(x,y)\to(a,b)} f(x,y) = l,$

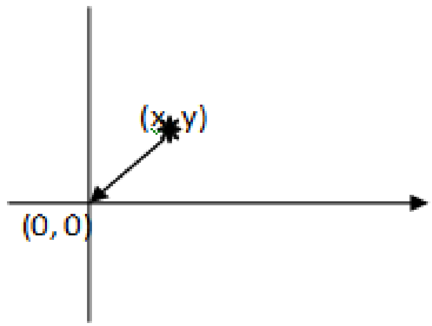
Meaning $\lim\limits_{(x,y)\to(a,b)}$ means x→ $a$ and y→b

# EXAMPLES

**1. Show that for a function f(x, y) = $\frac{xy}{x^2+y^2}$ $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.**

Soln.: Now **f(x, y) = $\frac{xy}{x^2+y^2}$ , actually (0, 0) is not defined.**

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.




Therefore $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

Along y =mx means, substitute y =mx keep x as it is, and convert for single variable x.

i.e $\lim_{x\to 0} \frac{x(mx)}{x^2+(mx)^2} = \lim_{x\to 0} \frac{mx^2}{x^2+m^2x^2} = \lim_{x\to 0} \frac{mx^2}{x^2\,(1+m^2)} = \lim_{x\to 0} \frac{m}{(1+m^2)}$

$= \frac{m}{(1+m^2)}$ which depends on m,

hence limit is not unique.

i.e for different values of m we get different limits and hence limit does not exist.

**Note: For all the examples, first path is straight is st.line y =mx, along this path if limit is in terms of m, we say limit does not exist, suppose it is zero or constant we choose another path.**

**2. Evaluate the $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6}$ if it exists.**

Soln.: Now $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6}$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.

Then $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6} = \lim_{x\to 0} \frac{xm^3x^3}{x^4+m^6x^6} = \lim_{x\to 0} \frac{xm^3x^3}{x^4+m^6x^6}$

$= \lim_{x\to 0} \frac{m^3x^4}{x^4(1+m^6x^2)} = m^3$, depends on m and so limit depends on m, hence limit is not unique.

$\therefore \lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6}$ does not exist.

## 3. Show that $\lim_{(x,y)\to)(0,0)} \frac{xy^2}{x^2+y^2} = 0$

**Soln.:** Now $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2}$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.

Then $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2} = \lim_{x\to 0} \frac{xm^2x^2}{x^2+m^2x^2} = \lim_{x\to 0} \frac{m^2x^3}{x^2(1+m^2)}$

$= \lim_{x\to 0} \frac{m^2x}{(1+m^2)} = 0$ not depends on m , i.e for any value for m limit is 0 only.

And along any other path also it is 0

$\therefore \lim_{(x,y)\to)(0,0)} \frac{xy^2}{x^2+y^2} = 0$

**4. Evaluate** $\lim_{(x,y)\to(0,0)} \frac{x-y^2}{x^2+y^2}$ **, if exists.**

**Soln.:** Now $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{x-y^2}{x^2+y^2}$

Let us choose the path along the curve $y^2 = mx$, i.e point (x,y) moves towards (0,0) along the curve $y^2 = mx$.

Then $\lim_{(x,y)\to(0,0)} \frac{x-y^2}{x^2+y^2} = \lim_{x\to 0} \frac{x-mx}{x^2+mx} = \lim_{x\to 0} \frac{x(1-m)}{x(x+m)}$

$= \lim_{x\to 0} \frac{1-m}{x+m} = \frac{1-m}{m}$ , depends on m, does not exist.

I

## 5. Evaluate $\lim_{(x,y)\to)(0,0)} \frac{xy^3}{x^2+y^6}$ , if exists.

**Soln.:** Now $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,( along the line y =mx.

Then $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6} = \lim_{x\to 0} \frac{xm^3x^3}{x^2+m^6x^6} = \lim_{x\to 0} \frac{xm^3x^3}{x^2+m^6x^6}$

$$= \lim_{x\to 0} \frac{m^3x^4}{x^2(1+m^6x^4)} = 0$$

$\therefore \lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^6}$ exists along the line y = mx

Let us choose another curve $y^3$ = mx, we get

$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6} = \lim_{x\to 0} \frac{xmx}{x^2+m^2x^2} = \lim_{x\to 0} \frac{mx^2}{x^2(1+m^2)}$

$$= \lim_{x\to 0} \frac{m}{(1+m^2)}$$

$= \frac{m}{(1+m^2)}$, depends on m, so limit does not exist.

## HOME WORK EXAMPLES

I

6. Evaluate $\lim_{(x,y)\to)(0,0)} \dfrac{x^2y^2}{x^4+y^4}$ , if exists.

7. Evaluate $\lim_{(x,y)\to)(0,0)} \dfrac{x^4y^4}{(x^2+y^4)^3}$ , if exists.

8. Evaluate $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y}{x^2+y^2}$ , if exists.

. A student scored 32% marks in science subjects out of 300. How much should he score in language papers out of 200 if he is to get overall 46% marks ?

- 72%
- 67%
- 66%
- 60%
- None of these

THANK YOU

BE SAFE AT HOME

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: Calculus–II and 3–D Geometry

III online Class

**Continuity of function of two variables**

By

Dr. M. M. Shankrikopp

HOD of Mathematics

Date: 5.5.2021

# CONTINUITY OF FUNCTION OF TWO VARIABLES

We define continuity of function of two variables in similar way as function of one variable.

In case of one variable we have f(x) is said to be continuous at x=a

if $\lim_{x \to a} f(x)$= f(a). i.e LHL = RHL= f(a)

in the same way we define continuity of f(x,y) at (a, b).

**Definition:** A function f(x, y) is said to be continuous at the point (a,b) if

$\lim_{(x,y)\to(a,b)} f(x,y)$= f(a, b)

i.e in other words

A function f(x, y) is said to be continuous at the point (a,b) if

i) f(a,b) is defined.

ii) $\lim_{(x,y)\to(a,b)} f(x,y)$ exists

iii) $\lim_{(x,y)\to(a,b)} f(x,y)$= f(a, b)

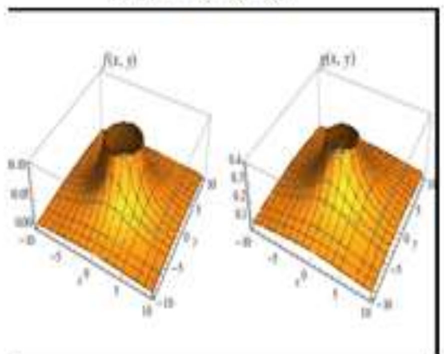
If any one of the above condition is not satisfied, it is discontinuous.

## Geometrical meaning of continuity of f(x,y)

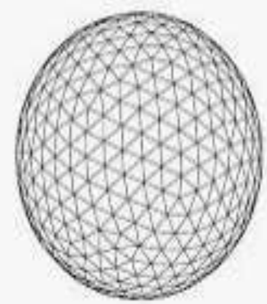
Now $z = f(x,y)$ is surface and it is continuous mans it has no hole on its surface.

For example:

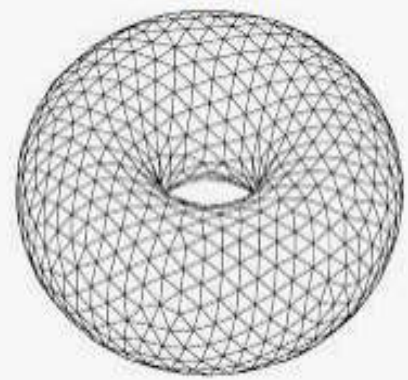
Discontinuous



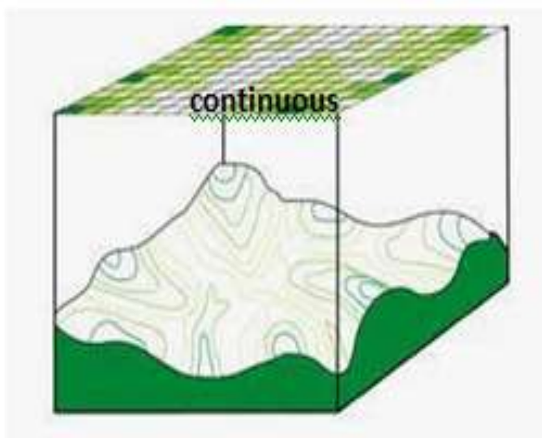
continuous



Discontinuous



Discontinuous




continuous

# Examples on continuity

**1. Show that the function**

$$f(x,y) = \begin{cases} \frac{2xy}{\sqrt{x^2+y^2}} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$$

is continuous at (0,0).

**Soln.:** Now given function is

$$f(x,y) = \begin{cases} \frac{2xy}{\sqrt{x^2+y^2}} & ,(x,y) \neq (0,0) \\ 0 & ,(x,y) = (0,0) \end{cases}$$

At $(x, y) \neq (0,0)$,

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2+y^2}}$$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.

Then $\lim_{(x,y)\to(0,0)} \frac{2xy}{\sqrt{x^2+y^2}} = \lim_{x\to 0} \frac{2xmx}{\sqrt{x^2+m^2x^2}} = \lim_{x\to 0} \frac{2mx^2}{x\sqrt{1+m^2}}$

$$= \lim_{x\to 0} \frac{2mx}{\sqrt{1+m^2}} = 0$$

And all along other paths also limit is 0 = f(0,0)

Thus $\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$

**2. If the function** $f(x,y) = \begin{cases} \dfrac{xy^2}{x^3+y^3}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0,0) \end{cases}$

Is the function f(x, y) continuous at (0, 0)?

**Soln.:** Now function $f(x,y) = \begin{cases} \dfrac{xy^2}{x^3+y^3}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0,0) \end{cases}$

Here, $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^3+y^3}$

Let us choose the path along straight line y = mx, i.e point (x,y) moves towards (0,0) along the line y =mx.

$$\lim_{(x,y)\to(0,0)} \frac{xm^2x^2}{x^3+m^3x^3} = \lim_{(x,y)\to(0,0)} \frac{m^2}{1+m^3} = \frac{m^2}{1+m^3}$$ depends on m,

limit does not exist

So function is discontinuous at (0,0).

## Home work.

**3. If the function** $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0,0) \end{cases}$

Is the function f(x, y) continuous at (0, 0)?

Thankyou

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: **C**alculus–II and 3–D Geometry

5th online Class

**Examples on Partial Derivatives**

By
Dr. M. M. Shankrikopp
HOD of Mathematics

Date: 7.5.2021

# EXAMPLES

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: **C**alculus–II and 3–D Geometry

6th online Class

**Higher Order Partial Derivatives**

By


Dr. M. M. Shankrikopp

HOD of Mathematics


Date: 11.5.2021

# Higher order Partial Derivatives

Before defining higher Order partial derivatives we know the following points

**NOTE:** (i) If $u = f(x,y)$ then (i) the derivative of u w.r.t. x treating y as constant is called as partial derivative of u w.r.t.x and denoted by $\frac{\partial u}{\partial x}$ or $u_x$ and (ii) the derivative of u w.r.t. y treating x as constant is called as partial derivative of u w.r.t.y and denoted by $\frac{\partial u}{\partial y}$ or $u_y$ where symbol $'\partial'$ is read as **'dov' or 'del'.**

(ii) The product rule, quotient rule and rule for composite function also same as function of one variable.

**We observe following comparisons:**

| Ordinary derivatives | | Partial derivatives | |
|---|---|---|---|
| 1. | $y = 3x^2 + 6x + 7$<br>$\frac{dy}{dx} = 6x+6+0 = 6x+6$ | 1. | $u = 3x^2y + 6xy^2 + 7$<br>$\frac{\partial u}{\partial x} = (6x)y + 6 .1.y^2 + 0 = 6xy + 6y^2$<br>(y treating as constant)<br>$\frac{\partial u}{\partial y} = 3x^2 + 6x(2y) + 0 = 3x^2 + 12xy$<br>(x treating as constant) |
| 2. | $y = e^{4x+3}$<br>$\frac{dy}{dx} = e^{4x+3}\, 4 = 4\, e^{4x+3}$ | 2. | $u = e^{4x+3y}$<br>$\frac{\partial u}{\partial x} = e^{4x+3y} \frac{\partial}{\partial x}(4x+3y)$<br>$= e^{4x+3y}\, 4 = 4\, e^{4x+3y}$<br>$\frac{\partial u}{\partial y} = e^{4x+3y} \frac{\partial}{\partial y}(4x+3y)$<br>$= e^{4x+3y}\, 3 = 3\, e^{4x+3y}$ |
| 3. | $y = \sin 5x$<br>$\frac{dy}{dx} = 5 \cos 5x$ | 3. | $u = \sin(xy)$<br>$\frac{\partial u}{\partial x} = \cos(xy) \frac{\partial}{\partial x}(xy)$<br>$= \cos(xy) .y = y \cos(xy)$<br>$\frac{\partial u}{\partial y} = \cos(xy) \frac{\partial}{\partial y}(xy)$<br>$= \cos(xy) .x = x \cos(xy)$ |
| 4. | In general we conclude that<br>If $y = g(v)$ and $v = f(x)$ then<br>$\frac{dy}{dx} = g^1(v) \frac{dv}{dx}$ | 4. | If $u = f(r)$ and r is a function of x & y<br>Then $\frac{\partial u}{\partial x} = f^1(r) \frac{\partial r}{\partial x}$<br>And $\frac{\partial u}{\partial y} = f^1(r) \frac{\partial r}{\partial y}$ |

## Higher Order Partial Derivatives

- If the partial derivative of the function is again continuous function we can go for next derivative. So it is possible to take the partial derivative of a partial derivative. This is just like getting second derivative of f(x).
- The higher order partial derivatives for a function of two variables are defined as follows.

If z =f(x,y) then,

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}(f_x) = f_{xx} \text{ or } f_{x^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}(f_y) = f_{yy} \text{ or } f_{y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(f_y) = (f_y)_x \text{ or } f_{yx}$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}(f_x) = (f_x)_y \text{ or } f_{xy}$$

Mixed partial derivatives (the last two)

Second order partial derivatives (all four)

Similarly

$$\frac{\partial^3 f}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f}{\partial x^2}\right) = f_{xxx}$$

$$\frac{\partial^3 f}{\partial y^3} = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial y^2}\right) = f_{yyy}$$ , $f_{xyy}$, $f_{xxy}$, $f_{yyx}$, $f_{yxx}$ are 3rd order partial order derivatives.

**For example**: If $u = x^3 - 3xy^2 + x + e^x \cos y + 1$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

**Soln.:** Now $u = x^3 - 3xy^2 + x + e^x \cos y + 1$ ------------------(1)

$\therefore \frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 1 + e^x \cos y$

Again diff. above eqn. p.w.r.t x we get

$\frac{\partial^2 u}{\partial x^2} = 6x - 0 + 0 + e^x \cos y = 6x + e^x \cos y$ -------------------(2)

diff. eqn. (1) p.w.r.t y we get

$\frac{\partial u}{\partial y} = 0 - 3x(2y) + 0 + e^x(-\sin y) + 0 = -6xy - e^x \sin y$

Again diff.w.r.t. y we get,

$\frac{\partial^2 u}{\partial y^2} = -6x - e^x \cos y$ ------------------------------------(3)

Adding (2) and (3) we get,

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x + e^x \cos y - 6x - e^x \cos y = 0$

# Symmetric functions and partial derivatives of such functions

A function f(x,y) is said to be symmetric if f(x, y) =f(y, x) and f(x,y,z) is symmetric if f(x,y,z) = f(y,z,x) = f(z,x,y)

i.e if we change x,y, z in cyclic order there is no change in function

**For example:**

(i) $x+y$, $x^2+y^2$, $\frac{x^2+y^2}{x+y}$, $x^2+xy+y^2$, etc. are symmetric functions of 2 variables.

(ii) $x^2+y^2+z^2$, , $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$, $x^3+y^3+z^3-3xyz$ etc. are symmetric functions of 3 variables.

**It is very important to note** that if given function is symmetric, then it is very easy to get partial derivative.

i.e if we get $u_x$, $u_{xx}$, $u_{xy}$ then directly we write $u_y$, $u_{yy}$, $u_{yx}$ etc. just by changing variables in cyclic order.

# EXAMPLES

**1. If $u = e^{ax-by}\sin(ax+by)$, show that $b\frac{\partial u}{\partial x} - a\frac{\partial u}{\partial y} = 2abu$**

**Soln.** Now $u = e^{ax-by}\sin(ax+by)$

$$\therefore \frac{\partial u}{\partial x} = e^{ax-by}(a)\sin(ax+by) + e^{ax-by}\cos(ax+by)\,a$$

$$= ae^{ax-by}(\sin(ax+by) + \cos(ax+by))$$

And $\frac{\partial u}{\partial y} = e^{ax-by}(-b)\sin(ax+by) + e^{ax-by}\cos(ax+by)\,b$

$$= be^{ax-by}(-\sin(ax+by) + \cos(ax+by))$$

$$b\frac{\partial u}{\partial x} - a\frac{\partial u}{\partial y} = ab\,e^{ax-by}[\sin(ax+by) + \cos(ax+by)] - ab\,e^{ax-by}[-\sin(ax+by) + \cos(ax+by)]$$

$$= 2ab\,e^{ax-by}[\sin(ax+by)]$$

$$= 2abu \text{ RHS}$$

**2. If $z = \sinh^{-1}\left(\frac{x}{y}\right)$ show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0.$**

**Soln.:** Now $z = \sinh^{-1}\left(\frac{x}{y}\right)$

$$\therefore \frac{\partial z}{\partial x} = \frac{1}{\sqrt{1+\left(\frac{x}{y}\right)^2}}\frac{1}{y} = \frac{y}{\sqrt{y^2+x^2}}\frac{1}{y} = \frac{1}{\sqrt{y^2+x^2}}$$

And $$\frac{\partial z}{\partial x} = \frac{1}{\sqrt{1+\left(\frac{x}{y}\right)^2}}\frac{x}{-y^2} = \frac{-y}{\sqrt{y^2+x^2}}\frac{x}{y^2} = \frac{-x}{y\sqrt{y^2+x^2}}$$

$$\therefore x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{x}{\sqrt{y^2+x^2}} + y\left(\frac{-x}{y\sqrt{y^2+x^2}}\right)$$

$$= \frac{x}{\sqrt{y^2+x^2}} - \frac{x}{\sqrt{y^2+x^2}}$$

$$= 0$$

$$\therefore x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$

# PUZZLE

8. There are 2 jugs with 4 litres and 5 litres of water respectively. The objective is to pour exactly 7 litres of water in a bucket. How can it be accomplished?

Water and jugs puzzle

THANKYOU

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1:**C**alculus–II and 3–D Geometry

13th online Class

**Examples on second Order Partial Derivatives**

By

Dr. M. M. Shankrikopp

HOD of Mathematics

Date: 25.5.2021

# Continued Examples on Second order partial derivatives in case of composite function

1. If $z = f(x, y)$ and $x = u\cos\alpha - v\sin\alpha$ and $y = u\sin\alpha + v\cos\alpha$ then prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}, \quad \alpha \text{ being constant}$$

**Soln.:** Now $z = f(x, y)$ and $x = u\cos\alpha - v\sin\alpha$ and $y = u\sin\alpha + v\cos\alpha$

i.e z is a composite function of u and v.

$$\therefore \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = \frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha$$

And $\frac{\partial^2 z}{\partial u^2} = \frac{\partial}{\partial u}\left(\frac{\partial z}{\partial u}\right) = \frac{\partial}{\partial u}(r)$ where $r = \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha$

$$= \frac{\partial r}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial r}{\partial y}\frac{\partial y}{\partial u}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha\right)\frac{\partial x}{\partial u} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\cos\alpha + \frac{\partial z}{\partial y}\sin\alpha\right)\frac{\partial y}{\partial u}$$

$$= \left(\cos\alpha\frac{\partial^2 z}{\partial x^2} + \sin\alpha\frac{\partial^2 z}{\partial x\partial y}\right)\cos\alpha + \left(\cos\alpha\frac{\partial^2 z}{\partial y\partial x} + \sin\alpha\frac{\partial^2 z}{\partial y^2}\right)\sin\alpha$$

$$\frac{\partial^2 z}{\partial u^2} = \cos^2\alpha\frac{\partial^2 z}{\partial x^2} + 2\sin\alpha\cos\alpha\frac{\partial^2 z}{\partial x\partial y} + \sin^2\alpha\frac{\partial^2 z}{\partial y^2} \text{ -----------(1)}$$

Next $\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = \frac{\partial z}{\partial x}(-\sin\alpha) + \frac{\partial z}{\partial y}\cos\alpha$

And $\frac{\partial^2 z}{\partial v^2} = \frac{\partial}{\partial v}\left(\frac{\partial z}{\partial v}\right) = \frac{\partial}{\partial v}(s)$ where $s = \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}(-\sin\alpha) + \frac{\partial z}{\partial y}\cos\alpha$

i.e $\frac{\partial^2 z}{\partial v^2} = \frac{\partial s}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial s}{\partial y}\frac{\partial y}{\partial v}$

$$= \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}(-\sin\alpha) + \frac{\partial z}{\partial y}\cos\alpha\right)\frac{\partial x}{\partial v} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}(-\sin\alpha) + \frac{\partial z}{\partial y}\cos\alpha\right)\frac{\partial y}{\partial v}$$

$$= \left(-\sin\alpha\frac{\partial^2 z}{\partial x^2} + \cos\alpha\frac{\partial^2 z}{\partial x\partial y}\right)(-\sin\alpha) + \left(-\sin\alpha\frac{\partial^2 z}{\partial y\partial x} + \cos\alpha\frac{\partial^2 z}{\partial y^2}\right)\cos\alpha$$

$$\frac{\partial^2 z}{\partial v^2} = \sin^2\alpha\frac{\partial^2 z}{\partial x^2} - 2\sin\alpha\cos\alpha\frac{\partial^2 z}{\partial x\partial y} + \cos^2\alpha\frac{\partial^2 z}{\partial y^2} \text{------------}(2)$$

Adding (1) and (2) we get, $\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = [\cos^2\alpha \frac{\partial^2 z}{\partial x^2} + 2 \sin\alpha \cos\alpha \frac{\partial^2 z}{\partial x \partial y} + \sin^2\alpha \frac{\partial^2 z}{\partial y^2}]$

$$+\sin^2\alpha \frac{\partial^2 z}{\partial x^2} - 2 \sin\alpha \cos\alpha \frac{\partial^2 z}{\partial x \partial y} + \cos^2\alpha \frac{\partial^2 z}{\partial y^2}$$

$$= \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$$

**Thus we have,** $\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$

2. **If u = f(x, y) where x = r cos$\boldsymbol{\theta}$, $\boldsymbol{y}$ = r sin$\boldsymbol{\theta}$ then show that**

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

***Soln***.: Now u = f(x, y) where x = r cos$\theta$, $y$ = r sin$\theta$

*i.e* u is composite function of r and $\theta$

$$\therefore \frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial r}=\frac{\partial u}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial u}{\partial y}\frac{\partial y}{\partial r}=\frac{\partial u}{\partial x}cos\theta+\frac{\partial u}{\partial y}sin\theta$$

i.e $\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}cos\theta+\frac{\partial u}{\partial y}sin\theta$--------------(1)

Again differentiate (1) p.w.r.t r again ,

$$\frac{\partial^2 u}{\partial r^2}=\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}\right)=\frac{\partial s}{\partial r}\quad \text{where } s=\frac{\partial u}{\partial r}=\frac{\partial u}{\partial x}cos\theta+\frac{\partial u}{\partial y}sin\theta$$

$$=\frac{\partial s}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial s}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\sin\theta\right)\frac{\partial x}{\partial r} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\cos\theta + \frac{\partial u}{\partial y}\sin\theta\right)\frac{\partial y}{\partial r}$$

i.e $\frac{\partial^2 u}{\partial r^2} = \left(\cos\theta\,\frac{\partial^2 u}{\partial x^2} + \sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y}\right)\cos\theta + \left(\cos\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \sin\theta\,\frac{\partial^2 u}{\partial y^2}\right)\sin\theta$

i.e $\frac{\partial^2 u}{\partial r^2} = \cos^2\theta\,\frac{\partial^2 u}{\partial x^2} + 2\sin\theta\cos\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \sin^2\theta\,\frac{\partial^2 u}{\partial y^2}$ ---------------------(P)

$Next\ \frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial \theta} = \frac{\partial u}{\partial x}(-r\sin\theta) + \frac{\partial u}{\partial y}(r\cos\theta)$

i.e $\frac{\partial u}{\partial \theta} = \mathrm{r}\left(-\frac{\partial u}{\partial x}\sin\theta + \frac{\partial u}{\partial y}\cos\theta\right)$ -----------------(2)

Again differentiate (2) p.w.r.t $\theta$ again ,

$\therefore\ \frac{\partial^2 u}{\partial \theta^2} = \frac{\partial}{\partial \theta}\left(\frac{\partial u}{\partial \theta}\right) = \frac{\partial}{\partial \theta}(t)$ where $t = \frac{\partial u}{\partial \theta}$

$= \frac{\partial t}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial t}{\partial y}\frac{\partial y}{\partial \theta}$ by chain rule

$$= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial \theta}\right)\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial \theta}\right)\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial}{\partial x}\left(\mathrm{r}\left(-\frac{\partial u}{\partial x}\sin\theta + \frac{\partial u}{\partial y}\cos\theta\right)\right)\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}\left(\mathrm{r}\left(-\frac{\partial u}{\partial x}\sin\theta + \frac{\partial u}{\partial y}\cos\theta\right)\right)\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial}{\partial x}(\mathbf{r}\,(-\frac{\partial u}{\partial x}\ sin\theta + \frac{\partial u}{\partial y}\ cos\theta)\,)\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}(\mathbf{r}\,(-\frac{\partial u}{\partial x}\ sin\theta + \frac{\partial u}{\partial y}\ cos\theta)\,)\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial}{\partial x}(-\frac{\partial u}{\partial x}\ \boldsymbol{r}\ sin\theta + \frac{\partial u}{\partial y}\ \boldsymbol{r}\ cos\theta)\,)\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}(-\frac{\partial u}{\partial x}\ \boldsymbol{r}\ sin\theta + \frac{\partial u}{\partial y}\ \boldsymbol{r}\ cos\theta)\,)\frac{\partial y}{\partial \theta}$$

$$= \frac{\partial}{\partial x}(-\frac{\partial u}{\partial x}\ y + \frac{\partial u}{\partial y}\ x)\,)\frac{\partial x}{\partial \theta} + \frac{\partial}{\partial y}(-\frac{\partial u}{\partial x}\ y + \frac{\partial u}{\partial y}\ x)\,)\frac{\partial y}{\partial \theta}$$

$$=[\,-\,\mathrm{y}\frac{\partial^2 u}{\partial x^2} + (-\frac{\partial u}{\partial x})\,0 + x\,\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial u}{\partial y}\ 1]\ (\text{-r}\ \sin\theta) +$$

$$[\,\text{-y}\ \frac{\partial^2 u}{\partial x\,\partial y} + \left(-\frac{\partial u}{\partial x}\right)1 + \ x\ \frac{\partial^2 u}{\partial y^2}\ + \frac{\partial u}{\partial y}\ (0)\,)\,]\ (\mathrm{r}\ \cos\theta)$$

$$= [\,-\,\mathrm{r}\ \sin\theta\,\frac{\partial^2 u}{\partial x^2} + \mathrm{r}\ \cos\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial u}{\partial y}\,]\ (\text{-r}\ \sin\theta)$$

$$+ [\,-\,\mathrm{r}\ \sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \left(-\frac{\partial u}{\partial x}\right)\mathbf{1} + \mathrm{r}\ \cos\theta\,\frac{\partial^2 u}{\partial y^2}\,)\,]\ (\mathrm{r}\ \cos\theta)$$

$$= [\,-\mathrm{r}\sin\theta\,\frac{\partial^2 u}{\partial x^2} + \mathrm{r}\cos\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial u}{\partial y}\,]\,(-\mathrm{r}\sin\theta)$$

$$+ [\,-\mathrm{r}\sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + \left(-\frac{\partial u}{\partial x}\right)1 + \mathrm{r}\cos\theta\,\frac{\partial^2 u}{\partial y^2}\,)\,]\,(\mathrm{r}\cos\theta)$$

$$= r^2\,\sin^2\theta\,\frac{\partial^2 u}{\partial x^2} - r^2\cos\theta\,\sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} - \mathrm{r}\sin\theta\,\frac{\partial u}{\partial y}$$

$$- r^2\cos\theta\,\sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} + r^2\,\cos^2\theta\,\frac{\partial^2 u}{\partial y^2} - \mathrm{r}\cos\theta\,\frac{\partial u}{\partial x}$$

$$= r^2\,\sin^2\theta\,\frac{\partial^2 u}{\partial x^2} + r^2\,\cos^2\theta\,\frac{\partial^2 u}{\partial y^2} - 2\,r^2\cos\theta\,\sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} - \mathrm{r}\sin\theta\,\frac{\partial u}{\partial y} - \mathrm{r}\cos\theta\,\frac{\partial u}{\partial x}$$

$$\therefore \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = \sin^2\theta\,\frac{\partial^2 u}{\partial x^2} + \cos^2\theta\,\frac{\partial^2 u}{\partial y^2} - 2\cos\theta\,\sin\theta\,\frac{\partial^2 u}{\partial x\,\partial y} - \frac{1}{r}\sin\theta\,\frac{\partial u}{\partial y} - \frac{1}{r}\cos\theta\,\frac{\partial u}{\partial x}$$

------------------------------(Q)

And $\frac{1}{r}\frac{\partial u}{\partial r} = \frac{1}{r}\cos\theta\,\frac{\partial u}{\partial x} + \frac{1}{r}\sin\theta\,\frac{\partial u}{\partial y}$ ------------------------------------(R)

Adding (P), (Q) and (R )we get

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = [\cos^2\theta \frac{\partial^2 u}{\partial x^2} + \sin^2\theta \frac{\partial^2 u}{\partial y^2} + 2\sin\theta\cos\theta \frac{\partial^2 u}{\partial x\,\partial y} \;]$$

$$+ [\sin^2\theta \frac{\partial^2 u}{\partial x^2} + \cos^2\theta \frac{\partial^2 u}{\partial y^2} - 2\cos\theta\sin\theta \frac{\partial^2 u}{\partial x\,\partial y} - \frac{1}{r}\sin\theta \frac{\partial u}{\partial y} - \frac{1}{r}\cos\theta \frac{\partial u}{\partial x}]$$

$$+ \frac{1}{r}\cos\theta \frac{\partial u}{\partial x} + \frac{1}{r}\sin\theta \frac{\partial u}{\partial y}$$

$$= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

**Thus we got** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$

# KLE's G. I. Bagewadi College Nipani
# Department of Mathematics

B.Sc II Semester 2020-21 Mathematics

Paper 2.1: **C**alculus–II and 3–D Geometry

22nd online Class

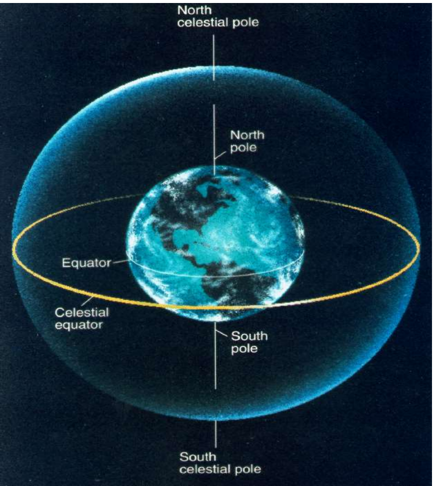
**Section of a sphere by the Plane**

By

Dr. M. M. Shankrikopp

HOD of Mathematics

Date: 8.6.2021



# Examples on finding equation of sphere passing through given points

1. **Find equation of sphere passing through origin and the points(0, 1, -1), (-1, 2, 0) and (-1,2, 3).**

Soln.: Let the required equation of the sphere be

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \text{----------}(1)$$

**It passes through the origin, we put x=0 , y =0, z=0 in (1)**

$0 + d = 0$

$\Rightarrow d = 0$ ---------------(2)

Next (1) passes through the point (0,1,-1), (-1,2,0) and (-1, 2, 3)

Therefore we have,

$0+1+1+2u(0) + 2v(1)+2w(-1)=0$

i.e $2+2v-2w=0$

i.e. **$1+u-w=0$------------(3)**

Next $1+4+0+2u(-1)+2v(2)+2w(0)=0$

i.e **$5 -2u+4v=0$----------(4)**

Also

$1+4+9+2u(-1) +2v(2) + 2w(3)=0$

i.e **$14-2u+4v+6w=0$ -------(5)**

**Substitute $-2u +4v =-5$ from (4) in (5) we get**

14 -5 +6w=0 => 6w = -9

=> **w = -3/2**

Put w in (3) we get, 1+u +3/2= 0

=>**u= -5/2**

From (4) we have

**5 -2u+4v=0**

**i.e 5 – 2(-5/2)+4v= 0**

**i.e.** 10 +4v =0

∴ v = -5/2

Then from(1), req. eqn.of sphere is

$$x^2+y^2+z^2+2(-5/2)x+2(-5/2)y+2(-3/2)z+d=0$$

$$x^2+y^2+z^2-5x-5y+2(-3/2)z+0=0$$

i.e $x^2+y^2+z^2+-5x-5y-3z=0$

**2. Find equation of sphere passing through origin and the points(-1, 1, 1), (1-1, 1) , (1,1,-1).**

Soln.: Let the required equation of the sphere be

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0 \text{----------} (1)$$

**It passes through the origin, we put x=0 , y =0, z=0, in (1)**

$0 + d = 0$

$\Rightarrow d = 0$ ---------------(2)

Next (1) passes through the point (-1,1,1), (1,-1,1) and (1, 1, -1)

Therefore we have,

$1+1+1+2u(1) + 2v(-1)+2w(1)=0$

i.e $3+2u-2v+2w=0$

i.e. **$u-v+w= -3/2$-----------------------(3)**

Similarly **we have $u+v-w= -3/2$-----(4)**

**And $-u+v+w= -3/2$---------------------(5)**

Adding (3) and (5) we get $2w = -6/2$

**$\therefore w = - 3/2$**

Similarly ,

From (4) and (5) we get $v = -3/2$

And from (3) and (4) we get $u = -3/2$

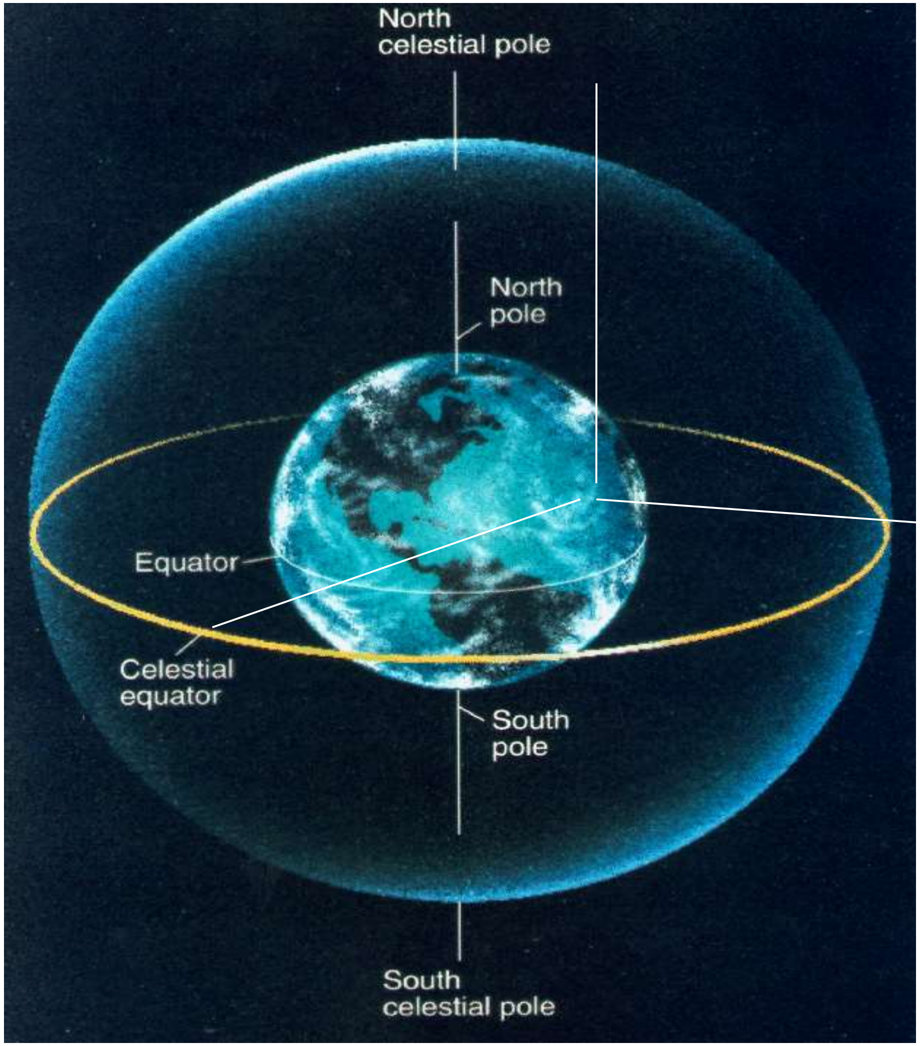
Then from(1), req. eqn.of sphere is

$$x^2+y^2+z^2 + 2(-3/2)x+ 2(-3/2)y+2(-3/2)z+0=0$$

i.e $x^2+y^2+z^2 - (3)x- 3y- (3)z =0$

i.e$x^2+y^2+z^2 -3x-3y -3z =0$

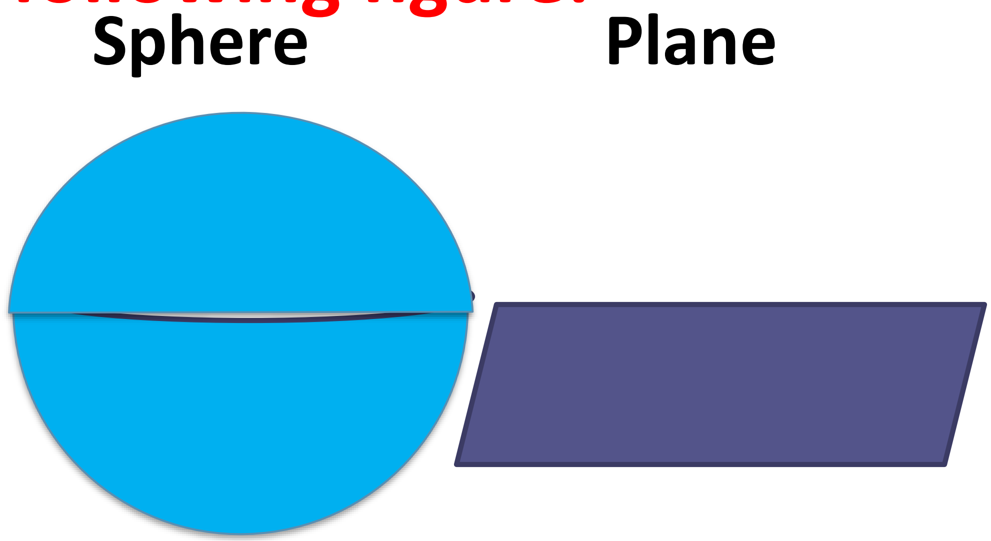
# Sphere with centre at origin



# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

We observe the following figure:



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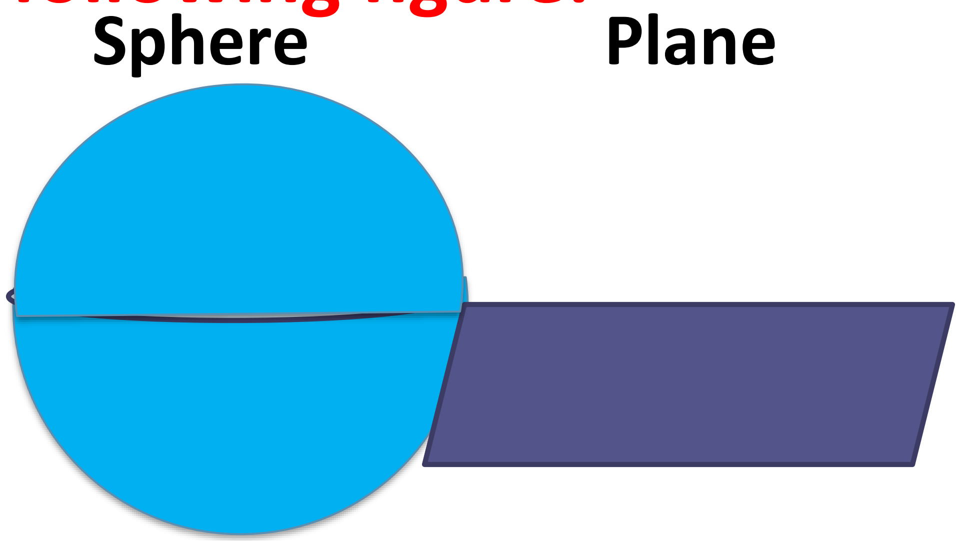

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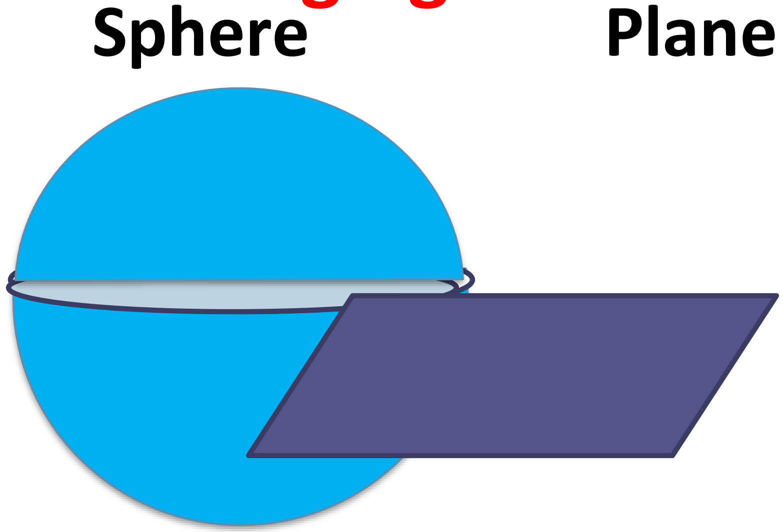

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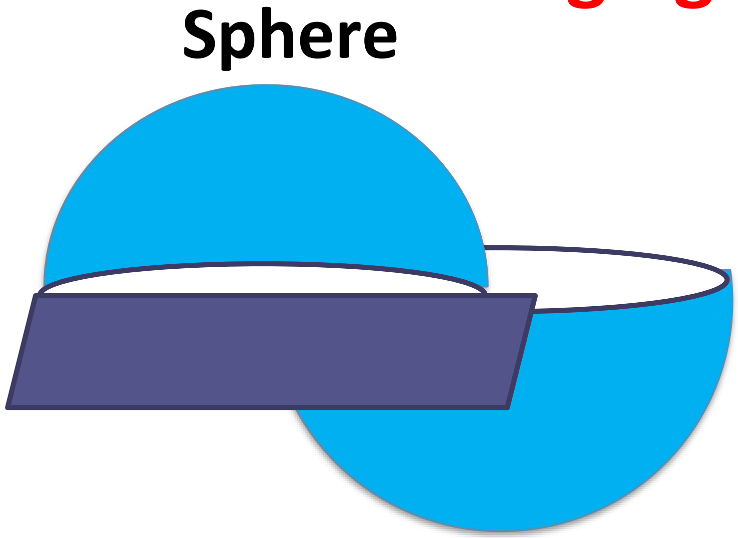

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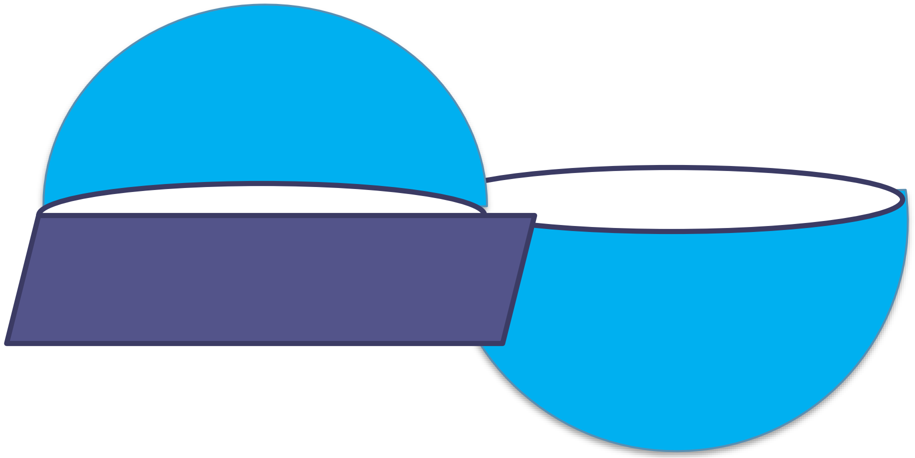
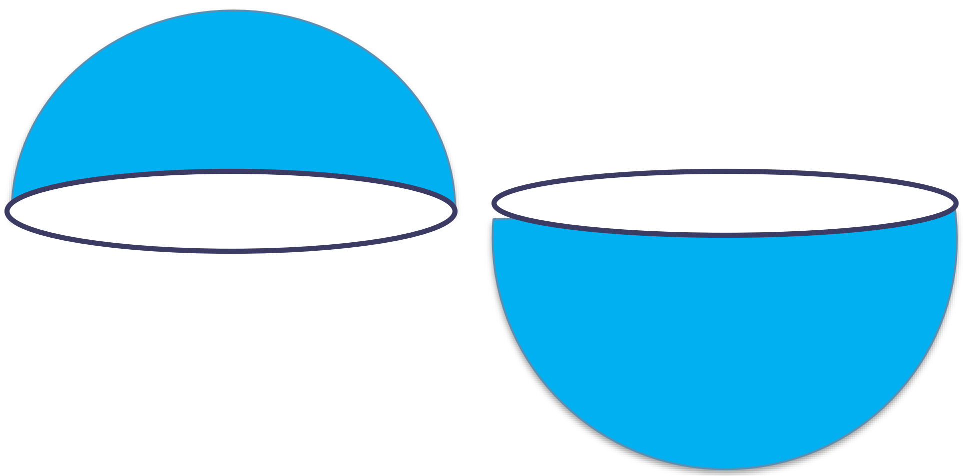


# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

We observe the following figure

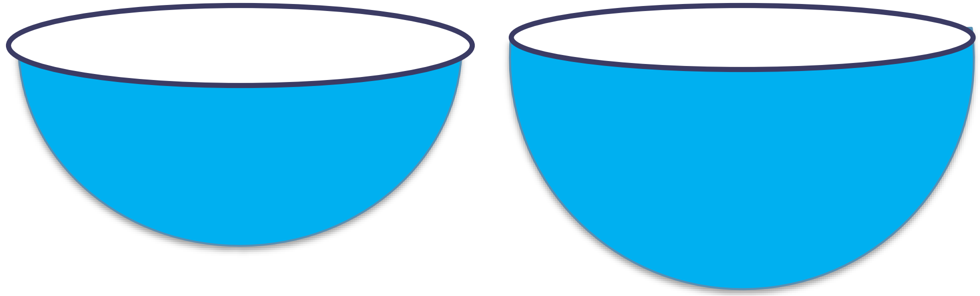
See, we get two sections in both surface of section is circle



# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then **what is the section?**

**We observe the following figure :**



# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

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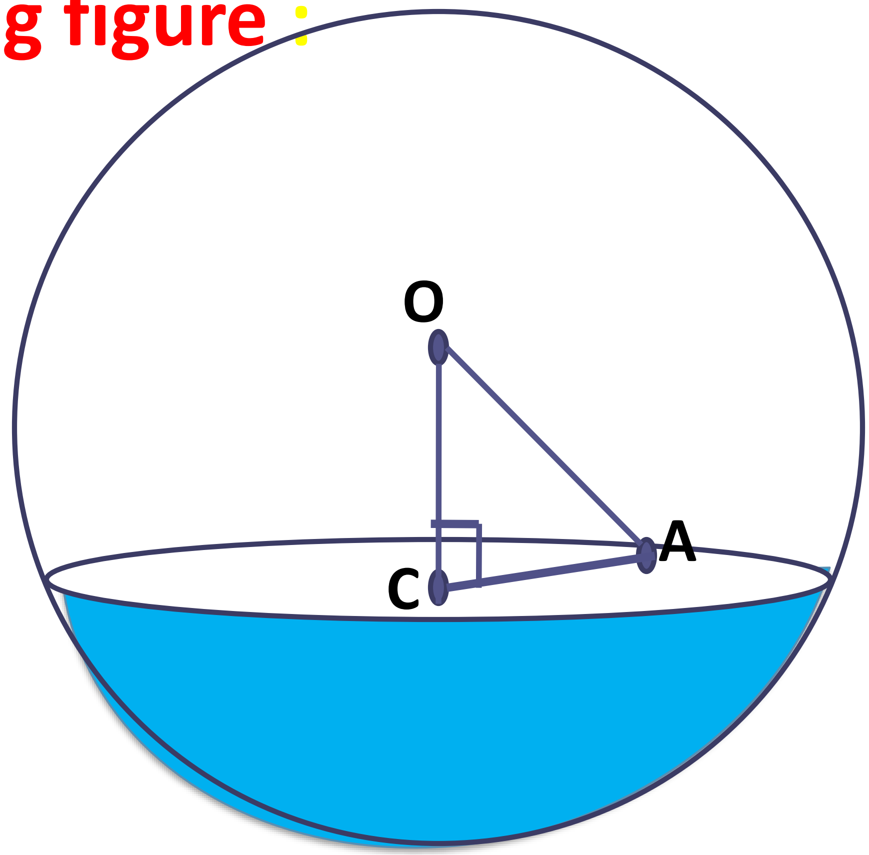
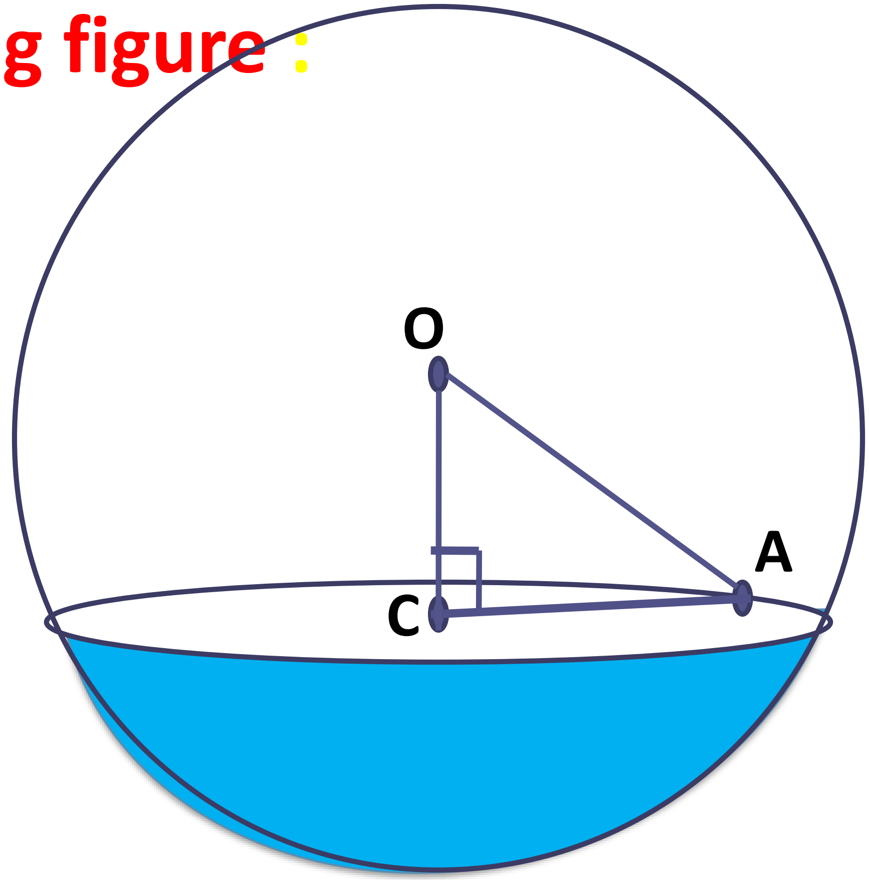


# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then **what is the section?**

**We observe the following figure :**



# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

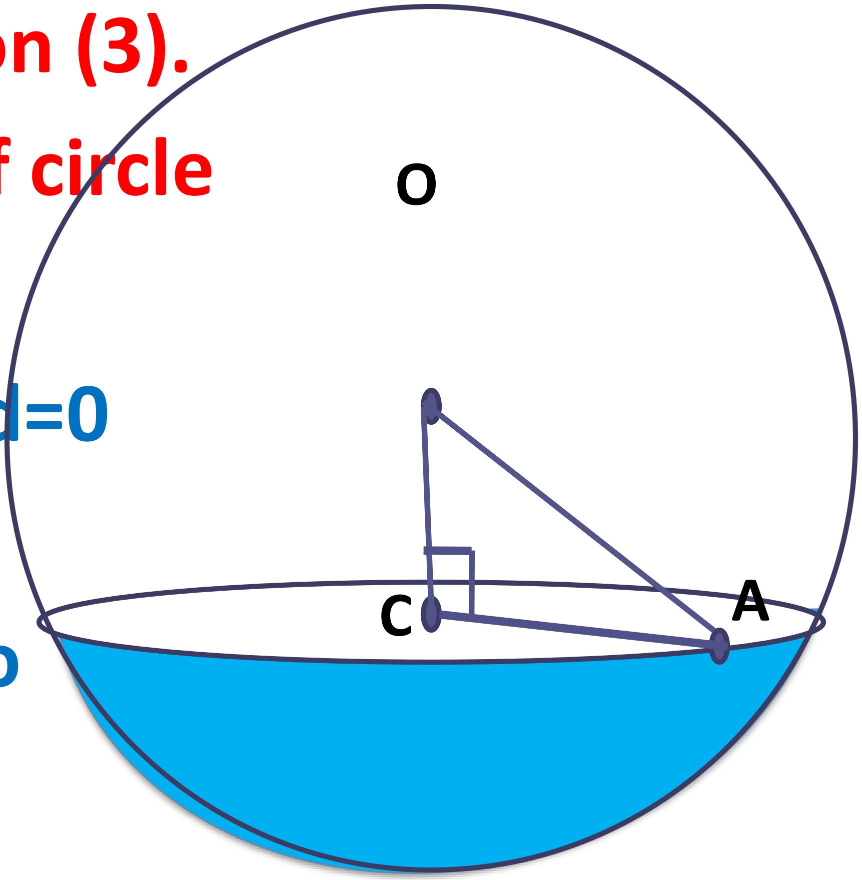
We obtain CA by equation (3).

Thus general equation of circle can be given by

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$

and $ax+by+cz+d_1=0$

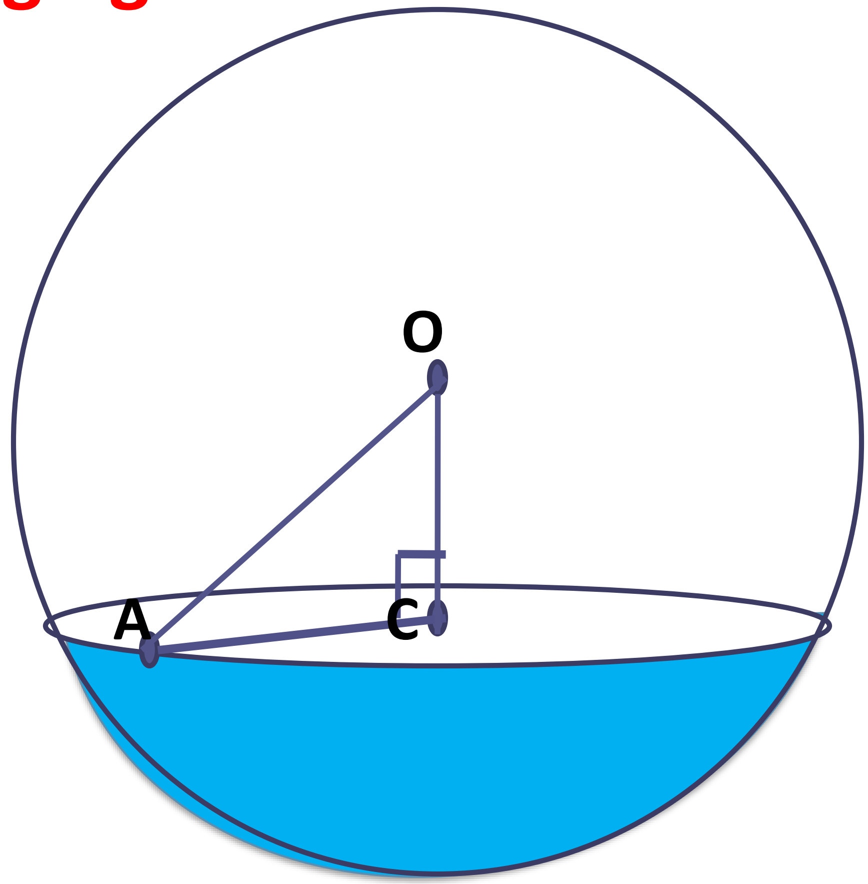
i.e combinedly these two gives circle.



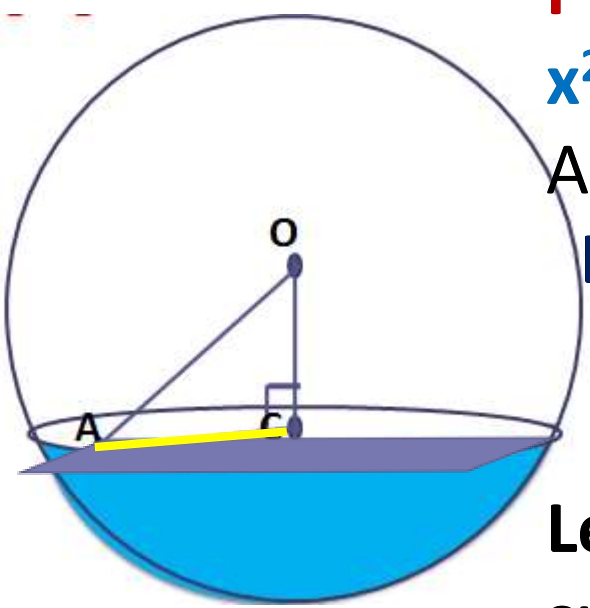
# Section of sphere by the Plane

If the plane $ax+by+cz+d=0$ cuts the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ then what is the section?

We observe the following figure :



**Theorem**: Prove that the section of the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ by the plane $ax+by+cz+d=0$ **is a circle and hence find its centre and radius**




**Proof**: Given Sphere is

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ ---------(1)

And given plane $ax+by+cz+d=0$-------(2)

**Let O(-u, -v, -w) be centre of sphere (1)**

**Let OC be the perpendicular drawn from O to the plane (2)**

**Let A be any point on the section of a sphere (1) by plan (2).**

Let CA be any line through C, hence CA is perpendicular to OC, and **OA is radius of the sphere** (b'cz, O is the centre of the sphere and A be any point on the surface of the sphere)

Hence $OA = \sqrt{u^2 + v^2 + w^2 - d}$ = constant

And OC = Length of Perpendicular from O(-u,-v,-w) to the plane (2)

$$OC = \left|\frac{a(-u)+b(-v)+c(-w)+d}{\sqrt{a^2+b^2+c^2}}\right| = \text{constant}$$

From the right angled triangle OCA, we have

$OA^2 = OC^2 + CA^2$ or $CA^2 = OA^2 - OC^2$ = constant

as OA and OC are constant

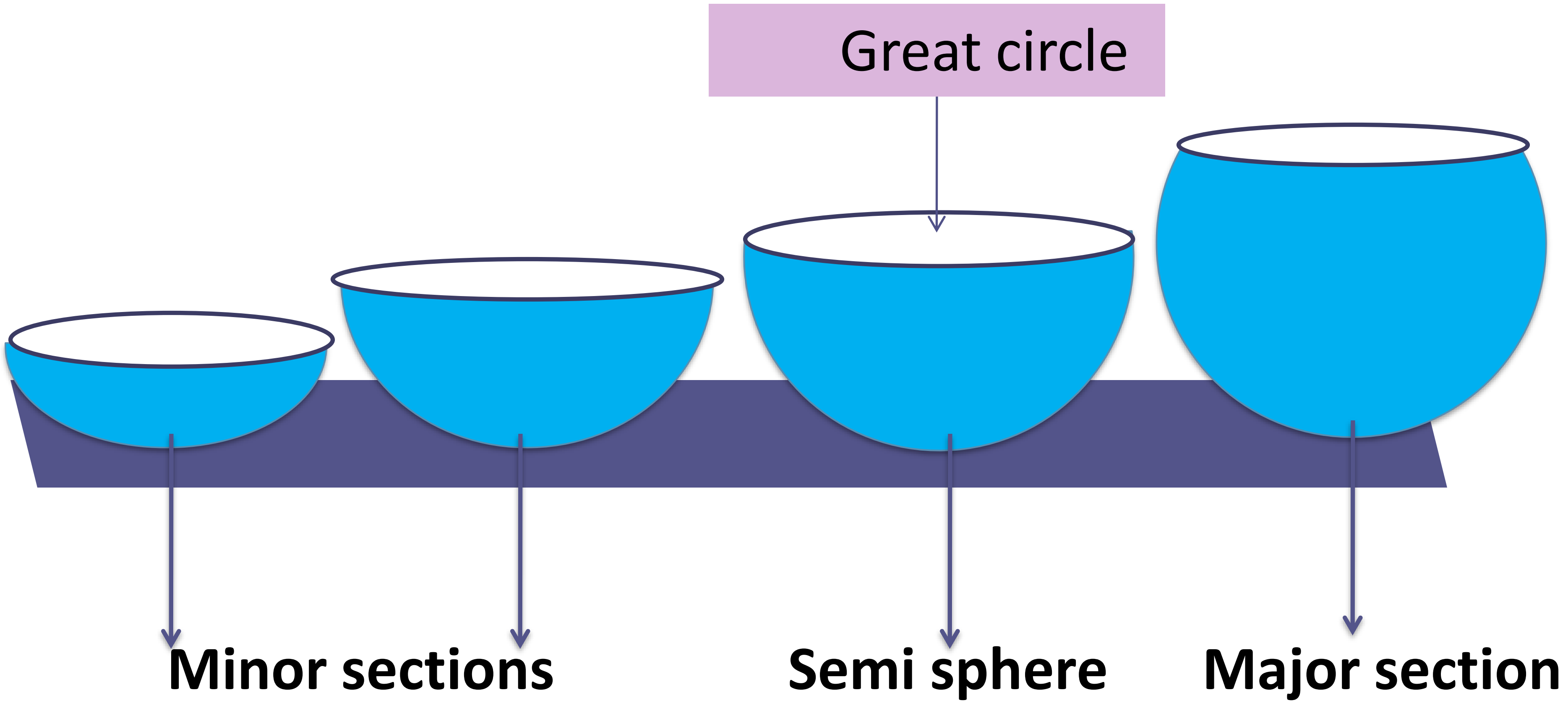
Hence distance of any point A from C is constant

This is true only when section is circle.

Thus the section of the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ by the plane $ax+by+cz+d=0$ **is a circle. And its radius is CA and centre is C.**

We have to find coordinates of C and length OC .

# Different Sections of sphere by the Plane




**For all these sections , surface is circle.**

**Note: (i) Hence equation of circle of section is given by**

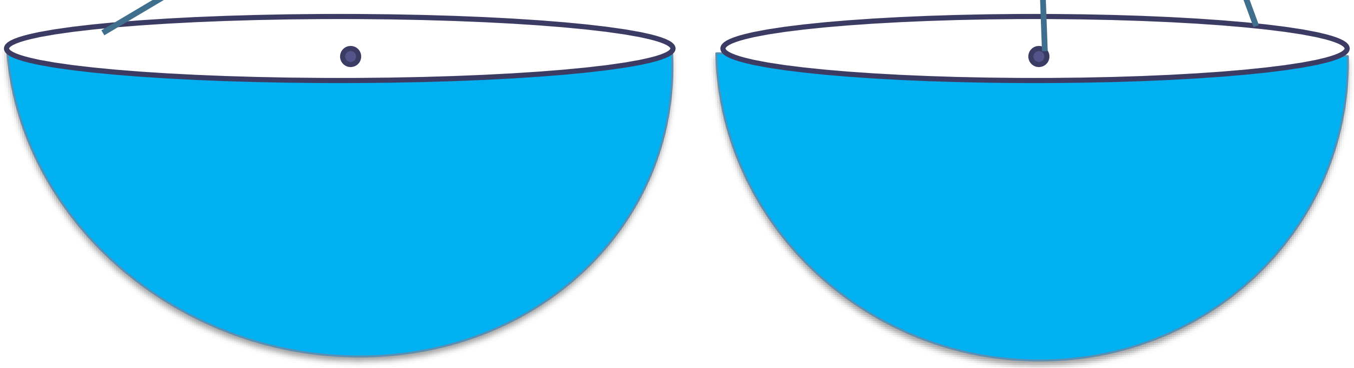
**S: $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ and Plane $ax+by+cz+p=0$**

# (ii) GREAT CIRCLE

- CIRCLE OF HEMISPHERE IS A GREAT CIRCLE

If the section of a sphere by plane passes through centre of sphere , then section is called as great circle .

CENTRE OF CIRCLE IS SAME AS CENTRE OF SPHERE.

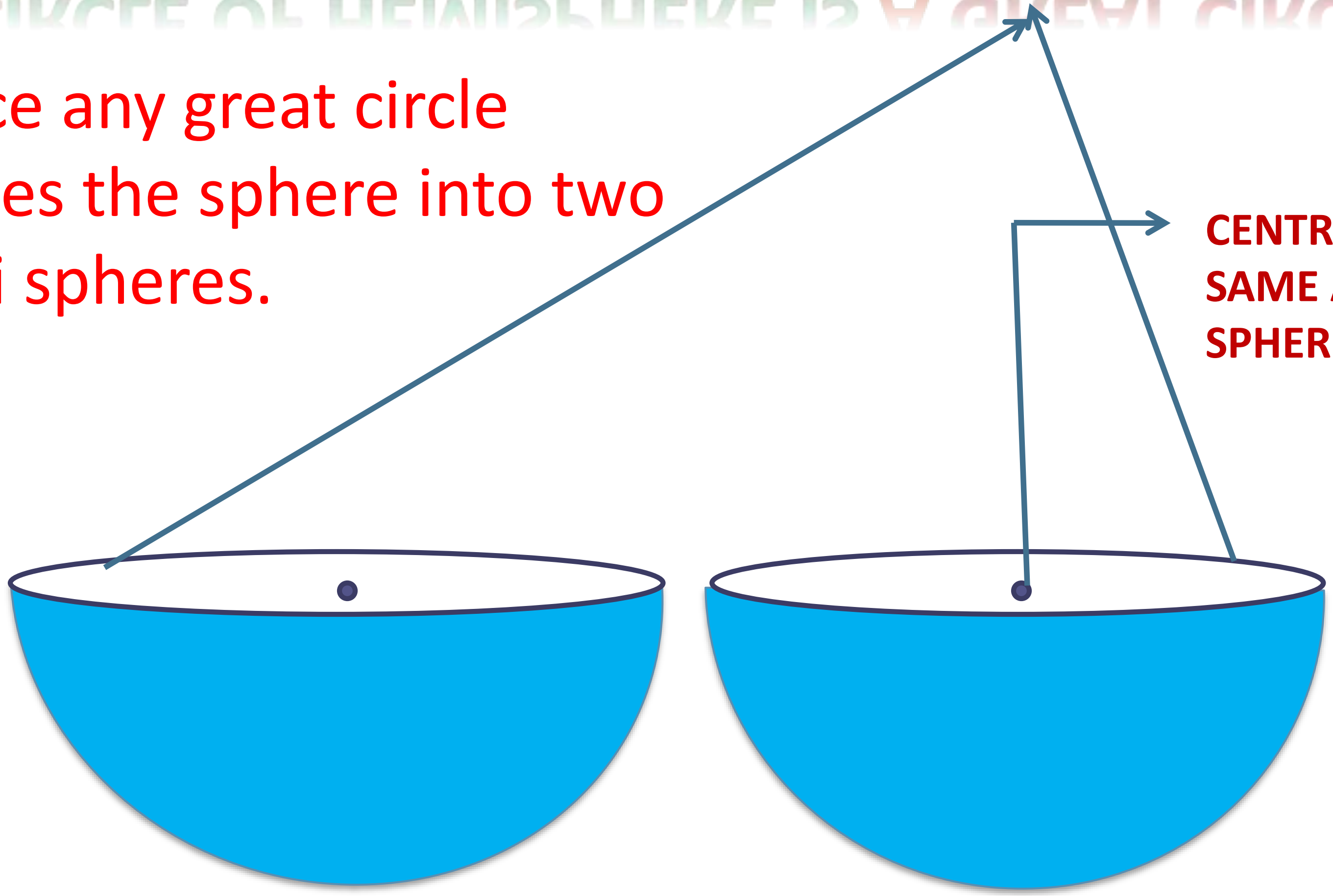


# (ii) GREAT CIRCLE

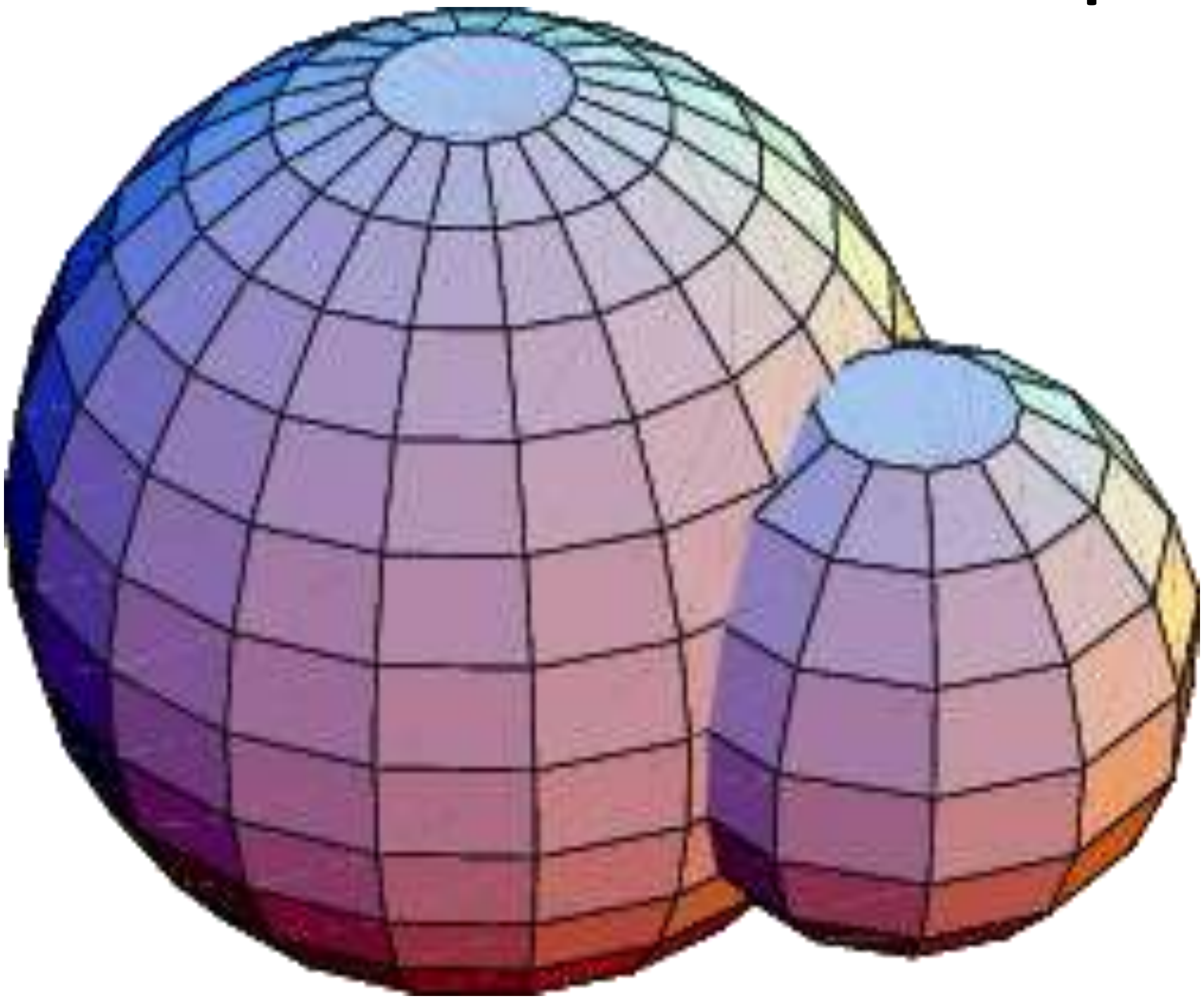
- CIRCLE OF HEMISPHERE IS A GREAT CIRCLE

Hence any great circle divides the sphere into two hemi spheres.

CENTRE OF CIRCLE IS SAME AS CENTRE OF SPHERE.



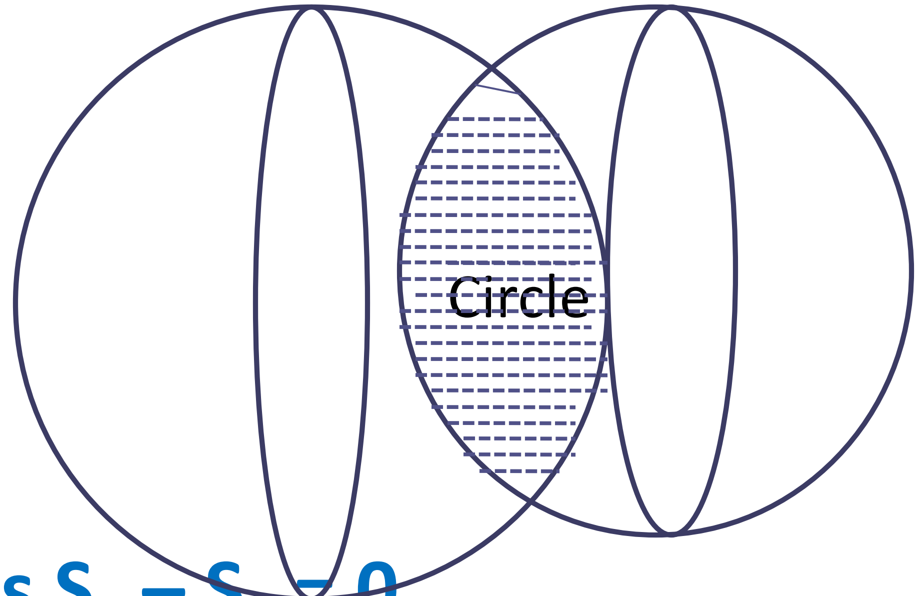
# Intersection of two spheres



## (iii). Intersection of two spheres whose equations are

$S_1 = x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ and

$S_2 = x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$



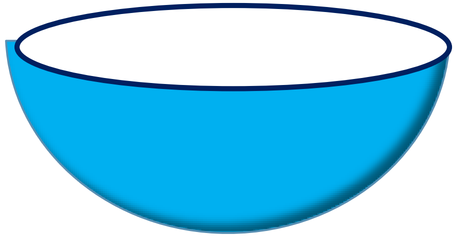

is taken as $S_1 - S_2 = 0$

i.e $2(u_1 - u_2)x + 2(v_1 - v_2)y + 2(w_1 - w_2)z + d_1 - d_2 = 0$

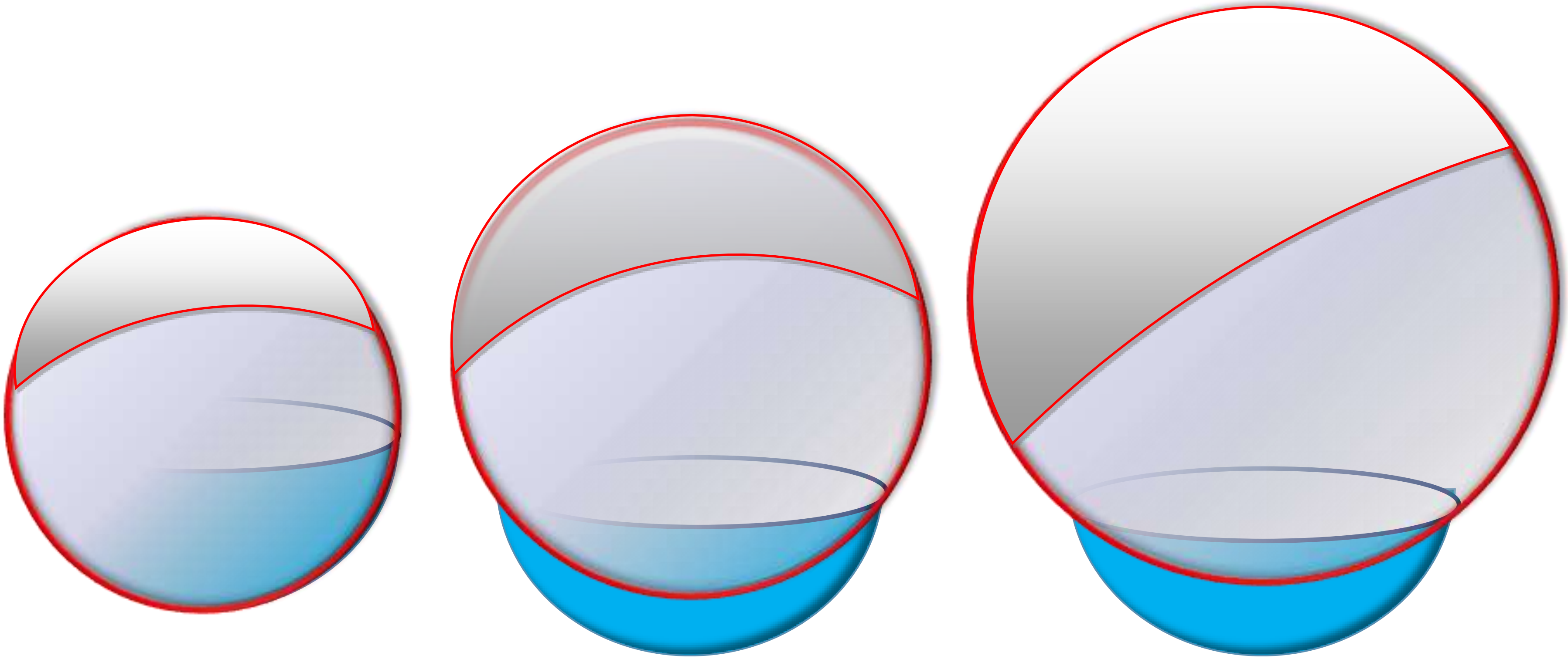
which is plane i.e circle.

4. **Spheres passing through the circle, $x^2+y^2+z^2+2ux+2vy+2wz+d=0$,**

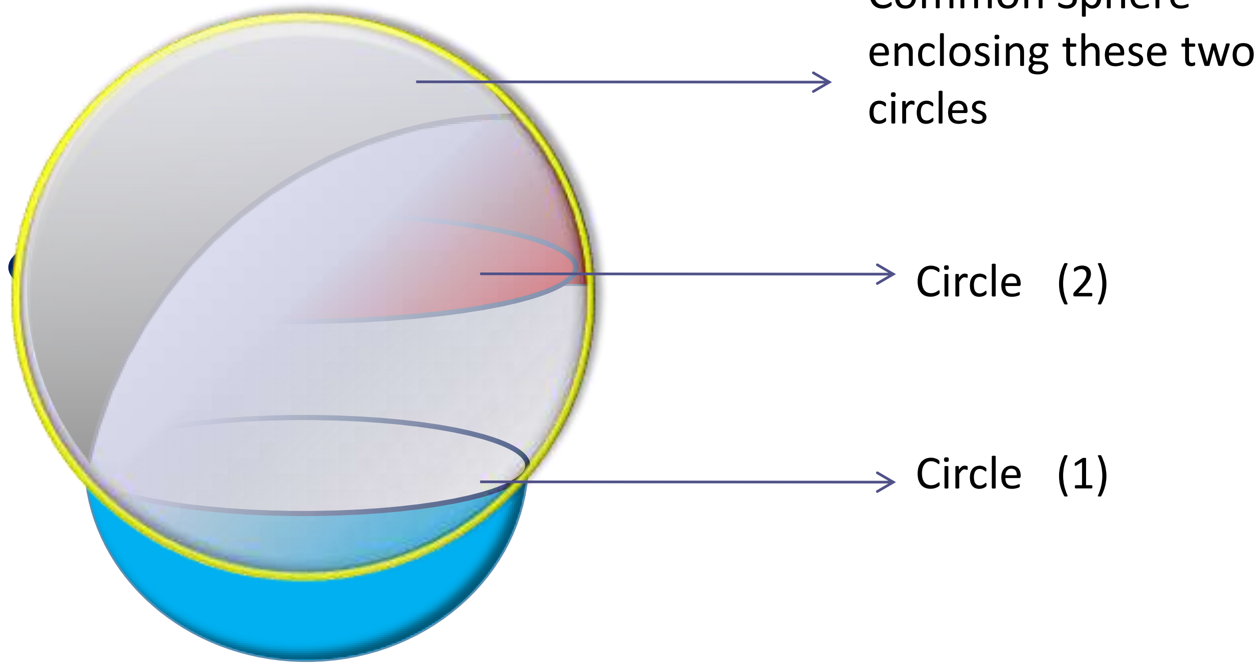
$$ax+by+cz+p=0$$



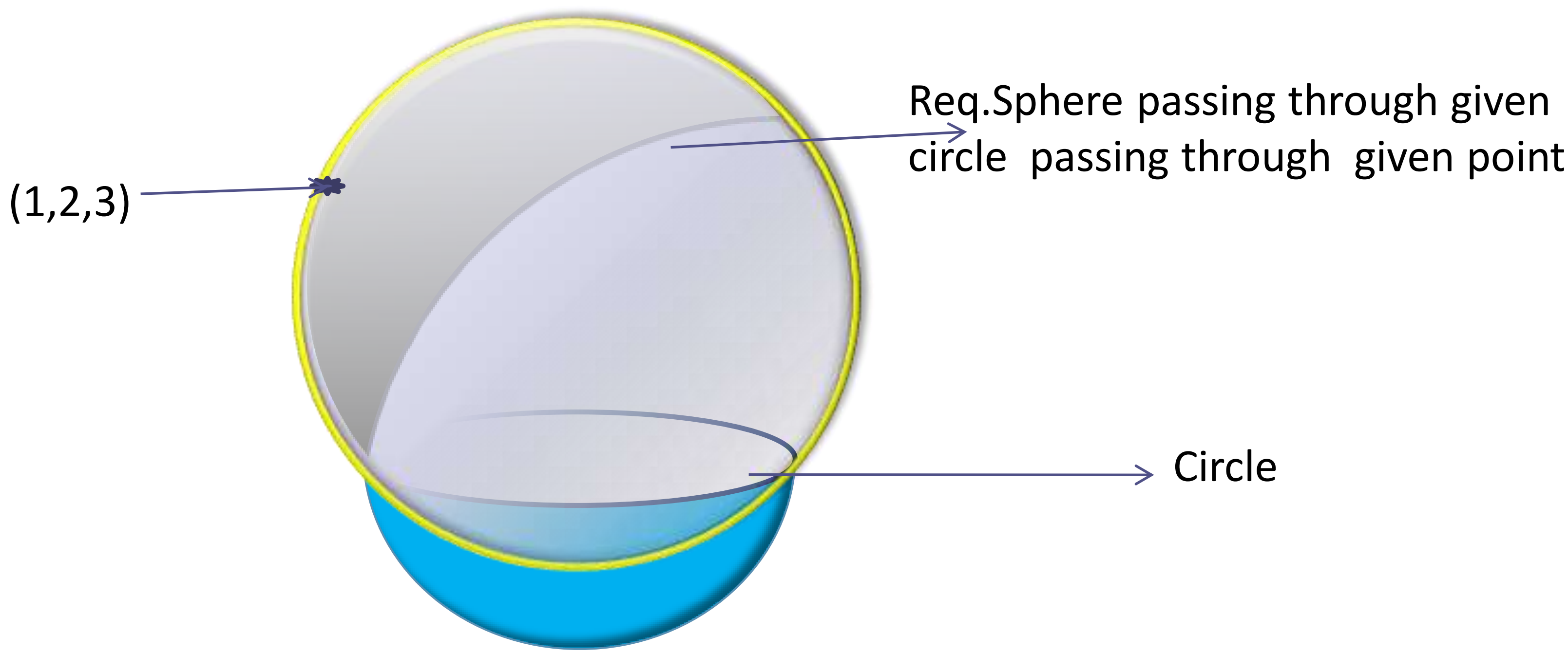
4. **Spheres passing through the circle,**
**S :$x^2+y^2+z^2+2ux+2vy+2wz+d=0$ and**
**P: $ax+by+cz+p=0$ is $S+\lambda P=0$**



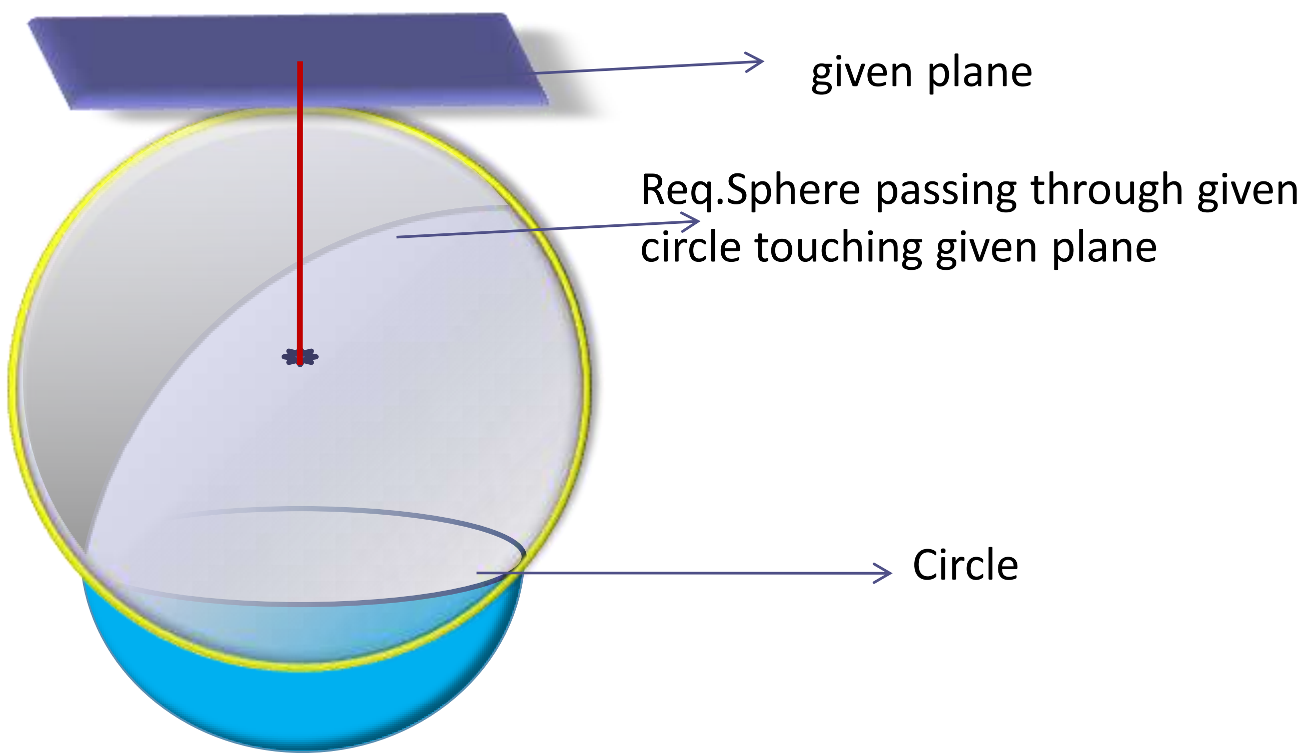
# 5. Sphere which is common for two circles



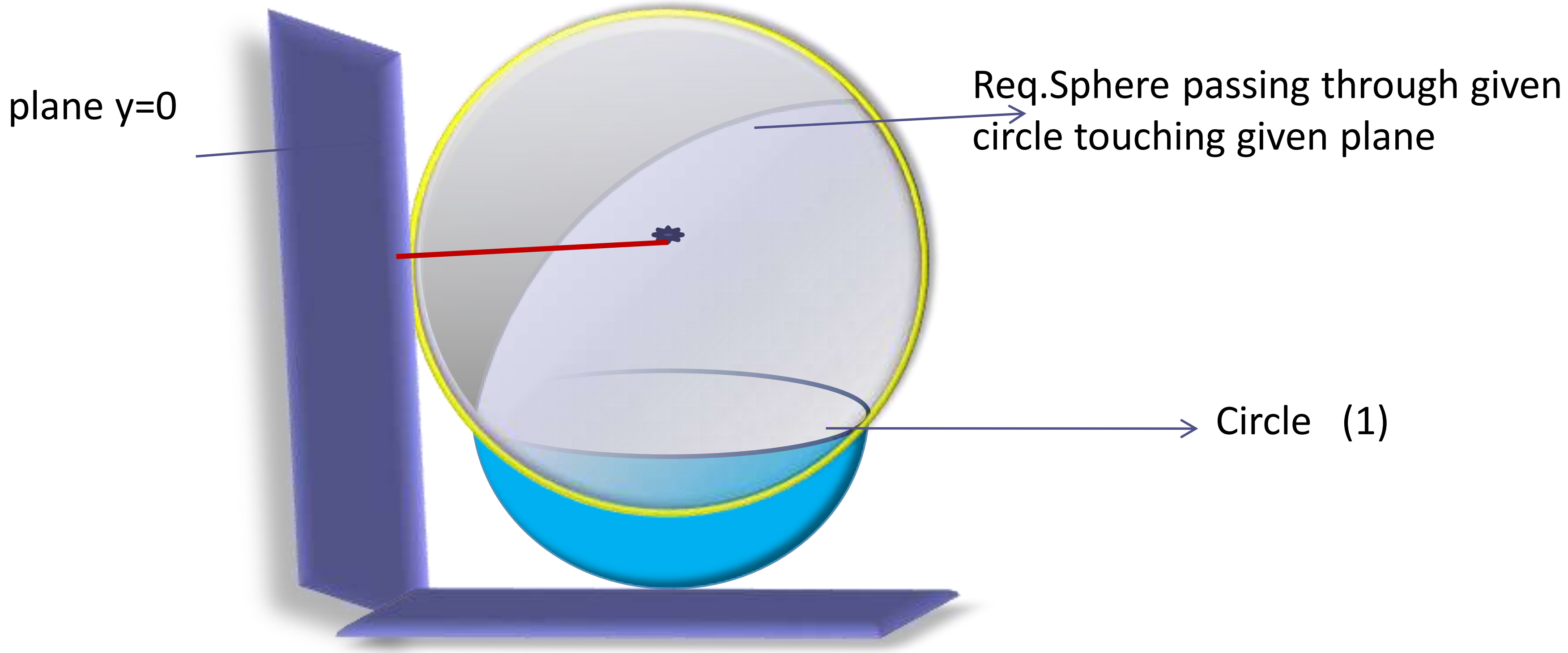
Eg . Find equation of **Sphere passing through circle $x^2+y^2+z^2=9$ and $2x+3y+4z=5$ and passing through the point (1,2,3)**



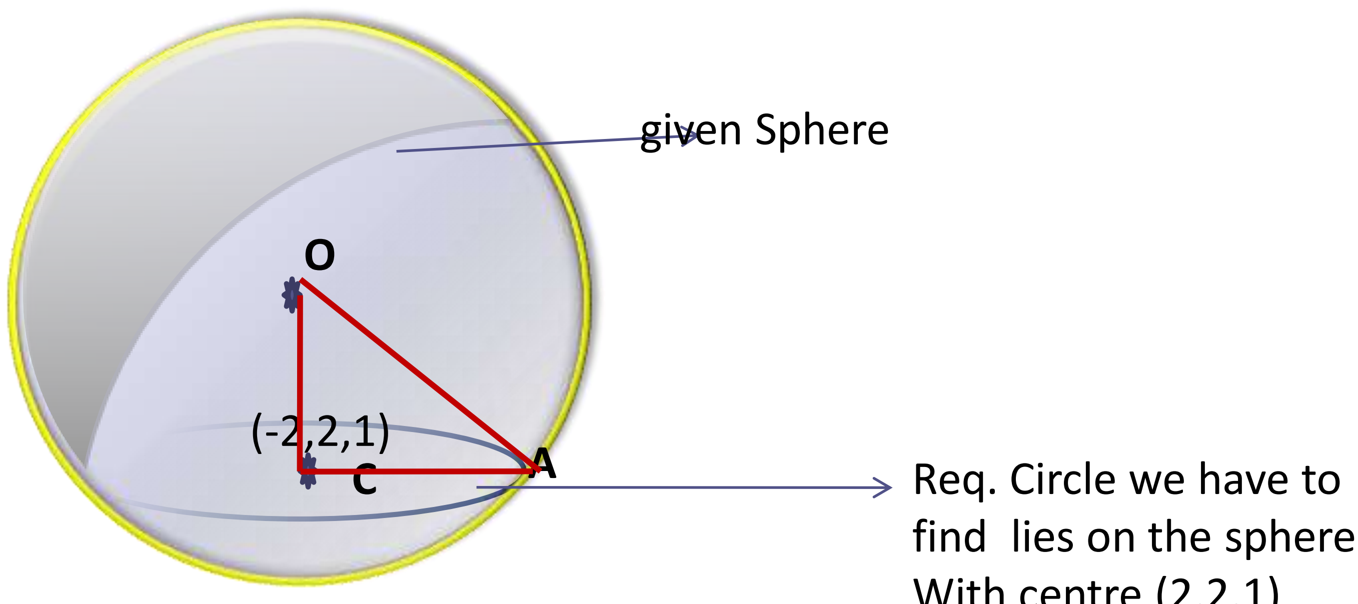
Eg . Find equation of **Sphere passing through circle $x^2+y^2+z^2-2x+2y+4z-3=0$ and $3x+4y=14$ and touching the plane $2x+y+z+4=0$**



Eg . Find equation of **Sphere passing through circle $x^2+y^2+z^2 -2x+2z=2$ and $y+z=7$ and touching the plane $y=0$**



Eg . Find equation to the circle which has centre at (-2,2,1) lies on the sphere $x^2+y^2+z^2+5x-7y+2z-8=0$.



# KLE's G I Bagewadi College, Nipani

Seminar Topic : Companies Act

Name: Abhilasha Koot

Class : B.Com I Sem

# THE COMPANIES ACT

The review and redrafting of the Companies Act, 1956 was taken up by the Ministry of Corporate Affairs on the basis of a detailed consultative process. The Companies Act, 2013 was passed by Lok Sabha on the 18th of December 2012 and passed by the Rajya Sabha on 8th August 2013 and is all set to replace the 57 year old Companies Act, 1956. The Companies Act, 2013 received the assent of the president on 29th August, 2013 and was notified in the Gazette of India on 30th August, 2013.

# THE BACKGROUND OF COMPANIES ACT, 2013

A brief background to the Introduction and status of the Companies Act 2013 is as under:  Companies (Amendment) Bill, 2003 had been introduced by Ministry of Corporate Affairs (MCA) (then Department of Company Affairs) in the Rajya Sabha on 7.5.2003.  Later on, a large number of changes were found to be necessary in the Bill. The Ministry of Corporate Affairs took up a comprehensive revision of the Companies Act, 1956 (the Act) in 2004.

- A ‗Concept Paper on new Company Law‘ was placed on the website of the Ministry on 4th August, 2004. The inputs received were put to a detailed examination in the Ministry. The Government also constituted an Expert Committee on Company Law under the Chairmanship of Dr. J.J. Irani on 2nd 4 December 2004 to advise on new Companies Bill. The Committee submitted its report to the Government on 31st May 2005

- Companies Bill 2008 was introduced by the Government in the Lok Sabha on October 23, 2008.

- Due to dissolution of the 14th Lok Sabha, the Companies Bill, 2008 lapsed. The Government decided to re-introduce the Companies Bill, 2008 as the Companies Bill, 2009, without any change except for the Bill year and the Republic year. The Ministry of Corporate Affairs had introduced the Companies Bill, 2009 in the Lok Sabha on August 3, 2009.  The 2009 Bill was referred to the Parliamentary Standing Committee on Finance on 9th September, 2009 which gave its report on 31st August, 2010.

- The standing committee, headed by the then finance minister, Yashwant Sinha, had given its recommendations on the Companies Bill, 2009, which has since been withdrawn (The Companies Act of 2013 incorporates 162 recommendations made by the Standing Committee.)  In view of numerous amendments to the Companies Bill, 2009 arising out of the recommendations of the Parliamentary Standing Committee on Finance and suggestions of the stakeholders, the Central Government withdrew the Companies Bill 2009 and introduced a fresh bill – The Companies Bill, 2011.

- The 2011 bill was introduced in Parliament on Wednesday, 14th December 2011.
- The Companies Bill, 2011 was referred to the Standing Committee on Finance on 5 th January, 2012 after an objection was raised against it in Parliament.  In the meanwhile, a corrigendum to the Companies Bill, 2011 had been issued that contained some changes of a substantive nature.  The Standing Committee Report came on 26th June 2012.

# THANKING YOU

# KLE’s G I Bagewadi College, Nipani

Seminar Topic :Amalgamation in the nature of merger

Name: Aishawarya Sanjay Jabade

Class : B.Com IV Sem

# What is Amalgamation?

- Amalgamation is defined as the combination of one or more companies into a new entity. It includes:
- Two or more companies join to form a new company
- Absorption or blending of one by the other
- Thereby, amalgamation includes absorption.
- However, one should remember that Amalgamation as its name suggests, is nothing but two companies becoming one. On the other hand, Absorption is the process in which the one powerful company takes control over the weaker company.
- Generally, Amalgamation is done between two or more companies engaged in the same line of activity or has some synergy in their operations. Again the companies may also combine for diversification of activities or for expansion of services
- Transfer or Company means the company which is amalgamated into another company; while Transfer Company means the company into which the transfer or company is amalgamated.

# Amalgamation in the Nature of Merger

- In this type of amalgamation, not only is the pooling of assets and liabilities is done but also of the shareholders’ interests and the businesses of these companies. In other words, all assets and liabilities of the transferor company become that of the transfer company. In this case, the business of the transfer or company is intended to be carried on after the amalgamation. There are no adjustments intended to be made to the book values. The other conditions that need to be fulfilled include that the shareholders of the vendor company holding atleast 90% face value of equity shares become the shareholders’ of the vendee company.

# Procedure for Amalgamation

- The terms of amalgamation are finalized by the board of directors of the amalgamating companies.
- A scheme of amalgamation is prepared and submitted for approval to the respective High Court.
- Approval of the shareholders’ of the constituent companies is obtained followed by approval of SEBI.
- A new company is formed and shares are issued to the shareholders’ of the transferor company.
- The transferor company is then liquidated and all the assets and liabilities are taken over by the transferee company.

# Why Amalgamate?

- To acquire cash resources
- Eliminate competition
- Tax savings
- Economies of large scale operations
- Increase shareholders value
- To reduce the degree of risk by diversification
- Managerial effectiveness
- To achieve growth and gain financially

# THANK YOU

# KLE’s G I Bagewadi College, Nipani

Seminar topic : Roles of RBI

Name: Akshata Hiremath

Class :M.Com IV sem

2019-2020

# Main Role and Functions of RBI

- **Monetary Authority**: Formulates, implements and monitors the monetary policy for A) maintaining price stability, keeping inflation in check ; B) ensuring adequate flow of credit to productive sectors.
- **Regulator and supervisor of the financial system:** lays out parameters of banking operations within which the country"s banking and financial system functions for- A) maintaining public confidence in the system, B) protecting depositors' interest ; C) providing cost-effective banking services to the general public.

- **Regulator and supervisor of the payment systems**: A) Authorises setting up of payment systems; B) Lays down standards for working of the payment system; C)lays down policies for encouraging the movement from paper-based payment systems to electronic modes of payments. D) Setting up of the regulatory framework of newer payment methods. E) Enhancement of customer convenience in payment systems. F) Improving security and efficiency in modes of payment

- **Manager of Foreign Exchange**: RBI manages forex under the FEMA- Foreign Exchange Management Act, 1999. in order to A) facilitate external trade and payment B) promote the development of foreign exchange market in India.
- **Issuer of currency:** RBI issues and exchanges currency as well as destroys currency & coins not fit for circulation to ensure that the public has an adequate quantity of supplies of currency notes and in good quality.
- **Developmental role :** RBI performs a wide range of promotional functions to support national objectives. Under this it setup institutions like NABARD, IDBI, SIDBI, NHB, etc.

- **Developmental role :** RBI performs a wide range of promotional functions to support national objectives. Under this it setup institutions like NABARD, IDBI, SIDBI, NHB, etc.
- **Banker to the Government:** performs merchant banking function for the central and the state governments; also acts as their banker.
- **Banker to banks:** An important role and function of RBI is to maintain the banking accounts of all scheduled banks and acts as the banker of last resort.
- An agent of Government of India in the IMF.

THANKYOU

# KLE's G I Bagewadi College, Nipani

Seminar Topic : Memorandum of Association

Name: Asmita Mengane

Class : B.Com I Sem

# MEMORANDUM OF ASSOCIATION

The Memorandum of Association or MOA of a company defines the constitution and the scope of powers of the company. In simple words, the MOA is the foundation on which the company is built. In this article, we will look at the laws and regulations that govern the MOA. Also, we will understand the contents of the Memorandum of Association of a company.

# OBJECT OF REGISTERING A MEMORANDUM OF ASSOCIATION OR MOA

The MOA of a company contains the object for which the company is formed. It identifies the scope of its operations and determines the boundaries it cannot cross.

It is a public document according to Section 399 of the Companies Act, 2013. Hence, any person who enters into a contract with the company is expected to have knowledge of the MOA.

It contains details about the powers and rights of the company.

# Format of Memorandum of Association (MOA)

- According to Section 4 of the Companies Act, 2013, companies must draw the MOA in the form given in Tables A-E in Schedule I of the Act. Here are the details of the forms:
- **Table A:** Form for the memorandum of association of a company limited by shares.
- **Table B:** Form for the memorandum of association of a company limited by guarantee and not having a share capital.
- **Table C:** Form for the memorandum of association of a company limited by guarantee and having a share capital.
- **Table D:** Form for the memorandum of association of an unlimited company.
- **Table E:** Form for the memorandum of association of an unlimited company and having share capital.

**Keep in mind the following aspects before submitting the MOA:**
**Print the MOA**

•Divide it into paragraphs

•Number the pages in sequence

•Ensure that at least seven people sign it (2 in the case of a private limited company and one in case of a One Person company).

•Have at least one witness to attest the signatures

•Enter particulars about the signatories and witnesses like address, description, occupation, etc.

Thanking you

# DATA COLLECTION

PRESENTED BY: VAISHALI MIRJE
M.COM III SEM

## ❑ Definition of Data Collection

**"Data collection is a process by which the researcher collects the information from all the relevant sources to find answers to the research problem, test hypothesis and evaluate the outcomes".**

## ❑ Methods of data collection

- **Primary Data**
- **Secondary Data**

○ **Explanation**

- **Primary Data**

**When the data are collected directly by the researcher for the first time is called as primary data. It is original in nature and is specific to a research problem. It is also called as first hand data.**

- **Secondary Data**

When the data is collected by someone else for his research work and has already passed through the statistical analysis is called the secondary data.

Thus, the secondary data is the second–hand data which is readily available from the other sources.

## ❑ Sources of Data

- **Primary Data**

**Methods of collection of primary data**

- ❖ Direct Personal Interview
- ❖ Indirect Oral Interview
- ❖ Information through agencies
- ❖ Mailed questionnaires
- ❖ Schedules sent through enumerators

- **Secondary data**

**These are readily available from the other sources and as such, there are no specific collection methods. The researcher can obtain data from the sources both internal and external to the organization.**

- **Internal sources of secondary data are:**
  - **Sales report**
  - **Financial statements**
  - **Customers details like name, age, contact details etc.**
  - **Company information**
  - **Reports and feedback from a dealer, retailer and distributor**
  - **Management Information System (MIS)**

- **External sources of secondary data:**
  - **Govt. censuses like the population census, agriculture census etc.**
  - **Information from other govt. departments like social security, tax records etc.**
  - **Business journals**
  - **Business magazines**
  - **Libraries**
  - **Internet**

## Tools of Data collection

**Different tools used for data collection may be ;**

- **Questionnaires**
- **Interviews**
- **Schedules**
- **Observation**

- **Questionnaires**

It may be defined as;

"A questionnaire is a systematic compilation of questions that are submitted to a sampling of population from which information is desired".

- **Merits**
- It is very economical
- It is a time saving process
- It covers the research in wide area
- It is most reliable in special cases
- **Demerits**
- Through this we get only limited responses
- Greater possibility of wrong answers
- Sometimes answers may be illegible
- Chances of receiving incomplete records are more

- **Interview**

  **It may be defined as;**

  **"Interviewing is fundamentally a process of social interaction".**

  - **Merits**
    - **Direct research**
    - **Deep research**
    - **Knowledge of past and future**
    - **Knowledge of special features**
    - **Examination of known data**
  - **Demerits**
    - **Misleading information**
    - **Defects due to interviewee**
    - **Incomplete research**

- **Schedules**

  **It may be defined as;**

  **"Schedule is the name usually applied to set of questions, which are asked and filled by an interviewer in a face to face situation with another".**

- **Points to be kept in mind while designing the schedules**
  - **Interviewer should not frame long, complex and defective questions.**
  - **Unrelated and unnecessary questions should not be asked.**
  - **Schedules should not contain personal and upsetting questions.**
  - **Its questions should be simple, clear and relevant to topic**
  - **Questions be suitable to respondents intelligence level.**

## Merits

- Higher percentages of responses
- Possible to observe personality factors
- Removal of Doubts
- It is possible to know about the defects of the interviewee

## ❑ Steps or procedures to collect data

- **Identify opportunities for collecting data**
- **Select opportunities and goals**
- **Plan an approach and methods**
  - **Who will the data be collected about?**
  - **What locations or geographical areas will the data be gathered about?**
  - **How should data be collected?**
    - **Qualitative data**
    - **Quantitative data**

- **What sources of data should be used to collect information?**
  - **Pre-existing or official data**
  - **Survey data**
  - **Interviews and focus group**
  - **Observed data**
- **How long will the data be collected?**

- **Collect data**
- **Analyze and interpret data**
- **Act on result**

## ❑ Ways to collect data

- **Surveys**
- **Online Tracking**
- **Transactional data tracking**
- **Online marketing analytics**
- **Social media monitoring**
- **Collecting subscription and registration data**
- **In store traffic monitoring**

# KLE's G I Bagewadi College, Nipani

Seminar Topic : Team Building

Name: Danamma Shedabale

Class :M.Com I Sem

2019-2020

# Team Building

- **Definition**: Team building is a management technique used for improving the efficiency and performance of the workgroups through various activities. It involves a lot of skills, analysis and observation for forming a strong and capable team. The whole sole motive here is to achieve the organization vision and objectives.

# How to Build a Great Team?

- Forming a great team requires a lot of skills and presence of mind. Usually, some managers specialize in team-building skills and are hired by the companies on this parameter.
- The manager responsible for team building must be able to find out the strengths and weaknesses of the team members and create the right mix of people with different skill sets. He must focus on developing strong interpersonal relations and trust among the team members.
- The manager must encourage communication and interaction among the team members and also reduce stress with the help of various team-building activities.
- He must clearly define the goals and objectives of the organization to the team members. He must also specify the role of each member in the team to direct them towards the achievement of the organizational goals

# Content: Team Building

- Process
  Advantages
  Disadvantages
  Example

# Team Building Process

- Team building is not a one-time act. It is a step by step process which aims at bringing a desirable change in the organization. Teams are usually formed for a particular task or project and are mostly for the short term

- **Identify the Need for Team Building**
- The manager has first to analyze the requirement of a team for completing a particular task. It should find out the purpose of the work to be performed, required skills for the job and its complexity before forming a team.
- **Define Objectives and Required Set of Skills**
- Next comes the chalking down of the organizational objectives and the skills needed to fulfil it.
- **Consider Team Roles**
- The manager considers the various aspects, i.e. the interactions among the individuals, their roles and responsibilities, strengths and weaknesses, composition and suitability of the possible team members.
- **Determine a Team Building Strategy**

- Now, the manager has to understand the operational framework well to ensure an effective team building. He must himself be assured of the objectives, roles, responsibilities, duration, availability of resources, training, the flow of information, feedback and building trust in the team.
- **Develop a Team of Individuals**
- At this stage, the individuals are collected to form a team together. Each member is made familiar with his roles and responsibilities within the team.
- **Establish and Communicate the Rules**
- The rules regarding the reporting of team members, meeting schedules, and decision making within the team are discussed. The individuals are encouraged to ask questions and give their views to develop open and healthy communication in the team.
- **Identify Individual's Strengths**
- Various team-building exercises are conducted to bring out the strengths of the individuals. It also helps in familiarizing the team members with each other's strengths and weakness.
- **Be a Part of the Team**
- At this point, the manager needs to get involved with the team as a member and not as a boss. Making the individuals realize their importance in the team and treating each member equally is necessary. The team members should see their manager as their team leader, mentor and role model

# Advantages of Team Building

- **Identify Strengths and Weaknesses**: Through team-building exercises, the strengths and weaknesses of each member can be identified. In day to day routine work, such an analysis cannot be done. These competencies can be used by the managers to form effective teams.
- **Direct Towards Vision and Mission**: Team building activities define the role and importance of the team for the organization to reach its vision. It makes the individuals understand the organization's goals, objectives, mission and vision very clearly and motivates them to contribute towards it.
- **Develops Communication and Collaboration**: Team building activities enhance the interpersonal relations of the team members. It makes individuals comfortable and familiar with one another. Collaboration develops trust and understanding among the team members.

- **Establishes Roles and Responsibilities**: It defines and clarifies the role of each member of a team. Moreover, the members are given individual responsibilities, along with the motivation of performing as a team.
- **Initiates Creative Thinking and Problem Solving**: In a team, individuals are motivated to give their views, opinions and solution to a particular problem. It leads to brainstorming and exploring their creative side.
- **Builds Trust and Morale**: By conducting team building activities, the organization makes the employees feel valued. It encourages them to develop their skills and build strong interpersonal relations, ultimately boosting the morale and trust of the team members

# Disadvantages of Team Building

- **Develops Conflict**: Sometimes, the team lacks coordination and understanding among its members. This leads to conflict and clashes within the team and hence decreases the efficiency and productivity. A lot of time is wasted in such conflict management.
- **Unproductive or Freeride Team Members**: At times, some of the team members do not contribute much to team performance. Such individuals are considered to be freeriding team members. They prove to be inefficient and less productive for the team.
- **May Lead to Non-Cooperation**: Every individual is different from one another. The team members sometimes lack cooperation and unity. This non-cooperation among the team members leads to wastage of efforts and hinders the performance of the team as a whole.
- **Difficult to Evaluate Individual Performance**: Whatever the result or the outcome the organization gets by team building is the team's achievement or failure. Usually, the organization overlooks the contribution of each member individually while rewarding the efforts of the whole team.

- **Involves Cost**: Team building activities require time and money. Moreover, a lot of time, cost and resources are consumed in ensuring coordination, balance, feedback, decision making and conflict management within the teams formed.
- **Accountability and Credibility Issues**: In case of failure, it becomes difficult to find out the reason. The team members sometimes do take up the accountability of their work, holding the other members to be responsible for the unfavourable outcome.
- In case of success, the team members get busy in taking up the credit themselves, ignoring the efforts of the whole team together.

# THANK YOU

# KLE's G I Bagewadi College, Nipani

Seminar topic :Commercial Banks

Name:Deeapa kadadevaramath

Class :M.Com IV sem
2019-2020

# What Is a Commercial Bank?

The term commercial bank refers to a financial institution that accepts deposits, offers checking account services, makes various loans, and offers basic financial products like certificates of deposit (CDs) and savings accounts to individuals and small businesses. A commercial bank is where most people do their banking.
Commercial banks make money by providing and earning interest from loans such as mortgages, auto loans, business loans, and personal loans. Customer deposits provide banks with the capital to make these loans.

**Commercial banks are of three types, which are as follows:**

**Public Sector Banks:**
**Private Sector Banks:**
**Foreign Banks:**

- **Public Sector Banks:**

Refer to a type of commercial banks that are nationalized by the government of a country. In public sector banks, the major stake is held by the government. In India, public sector banks operate under the guidelines of Reserve Bank of India (RBI), which is the central bank. Some of the Indian public sector banks are State Bank of India (SBI), Corporation Bank, Bank of Baroda, Dena Bank, and Punjab National Bank.

- **Private Sector Banks:**

Refer to a kind of commercial banks in which major part of share capital is held by private businesses and individuals. These banks are registered as companies with limited liability. Some of the Indian private sector banks are Vysya Bank, Industrial Credit and Investment Corporation of India (ICICI) Bank, and Housing Development Finance Corporation (HDFC) Bank

- **Foreign Banks:**

Refer to commercial banks that are headquartered in a foreign country, but operate branches in different countries. Some of the foreign banks operating in India are Hong Kong and Shanghai Banking Corporation (HSBC), Citibank, American Express Bank, Standard & Chartered Bank, and Grindlay’s Bank. In India, since financial reforms of 1991, there is a rapid increase in the number of foreign banks. Commercial banks mark significant importance in the economic development of a country as well as serving the financial requirements of the general public.

THANK YOU

# KLE's G I Bagewadi College, Nipani

Seminar Topic : Single Entry System

Name: Ganesh Deshinge

Class : B.Com I Sem

# SINGLE ENTRY SYSTEM

# Meaning and definition:

The single entry system is defined as "a system of maintaining business transactions in a manner convenient to a particular trader; where the principle of double entry system is not applied for all the transactions."

It is incomplete, unscientific, inaccurate and unsatisfactory system of recording the business transactions.

Single entry system is a system of book-keeping in which each and every aspect is not kept.

Single entry system is a system which is not a complete double entry system.

It is a system in which every debit has no corresponding credit or vice-versa.

## Features or essentials or characteristics of single entry system:

1. It is incomplete, unscientific, inaccurate and unsatisfactory system.
2. Each and every aspect is not to be recorded.
3. Under this system personal accounts of debtors and creditors and cash and bank accounts are commonly maintained.
4. For majority transactions only one aspect is recorded and so, this system is popularly called as single entry system.
5. No hard and fast rules are followed.
6. It is simple and easy method.

7. It is less costly method.
8. It is suitable to petty businessmen.
9. It is less time consuming method.
10. It is not possible to prepare the trial balance and thereby we cannot verify the arithmetical accuracy of books of accounts.
11. It is not possible to ascertain the correct profit or loss of the business.
12. It is not possible to ascertain the correct financial position of the business.
13. It gives much scope for misappropriation and fraud.

# Merits or advantages:

1. It is simple and easy method.
2. It is less costly method.
3. It is less time consuming method.
4. It is suitable to petty businessmen

# Demerits or disadvantages:

1. It is incomplete, unscientific, inaccurate and unsatisfactory system.
2. It is not possible to prepare the trial balance and thereby we cannot verify the arithmetical accuracy of books of accounts.
3. It is not possible to ascertain the correct profit or loss of the business.
4. It is not possible to ascertain the correct financial position of the business.
5. It is not suitable to big businessmen.
6. It gives much scope for misappropriation and fraud.

## Types or Methods of single entry system:

On the basis of books of accounts maintained, single entry system can be divided in to three types. They are:

1. **Pure single entry system:**

   It is a system under which only the personal accounts of debtors and creditors are maintained. And no subsidiary books are maintained.

2. **Simple single entry system:**

   It is a system under which only one subsidiary book, namely, the cash book and a ledger containing only the personal accounts of debtors and creditors are maintained.

**3. Quasi or semi- single entry system:**

It is a system under which more than one subsidiary book ( usually, the cash book, the purchases book, the sales book, the purchases returns book and the sales returns book) and a ledger containing only the personal accounts of debtors and creditors are maintained. The other the personal accounts, real accounts and the nominal accounts are not maintained.

# THANK YOU

# KLE’s G I Bagewadi College, Nipani

Seminar Topic : Mental Health

Name :Gopal Shingade

Class : M.Com IV Sem

# Mental health

- Mental health is a vital component of the total health of an individual because our entire thought process takes place in mind, ideas originate in mind and all kinds of directions are issued from mind which guide, shape and regulate communication, conduct and behavior and determine personal and social functioning as well as adjustment

# Definition of mental health

- According to Maslow (1970) people who have fulfilled their potentialities to the greatest degree will lead us to the formulation of a ‘positive psychology’ and will rid us from the negative approaches. He is always concerned to study the best, the healthiest and the most mature side of human nature.

# Factors influencing mental health

- 1. **Individual factors include a person's biologic make up having a sense of harmony in one's life, vitality, finding meaning in life, emotional resilience or hardiness, spirituality, having positive identity.**
- 2. **Interpersonal factors include effective communication, helping others, intimacy and maintaining a balance of separateness and connection (sense of belongingness), family and social support.**

- 3. **Social-cultural factors include having a sense of communication, access to adequate resources, intolerance of violence, social organization, time orientation, environmental control.**
- 4. **Self-esteem plays a significant role in determining mental health, people with high self-esteem experience less stress and strain and shoulder their responsibilities very well.**

- 5. **Internal locus of control is associated with mental health. They take responsibility for their own actions and view themselves as having control over their destiny. They are managed by themselves rather than by external factors.**

- 6. **Emotional intelligence is positively higher related with general health, healthy coping style, empathy, happiness, whole constructs like alexithymia, neuroticism, stressful events and mood fluctuations are negatively correlated.**

# THANK YOU

# KLE’s G I Bagewadi College, Nipani

Seminar Topic :Departmental Accounts

Name: Karishma Borgave

Class : B.Com I Sem

# DEPARTMENTAL ACCOUNTS

# Meaning:

A big business concerns dealing in different kinds of goods or services is usually divided into a number of departments.

A business having a number of departments each specializing in a particular line of activity is called departmental undertaking.

Under one management and under one roof the different goods and services are rendered is called departmental undertaking.

The accounts relating to different goods or services are called departmental accounting

# NEED FOR DEPARTMETNAL UNDERTAKING (OBJECTIVES OR ADVANTAGES):

1. To ascertain the result of each department
2. To compare the trading result of one department with the another department
3. To take necessary steps either to improve the department which is under loss or to close down all together the department which is under loss.

4. To evaluate the performance of each department

5. To reward the departmental managerial staff on the basis of trading results.

6. To have effective managerial control over the working of each department

# APPORTIONMENT OR ALLOCATION OF COMMON EXPENSES:

Apportionment of expenses means allocation of common expenses among the different departments on suitable basis

# APPORTIONMENT OR ALLOCATION OF COMMON EXPENSES:

| Sl. No | BASIS OF ALLOCATION | COMMON EXPENSES |
|---|---|---|
| 01 | Net purchase ratio ( Total purchases minus Purchase returns or returns outwards) | Carriage inwards, fright, octroi, duty etc |
| 02 | Net sales ratio (Total sales minus sales returns or returns inwards) | Commission on sales, discount allowed, carriage outwards, bad debts, RDD, RFDD, advertisement, sales tax etc |
| 03 | Staff appointed ratio (No of employees) | Salary, wages, labour welfare, canteen expenses etc. |

| | | |
|---|---|---|
| **04** | **Space or area occupied** | **Rent, rates, taxies, insurance on building, repairs of building, depreciation on building etc.** |
| 05 | Closing stock ratio | Insurance on goods, godown rent etc |
| 06 | Time spent or time devoted ratio | Salary of works manager |
| 07 | Value of assets ratio | Depreciation, repairs, maintenance of assets |
| 08 | Units consumed ratio | Lighting and heating, power, motive power, electricity etc. |

# IMPORTANT NOTES:

1. If the basis for allocation in not given in the problem, then those expenses should be apportioned on the basis of sales ratio ( turnover ratio)
2. However, there are certain common indirect expenses which cannot be apportioned on any one of the above basis. Such expenses should be directly recorded in the General Profit and Loss Account, which is meant for the entire organization. Such expenses are bank interest, accountancy charges, audit fees, income tax, and insurance on comprehensive policy.

# APPORTIONMENT OR ALLOCATION OF COMMON Incomes:

| Sl. No | BASIS OF ALLOCATION | COMMON INCOMES |
|---|---|---|
| 01 | Net purchase ratio | Discount received, commission earned |
| 02 | According to given ratio or allocated equally | Interest from bank, interest on investments etc. |

# INTERDEPARTMENTAL TRANSFER OR TRANSACTIONS:

Transfer of goods and services from one department to another department is called interdepartmental transactions.

# Treatment:

1. Interdepartmental transfer of goods from one department to another department is appearing in the trading account. The entry is:

   Receiving Department A/c ..............................Dr

   To Giving Department A/c

2. Interdepartmental transfer of service from one department to another department is appearing in the profit and loss account. The entry is:

   Receiving Department A/c ..............................Dr

   To Giving Department A/c

PURCHASES BOOK: It is a book meant for recording only credit purchases of goods.

Note: 1. Cash purchases of goods are appearing in the cash book.

2. Credit purchases of the things other than the goods are appearing in the journal proper.

SALES BOOK: It is a book meant for recording the credit sales of goods only.

Note: 1. Cash sales of goods are appearing in the cash book.

2. Credit sales of the things other than the goods are appearing in the journal proper.

THANK YOU

# KLE's G I Bagewadi College, Nipani

Seminar Topic : Nature of Contract

Name : Kiran Suresh Naik

Class : B.Com V Sem

# Business Law or Mercantile Law

## Introduction:

Man is a social being. He lives in society with his fellow beings. When living so, he has to observe a code of conduct or a set of rules for peaceful living and welfare of the whole society.

**These rules of conduct, when recognised by the State and enforced by it on people are termed as Law.**

Such law is not static. It changes when circumstances and conditions in the society change. Law is therefore dynamic.

# Need for the Knowledge of Law:

- Ignorance of law is not an excuse. Though it is not possible for a common man to learn every branch of law, yet he must know at least the general principles of the law of his country.
- Knowledge of Mercantile Law or Business Law is essential to people engaged in various economic and commercial activities i.e. business activities.
- The general knowledge of mercantile law will certainly help businessmen to solve their business problems and avoid conflicts with others.

# Meaning and Scope:

- The term ‘**Mercantile or Business Law**’ may be defined as that branch of law which deals with the rights and obligations arising out of mercantile or business transactions between businessmen.
- It consists of those rules that govern and regulate trade, commerce and industry. It is one of the important branches of Civil Law. Mercantile law is also known as Business Law.

- The scope of mercantile law is very wide and varied. It includes law relating to contracts, partnership, companies, sale of goods, negotiable instruments, carriage of goods, insolvency and arbitration and applies not only to businessmen but also to bankers and other professional men as well as to common people. Hence it is also known as Business Law.

# Law of Contract

## Introduction:

- Law of Contract is one of the most important branches of mercantile law. It is the foundation of modern business.
- In business, promises are made at one time and are performed at another time. To see that the promises made are duly performed by the parties to the Contract and to carry on the business smoothly, the law of Contract came into force.
- The law of Contract lays down the rules relating to promise, their formation, their performance and their enforceability.

# Indian Contract Act 1872

- It determines the circumstances in which promise made by the parties to a contract shall be legally binding on them.
- All of us enter into a number of contracts everyday knowingly or unknowingly.
- Each contract creates some right and duties upon the contracting parties. Indian contract deals with the enforcement of these rights and duties upon the parties.

# Meaning of Contract:

- According to Section 2(h) of the Indian Contract Act 1872, “A contract is an agreement enforceable by law”.
- According to Fredrick Pollock, “Every agreement and promise enforceable by law is a contract”.
- A contract is legally binding agreement between two or more persons.
- A contract is an agreement between two or more parties which is enforceable at law.

From the above definitions, a contract essentially consists of two elements:

1. an agreement and
2. its legal enforceability i.e. legal obligation.

# KLE’s G I Bagewadi College, Nipani

Seminar Topic : Reconciliation of Profits

Name: Komal Bhimgouda Patil

Class : B.Com VI Sem

# Reconciliation of Cost and Financial Accounts

Meaning

In business concern where Non-integrated Accounting System is followed. cost and financial accounts are maintained separately, the difference between the end result of these two are required to be reconciled. Reconciliation of cost and financial accounts mean tallying the profit or loss revealed by both set of accounts. The chief aim is to find out the reasons for the difference between the results shown by Cost Accounts and Financial Accounts.

Reasons for the Difference

The various reasons which create difference between cost and financial profit or loss shown by the two set of books may be listed under the following heads :

(1) Items shown only in Financial Accounts
(2) Items shown only in Cost Accounts
(3) Absorption of Overheads
(4) Methods of Stock Valuation
(5) Abnormal Loss and Gains

- (1) Items shown only in Financial Accounts: Some items of income and expenses which are included
- only in financial accounts but are not shown in cost accounts and vice versa. The following items are shown
- in financial accounts but not in cost accounts:
- (A) Income:
- (1) Profit on sale of fixed assets
- (2) Interest received on investment
- (3) Dividend received on investment
- (4) Rent, brokerage and commission received
- *Reconciliation of Cost and Financial Accounts*
- (5) Premium on issue of shares
- (6) Transfer fees received.
- (B). Expenditure:
- (1) Loss on sale of fixed assets, e.g., Plant, Machinery, Building etc.
- (2) Interest paid
- (3) Discount paid
- (4) Dividend paid
- (5) Losses due to scrapping of plant and machinery
- (6) Penalties and,fines
- (7) Expenses of shares' transfer fees
- (8) Preliminary expenses written off
- (9) Damages payable at law.

- (2) Items shown only in Cost Accounts: There are some items which are recorded only in Cost Accounts
- but are not included in financial accounts, national interest on capital, notional rent of premises owned, salary
- to proprietor etc. are not recorded in financial account because the amount is not actually spent or paid. These
- expenses reduced the profit in cost account while in financial account it may be the reverse effect.
- (3) Absorption of Overheads : In financial accounts actual amount of expenses paid are recorded
- while in cost accounts overheads are charged at predetermined rates. If overhead charged are not equal to
- the amount of overhead incurred the under or over absorption of overhead leads to difference in profits of
- two accounts.
- (4) Methods of Stock Valuation: The term stock refers to opening or closing stock of raw materials,
- work in progress and finished goods. In financial accounts stocks are valued at cost price or market price
- whichever is lower. In Cost Account; stock of raw materials can be valued on the basis of FIFO, LIFO and
- Simple Average Method etc., and work in progress may be valued at Prime Cost or Work Cost. Finished
- stocks are generally valued on the basis of cost of production. Thus, the adoptation of different method of
- valuation of stock leads to difference in profits of two sets of accounts.
- (5) Abnormal Losses and Gains: Different items of abnormal wastages, losses or gains which are
- included in financial accounts but are not recorded in cost accounts. Thus, the figures of abnormal losses and
- gains may affect the results in financial accounts alone.

# THANK YOU

# Evaluating a Company's External Environment


Madhuri Kadapure
M.Com I Sem

# Thinking about Company's situation

- Thinking about external environment → Form a strategic vision (where)
- Thinking about internal environment → Form a strategic vision (where)

Form a strategic vision (where) → Identify promising strategic options → Select the best strategy & business model

# The strategically relevant components of a company's external environment

## Company's macro environment

all relevant factors & influences outside the company's boundaries

Relevant influences can sometimes impact on company's business situation, direction, & strategy.

ex) Cigarette company : antismoking ordinances

Motor vehicle company : fuel mileage, gasoline prices

Drug company : longer life expectancies

# Company's industry & competitive environment

## Task is much more focused

- Question 1. what are the industry's dominant economic features?
- Question 2. what competitive forces & how strong is each force?
- Question 3. What forces are driving industry change & impact on competitive intensity?
- Question 4. what market positions industry rivals occupy?
- Question 5. what strategic moves are rivals make next?
- Question 6. what are the key factors for future success?
- Question 7. the outlook offer a good opportunity to earn profits?

# Q1.What are the industry's dominant economic features?

- **Economic features are defined by such factors as market size & growth rate.**

**the number & sizes of buyers & sellers**

**the geographic boundaries of the market**

**the pace of technological change**

**the extent of vertical integration**

**cf.) Table 3.1 – Textbook**

**When an industry is characterized by important learning/experience curve effects, industry members are strongly motivated to adopt volume increasing strategies to capture the resulting cost-saving economies and maintain their competitiveness**

# Q.2 Is the industry competitively attractive or unattractive?

## ◆The Five-Forces Model of competition

# Competitive rivalry within an industry >>An ongoing and dynamic confrontation

| Threat of New entrance | Threat of Substitute product | Bargaining Power of suppliers | Bargaining Power of Buyers |
|---|---|---|---|
| >size of the pool.<br><br>>whether the Candidates face High or low Entry barriers | >available and attractive Price.<br>>the buyer Substitute image.<br>>the cost that Buyers incur in Switching to The substitutes. | >Influence the Terms and Condition of Supply.<br><br>>Industry Members-suppliers collaboration | >Negotiate Favorable terms<br><br>>Strategic Partnerships |

# Rivalry among competitive sellers

## ◆ Rivalry is generally stronger when:

- Competing seller are active in making fresh moves to improve their market standing and business performance.
- Buyer demand is growing slowly
- Buyer demand falls off and sellers find themselves with excess capacity and/or inventory.
- The number of rivals increases and rivals are of roughly equal size and competitive capability.
- Buyer costs to switch brands are low.
- The products of rival sellers are commodities or else weakly differentiated.
- One or two rivals have powerful strategies and other rivals are scrambling to stay in the game.

# Rivalry among competitive sellers

## ◆ Rivalry is generally weaker when:

- Industry members move only infrequently or in a nonaggressive manner to draw sales and market share away from rivals.
- Buyer demand is growing rapidly.
- The products of rival sellers are strongly differentiated and customer loyalty is high.
- Buyer costs to switch brands are high.
- They are fewer than 5 sellers or else so many rivals that any one company’s actions have little direct impact on rivals’ business.

# Threat of new entrance

- **Entry threats are generally stronger when:**
  - The pool of entry candidates is large and some have resources that would make them fomidable market contenders.
  - Entry barriers are low or can be readily hurdled by the likely entry candidates
  - When existing industry members are looking to expand their market reach by entering product segments or geographic areas where they currently do not have a presence.
  - Newcomers can expect to earn attractive profits.
  - Buyer demand is growing rapidly.
  - Industry members are unable to strongly contest the entry of newcomers.

# Threat of new entrance

◆ **Entry threats are generally weaker when:**

- The pool of entry candidates is small.
- Entry barriers are high.
- Existing competitors are struggling to earn healthy profits.
- The industry’s outlook is risky or uncertain.
- Buyer demand is growing slowly or is stagnant
- Industry members will strongly contest the efforts of new entrants to gain a market foothold.

# Threat of substitute products

- **Competitive pressures from substitute products are generally stronger when:**
  - Good substitutes are readily available or new ones are emerging.
  - Substitutes are attractively priced.
  - Substitutes have comparable or better performance features.
  - End users have low costs in switching to substitutes.
  - End users grow more comfortable with using substitutes.

# Threat of substitute products

- **Competitive pressures from substitute products are generally weaker when:**

  - Good substitutes are not readily available or don't exist.
  - Substitutes are higher priced relative to the performance they deliver.
  - End users have high costs in switching to substitutes.

# Bargaining Power of suppliers

- **Supplier bargaining power is generally stronger when:**
  - Industry members incur high costs in switching their purchases to alternative suppliers.
  - Needed inputs are in short supply (which gives suppliers more leverage in setting prices).
  - A supplier has differentiated input that enhances the quality or performance of sellers' products or is a valuable or critical part of sellers' production process.
  - There are only a few suppliers of a particular input.
  - Some suppliers threaten to integrate forward into the business of industry members and perhaps become a powerful rival.

# Bargaining Power of suppliers

- **Supplier bargaining power is generally weaker when:**
  - The item being supplied is a commodity, that is, an item readily available from many suppliers at the going market price.
  - good morning
  - Seller switching costs to alternative suppliers are low.
  - Good substitute inputs exist or new ones emerge.
  - There is a surge in the availability of supplies.
  - Industry members account for a big fraction of suppliers’ total sales and continued high volume purchases are important to the well-being of suppliers.
  - Industry members are a threat to integrate backward into the business of suppliers and to self-manufacture their own requirements.

# Bargaining Power of buyers

- **Buyer bargaining power is generally stronger when:**
  - Buyer switching costs to competing brands or substitute products are low.
  - Buyers are large and can demand concessions when purchasing large quantities.
  - Large volume purchases by buyers are important to sellers.
  - Buyer demand is weak or declining.
  - There are only a few buyers
  - Identity of buyers adds prestige to the seller's list of customers.
  - Quantity and quality of information available to buyers improves.

# Bargaining Power of buyers

## ◆ Buyer bargaining power is generally weaker when:

- Buyers purchase the item infrequently or in small quantities.
- Buyer switching costs to competing brands are high.
- There is a surge in buyer demand that creates a seller's market.
- A seller's brand reputation is important to a buyer.
- A particular seller's product delivers quality or performance that is very important to buyer and that is not matched in other brands.
- Buyer collaboration or partnering with selected sellers provides attractive win-win opportunities.

# Q3. what forces are driving industry change and what impacts will they have?

Driving forces

Driving industry participants

How the industry Landscape will Be altered

# Driving forces

- 1. Change in the long term industry growth rate
- 2. Increasing globalization
- 3. Emerging new Internet capabilities and applications
- 4. Change in who buy the product and how they use it
- 5. Product innovation
- 6. Technological change and manufacturing process innovation.
- 7. Marketing innovation

# Driving forces

- 8. Entry or exit of major firms
- 9. Diffusion of technical know-how across more companies and
- more countries
- 10. Change in cost and efficiency
- 11. Growing buyer preferences for differentiated products
- 12. Reductions in uncertainty and business risk
- 13. Regulatory influences and government policy change
- 14. Changing societal concerns, attitudes, and lifestyles

# Driving forces analysis

- 1)identifying what the driving forces are
- 2)assessing whether the drivers of change are, on the whole, acting to make the industry more or less attractive
- 3)determining what strategy changes are needed to prepare for the impacts of the driving forces.

Making strategy adjustments to take the impact of the driving Forces into account

- driving forces has practical value and is basic to the task of thinking strategically about where the industry is headed and how to prepare for the anticipated changes.

# Q4. What market positions do rivals occupy, who is strongly positioned and who is not?

- **Strategic group mapping**

  **: very useful tool for comparing the market positions of each firm separately or grouping them into like positions**

# Using strategic group maps to assess the market positions of key competitors

**Strategic group's composition : competitor in the industry & positions in market**

guidelines for mapping the positions▼

1) The two variables selected as axes for the map should not be highly correlated

2) the variables chosen as axes for the map should expose big differences in how rivals position themselves to compete in the markets.

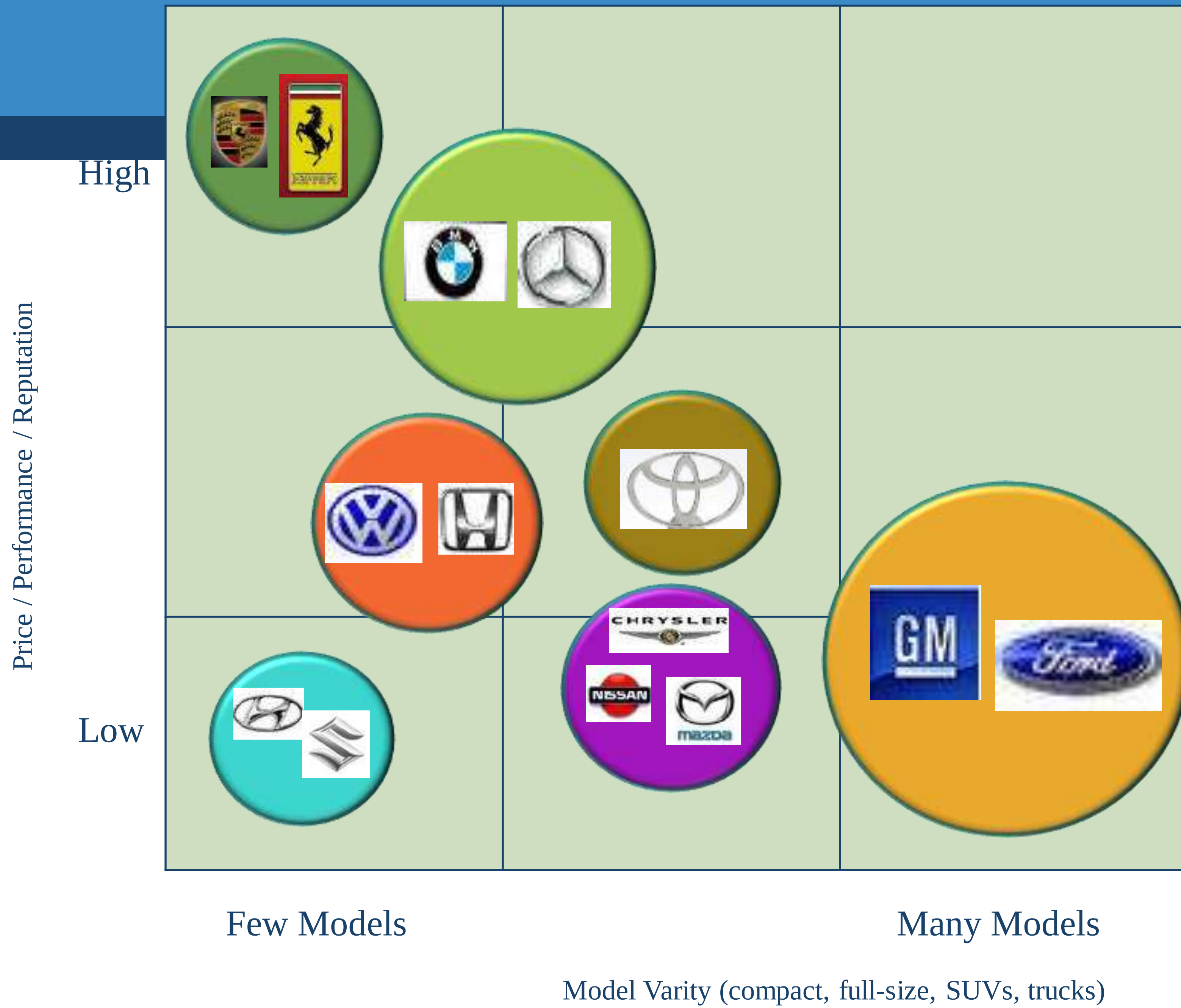
3) the variables used as axes don't have to be either quantitative or continuous

4) drawing the sizes of the circles on the map proportional to the combined sales of the firms

5) it is advisable to experiment with different pairs of competitive variables

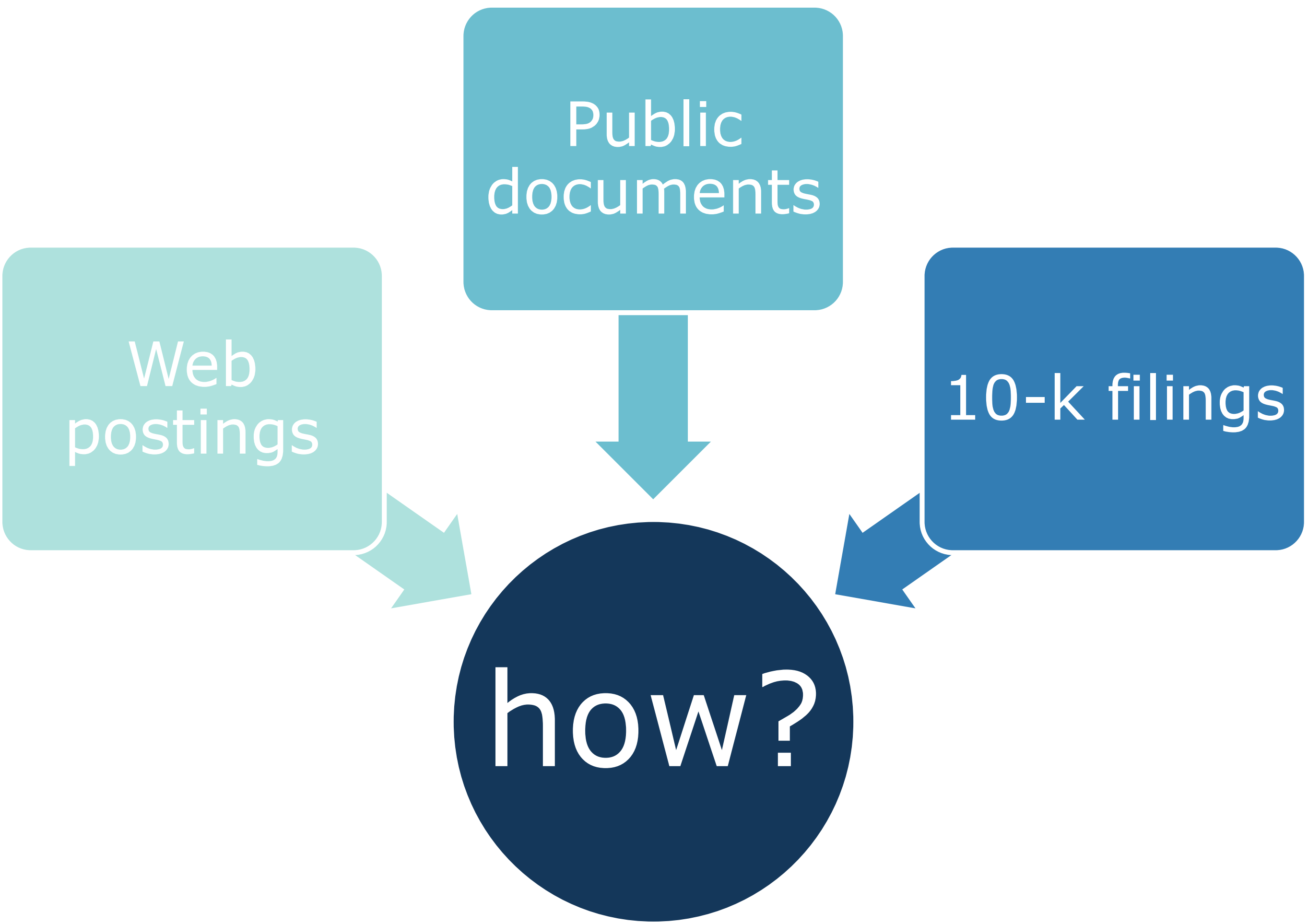
## *What can be learned from strategic group maps?*

- ✓ recognize close competitors and which are distant competitors.
- ✓ In strategic position map, all positions are not always mean equal attractive position.


High
Low
Price / Performance / Reputation
BMW
CHRYSLER
NISSAN
mazda
GM
Ford
Few Models
Many Models
Model Varity (compact, full-size, SUVs, trucks)

# Q5. what strategic moves are rivals likely to make next?

## ◆Identifying Competitor's Strengths and Weaknesses



## ◆Predicting Rivals' Next Moves

Good feels for each rival's situation

↓

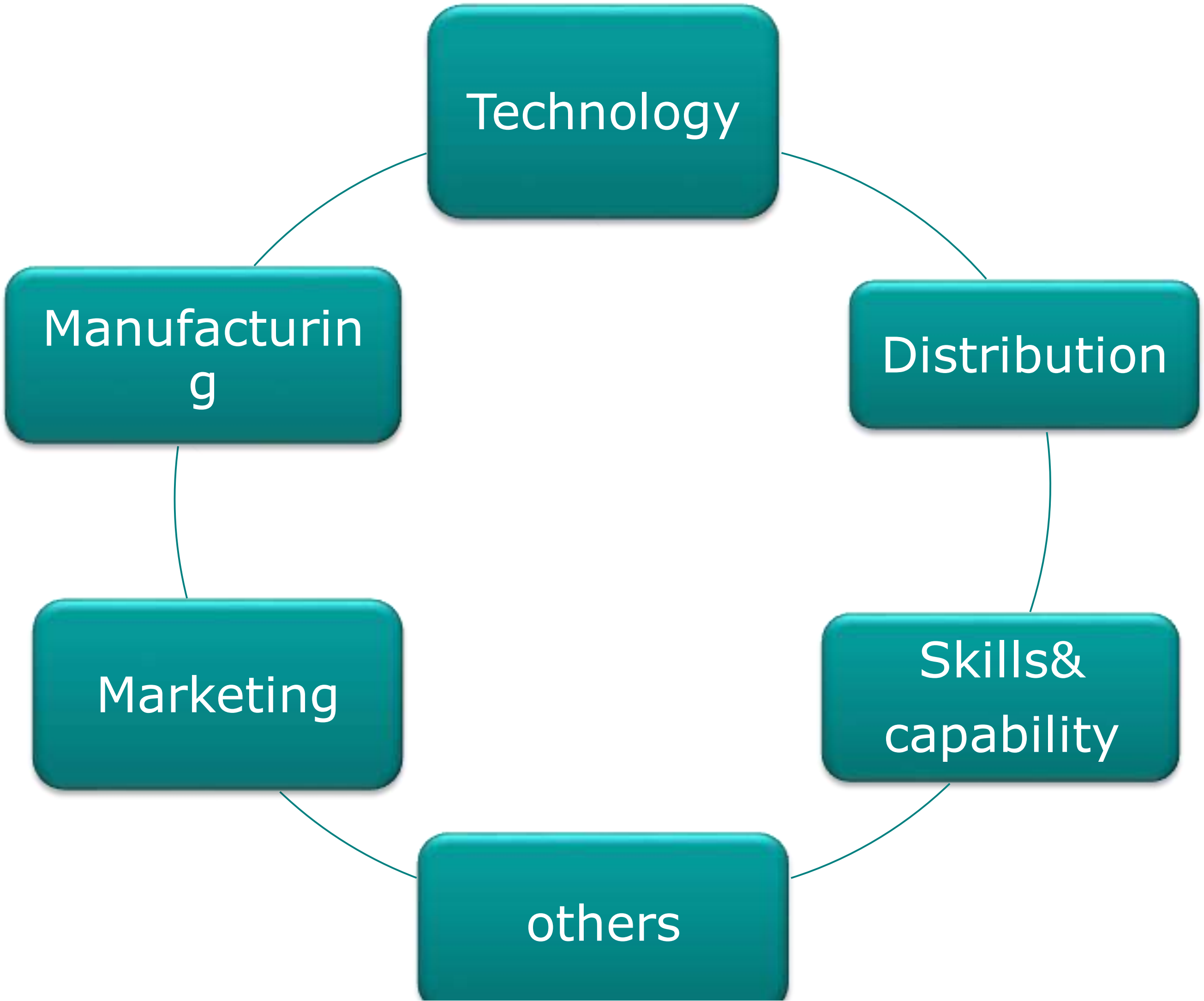
How its managers think

↓

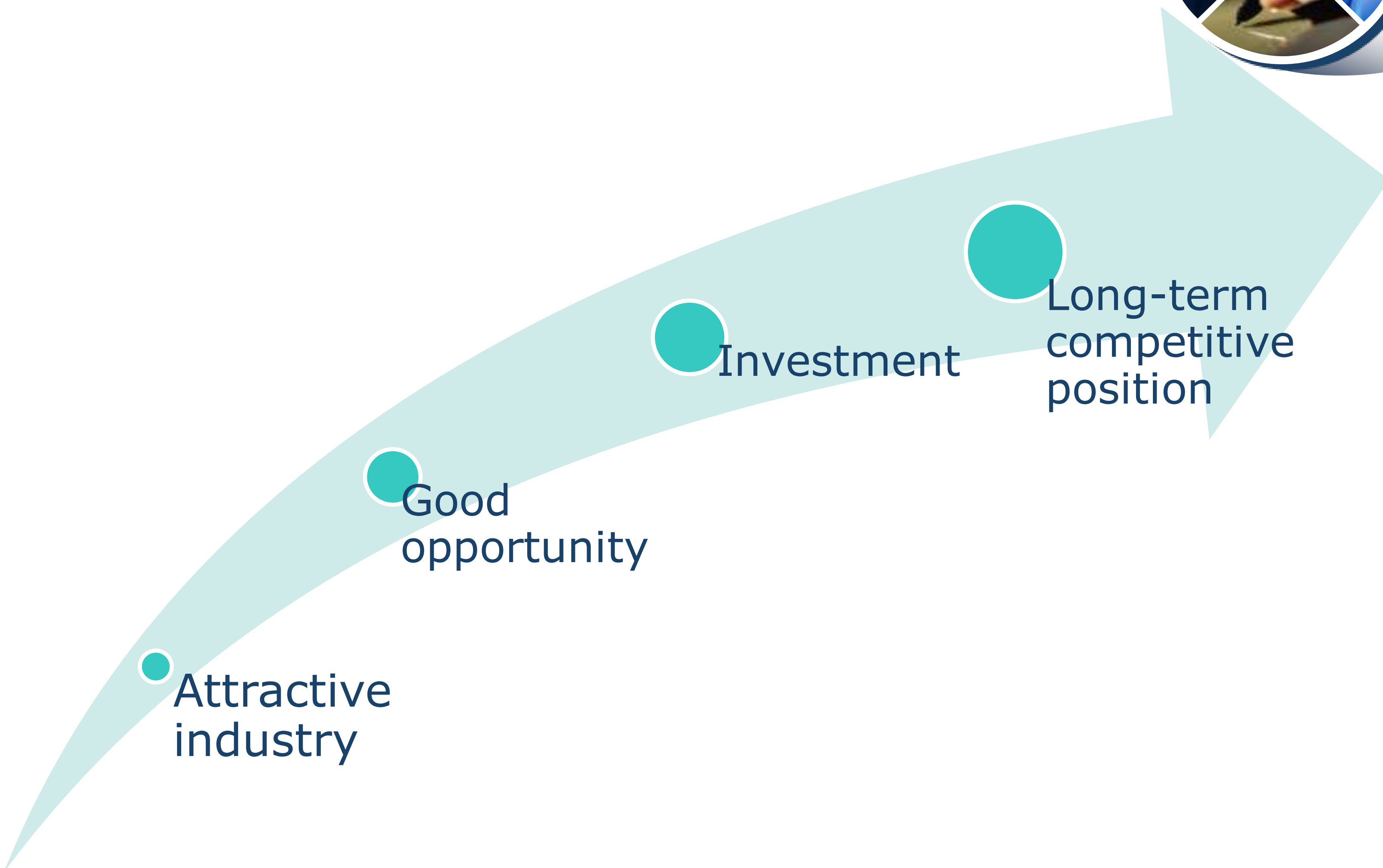
What the rival's best strategic options are

# Q6. what are the key factors for future competitive success?

- **Common types of industry Key Success Factors**



# Q.7 Does the outlook for the industry offer the company a good opportunity to earn attractive profits?



# summary

1. What are the industry’s dominant economic features
2. What kinds of competitive forces are industry members facing, And how strong is each force
3. What forces are driving changes in the industry, and What impact will these changes have on competitive Intensity and industry profitability
4. What market positions do industry rivals occupy

# summary

5 What strategy moves are rivals likely to make next?

6 What are the key factors for future competitive success?

7 Dose the outlook for the industry present the company With sufficiently attractive prospects for profitability?


# Thank You !

# KLE's G I Bagewadi College, Nipani

Seminar Topic : Retail Pricing

Name: Mayuri Y Khavare

Class : B.Com III Sem

# RETAIL PRICING

**Introduction:**

One of the four major elements of the marketing mix is price. It is one of the four P's. Price, Product, Promotion and Place, or where the product is distributed.

The price is a very significant factor in determining the other elements of the marketing mix. Price determines the consumer group that will be targeted, as well as the advertising and promotion and distribution.

We as customers, often get to read advertisements from various retailers saying, “Quality product for right price!” This leads to following questions such as what is the right price and who sets it? What are the factors and strategies that determine the price for what we buy?

The core capability of the retailers lies in pricing the products or services in a right manner to keep the customers happy, recover investment for production, and to generate revenue.

**What is Retail Pricing?**

The price at which the product is sold to the end customer is called the retail price of the product. Retail price is the summation of the manufacturing cost and all the costs that retailers incur at the time of charging the customer.

Companies do their pricing in a variety of ways. In small companies prices are often set by the boss. In large companies pricing is handled by division and product line managers. To the manufacturers, price represents the quantity of money received by the firm or seller for its products. To a customer it represents a monetary sacrifice.

Pricing is one of the most important elements of the marketing mix, as it is the only element of the marketing mix, which generates a turnover for the organisation. The other 3 elements of the marketing mix are the variable cost for the organisation.

**Importance of Retail Pricing:**

1. Price if very essential to both seller and the buyer in the market place because in money economy without price there cannot be marketing.
2. In a competitive market economy, price is determined by free play of demand and supply of a commodity and it determines the nature of market competition.
3. Price influences purchasing decision of a consumer.
4. Price is only revenue factor in marketing mix and rest are cost factors.

5. Pricing is an instrument for dealing with the competitors in the market.
6. Price reflects purchasing power of currency.
7. Price can determine the general living standards.
8. Pricing decisions have impact on financial objectives of the firm.

**Pricing Objectives:**

A business will have some long-term objectives and some short term objectives. While determining the objectives of pricing policy, marketers must take into account the reactions of number of parties such as customers, competitors, dealers, Government, public and so on. The following are the some of the objectives of pricing policy

1. **Profit Optimisation and Maximisation:** Profit is one of the major objectives of pricing. Firms usually adopt optmisation of profits rather than maximisation as the objective because they consider profit over a long period to be more effective than its maximisation.
2. **Predetermined Return on Investment:** It is obvious fact that every business should earn a fair rate of return on the capital employed. They estimate the current demand and costs associated with different alternative prices and then select the price that ensures maximum current profits. Return on Investment (ROI) or cash flow.

3. **To Meet Challenges of Consumerism:** Often consumer- grievances focus on fairness of pricing. Therefore enlisting consumer confidence in price structure should be important objective of pricing management.
4. **To Survive and Exploit Competitive Position:** Due to severe competition many firms face difficulties in the market place. As long as prices cover fixed costs and variable costs the company stays in business. Also to exploit competition strategy of price cutting or low pricing is resorted to defend company’s market share against competitors challenge. The price leader who maintains stable prices in the industry dominates other firms.

5. **Retail Price Maintenance:** The price policy aims to ensure that, retailers would sell the products at fixed rate throughout the market by an agreement to maintain resale price at a desired rates.

## B. Determination of Retail Pricing:

Service providers may have different methods or approaches in fixing the price for their service. Such approach method depends on the nature of service that is delivered. Following are some of the methods.

# THANK YOU

# KLE's G I Bagewadi College, Nipani

Seminar Topic : Company Final Accounts

Name: Pooja Umesh Swami

Class : B.Com III Sem

# Company Final Accounts

- Financial statements are the basic and formal annual reports through which the corporate management communicates financial information to its owners and various other external parties which include investors, tax authorities, government, employees, etc. These normally refer to: (a) the balance sheet (position statement) as at the end of accounting period, and (b) the statement of profit and loss of a company. Now-a-days, the cash flow statement is also taken as an integral component of the financial statements of a company.

# Nature of Financial Statements

The following points explain the nature of financial statements:

1. Recorded Facts: Financial statements are prepared on the basis of facts in the form of cost data recorded in accounting books. The original cost or historical cost is the basis of recording transactions. The figures of various accounts such as cash in hand, cash at bank, trade receivables, fixed assets, etc., are taken as per the figures recorded in the accounting books. The assets purchased at different times and at different prices are put together and shown at costs. As these are not based on market prices, the financial statements do not show current financial condition of the concern.
2. Accounting Conventions: Certain accounting conventions are followed while preparing financial statements. The convention of valuing inventory at cost or market price, whichever is lower, is followed. The valuing of assets at cost less depreciation principle for balance sheet purposes is followed.

- 3. Postulates: Financial statements are prepared on certain basic assumptions (pre-requisites) known as postulates such as going concern postulate, money measurement postulate, realisation postulate, etc. Going concern postulate assumes that the enterprise is treated as a going concern and exists for a longer period of time. So the assets are shown on historical cost basis. Money measurement postulate assumes that the value of money will remain the same in different periods.

- 4. Personal Judgements: Under more than one circumstance, facts and figures presented through financial statements are based on personal opinion, estimates and judgements. The depreciation is provided taking into consideration the useful economic life of fixed assets. Provisions for doubtful debts are made on estimates and personal judgements. In valuing inventory, cost or market value, whichever is less is being followed. While deciding either cost of inventory or market value of inventory, many personal judgements are to be made based on certain considerations. Personal opinion, judgements and estimates are made while preparing the financial statements to avoid any possibility of over statement of assets and liabilities, income and expenditure, keeping in mind the convention of conservatism

# THANKING YOU

# KLE’s G I Bagewadi College, Nipani

Seminar Topic : RETAIL MANAGEMENT

Name: Pooja Umesh Swami

Class : B.Com III Sem

# Role and Importance of Retailer in the Channel of Distribution:

Retailers have an important place in the distribution channel as they sell goods to final consumer. They play useful as well as an important role in distribution channel as the last link.

In the absence of retailers, the consumers cannot find their necessary goods at a single convenient shop. They need to visit many shops to meet their needs. Similarly it is difficult for middlemen and manufacturer to directly reach the consumer.

The role and importance of retailer can be understood by services they provide to producers, wholesalers and consumers.

## Services of Retailer to Producers and Wholesalers:

a. They are the source for wider distribution of goods to consumers.
b. They share vital information about consumers, their preferences, likes, dislikes, price etc.
c. Assume risk by storing goods in advance.
d. Ensure increase in sales volume by reaching large number of customers.
e. Provide storing facility and reduce cost of operations for middlemen.
f. Undertake distribution on behalf of middlemen.

## Services of Retailer to Consumers:

a. Making readily available of varieties of goods.
b. Supply of necessary and fresh goods.
c. Selection facility that is offering wide variety and give choice of selection.
d. Credit facility.
e. Home delivery.
f. Information of arrival of new goods.
g. Offering help and advice in shopping.
h. Facility of comfortable shopping.

The services they provide to producers, wholesalers and consumers make it clear that retailers are present in the distribution channel as bridge between producers or wholesalers and consumers.

The importance and role of retailers can be understood from the following points:

# Role and Importance of Retailers:

1. Link and communication between manufacturer and consumer:
2. He acts as an expert while distributing the goods:
3. Create Time and Place Utility:
4. Comfort and pleasant of shopping goods:
5. Service to manufacturers and middlemen:
6. Provision of storage and warehousing:
7. Increase in productivity:
8. Increase in standard of living:
9. Increase in employment opportunities:
10. Increase in GDP:
11. Retail as a separate branch of study:

## Role and Importance of Retailers:

1. Link and communication between manufacturer and consumer:
2. He acts as an expert while distributing the goods
3. Create Time and Place Utility
4. Comfort and pleasant of shopping goods
5. Service to manufacturers and middlemen
6. Provision of storage and warehousing
7. Increase in productivity
8. Increase in standard of living
9. Increase in GDP
10. Retail as a separate branch of study

# Functions of Retailer in the Distribution Channel:

1. **Assortment of Products:**
2. **Breaking Bulk:**
3. **Holding Inventory:**
4. **Convenient Locations and Timings:**
5. **Providing Services:**
   a. Credit facility.
   b. Free home delivery.
   c. Service guarantee.
   d. Information about variety of goods.
   e. Exchange/Return guarantee etc.
6. **Recording and providing feedback**
7. **Increasing the value of products and services:**

6. **Recording and providing feedback**
7. **Increasing the value of products and services:**

## Challenges of Retail Sector:

**Challenges Faced By Organized Retailers**

Organized retail in India is little over a decade old. It is largely an urban phenomenon and the pace of growth is still slow. Some of the reasons for this slow growth are: -

1. **Belief in unorganized retail:** The very first challenge facing the organized retail industry in India is competition from the unorganized sector. Traditionally retailing has established in India for centuries. It is a low cost structure, mostly owner operated, has negligible real estate and labour costs and little or no taxes to pay. Consumer familiarity that runs from generation to generation is one big advantage for the traditional retailing sector.

2. **Retailing is not considered as industry**: Lack of recognition as an industry hampers the availability of finance to the existing and new players. This affects growth and expansion plans.

3. **Rocket high prices of real estate**: Real estate prices in some cities in India are amongst the highest in the world. The lease or rent of property is one of the major areas of expenditure; a high lease rental reduces the profitability of a project.

4. **High stamp duties:** In addition to the high cost of real estate the sector also faces very high stamp duties on transfer of property, which varies from state to state (12.5% in Gujarat and 8% in Delhi). The problem is compounded by problems of clear titles to ownership, while at the same time land use conversion is time consuming and complex as is the legal process for settling of property disputes.

THANKING YOU

# STRATEGIC MANAGEMENT

SEMINAR

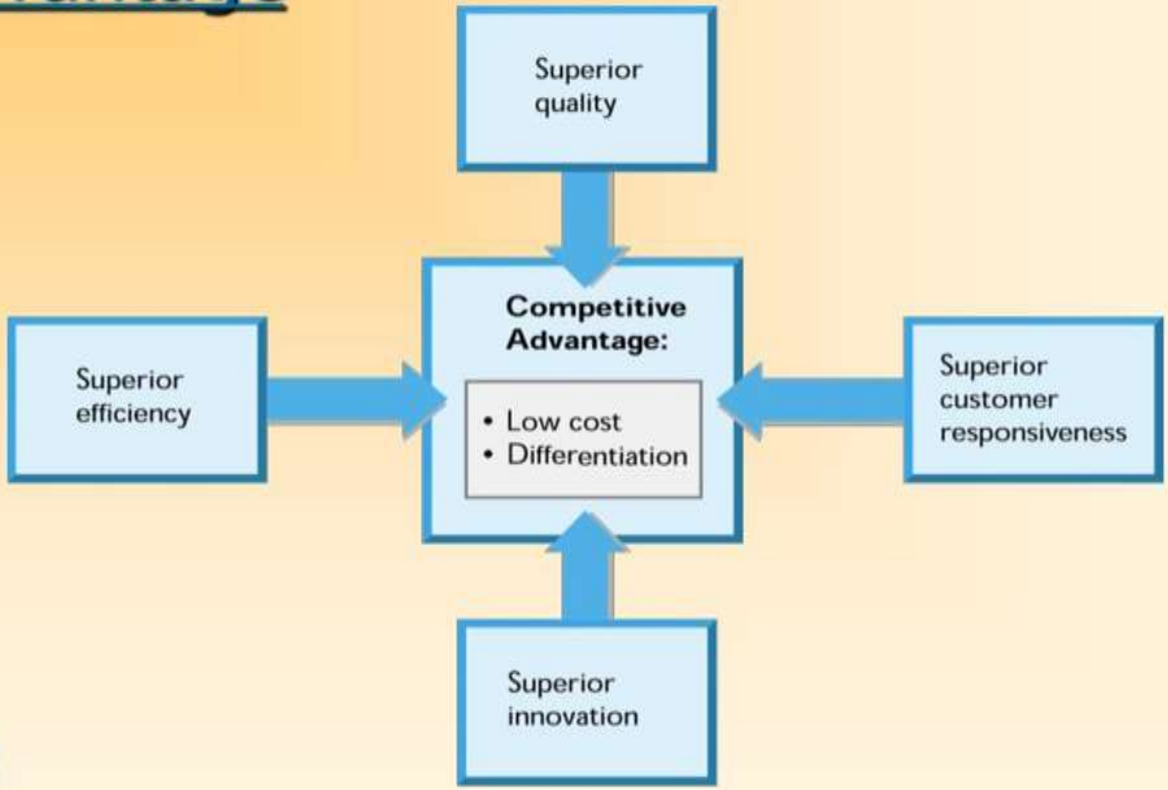
BY MS. SONALI S KADAM

M.COM I SEMISTER

# COMPETITIVE ADVANTAGE

A Competitive advantage is an advantage over competitors gained by offering consumers greater value, either by means of lower prices or by providing greater benefits and service that justifies higher prices.

# Generic Building Blocks of Competitive Advantage



# RESOURCES AND CAPABILITIES

**A 'Resources and Capabilities' analysis is a study about the potential of a company. Instead of focusing on its results, it highlights the tools and internal opportunities a company could use for maximizing its outcome.**

## Resources are

1. **Financial Resources**
2. **Human Resources**
3. **Natural Resources**
4. **Technological Resources**

## CAPABILITIES

**Capabilities are the firm's capacity to deploy resources that have been purposely integrated to achieve a desired end state.**

### Capabilities are

- **Teamwork**
- **Good Relationship**
- **Adaptability**
- **Adjustment**
- **Cost Leadership**
- **Focus or concentration**

# BE & CG
# unit 4 :Managing ethical Dilemmas in Business:

Prepared by

Name: Shruti k Ammannavar

Class:Mcom || sem

REGno:MC191822

# Contents of unit

- Meaning, Nature & Significance of ethical dilemmas
- Ethical dilemmas vs ethical issues
- Ethical in marketing
- Ethical in Finance & Accounting Practice
- HRM Practice & ethical implication
- Ethical issues relating to IT
- Ethical in global bussiness

# Meaning of Ethical Dilemmas

- An ethical dilemma or ethical paradox is a decision making problem between two possible moral imperatives , neither of which is unambiguously acceptable or preferable.

- An ethical dilemma is a moral situation in which a choice has to be made between two equally undesirable alternatives.

# Nature & Significance of ED

- It is critically evaluates the situation in which a different choice to be made between two or more alternatives especially equally undesirable ones.
- In ED no matter what course of action is taken , some ethical principles are to be compromise
- ED helps to distinguish between values , ethics, moral values, laws & policies. And business entity can prefers necessary aspects.
- There must be different course of action choose from alternative course of action are required in critical situation.
- ED also makes distinguish which personal & professional ethics & also explains which one we have to follow up in various situation.
- ED helps to understand the situation & guide to the relevant body to choose the right one.

# ED v/s Ethical issues:

- Ed is when one is faced two or more multiple option of choice or a confusion of understanding based on ethics.
- But ethical issues when a matter has both aspects right & wrong, if it is practising unethical activities . Then it will be considered as ethical issues.

# Ethics in marketing:

- Ethics marketing is a process through which companies generates the customer interest in product or services & builds strong customer relationship . It creates value for all stakeholders by incorporating social & environmental.
- Ethics in marketing refers to the applications of ethical values in production or product , pricing , placing or distribution , promotion or advertisement, services after sales etc.

# Why a company needs ethics in marketing?

- When a company adopts ethics in marketing  that helps to increase the no of customer & positive attitudes towards the company which helps to reduce the advertisement cost.
- Ehics in marketing builds good image about the company in the minds of customer, employees or all the stakeholders.

# Ethical issues in marketing :

- Low quality product or services
- Fixation of high price
- Late or delay in distribution
- Misleading advertisement
- No service after sales
- Misguiding customers

# Ethical Practices in marketing :

- Providing good quality product or services
- Safe delivery of product
- Fixing the reasonable price
- No marketing of the expired goods
- True advertisement
- Consumer education
- Discloser of consumer responsibilities
- Prevent black market activities

# Ethics in Finance & Accounting :

- Allocation of financial assets in profitable projects
- Providing reasonable  salary, wages to the employees
- Regular payment of all taxes
- Effective use of government subsidies
- Some portion of fund to reserve for CSR
- Focusing on profit & wealth maximisation
- Discloser of financial information to the concerned bodies

# Ethics in accounting practices:

Perceptive about a company is highly depends upon the  final accounts of the company . Every companies financial statements  are prepared  on the bases of accounting records. That's why here accounting plays important role.

# Ethics in  accounting practice

- Recording in every aspect of the financial activities in concerned books of account
- Maintaining all the books of accounts already prescribed by the concerned bodies
- Follow up all the rules & regulation & guidelines relating to the accounts
- Providing all the recorded information to the auditor as well as concerned bodies
- Proper allocation & classification of financial transactions
- Do not creating & recordind fake transaction

# HRM Practices and ethical implications:

- Managing ethically all the manpower of the company is known as ethical human resources management or application of ethical values in human resource management is called as the ethical human resource management.

# Ethical implication in HRM practices:

- Fair working hour
- Good training facilities
- Organising career development programmes
- Good infrastructure for the employees
- Providing direction with integrity
- Equality
- Job security
- Awarding the employee for work excellent

# Ethical issues relating to IT:

1] Computer system crime relating ethical issues:

. Information theft

. Financial fraud

. Hacking the computer system

. Destroying information

. Creation fake account & user

2]Intellectual property related issues:

.Copying others name (Artel-Aircel)

.Copying designs, images, invention etc

.Copying symbols or logo

# Ethics in global business:

- Normally global business activity. It is operating its operation in more than 20 countries , but in such kind of business activities we can not find the ethics. There are so many reasons for unethical behaviour like religion, language, political interests, traditions , culture etc.

# Importance or needs of ethics in global business:

*It needed for sustainable development of the host country :

A)Environmental protection B)Effective use of resources C) pollution control D)Producing quality product

*Its needed to prefer the regionality

*Coordination between home country& host country employees.

*It helps to earn goodwill in the host country

*It helps to earn maximum profit

*It helps to grab the market

# THANK YOU

# BUSINESS ETHICS & CORPORATE GOVERNANCE

SEMINAR

BY MS. SONALI S KADAM

M.COM II SEMISTER

# SOURCES OF BUSINESS ETHICS

1. Genetic Inheritance
2. Religion
3. Legal System
4. Philosophical System
5. Code of Conduct
6. Cultural Experiences

# Ethical Values for Business Success

**(By Mahatma Gandhi)**

1. Honesty
2. Integrity
3. Responsibility
4. Quality
5. Trust
6. Respect
7. Leadership
8. Corporation, Citizenship
9. Shareholders Value
10. Transferency & Accountability
11. Equality
12. Excellence
13. Peace

# HUMAN RESOURCE DEVELOPMENT

SEMINAR

BY MS. SONALI S KADAM

M.COM II SEMISTER

# Trade Unions

## Meaning

**Labour unions or trade unions are organizations formed by workers from related fields that work for the common interest of its members. They help workers in issues like fairness of pay, good working environment, hours of work and benefits.**

# Forms of Trade Union

I. Classical

II. Neo Classical

III. Revolutionary

# Objectives of Trade Union

1. Wages and Salaries
2. Working Conditions
3. Discipline
4. Personal Policies
5. Welfare
6. Employee
7. Negotiating Machinery
8. Safeguarding organisation health and interest of industry

# KLE's G I Bagewadi College, Nipani

Seminar Topic : Retail Marketing Mix

Name: Prabhavati Shivanand Patil

Class : B.Com III Sem

# Introduction:

- Retail marketing includes activities of selling goods or service to final consumer for personal, non-business use. Any organisation selling to final consumer, whether a manufacturer, wholesaler, retailer is doing retailing. It does not matter how goods or services are sold i.e., by person, mail vending machine, internet, mobile etc., where they are sold, in store, on street or in the consumer home.
- **Retail marketing primarily undertakes following activities**:
- Identify the customer and understand his needs.
- Store the needed merchandise or goods.
- Attractive presentation of goods for easy identification and convenience.
- Provide necessary comfort in purchase i.e., location, price, service etc.

# Retail Marketing Mix:

- Retail is concerned with distribution of goods to ultimate or final customer for consumption and use. Marketing activity of Retail must be aimed at convenient and easy availability of desired product. It must undertake these activities and functions that ensure timely delivery of desired products. For this purpose Retail marketing mix must have following mixes in its marketing components.
- Place
- Presentation
- People
- Promotion
- Product
- Price

- **1. Place:**
- Convenience of shopping and easy availability of the product ensures successful marketing of retail products. Retail products which are consumed regularly must be made available conveniently. In case of store based retailing, store should be placed at a convenient location that ensures easy accessibility to customers. The goods and services must be available at a time when customer needs it. Eg. Diary products, News Papers, Bakery needs in the morning, ATM, Gas station etc., work 24 x 7. Depending on the nature of the product or service, they must be made available to customers in a store and they must be displayed attractively.

**2. Presentation:**

Proper presentation of the products will attract the visiting customers and enables them select the product of his /her choice easily. Presentation as a mix of retail marketing has the following elements.

Visual Merchandising

Space Allocation

Fixture Layout and

Assortment and Category Management

**Visual Merchandising:** It is an important element of category management. The product must be displayed in such a way that it should attract the attention of the visitors and make them buy. Retailer must utilize the space to give visual look to the products.

**Space Allocation:** Retail store has limited space and the available space has to be allocated to different products, brands and ensure adequate variety in each of line of product. The space allocation must offer;

Easy to locate the product.

Attract the customer towards the Rack or Shelf space

Provide convenience to customer to pick the product

- **3. People:**
- Sales staff on each counter i.e., frontline staff will play an important role in promoting and selling the product that is desired by the customer. People engaged in retailing must have the knowledge of product and customer. He must patient to address to queries of the customers. Proper training should be given to employees to improve his ability and skill to handle different customers and convert their desire into demand.

- **4. Promotion**:
- Promotion as a retail marketing mix creates awareness of the products available with the store. Arouse interest among the customers with offers, packages, presentation etc., and create desire among the customers to buy the product with attractive ads and sales promotion techniques. Various tools of promotion mix are advertising, Personal selling, Sales promotion and Publicity and public relations.
- **5. Product**:
- Store must offer wide variety of products to meet the need and taste of visiting customers. Depending upon the type of store (Specialty, Convenience, Departmental stores etc.) the organisation must have large variety of merchandise to satisfy the need of the customers.
- Quality of product also plays an important role. Assured quality and Guarantee will ensure continued support of customers. It enhances the goodwill of the organisation.

## 6. Price:

Price always goes along with the quality. Pricing is the only element in a marketing mix that produces revenue and the other elements result in cost. Price is an integral element of the retail marketing mix. Prices in retail depend upon many factors. Product pricing strategy changes from time to time according to market design. It is important to the any retailer to keep the cost of his item competitive to attract buyers.

THANKING YOU

# KLE’s G I Bagewadi College, Nipani

Seminar Topic : Financial management

Name: Shreya Utture

Class : B.Com IV Sem

# Financial Management

## Introduction:

In our present day economy, finance is defined as the provision of money at the time it is required. Every enterprise, whether big or small, needs finance to carry on its operations and to achieve its targets.

In fact, finance is so indispensable today that it is rightly said to be the lifeblood of an enterprise. Without adequate finance, no enterprise can possibly accomplish its objectives.

Finance has become so much important for every business enterprise that all managerial activities are connected with it.

Finance deals with inflow and out flow of cash with rapid growth of complex industrial structure.

The importance of financial function has increased so that it has given birth to a separate subject known as "**Financial Management**".


Bro whats financial management?
A man found $100
He went to 5 Star hotel for dinner
His bill was $3000
When he said he has only $100
manager handed him to police.
He gave $100 to police and went free...
Its called FINANCIAL MANAGEMENT!

## Meaning:

Financial Management is the part of the management activity which is concerned with the planning and controlling of firm's financial resources.

It deals with finding out various sources for raising funds for the firm. The sources must be suitable and economical for the needs of the business.

The proper use of funds also forms a part of financial management. Financial management is recent origin. This subject is still developing and has not yet acquired a body of knowledge of its own.

Financial Management is that specialised function of general management which is related to the procurement of finance and its effective utilisation for the achievement of common goal of the organisation.

## Definitions:

**Prof. Ezre Soloman:** "Financial Management is concerned with the efficient use of an important economic resource, namely capital funds."

**J. F. Bradley:** "Financial Management is the area of business management devoted to judicious use of capital and careful selections of sources of capital in order to enable a business firm to move in the direction of reaching its goals."

**Howard and Upton:** "It is the application of general managerial principles to the area of financial decision making".

From the above definitions it is clear that financial management is that specialized activity which is concerned with the collection or raising of finance and its effective utilization for the attainment of common objectives of the business enterprise.

It includes financial planning, financial administration and financial control. In short, financial management is mainly concerned with the proper management of funds.

## Functions of Financial Management:

Following are the important functions of Financial Management

According to the modern approach of financial management the scope and functions of financial management has changed from mere raising of funds to financial decision making process leading to wise use of funds.

For the sake of convenience functions may be broadly classified into two categories such as executive functions and incidental functions.

## I. Executive Functions:

a. **Financial Forecasting:**
b. **Raising of Funds:**
c. **Managing Flow of Funds:**
d. **Cost Control:**
e. **Allocation of Net Profit or Management of Income:**
f. **Analysis and Appraisal of Financial Performance:**

**II. Routine of Incidental Functions:** These functions mainly comprise the work of routine nature which is necessary for the execution of financial decisions at the executive level. Some of the important routine functions are as follows:

a. Record keeping and reporting
b. Preparation of various financial statements
c. Cash planning and management
d. Credit management i.e. negotiations with banks and financial institutions.
e. Custody and safeguarding the different financial securities etc.
f. Providing top management with information on current and prospective financial conditions of the business.
g. To keep track of stock exchange quotations and behaviour of stock market prices.

Thus, from the above discussion it can be concluded that, the financial manager is not only responsible for maintaining financial health of the organisation but also increase the economic welfare of the shareholders by utilizing the funds in an effective manner.

# KLE’s G I Bagewadi College, Nipani

Seminar Topic : Income from Business

Name: Yuvaraj Kamate

Class : B.Com VI Sem

# Income From Business or Profession:

## Business:

As per Section 2(13), the term Business includes any trade, commerce or manufacture or any adventure or any concern in the nature of trade, commerce or manufacture.

Business means any activity carried on with a profit motive.

**Profession: Sec.2(36):**

Profession refers to an occupation which requires intellectual skill or manual skill controlled by the intellectual skill of a person engaged in such occupation.

Examples:

1. Doctors
2. Lawyers (Advocates),
3. Engineers (Architects)
4. Auditors (Chartered Accountants) etc.

# Professionals:

Professionals are those persons who are engaged in the profession.

Examples:

1. Doctors
2. Lawyers or solicitors
3. Chartered Accountants
4. Engineers etc.

# Vocation:

It is only the way of living or an activity for which one has special features of fitness.

Examples:

1. Music
2. Dancing
3. Brokerage
4. Insurance agency etc.

# Methods of Accounting: Sec. 145

## 1) Cash System Of Accounting:

Under this system of accounting the actual cash received and actual cash paid are to be recorded in the books of accounts.

But credit transactions and outstanding items are not be recorded.

**2)Mercantile System of Accounting:**

Under this system of accounting the cash as well as credit transactions including all outstanding items are to be recorded in the books of accounts to find out the income or loss from Business or Profession.

**Computation of Income from Business (If Profit & Loss A/c is given) :**

| Particulars | Rs. | Rs. |
|---|---|---|
| Net profit as per P&L A/c | - | Xxx |
| Add: Expenses Disallowed: | | |
| 1 All Reserves & Provisions (Reserve for Bad Debts, provision for Income Tax, Provision for Repairs, Provision for Gratuity, Depreciation Fund etc.) | xxx | |
| 2 All taxes (Income Tax, Advance Income Tax, Fringe Benefit Tax, Wealth Tax etc. except Sales Tax, Excise Duty and Local Taxes of premises used for Business.) | xxx | |
| 3 Rent Paid to Staff | xxx | |
| 4 All capital expenditures (Purchase of fixed assets, expenses incurred at the time of purchase of assets) except on Scientific Research. | xxx | |
| 5 All capital losses and loss on sale of assets | xxx | |

| | | |
|---|---|---|
| 6 All types of Charities & Donations. | xxx | |
| 7 All Expenses relating to other head of Income | xxx | |
| (Repairs, Taxes, Fire Insurance on house property) | | |
| 8 All Personal Expenses (Drawings, Salary to Proprietor, Interest on Capital, Medical Expenses etc.) | xxx | |
| 9 Depreciation Debited to P&L A/c (Treated Separately) | xxx | |
| 10 Gifts & Presents given to friends and relatives | xxx | |
| 11 All types of Fine or Penalty | xxx | |

| | |
|---|---|
| 12 Any Payment to Partner | xxx |
| 13 Any Salary or Interest payable outside India unless tax is deducted at source | xxx |
| 14 Past Losses | xxx |
| 15 Difference in Trial Balance. | xxx |
| 16 LIC Premium | xxx |
| 17 Amount invested in Saving Certificates | xxx |
| 18 Speculation Losses | xxx |
| 19 Legal Expenses on Criminal Cases | xxx |

| | | |
|---|---|---|
| 20 Legal Expenses on Acquisition of Assets | xxx | |
| 21 Loss by Theft from Residence | xxx | |
| 22 Expenses on Illegal Business | xxx | |
| 23 Employer's contribution to URPF | xxx | |
| 24 Cost of patent rights, technical Know-How | xxx | |
| 25 Preliminary Expenses & Goodwill Written off | xxx | |

| | | |
|---|---|---|
| **26 Any Expenses above Rs. 20,000 paid in cash** | xxx | |
| **27 Any Expenditure on Advertisement in the souvenir, pamphlets or Magazines' of political parties** | xxx | |
| **28. Over valuation of opening stock** | xxx | |
| **29. Under valuation of closing stock** | xxx | xxx |
| | | xxx |

# THANK YOU

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# Teaching and Learning through PPT by Teachers and Students

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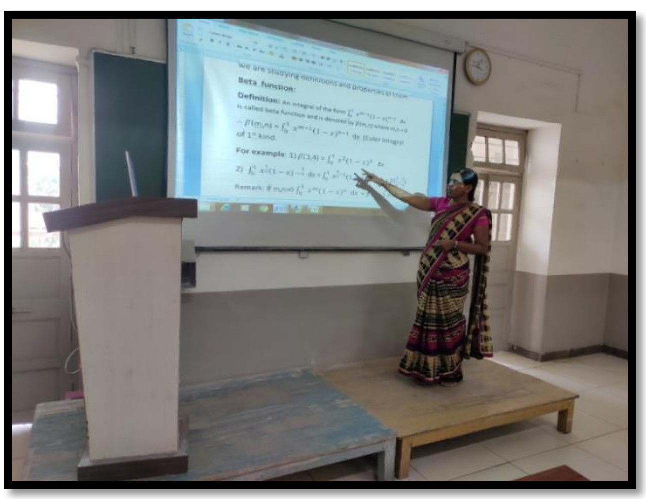
## DEPARTMENT OF MATHEMATICS

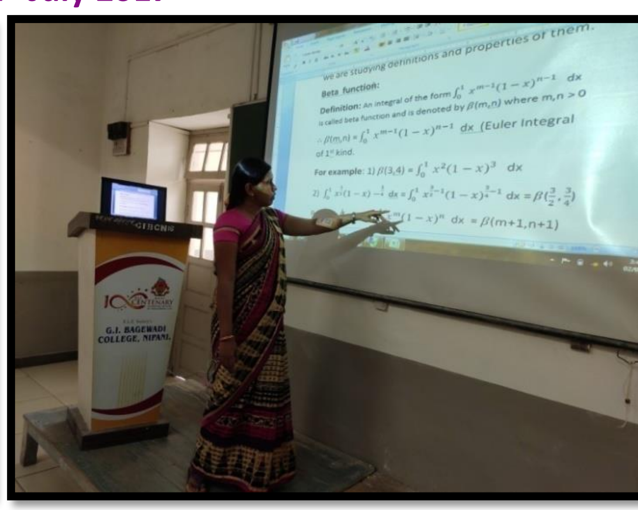
## LECTURE BY FACULTY USING PPT



**Dr. Smt.M.M.Shankrikopp Class: B.Sc I topic: "Limits & Continuity"**

**Date: 31st July 2017**



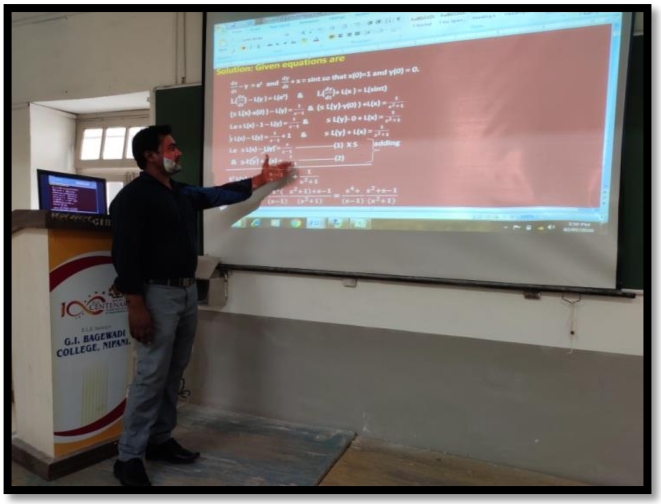


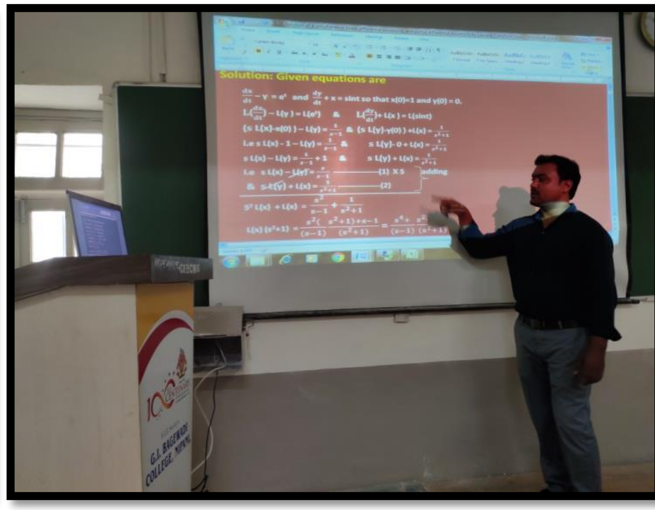

**Miss G.L.Karaguppi Class : B.Sc V sem. Date: 13th Sept.2017**

**Topic: Solving examples on Beta, Gamma functions by on**

EPARTMENT OF MATHEMATICS

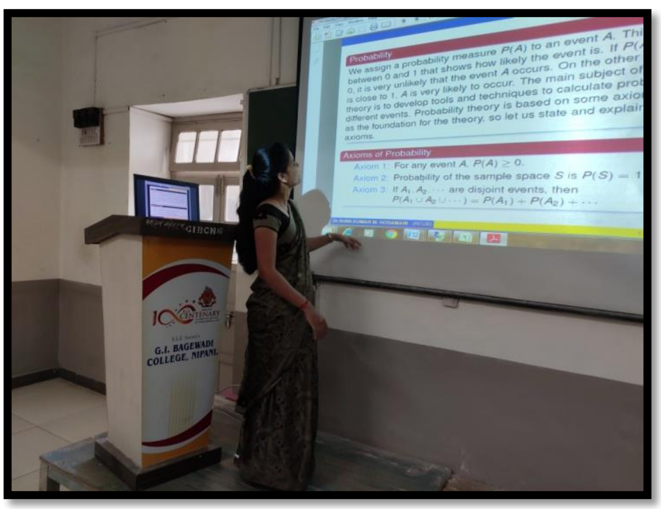
# LECTURE BY FACULTY USING PPT

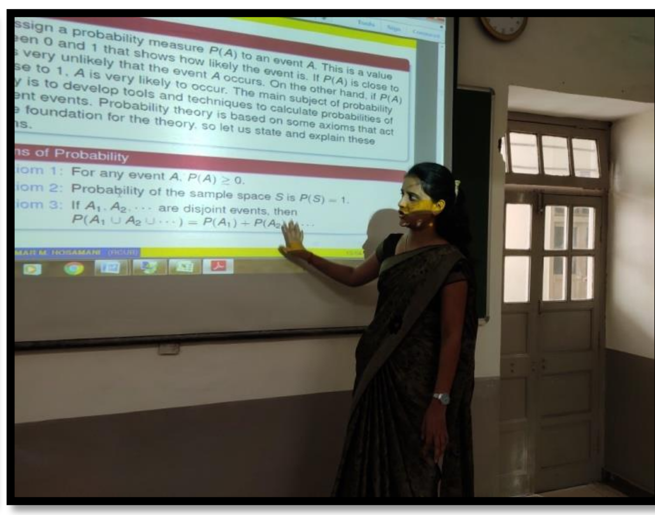



**Mr. J. N.Magadum** **Class: B.Sc VI Sem.** **Date 3rd Jan 2018**

**Topic: Solving Simultaneous LDE**



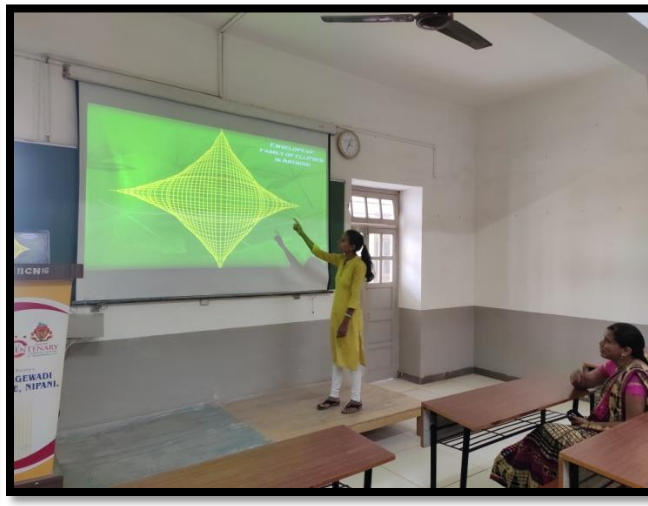
**Miss V.U.Khot** **Class : M.Sc. IV Sem.** **Date: 1st Feb 2018**

**Topic: Introduction to “Probability Theory”**

## DEPARTMENT OF MATHEMATICS

# Topper as a teacher

**Topic:" Envelope of a curve"** **Class: B.Sc II Sem**

**Name: Sangita MoreDate: 14/03/2017**

**Class: B.Sc II Sem** **Time: 2.00pm to 3.00pm**

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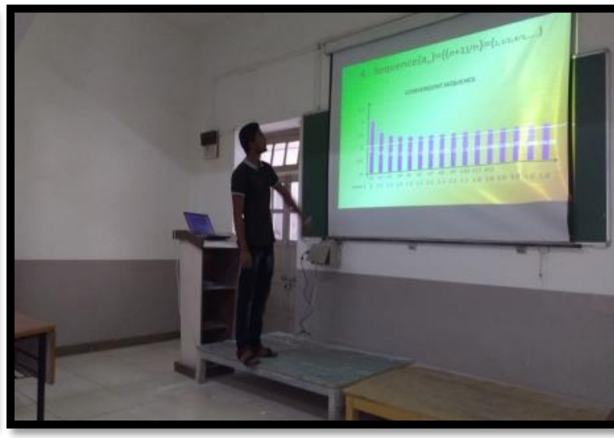
## SEMINARS USING PPT

**Name: Sri. ShivaprasadToli,**

**Class :B.Sc III Sem.**

**Topic: Graphical Representation of Sequence**

**Date: 30.7.2019**

K.L.E. Society's

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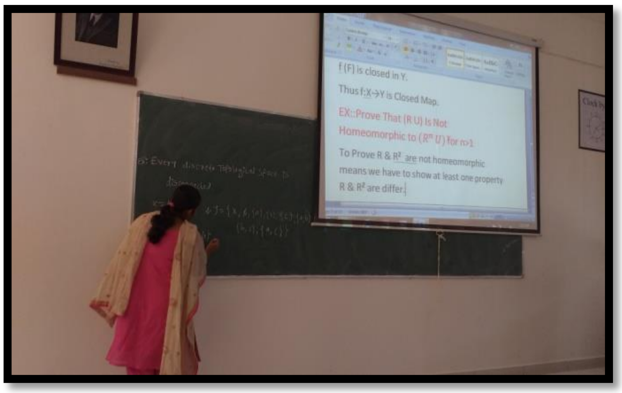
## SEMINARS USING PPT

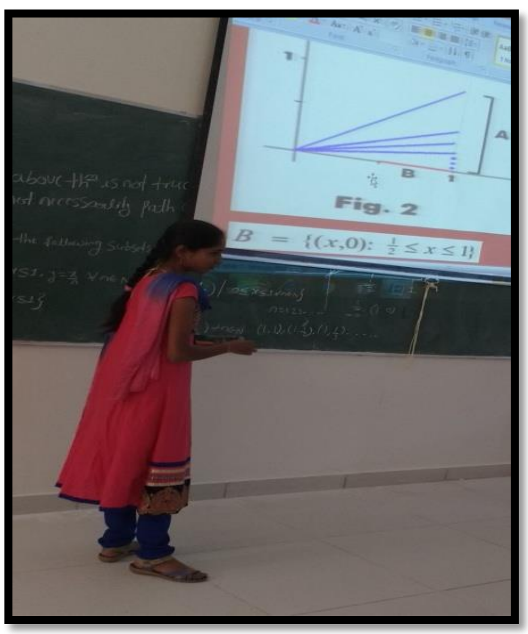



Name: Miss Laxmi Mantrennavar,
Miss SupriyaKaramle

Class: M.SC.II

Topic: Homeomorphism of topological spaces'

Date:



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**LECTURE THROUGH PPT**

**Name of the Teacher : Smt. Geeta Kamate** **Class: B.Sc I sem.**

**Topic: Relations and Functions** **Date: 31.07.2019**

**Name of the Teacher : Sri. S. A. Chougale** **Class: Certificate Course students**

**Topic: Problems on Train** **Date:18.01.2018**

## DEPARTMENT OF NSS

## NSS officer Dr. S.M. Ryamane,HOD of Marathi, is presenting NSS activities at KLE Head Office Belagavi, through PPT

## DEPARTMENT OF ECONOMICS

**IQAC Coordinator Dr. B.S.Kamble, HOD of Economics, Participated as a Resource Person for one day Workshop on IQAC and ---------at Muddebihal College**

## DEPARTMENT OF MATHEMATICS

**Miss PradnyaBhivase, Miss Parvati Chougule and Miss Ayushi Kadam of B.Sc final Year, attended PPT competition, held at GSS College, Belagavi on 28.2.2020, and got First Place.**

**Topic:Graphical Representation of Sequence**

# DEPARTMENT OF LIBRARY AND INFORMATION CENTRE

Department of Library &Information Centre of the Information was organized six days orientation programme on “Library Information” from 22-07-2019 to 27-07-2019 for all UG & PG first year students.

It was organized in six sessions. In the beginning of the orientation programme Dr. Anand Y. Kenchakkanavar, librarian conducted **Library Orientation Programme using ICT**

Librarian introduce the location of Check Point & Property Counter, Circulation Section, OPAC Station, Display rack for new arrival books /Journals, Librarian Office, Staff and Ladies Reading room, News paper section, Reference Section, Back Volume of Journals and Xerox Section.

## DEPARTMENT OF MATHEMATICS

### Topper as a Teacher

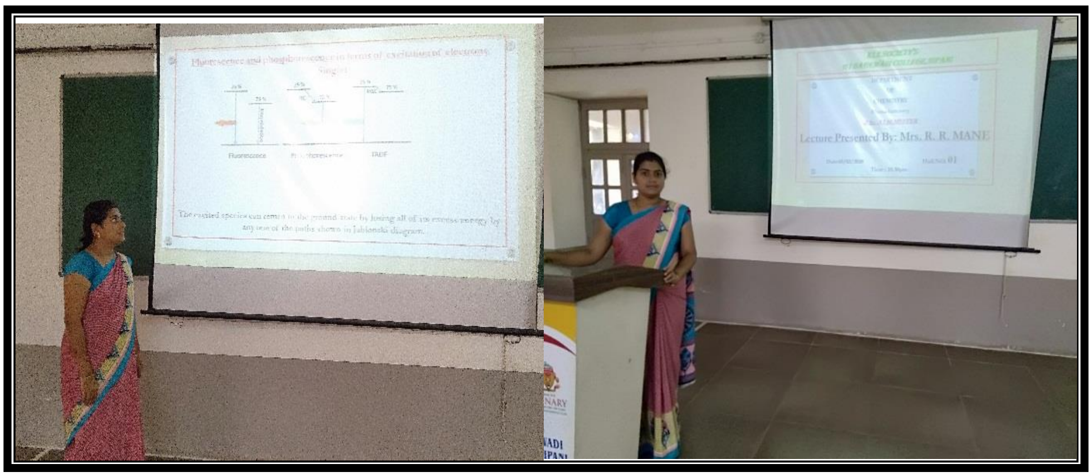
**Name of the Student: Mr.Sukshay Padre, of B.Sc.III sem.**

**Class Engaged :B.Sc I sem., Date: 10.4.21,**

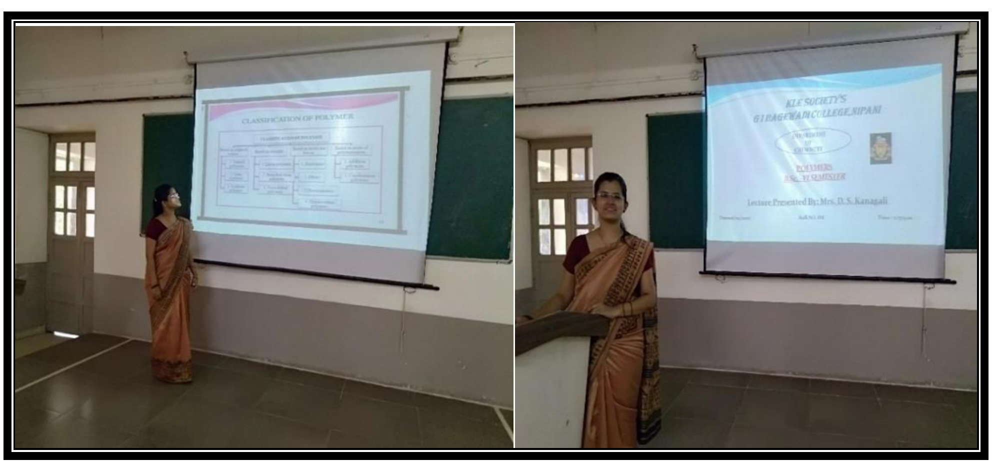
**Topic:Graphical Representation of Continuity and Differentiability**

# Department of Chemistry


By Mr. S.M.Narawade on the topic “Aromatic Amines” on 15/02/2020 at 2pm.

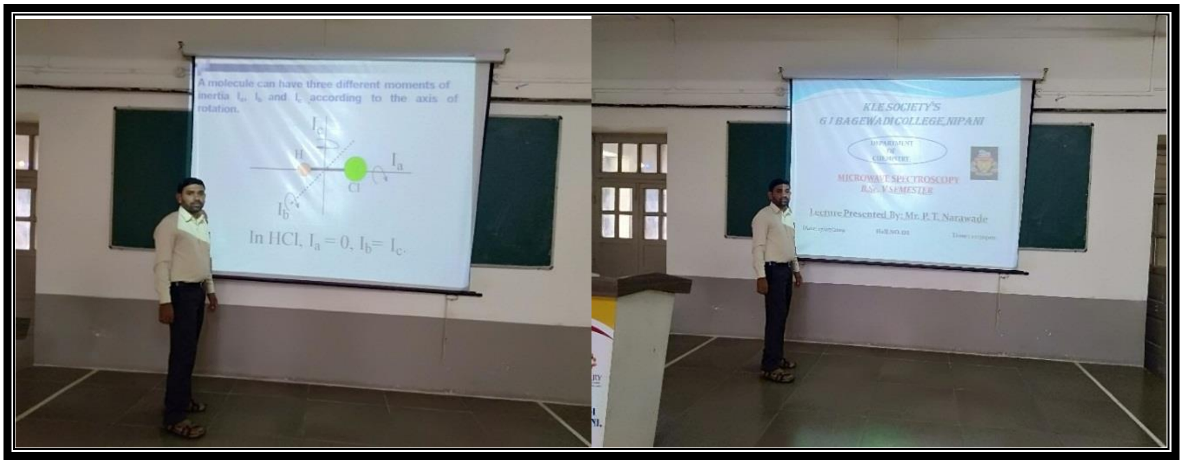



By Mrs. R. R. Mane on the topic “Photochemistry” on 10/02/2020 at 10.30am.

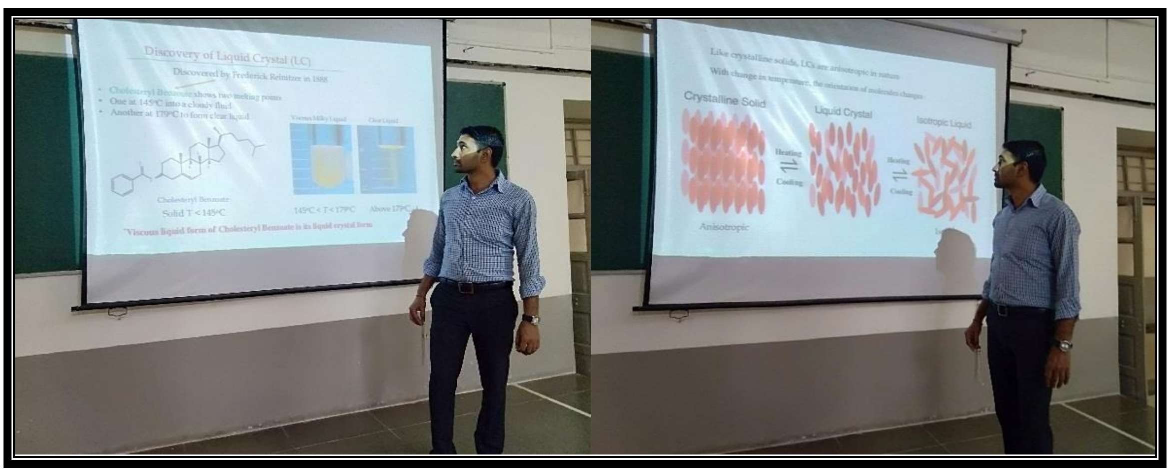



ByMrs. D. S. Kanagali on the topic “Polymer “on 06/02/2020at 12.30pm.

# Department of Chemistry

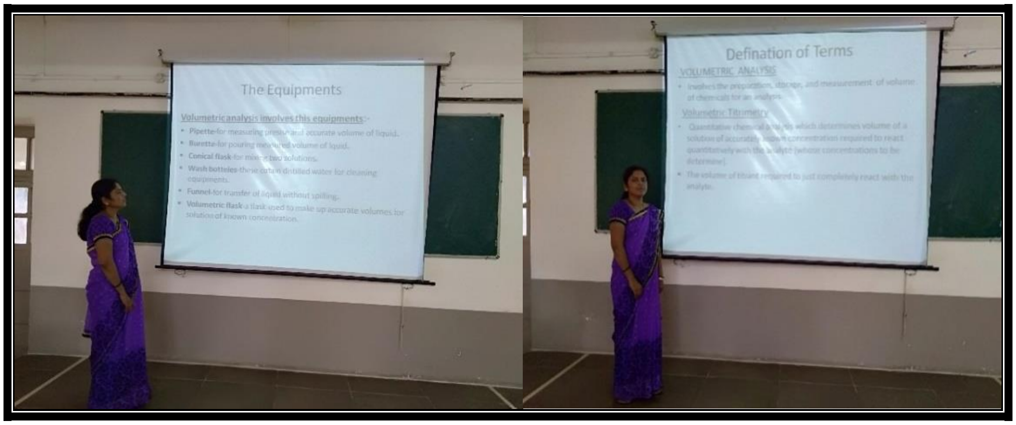



Mr. P. T. Narawade on the topic "Microwave Spectroscopy "on 17/07/2019 at 12.30pm.

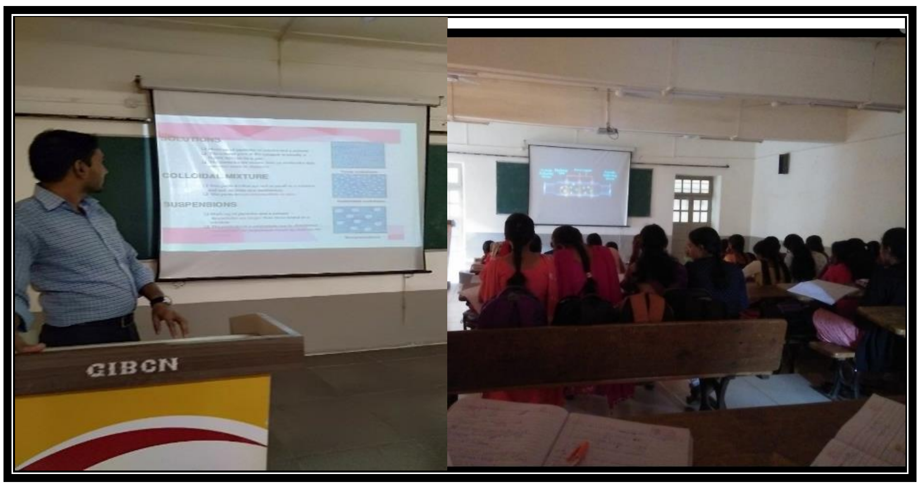



By Mr. S.M.Narawade on the topic "Liquid Crystal" on 10/02/2020 at 2pm

# Department of Chemistry




Mrs. D. D. Bhoite on the topic "Volumetric analysis" on 06/02/2020 at 12.30pm.




Mr. S.M.Narawade on the topic "Colloids" on 09/03/2020 at 2pm


Mrs. D. S. Kanagali on the topic "Chemical Kinetics "On 29/01/2020 at 10.30AM

## DEPARTMENT OF ZOOLOGY

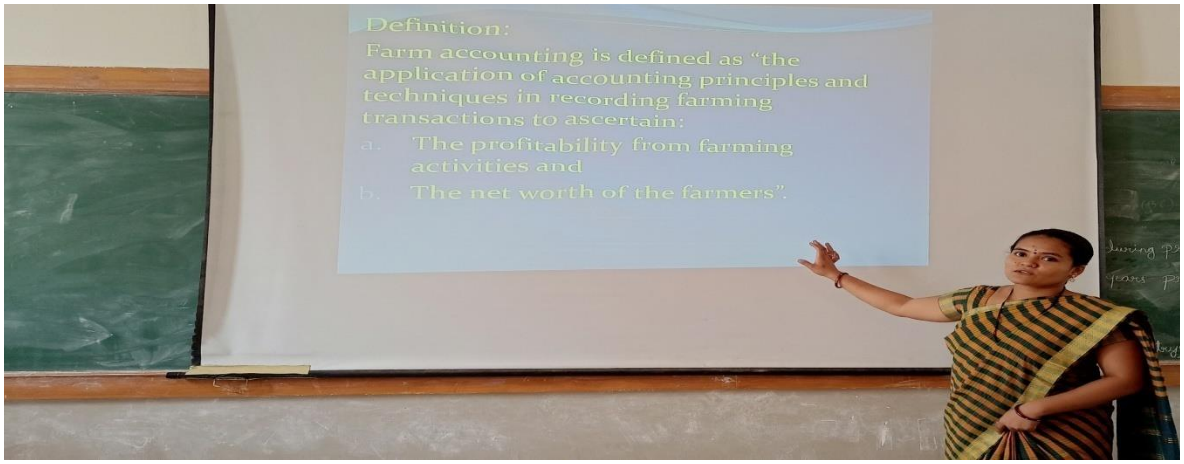
**Mr.AbhishekNarayankar of III Sem Presenting PPT on Placenta topic**

**Miss Pratiksha Suryavanshi of B.Sc V Sem teaching on the topic Ecosystem**

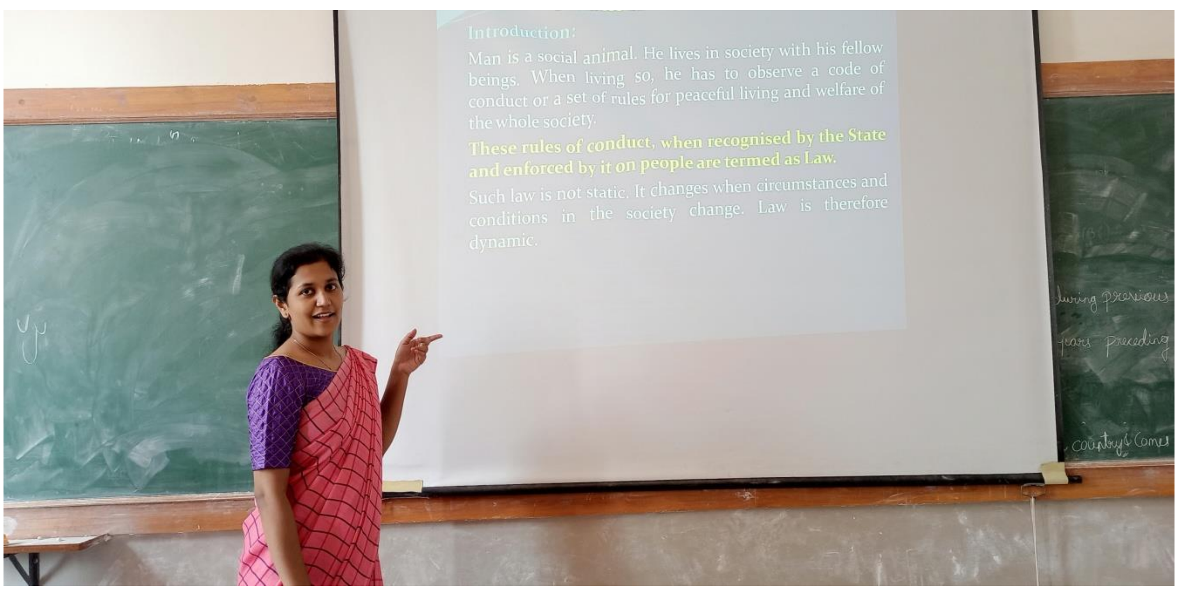
# Department Of Commerce

## LECTURE BY FACULTY BY USING PPT




**Smt. Priyanka Kamate** **Class: B.com I sem** **Date:10/02/2017**

**Topic: Farme Accounting**

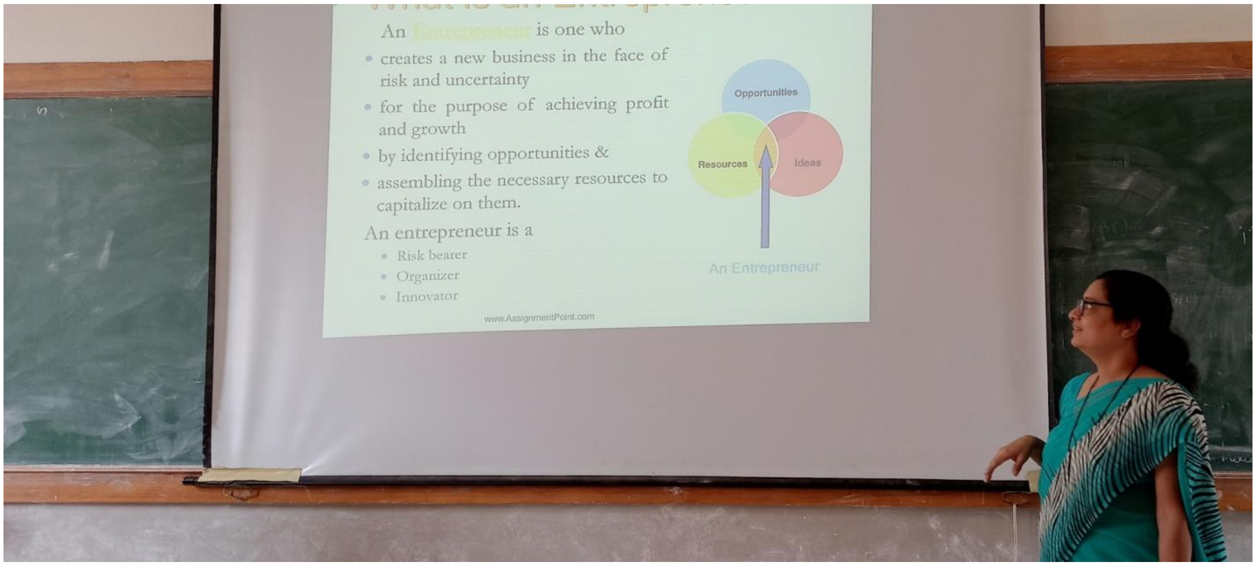



**Miss. Shruti Mirje** **Class: B.com III sem** **Date:11/12/2018**

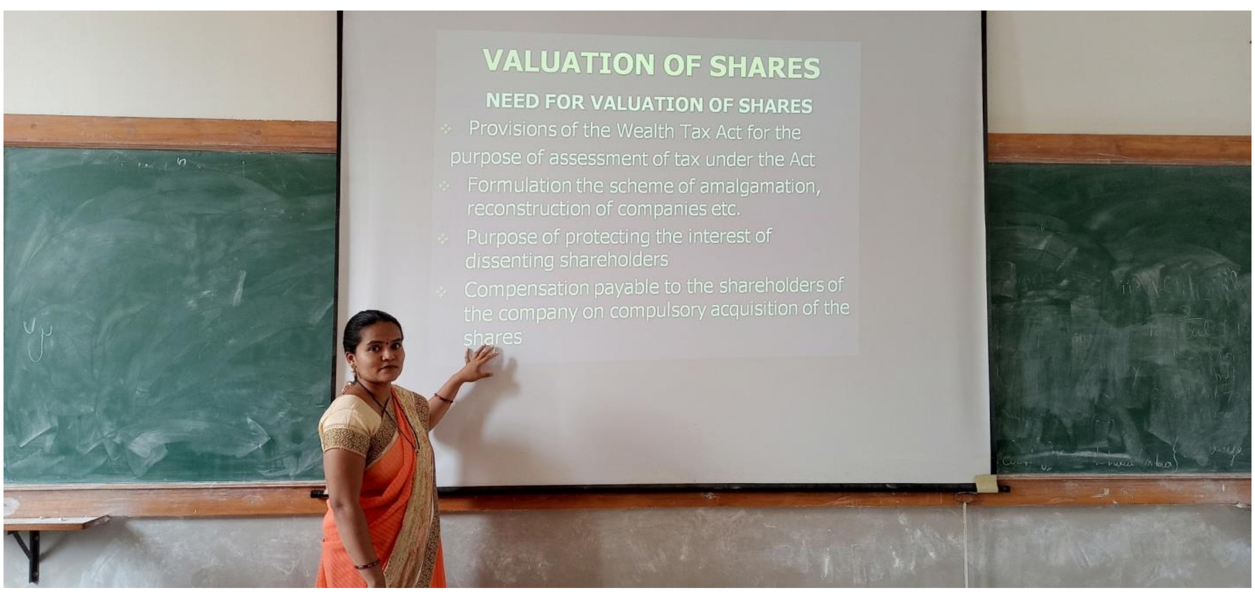
**Topic: Banking law**

# Department Of Commerce

## LECTURE BY FACULTY BY USING PPT



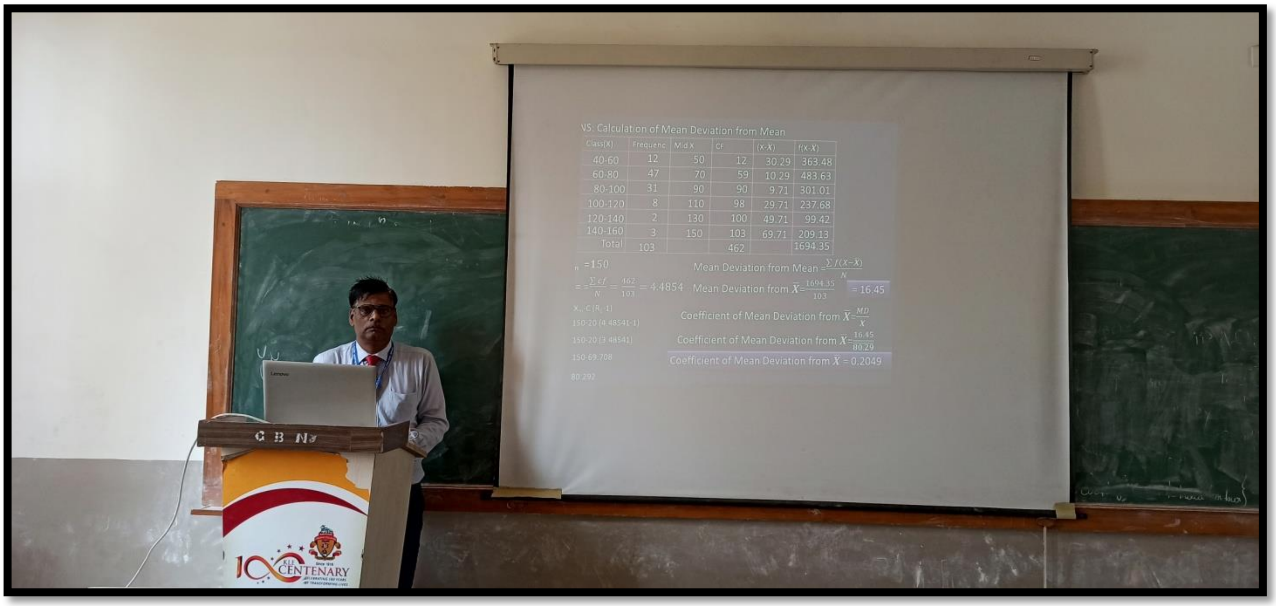

**Smt. S. A. Deshpande** **Class: B.com II sem** **Date:30/12/2019**
**Topic: Marketing**




**Smt. Priyanka Kamate** **Class: B.com IV sem** **Date:20/01/2020**
**Topic: Valuation of Share**

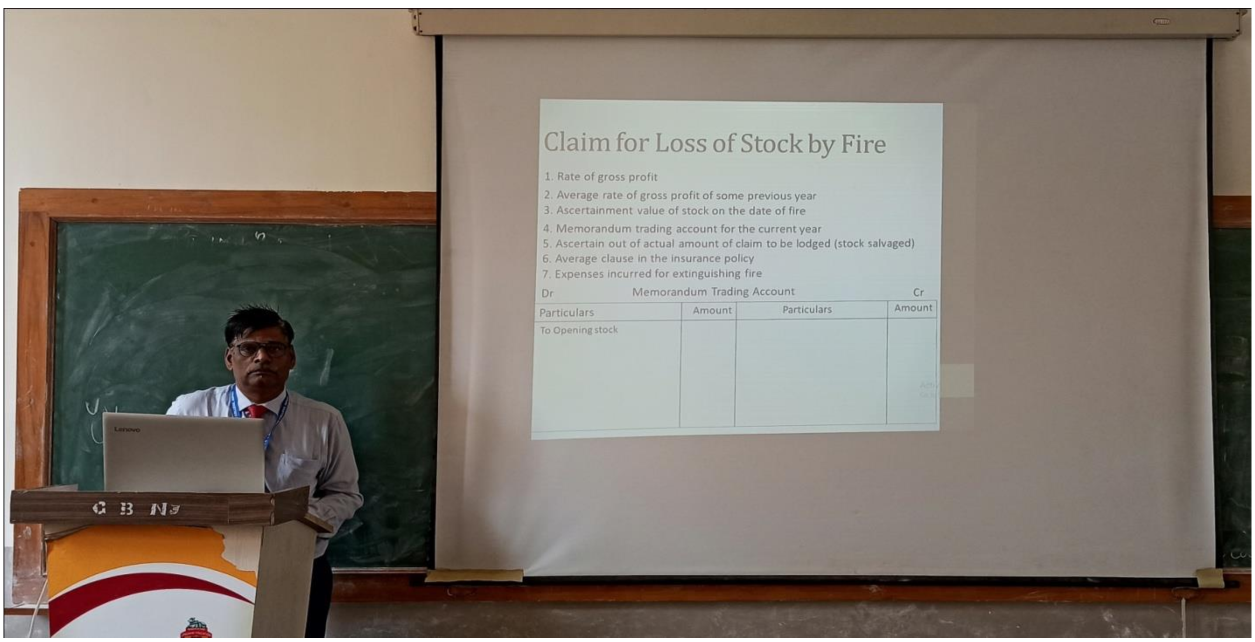
# Department Of Commerce

## LECTURE BY FACULTY BY USING PPT




Shri. B.G. Kankanwadi Class: B.com V sem Date: 05/01/2021

Topic: Calculation of Mean Deviation




Shri. B.G. Kankanwadi Class: B.com I sem Date: 29/01/2021

Topic: Classification of loss of stock

Department of Commerce


Miss.Pallavi Anure Class: B.com I sem Date:09/02/2021

Topic: Macro Environment

K.L.E. Society's

G.I.Bagewadi Arts, Science and Commerce College, Nipani-591237

Accredited at 'A' level by NAAC with CGPA 3.35

(Affiliated to Rani Channamma University, Belagavi, Karnataka, India)

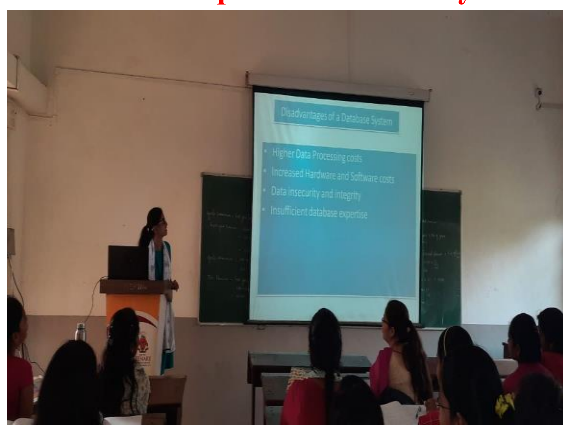
Website: www.klegibnpn.edu.in E-mail: klegib_npn@yahoo.co.in Ph.: 08338-220116

# Department Of Commerce

## ICT USED BY STUDENTS

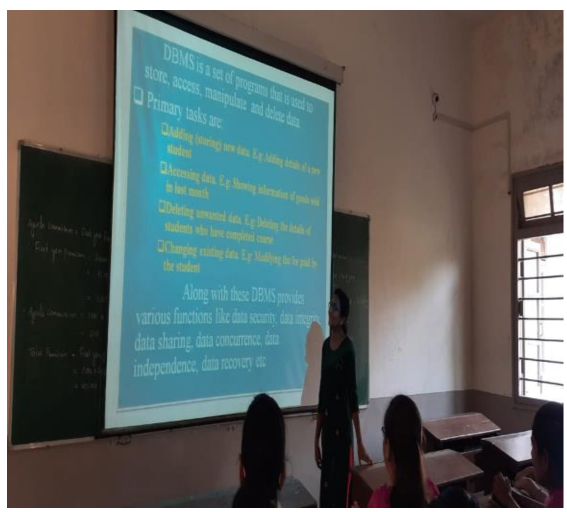
**Class : B.Com III Sem** **Topic: Data Based System**


**Name : Komal Gadakari**




**Name: Komal Kallimani**

**Class : B.Com III Sem** **Topic: Data Based Management System**

**Name :Mahin Hawaldar** **Name: Anagha Mohite**


K.L.E. Society's

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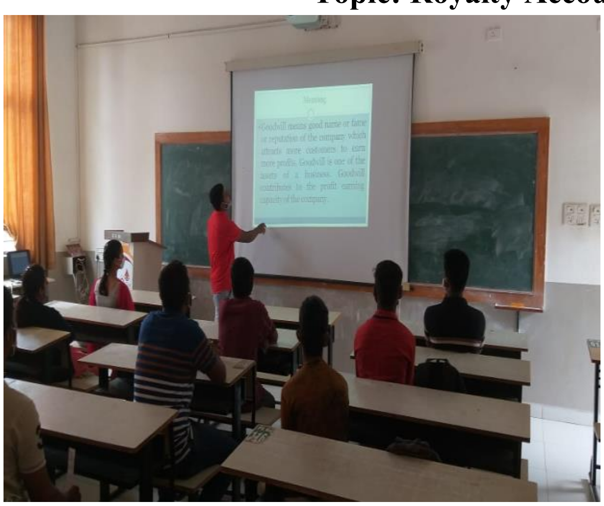
# Department Of Commerce

## ICT USED BY STUDENTS


**Name: Prajakta Malgave**
**Date: 08/08/2018**
**Class : B.Com III Sem**
**Topic: Royalty Accounts**



**Name : Nikunj Potadar**
**Date : 03/08/2018**
**Class: B.Com I Sem**
**Topic: Menaning & Objective of Farm Accounting**

**Name: Satish Chavan**
**Date: 06/08/2018**
**Class: B.Com III Sem**
**Topic : Valuation of Shares**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**Website: www.klegibnpn.edu.in E-mail: klegib_npn@yahoo.co.in Ph.: 08338-220116**


# Department Of Commerce

## ICT USED BY STUDENTS



**Name : Dannamma Shedabale**
**Class: M.Com I Sem**
**Date: 30/08/2019**
**Topic: Vision and Mission**

**Name: Rohini Chonch**
**Class: M.Com II Sem**
**Date:26/02/2020**
**Topic: Sources of Ethics**

K.L.E. Society's

**G.I.Bagewadi Arts, Science and Commerce College, Nipani-591237**

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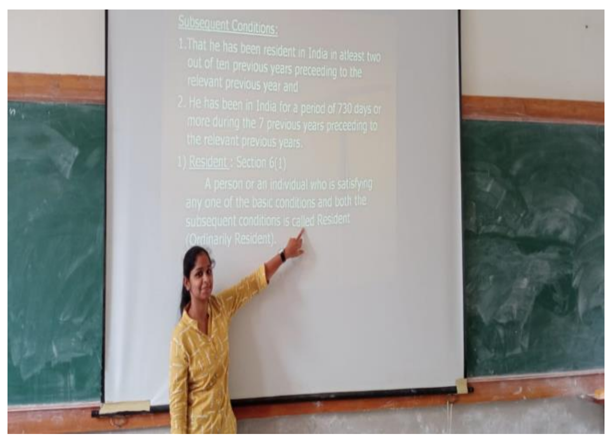
**Website: www.klegibnpn.edu.in E-mail: klegib_npn@yahoo.co.in Ph.: 08338-220116**

# Department Of Commerce

## ICT USED BY STUDENTS 2017-18


**Name :Apoorva Kamate**
**Class: B.Com I Sem**
**Date: 14/07/2017**
**Topic: Single Entry System**

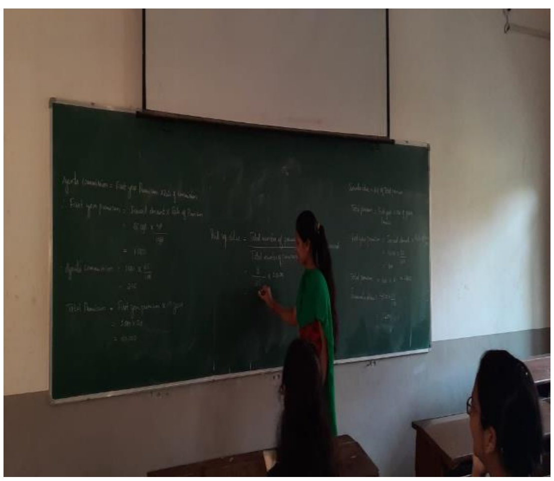



**Name: Shreya Utture**
**Class: 28/07/2017**
**Date: B.Com V Sem**
**Topic: Residential Status**

# Department Of Commerce

## PROBLEM SOLVING METHODS

**Class: B.com III Sem** **Subject: Commercial Arithmetic**



**Name: Rutuja Patil**

**Name: Shreya Khot**

**Name: Vinod Hipparagi**

**Name: Sahil Shrikhande**

# Department Of Commerce

## SEMINAR BY STUDENTS

2018-2019

Name : vidhyashree marennavar
Class: M.com I sem
Topic: Needs and Process of Motivation
Date: 17/11/2018

Name: Akshata Hiremath
Class: M.com II sem
Topic: Elements of Business Environment
Date: 12/02/2018

Name : Sonali Kadam
Class: M.com I sem
Topic: ETOP internal analysis
Date: 12/11/2018

# Department Of Commerce

## SEMINAR BY STUDENTS

2018-19

Name :Prajyoti Koth
Class: B.com II sem
Topic: Limited liability partnership
Date : 30/01/2019

Name: Asmita Magadum
Class: B.Com I Sem
Topic : Meaning, features of Companies
Date: 11/07/2018

# Department Of Commerce

## SEMINAR BY STUDENTS

2019-20

Name : Akash Shinge
Class: B.Com V sem

Name: Niranjan Chavan
Class: B.Com V Sem

Topic: Tools of Financial Analysis
Date: 11/09/2019

Topic : Deductions from Gross Total Income
Date: 14/08/2019

Name: Aarushi Rangole
Class: B.Com I sem
Topic: Meaning and features of companies
Date: 11/09/2019

# Department Of Commerce

## SEMINAR BY STUDENTS

2019-20

Name: Madhuri Kadapure
Class: M.Com I sem
Topic: External Environment
Date: 03/09/2019

Name: Rachana V
Class: M.Com I sem
Topic: BE & CG
Date: 24/09/2019

Name: Shruti Pujari
Class: M.Com II sem
Topic: Business Ethics
Date: 17/02/2020

# Department Of Commerce

## SEMINAR BY STUDENTS

2017-18

Name : Akshay Babannavar
Class: B.Com I sem
Topic: Memorandam of Association
Date:06/09/2017

Name : Malu Banne
Class: B.Com III sem
Topic:Methods of valuation
Date:09/09/2017

# Department Of Commerce

## SEMINAR BY STUDENTS

2017-18

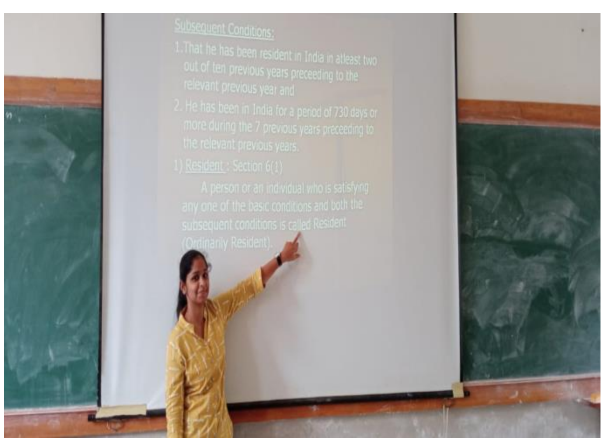

Name : Kaveri Heddurshetti
Class: M.Com III sem
Topic: Personality Development
Date:07/09/2017

Name :Megha Anagali
Class: M.Com VI sem
Topic: Special Economic Zones
Date:24/03/2018

# Department Of Commerce

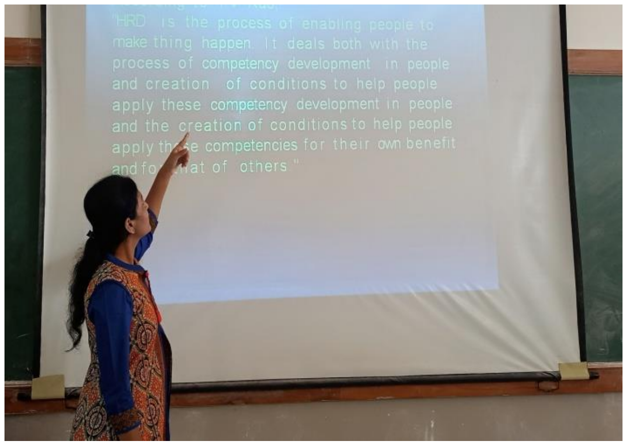
## SEMINAR BY STUDENTS

2016-17




Name : Shreya Utture

Class: B.Com IV sem
Topic: Financial Management
Date:13/01/2017




Name : Shruti Mirje
Class: M.Com IV sem
Topic: Introduction to Environmental Accounting
Date:17/03/2017

# Department Of Commerce

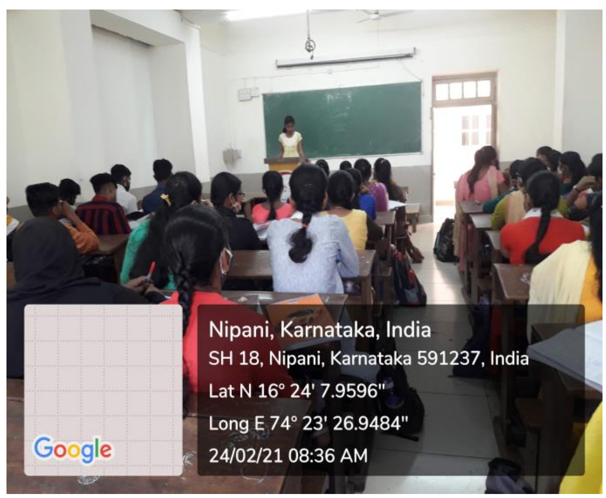
## Seminar By Students

2020-21



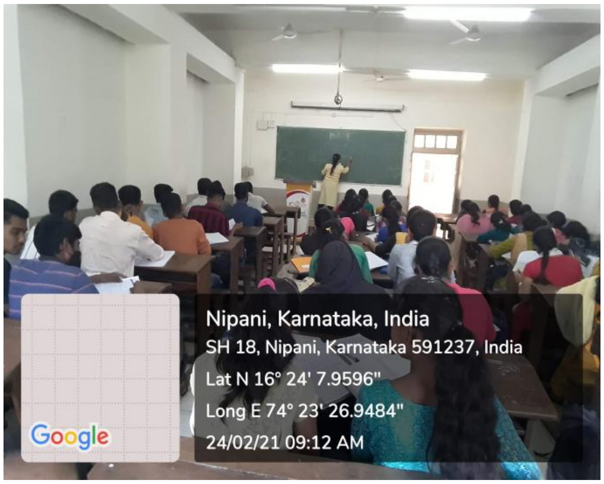
Name : Amruta Ghatage
Class: B.Com I sem Div: A
Topic: Economic Environment
Date: 23/02/2021

Name: Mithali Chinchali
Class: B.Com I Sem Div: A
Topic : Legal Environment
Date: 23/02/2021


Name: Laxmi Amble
Class: B.Com I sem Div: A
Topic: Economic System
Date: 23/02/2021

Name: Rasika Khot
Class: B.Com I sem Div: B
Topic : Royalty Account
Date: 24/02/2021



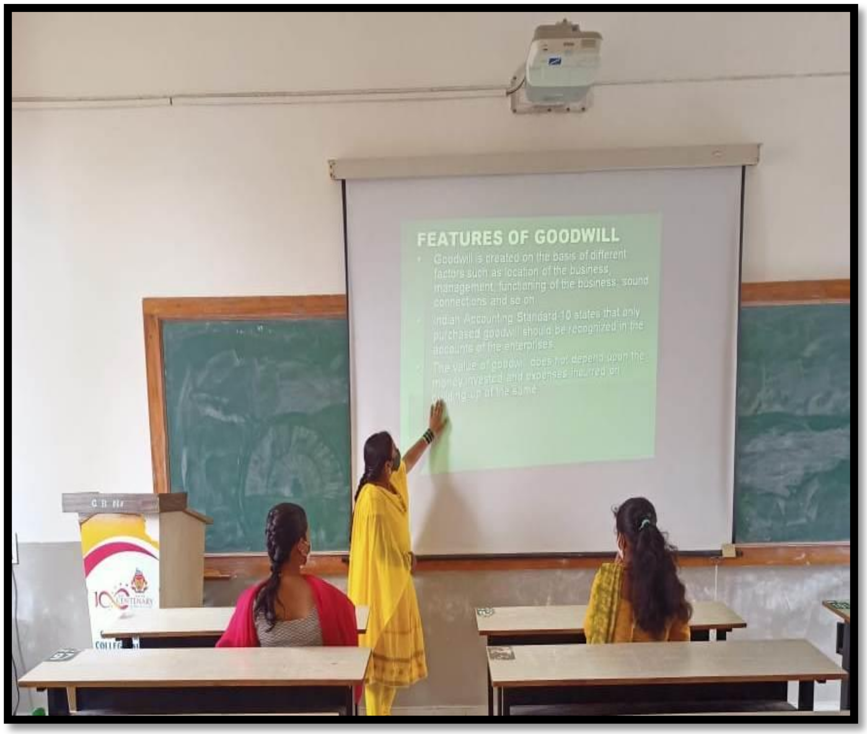


Name: Shweta Murale
Class: B.Com I sem  Div: B
Topic: Problem on Royalty Accounts
Date: 24/02/2021

Name: Laxmi P Amble
Class: B.Com I sem
Topic: Technological Environment
Date: 23/02/2021


Name: Shreya Khot
Class: B.Com III sem Div: B
Topic: Problem on Bank Accounts
Date: 24/02/2021

Name: Komal Gadakari
Class: B.Com III Sem Div: A
Topic : Problem on Valuation of Goodwill

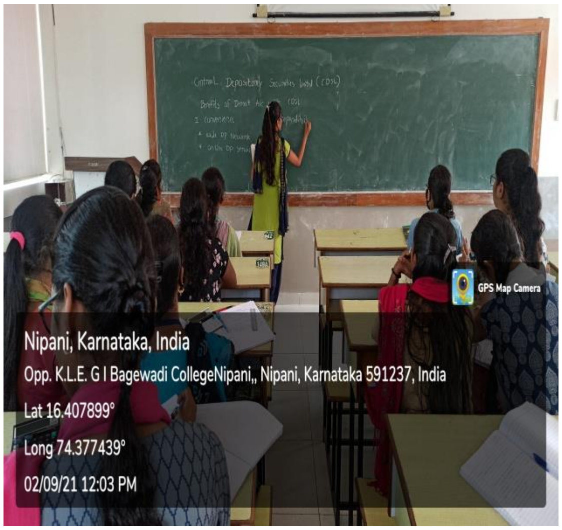


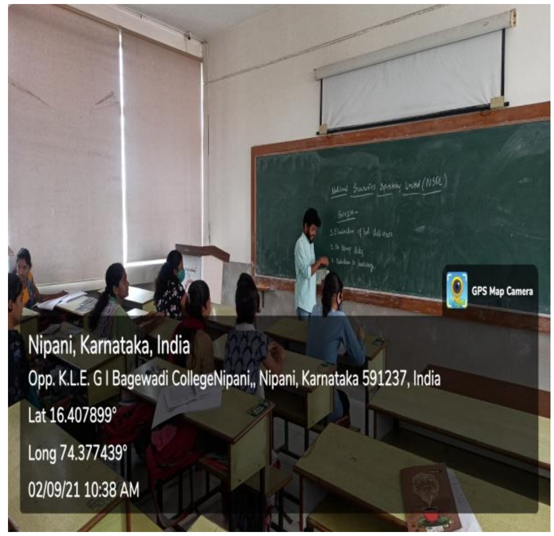

**Name :Abuli Todakar**
**Class: M.Com I Sem**
**Date:**
**Topic**: Valuation of Goodwill

# Department Of Commerce

## SEMINAR BY STUDENTS

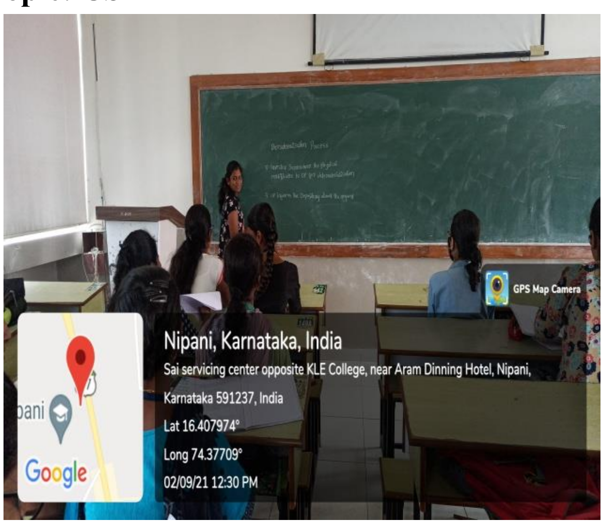
2020-21



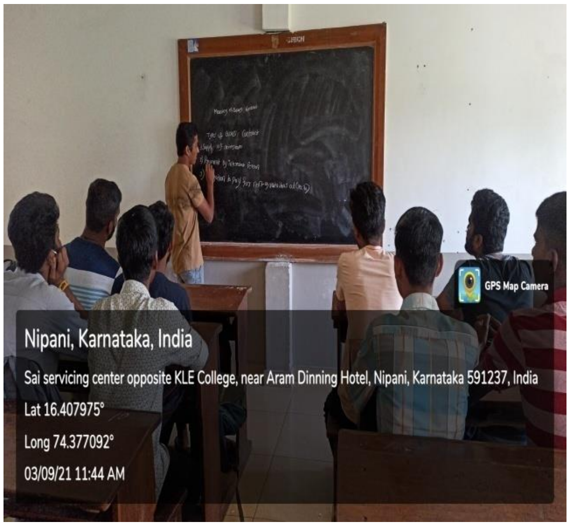




**Name : Ashwini pol**
**Class: B.Com VI Sem**
**Date: 2/9/2021**
**Topic: CSDL**

**Name : Kiran Naik**
**Class: B.Com VI Sem**
**Date: 2/9/2021**
**Topic: NSDL**




**Name : Pooja Chougule**
**Class: B.Com VI Sem**
**Date: 2/9/2021**
**Topic: Dematerialisation Process**




**Name : Suparshawa Desai**
**Class: B.Com IV Sem**
**Date: 3/9/2021**
**Topic: Quasi Contact**


**Name : Kaveri Suryavanshi**
**Class: B.Com IV Sem**
**Date: 8/9/2021**
**Topic :Rights to information under the Act**


**Name : Shashikala Vibhute**
**Class: B.Com IV Sem**
**Date: 4/9/2021**
**Topic: Difference between Indemnity & Guarantee**


**Name : Sukanya Jamkhandi**
**Class: B.Com IV Sem**
**Date: 4/9/2021**
**Topic: Rights of Security**



**Name : Aniket Ammnava**
**Class: B.Com IV Sem**
**Date: 2/9/2021**
**Topic: Types of Guarantee**

**Name: Shilpa Chikkode**
**Class: B.com II Sem**
**Date:7/9/2021**
**Topic: Participants in Securities Market**

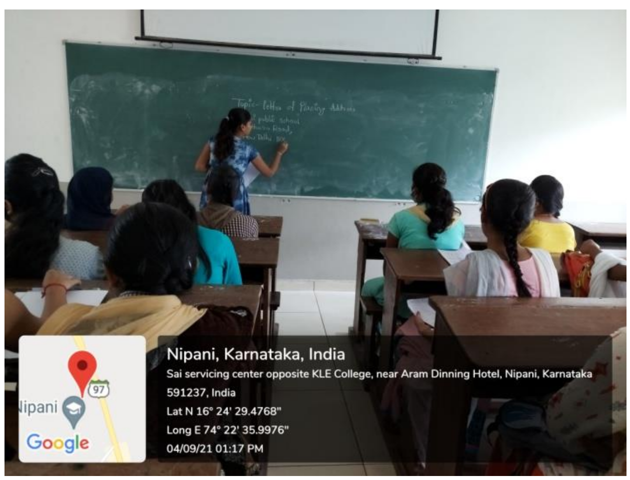


**Name: Ammanulla Nadaf**
**Class: B.com II Sem**
**Date: 7/9/2021**
**Topic: Types of Investment**

**Name: Revati Ankali**
**Class: B.com II Sem**
**Date: 7/9/2021**
**Topic: Types of FIS**


**Name: Sakshi Ankali**
**Class : B.Com II Semester**
**Date: 07.09.2021**
**Topic: Objectives of HRM**


**Name: Kaveri Borgalli**
**Class : B.Com IV Semester**
**Date: 08.09.2021**
**Topic: Components of Communication**




**Name: Padmavati Shilepatil**
**Class : B.Com IV Semester**
**Date: 04.09.2021**
**Topic: Order Placement Letters**

# Department Of Commerce

## SEMINAR BY STUDENTS

## YEAR 2020-21

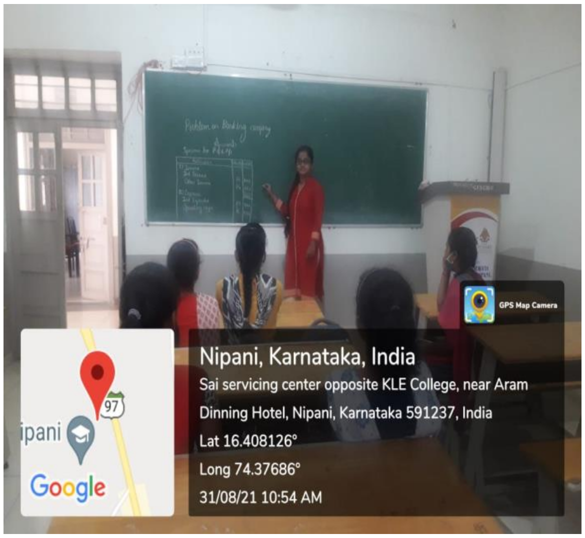

Name : Suparshwa Desai
Class: B.Com III sem
Topic: Fair Value of Shares
Date: 15/01/2021

Name: Sahil Shrikhande
Class: B.Com III sem
Topic: Valuation of Goodwill
Date: 24/02/2021

Name : Ganesh Deshinge
Class: B.Com III sem
Topic: Valuation of Goodwill
Date : 17/02/2021

Name: Shubham Thane
Class: B.Com V Sem
Topic : Income From Salary
Date: 21/12/2021

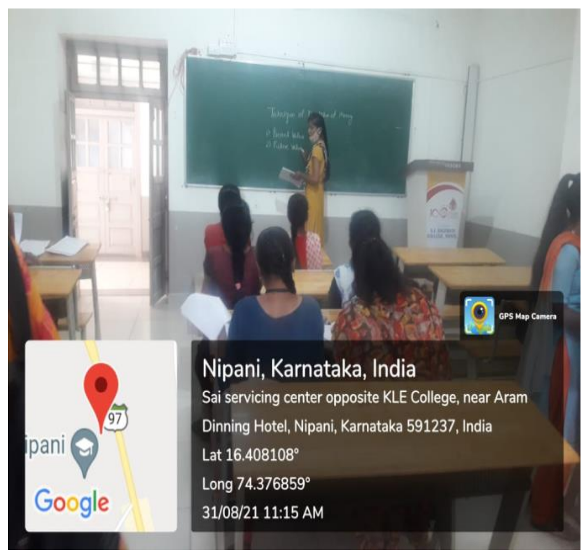
# Department Of Commerce




Name: Miss. Shreya Khot
Class: B.Com IV Sem
Topic: Problem on Banking Company
Date: 31/08/2021


Name: Miss. Rutuja walake
Class: B.Com VI Sem
Topic: Problem on Process Costing
Date: 31/08/2021

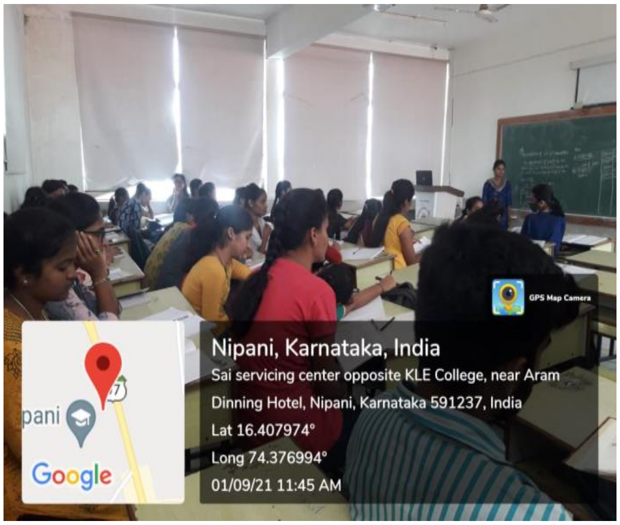

Name: Miss. Rutuja Kasar
Class: B.Com IV Sem
Topic: Methods of Forensic Accounting
Date: 31/08/2021




Name: Miss. Adarsha Joke
Class: B.Com IV Sem
Topic: Forensic Accounting
Date: 31/08/2021

# Department Of Commerce

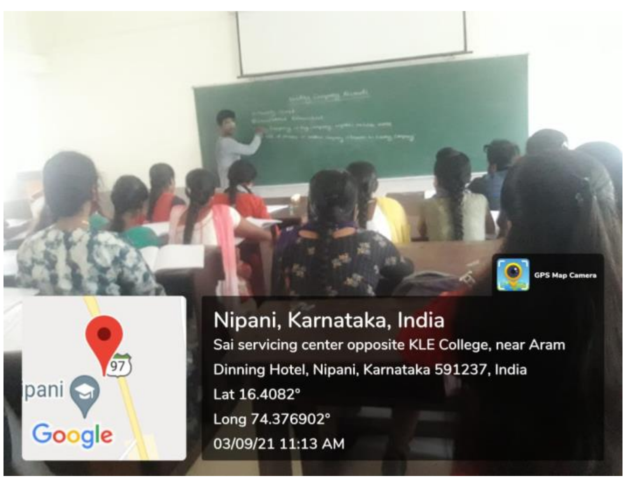
## SEMINAR BY STUDENTS





Name: Miss. Nandini D Patil
Class: B.Com VI Sem
Topic: Problem on Contract Costing
Date: 01/09/2021

Name: Mr. Shubham Thane
Class: B.Com VI Sem
Topic: Problem on Process Costing
Date: 06/09/2021





Name: Mr. Sahil Shrikhande
Class: B.Com IV Sem
Topic: Holding Company Accounts
Date: 03/09/2021

Name: Miss. Ashwini Halagadgi
Class: B.Com VI Sem
Topic: Problem on Process Costing
Date: 03/09/2021

K. L. E. Society's

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Website : www.klegibnpn.edu.in ☎ (08338) 220116 E-mail : klegib_npn@yahoo.co.in

# PDF STUDY MATERIALS

**K.L.E Society's**

**G.I. Bagewadi Arts, Science & Commerce College, NIPANI**

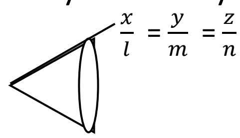
**DEPARTMENT OF MATHEMATICS**

**B.Sc. II Sem. Paper II**

***Lecture on Cone April 2020***

**Theorem:** General equation of the cone with vertex at origin is homogenous of second degree of the type $ax^2+by^2+cz^2+2hxy+2gzx+2fyz=0$.

**Cor.1**: If $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is a generator for the cone $ax^2+by^2+cz^2+2hxy+2gzx+2fyz=0$ then prove that D.R's must satisfy eqn. of cone.




Proof: Given generator is If $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}=r$

$\therefore$ any point on the generator is (lr, mr, nr), which lies on the cone

$ax^2+by^2+cz^2+2hxy+2gzx+2fyz=0$---(1)

⇨ This point (lr, mr, nr) must satisfy (1)

$\therefore$ we havea$(lr)^2+b(mr)^2+c(nr)^2+2h(mr)(nr)+2g(nr)(mr)+2f(mr)(lr)=0$

i.e $r^2(a(l)^2+b(m)^2+c(n)^2+2hlm+2gln+2fmn)=0$

But $r^2\neq0$, $\therefore$ $al^2+bm^2+cn^2+2hlm+2gln+2fmn=0$

i.e D.R's satisfy the eqn. of cone.

**Cor.2:** General equation of the cone with vertex at origin and passing through coordinate axis is $hxy+gzx+fyz=0$.

**Proof:** Let General equation of the cone with vertex at origin be $ax^2+by^2+cz^2+2hxy+2gzx+2fyz=0$.---(1)

If (1) passes through coordinate axis (they areas generators) then D.R.'s of x-axis, y-axis and z-axis must satisfy eqn. (1) by cor.(1)

But D.R's of x-axis are 1,0,0 , they satisfy eqn. (1) => $a(1)^1+0+0+0+0+0=0$ , => a=0

Similarly D.R's of y-axis are 0,1,0 and z-axis 0,0,1 must satisfy (1)

⇨ b=0 and c=0

$\therefore$ eqn. (1) becomes $0+0+0+2hxy+2gzx+2fyz=0$.

i.e $hxy+gzx+fyz=0$.

**Thus equation of the cone with vertex at origin and passing through coordinate axis is $hxy+gzx+fyz=0$.**

Examples:

1. **Prove that the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ , where $l^2+2m^2-3n^2=0$ is a generator for the cone $x^2+2y^2-3z^2=0$.**

   **Proof:** We know that if $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is a generator for the cone $x^2+2y^2-3z^2=0$ then by cor. (1), D.R's l, m, n of the generator must satisfy the eqn of cone.

   $\therefore$ we have $l^2+2m^2-3n^2=0$ which is true.

**2. Find eqn. of cone generated by the line through (1,2,3)whose D.C.'s satisfy the eqn. $2l^2+3m^2-4n^2=0$.**

**Soln.:** Eqn of generator passing through the point (1,2,3) with D.C's l, m, n is

$\frac{x-1}{l} = \frac{y-2}{m} = \frac{z-3}{n}$ = r (say)

⇨ $l = \frac{x-1}{r}$ , $m = \frac{y-2}{r}$, $n = \frac{z-3}{r}$

By cor. (1) we know that D. C's of generator satisfy the eqn. of cone and that eqn. is given by

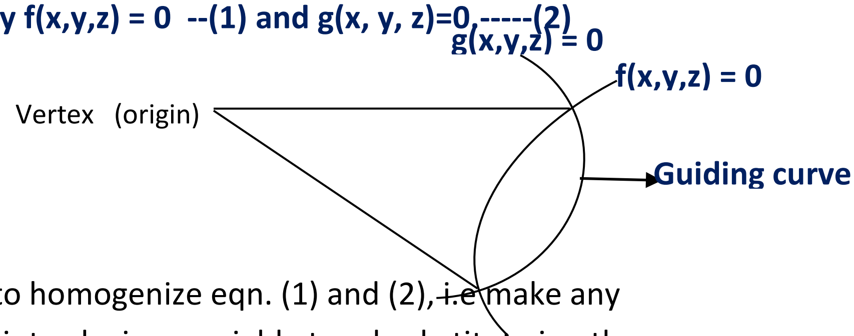
$2l^2+3m^2-4n^2 = 0$ -----------(1)

Substitute l, m, n in (1) we get $2\left(\frac{x-1}{r}\right)^2+3\left(\frac{y-2}{r}\right)^2-4\left(\frac{z-3}{r}\right)^2 = 0$

i.e $2(x-1)^2 + 3(y-2)^2 - 4(z-3)^2 = 0$.

i.e $2x^2 + 3y^2 - 4z^2 - 4x - 12y + 24z - 22 = 0$ which is required eqn. of cone.

**Finding eqn of cone with vertex at origin and passing through the guiding curve as intersection of curves given by f(x,y,z) = 0 --(1) and g(x, y, z)=0 -----(2)**




To find eqn. of cone we have to homogenize eqn. (1) and (2), i.e make any One eqn. as homogeneous by introducing variable t and substitute in other.

We come to know by following examples: (these examples are important for 2 marks)

3. Find Find eqn. to the cone with vertex at origin which passes through the curve given by $ax^2+by^2+cz^2 = 1$ and $\alpha x^2 + \beta y^2 = 2z$.

**Soln.**: Given curve is intersection of $ax^2+by^2+cz^2 = 1$ --------(1)

$\alpha x^2 + \beta y^2 = 2z$.------(2)

We have make both homogeneous by introducing 3rd variable 't'

$ax^2+by^2+cz^2 = t^2$ -------(3) homo. of degree 2.

$\alpha x^2 + \beta y^2 = 2zt$.---------(4) homo. of degree 2.

From (4), $t = \frac{\alpha x^2 + \beta y^2}{2z}$, substitute this t in (3) we get $ax^2+by^2+cz^2 = \left(\frac{\alpha x^2 + \beta y^2}{2z}\right)^2$

i.e , $4z^2(ax^2+by^2+cz^2) = (\alpha x^2 + \beta y^2)^2$ which is the Req. eqn. of the cone.

4. Find eqn. to the cone with vertex at origin which passes through the curve given by $x^2+y^2+z^2+x-2y+3z-4=0$ and $x^2+y^2+z^2+2x-3y+4z-5=0$

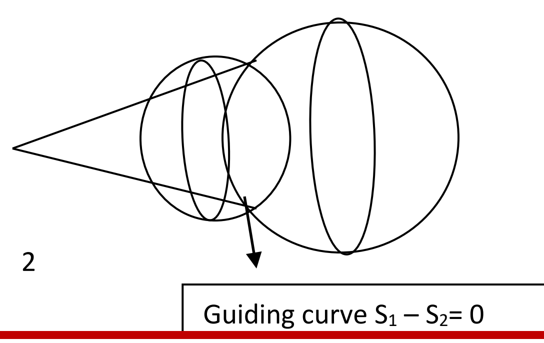
**Soln.:.** The curve of intersection of

$S_1 : x^2+y^2+z^2+x-2y+3z-4=0$ --------(1)

$S_2 : x^2+y^2+z^2+2x-3y+4z-5=0$-------(2)

is $S_1 - S_2 = 0$

i.e $x - y + z = 1$ ----(3)



Homogenizing (1) and (3) we get, $x^2+ y^2+z^2 + xt- 2yt +3zt -4t^2 = 0$ ----(4) (b'cz this eqn. of degree 2)

& $x- y +z =t$ --------------------------------(5) (b'cz this eqn. of degree 1)

Substitute $t=x-y+z$ in (4) we get $x^2+ y^2+z^2 +( x- 2y +3z)(x-y+z) -4(x-y+z)^2 = 0$ req.eqn.

**5.** Find eqn. to the cone with vertex at origin and base as a circle $x=a$ , $y^2+z^2 = b^2$. (this is a guiding curve, circle lies on the yz plane)

**Soln.:** The guiding curve is intersection of $x=a$------------(1)

**and** $y^2+z^2 = b^2$-------(2)

Homogenizing (1) and (2) we get, $x=at$ => $t = \frac{x}{a}$ ------(3)

**and** $y^2+z^2 = b^2 t^2$-------(4)

Substitute (3) in (4) we get $y^2+z^2 = b^2 (\frac{x}{a})^2$

**i.e** $a^2(y^2+z^2) = b^2x^2$, which is req. eqn. of cone.

**6.** Find eqn. to the cone with vertex at origin and base is $x^2+y^2 = 4$ and $z=2$. (this is a guiding curve, circle lies on the xy plane)

**Try this same as example (5)**

**7.** Find eqn. to the cone with vertex at (0,0,0) which passes through the curve of intersection of $x^2+ y^2+z^2 + x- 2y +3z -4 = 0$ and $x-y+z=2$.

**Try this also. Homogenize these two eqns. And replace t between them we get.**

**8.** Find eqn. to the cone with vertex at (0,0,0) which passes through the curve of intersection of plane $lx+my+nz=p$ and $ax^2+b y^2+cz^2 = 1$.

**Soln.**: Guiding curve is intersection of $lx+my+nz=p$ ----------(1)

**and** $ax^2+b y^2+cz^2 = 1$.-----(2)

Homogenizing (1) and (2) we get, $lx+my+nz=pt$ --(3) (b'cz this eqn. of degree 1)

$ax^2+b y^2+cz^2 = t^2$ ---(4) (b'cz this eqn. of degree 2)

Substitute $t = \frac{lx+my+nz}{p}$ from (3) in (4) we get $ax^2+by^2+cz^2 = (\frac{lx+my+nz}{p})^2$

i.e $p^2(ax^2+by^2+cz^2 ) = (lx + my + nz)^2$, which is req. eqn. of cone

**9.** Find eqn. to the cone with vertex at (0,0,0) which passes through the curve of intersection of $ax^2+by^2 = 2z$ plane $lx+my+nz=p$ .

**Try this example.**

**10.** Find eqn. to the cone with vertex at (0,0,0) which contains the curve given by(guiding curve) $x^2 - y^2 + 4ax=0$ plane $x+y+z=6$ .

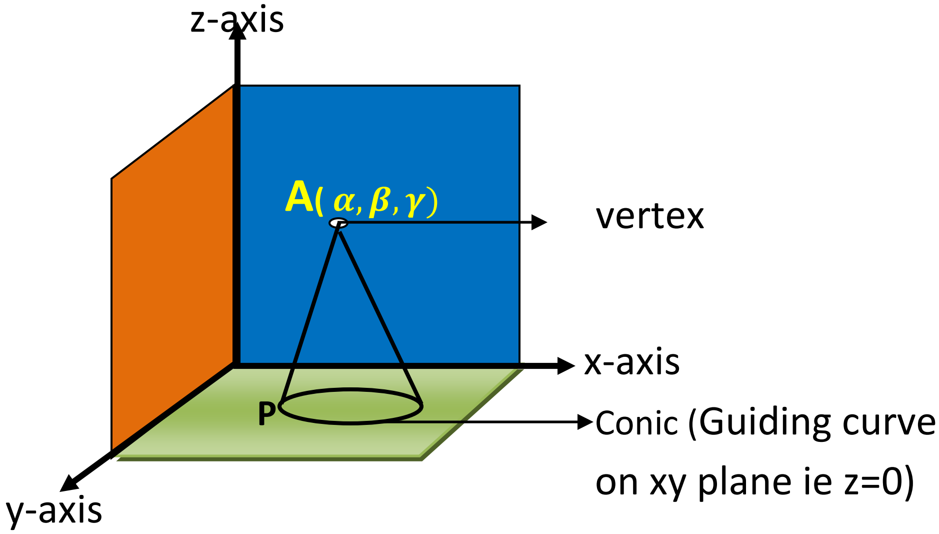
Hint: Homogenizing, $x^2 - y^2 + 4axt=0$ and $x+y+z=6t$ => $t = \frac{x+m+z}{6}$ , put in

**Theorem: To find eqn. of cone with given vertex ( $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$) and conic as guiding curve.**

**Proof:** Let A( $\alpha, \beta, \gamma$) be vertex of the cone and let $ax^2+by^2+2hxy+2gx+2fy+c=0 : z=0$ -------(1)

be given conic which is guiding curve.

**[i.e guiding curve is conic (it may be ellipse or circle) lies on xy plane i.e z=0, thatswhy in the conic z term is not there. Similarly, if conic lies on yz plane , i.e x=0, then conic contains noly y and z terms and so on.]**




Any line **AP** through A( $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$) having D.R.'s l, m, n can be written as $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ **-------(2)**

But this line intersect xy-plane at z=0 ( b'cz on xy plane z coordinate is zero, its eqn. is z=0)

$\therefore$ **we have** $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{0-\gamma}{n}$

⇨ $\frac{x-\alpha}{l} = \frac{0-\gamma}{n}$ and $\frac{y-\beta}{m} = \frac{0-\gamma}{n}$

⇨ $x = \boldsymbol{\alpha} - \frac{l\gamma}{n}$, $y = \boldsymbol{\beta} - \frac{m\gamma}{n}$, z = 0, are coordinates of P, which lies on conic (1), as shown in fig.

substitute in (1) we get ,

$a(\alpha - \frac{l\gamma}{n})^2 + + 2h(\alpha - \frac{l\gamma}{n})(\beta - \frac{m\gamma}{n}) + b(\beta - \frac{m\gamma}{n})^2 + 2g(\alpha - \frac{l\gamma}{n}) + 2f(\beta - \frac{m\gamma}{n}) + c = 0$

i.e $a(\alpha - \gamma\frac{l}{n})^2 + + 2h(\alpha - \gamma\frac{l}{n})(\beta - \gamma\frac{m}{n}) + b(\beta - \gamma\frac{m}{n})^2 + 2g(\alpha - \gamma\frac{l}{n}) + 2f(\beta - \gamma\frac{m}{n}) + c = 0$**---(3)**

But from (2) $\frac{x-\alpha}{l} = \frac{z-\gamma}{n}$ and $\frac{y-\beta}{m} = \frac{z-\gamma}{n}$

⇨ $\frac{x-\alpha}{z-\gamma} = \frac{l}{n}$ and $\frac{y-\beta}{z-\gamma} = \frac{m}{n}$ i.e $\frac{l}{n} = \frac{x-\alpha}{z-\gamma}$ and $\frac{m}{n} = \frac{y-\beta}{z-\gamma}$

Substitute these values in (3) we get

$a(\alpha - \gamma\frac{x-\alpha}{z-\gamma})^2 + 2h(\alpha - \gamma\frac{x-\alpha}{z-\gamma})(\beta - \gamma\frac{y-\beta}{z-\gamma}) + b(\beta - \gamma\frac{y-\beta}{z-\gamma})^2 + 2g(\alpha - \gamma\frac{x-\alpha}{z-\gamma}) + 2f(\beta - \gamma\frac{y-\beta}{z-\gamma}) + c = 0$

LCM is $(z - \boldsymbol{\gamma})^2$,

**∴ we get**

$a[\alpha (z-\gamma)-\gamma (x-\alpha)]^2+2h[\alpha (z-\gamma)-\gamma (x-\alpha)]\,[\beta (z-\gamma)-\gamma (y-\beta)]$
$+b\,[\beta (z-\gamma)-\gamma (y-\beta)]^2+2g\,[\alpha (z-\gamma)-\gamma (x-\alpha)](z-\gamma)$
$+2f[\beta (z-\gamma)-\gamma (y-\beta)]\,(z-\gamma)+c\,(z-\gamma)^2=0$

Simplifying we get

$a(\alpha z-\gamma x)^2+2h(\alpha z-\gamma x)(\beta z-\gamma y)+b(\beta z-\gamma y)^2$
$+2g(\alpha z-\gamma x)(z-\gamma)+2f(\beta z-\gamma y)(z-\gamma)+c\,(z-\gamma)^2=0$

Which is req. eqn. of cone with vertex as A$(\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma})$.

**NOTE: Remember the procedure which we are applying for examples, not direct formula, whatever the theory is there same we have to apply for example.**

**Examples:**

1. Find eqn. to the cone with vertex at (1,2,3) and whose generating line pass through the conic $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and z=0 .(i.e guiding curve is ellipse lies on xy plane)

**Soln.: Given conic** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and z=0 ------------------(1)

Any line **AP** through A$(\mathbf{1},\mathbf{2},\mathbf{3})$ having D.R.'s l, m, n can be written as $\frac{x-\mathbf{1}}{l}=\frac{y-\mathbf{2}}{m}=\frac{z-\mathbf{3}}{n}$ **-------(2)**

But this line intersect xy-plane at z=0 ( b'cz on xy plane z coordinate is zero, its eqn. is z=0)

**∴ we have** $\frac{x-\mathbf{1}}{l}=\frac{y-\mathbf{2}}{m}=\frac{0-\mathbf{3}}{n}$

⇨ $\frac{x-\mathbf{1}}{l}=\frac{0-\mathbf{3}}{n}$ and $\frac{y-\mathbf{2}}{m}=\frac{0-\mathbf{3}}{n}$

⇨ $x=\mathbf{1}-\mathbf{3}\frac{l}{n}$, $y=\mathbf{2}-\mathbf{3}\frac{m}{n}$, z = 0, are coordinates of P, which lies on conic (1),

substitute in (1) we get ,

$\frac{(1-\mathbf{3}\frac{l}{n})^2}{a^2}+\frac{(\mathbf{2}-\mathbf{3}\frac{m}{n})^2}{b^2}=\mathbf{1}$ **-------------------------------------------------------------------------------------------(3)**

But from (2) $\frac{x-\mathbf{1}}{l}=\frac{z-\mathbf{3}}{n}$ and $\frac{y-\mathbf{2}}{m}=\frac{z-\mathbf{3}}{n}$

⇨ $\frac{x-\mathbf{1}}{z-\mathbf{3}}=\frac{l}{n}$ and $\frac{y-\mathbf{2}}{z-\mathbf{3}}=\frac{m}{n}$ i.e $\frac{l}{n}=\frac{x-\mathbf{1}}{z-\mathbf{3}}$ and $\frac{m}{n}=\frac{y-\mathbf{2}}{z-\mathbf{3}}$

Substitute these values in (3) we get

$\frac{1}{a^2}(1-\mathbf{3}\frac{x-\mathbf{1}}{z-\mathbf{3}})^2+\frac{1}{b^2}(\mathbf{2}-\mathbf{3}\frac{y-\mathbf{2}}{z-\mathbf{3}})^2=1$

LCM is $(z-\mathbf{3})^2$,

**∴ we get**

$\frac{1}{a^2}[(z-3)-\mathbf{3}(x-1)]^2+\frac{1}{b^2}[2(z-3)-3(y-2)]^2=(z-3)^2$

Simplifying we get, $b^2[z-3x]^2+a^2[2z-3y]^2=a^2b^2(z-3)^2$

which is req. eqn. of cone with vertex at A$(\mathbf{1},\mathbf{2},\mathbf{3})$.

2. Find eqn. to the cone with vertex at (0,0,3) and guiding curve $x^2+ y^2 =4$ and z=0 .(i.e guiding curve is circle lies on xy plane)

**Soln.: Given conic** $x^2+ y^2 =4$ and z=0 ------------------(1)

Any line **AP** through A$(\mathbf{0},\mathbf{0},\mathbf{3})$ having D.R.'s l, m, n can be written as $\frac{x-\mathbf{0}}{l}=\frac{y-\mathbf{0}}{m}=\frac{z-\mathbf{3}}{n}$ **-------(2)**

But this line intersect xy-plane at z=0 ( b'cz on xy plane z coordinate is zero, its eqn. is z=0)

∴ **we have** $\frac{x-\mathbf{0}}{l}=\frac{y-\mathbf{0}}{m}=\frac{0-\mathbf{3}}{n}$

⇨ $\frac{x-\mathbf{0}}{l}=\frac{0-\mathbf{3}}{n}$ and $\frac{y-\mathbf{0}}{m}=\frac{0-\mathbf{3}}{n}$

⇨ x = $-\mathbf{3}\frac{l}{n}$, y= -- $\mathbf{3}\frac{m}{n}$, z = 0, are coordinates of P, which lies on conic (1),

substitute in (1) we get ,

$(-\mathbf{3}\frac{l}{n})^2+(-\mathbf{3}\frac{m}{n})^2=4$ ------------------------------------------------------------------------------------**(3)**

But from (2) $\frac{x}{l}=\frac{z-\mathbf{3}}{n}$ and $\frac{y}{m}=\frac{z-\mathbf{3}}{n}$

⇨ $\frac{x}{z-\mathbf{3}}=\frac{l}{n}$ and $\frac{y}{z-\mathbf{3}}=\frac{m}{n}$ i.e $\frac{l}{n}=\frac{x}{z-\mathbf{3}}$ and $\frac{m}{n}=\frac{y}{z-\mathbf{3}}$

Substitute these values in (3) we get

$(-\mathbf{3}\frac{x}{z-\mathbf{3}})^2+(-\mathbf{3}\frac{y}{z-\mathbf{3}})^2=4$

LCM is $(z-\mathbf{3})^2$,

∴ **we get**

$[-\mathbf{3}(x)]^2+[-3(y)]^2=4(z-3)^2$

Simplifying we get, $9x^2+9y^2=4(z-3)^2$

which is req. eqn. of cone with vertex at A$(\mathbf{1},\mathbf{2},\mathbf{3})$.

3. Find eqn. to the cone with vertex at (0,0,1) and guiding curve $x^2+ y^2 =1$ and z=0 .(i.e guiding curve is circle lies on xy plane)
4. Find eqn. to the cone with vertex at (1,2,3) and guiding curve $x^2+ y^2 =9$ and z=0 .(i.e guiding curve is circle lies on xy plane)

   **Try these two examples**
5. Find eqn. to the cone with vertex at (1,1,0) and guiding curve $x^2+ z^2 =4$ and y=0 .(i.e guiding curve is circle lies on xz plane)

**Soln.: Given conic** $x^2+ z^2 =4$ and y=0 ------------------(1)

Any line **AP** through A$(\mathbf{1},\mathbf{1},\mathbf{0})$ having D.R.'s l, m, n can be written as $\frac{x-\mathbf{1}}{l}=\frac{y-\mathbf{1}}{m}=\frac{z-\mathbf{0}}{n}$ **-------(2)**

But this line intersect xz-plane at y=0 ( b'cz on xz plane y coordinate is zero, its eqn. is y=0)

∴ **we have** $\frac{x-\mathbf{1}}{l}=\frac{0-\mathbf{1}}{m}=\frac{z-\mathbf{0}}{n}$

⇨ $\frac{x-\mathbf{1}}{l} = \frac{0-\mathbf{1}}{m}$ and $\frac{z-\mathbf{0}}{n} = \frac{0-\mathbf{1}}{m}$

⇨ x = $\mathbf{1}-\frac{l}{m}$, z= $\frac{-n}{m}$, y = 0, i. e x = $1-\frac{l}{m}$, y = 0, z= $\frac{-n}{m}$, are coordinates of P, which lies on conic (1),

substitute in (1) we get ,

$(\mathbf{1}-\frac{l}{m})^2 + (\frac{-n}{m})^2 = 4$ ----------------------------------------------------------------------------------------**(3)**

But from (2) $\frac{x-\mathbf{1}}{l} = \frac{y-\mathbf{1}}{m}$ and $\frac{y-\mathbf{1}}{m} = \frac{z-\mathbf{0}}{n}$

⇨ $\frac{x-1}{y-1} = \frac{l}{m}$ and $\frac{z}{y-1} = \frac{n}{m}$ i.e $\frac{l}{m} = \frac{x-1}{y-1}$ and $\frac{n}{m} = \frac{z}{y-1}$

Substitute these values in (3) we get

$(1-\frac{x-1}{y-1})^2 + (-\frac{z}{y-1})^2 = 4$

LCM is $(y-1)^2$,

**∴ we get**

$[y-x]^2 + [-z]^2 = 4(y-1)^2$

Simplifying we get, $x^2+y^2-2xy+z^2 = 4(y-1)^2$, i.e $x^2 - 3y^2 + z^2 - 2xy + 8y - 4 = 0$

which is req. eqn. of cone with vertex at A$(\mathbf{1},\mathbf{1},\mathbf{0})$.

**Another type of examples:(Important for 5 marks)**

**In this type they will give one eqn. and ask to prove it is cone and also to find vertex.**

**Procedure:**

**Given eqn.is interms of x,y and z, hence let it be f(x,y,z)=0, which is general eqn. of second degree.**

**Introduce variable 't' and make the eqn. as homogeneous of second degree in x,y,z and t**

**Let it be F(x, y, z, t)=0----------------------------------(1)**

**Differentiate (1) partially w.r.t x, y, z, and t,**

**We get four equations namely $\frac{\partial F}{\partial x} = 0$ ------------(2), $\frac{\partial F}{\partial y} = 0$ ------------(3),**

**$\frac{\partial F}{\partial z} = 0$ ------------(4), $\frac{\partial F}{\partial t} = 0$ ------------(5), in all these put t=1 first and solve any three of the above eqns. For x, y and z and substitute in the remaining eqn. it must be satisfied then we say give represent cone with (x,y,z)as vertex.**

**You come to know by following example**

**Examples:**

1. Prove that eqn. $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$ represents cone & find its vertex.

**Soln.:** Given eqn. be $f(x, y, z) = 4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$--------------**(1)**

Make it homogeneous of second degree by introducing t

i.e. $F(x, y, z, t) = 4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12xt - 11yt + 6zt + 4t^2 = 0$----------------------------**(2)**

Differentiate (2) partially w.r.t x, y, z, and t, We get $\frac{\partial F}{\partial x}$ = 0 , 8x -0 + 0 + 2y +0 + 12t – 0 +0 +0 +0=0

i.e $\frac{\partial F}{\partial x}$ = 0 => 8x + 2y +12t =0, i.e 8x + 2y +12 =0 ---------------------(3) i.e put t=1

Next, $\frac{\partial F}{\partial y}$ = 0 => -2y +2x -3z - 11t =0 , i.e 2x-2y – 3z -11 = 0------------------(4) by sub. t=1

$\frac{\partial F}{\partial z}$ = 0 => 4z – 3y + 6t = 0, i.e 4z – 3y + 6 = 0---------------------(5), t=1

$\frac{\partial F}{\partial t}$ = 0 => 12x -11y +6z +8t=0 i.e 12x -11y +6z +8=0 -------------(6), t=1

(3) – 4(4) gives 8x + 2y +12- (8x – 8y -12z -44) =0

i.e 10y +12z +56=0 i.e 6z + 5y +23=0------------------------------------(7)

Next, 3(4) – 2(7) gives

12z – 9y +18 –(12z+10y+56)=0

i.e -17y -34 =0 => **y = -2**

Put y = - 2 in (5) we get, 4z – 3(-2) + 6= 0 i.e **z = -3**, and from (3) we get **x =-1**

We used only eqns, (3), (4) and (5) and got **x =-1, y = -2, z = -3** , substitute these in (6)

12(-1) – 11(-2) + 6(-3) +8 = -12+22-18+8 = -30 +30 =0, so it is satisfied by these values.

⇨ Eqn. (1) represents cone with **(-1, -2, -3)** as vertex.

**2.** Prove that eqn. $2x^2 +2 y^2 +7z^2$ – 10yz - 10zx +2x +2y +26z -9 =0 represents cone & find its vertex.

**Soln.:** Given eqn. be f(x, y, z) = $2x^2 +2 y^2 +7z^2$ – 10yz - 10zx +2x +2y +26z -17 =0 **--------------(1)**

Make it homogeneous of second degree by introducing t

i.e. F(x, y, z, t) =$2x^2 +2 y^2 +7z^2$ – 10yz - 10zx +2xt +2yt +26zt $-17t^2$ =0 ---------------------------**(2)**

Differentiate (2) partially w.r.t x, y, z, and t,

We get $\frac{\partial F}{\partial x}$ = 0 , 4x -10z +2t =0 i.e 4x -10z +2 =0 ( put t=1) -----------(3)

Next, $\frac{\partial F}{\partial y}$ = 0 => 4y -10z +2t=0 , i.e 4y -10z +2=0----------------------(4) by sub. t=1

$\frac{\partial F}{\partial z}$ = 0 => 14z – 10y - 10x+ 26t = 0, i.e 14z – 10y - 10x+ 26 = 0---------------------(5), t=1

$\frac{\partial F}{\partial t}$ = 0 => 2x +2y +26z -34t=0 i.e 2x +2y +26z - 34=0 -------------(6), t=1

(3) – (4) gives 4x -10z +2 – (4y-10z+2)=0

i.e 4x – 4y =0 ie x – y =0 i.e x =y -------------------------------------(7)

Put x =y in (5) , we get 14z – 10y - 10y+ 26 = 0

Ie 14z -20y +26 =0 i.e -10y + 7z +13=0--------------------(8)

Then 5(4) + 2(8) gives 20y -50z +10 +(-20y +14z +26)=0

-36z +36=0 => z = 1

Put z =1 in (4) we get 4y – 10 +2 =0 => y =2 => x =2 from (7)

Put x=2, y = 2 and z=1 in (6) we get

2(2)+2(2)+26(1)-18=4 + 4 +26 - 34 =34-34 =0

∴ (6) is satisfied by (2,2,1)

⇨ Eqn. (1) represents cone with **(2,2,1)** as vertex.

**3.** Prove that eqn. $2x^2 -8xy - 4yz - 4x -2y +6z + 35 = 0$ represents cone & find its vertex.

**Soln.:** Given eqn. be $f(x, y, z) = 2x^2 -8xy - 4yz - 4x -2y +6z + 35 = 0$ **--------------(1)**

Make it homogeneous of second degree by introducing t

i.e. $F(x, y, z, t) = 2x^2 -8xy - 4yz - 4xt -2yt +6zt + 35t^2 = 0$ ----------------------------**(2)**

Differentiate (2) partially w.r.t x, y, z, and t,

We get $\frac{\partial F}{\partial x} = 0$ => $4x -8y -4t=0$ i.e $4x -8y -4 =0$ ( put t=1) -----------(3)

Next, $\frac{\partial F}{\partial y} = 0$ => $- 8x -4z -2t=0$ , i.e $8x +4z +2=0$----------------------(4) by sub. t=1

$\frac{\partial F}{\partial z} = 0$ => $- 4y + 6= 0$, => $y = \frac{3}{2}$

$\frac{\partial F}{\partial t} = 0$ => $- 4x -2y +6z +70t=0$ i.e $- 4x -2y +6z +70=0$ -------------(6), t=1

Put $y = \frac{3}{2}$ in (3) we get $4x - 8(\frac{3}{2}) -4=0$, => $x = 4$

Put x=4 in (5) we get, 32+4z+2=0 => 4z = -34 ∴ $z = -\frac{17}{2}$

Put x=4, $y =\frac{3}{2}$ and $z=\frac{-17}{2}$ in (6) we get

$-16 - 2(\frac{3}{2}) + 6(-\frac{17}{2}) +70 = -16-3 -51 +70 = -70 + 70 = 0$ ∴ (6) is satisfied by $(4, \frac{3}{2}, -\frac{17}{2})$

⇨ Eqn. (1) represents cone with $(4, \frac{3}{2}, -\frac{17}{2})$ as vertex.

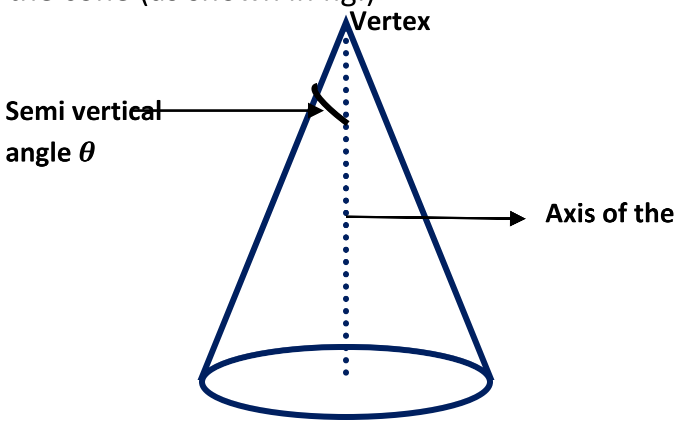
**4.** Prove that eqn. $x^2 - 2y^2 +3z^2 - 4xy +5yz -6zx + 8x -19y -2z - 20 = 0$ represents cone & find its vertex.

**5.** Prove that eqn. $2y^2 - 8xy - 8yz -4zx + 6x -4y -2z +5 = 0$ represents cone & find its vertex.

**Try (4) and (5) as home work**

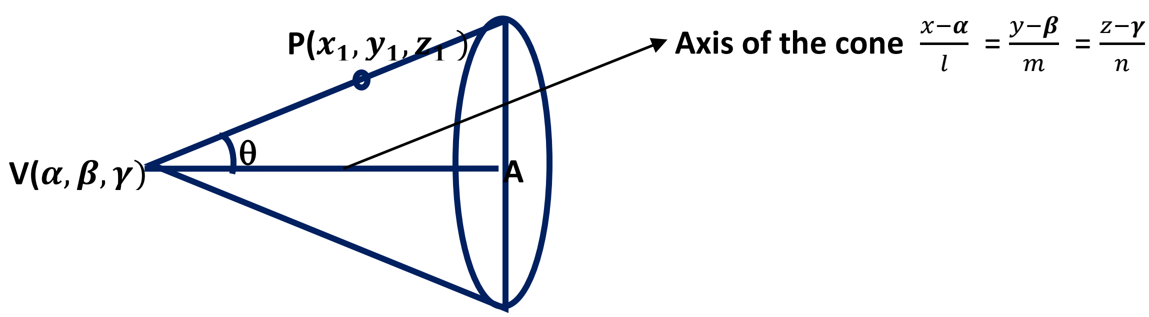
## Right circular Cone:

**Definition(important for 2. Marks):** A surface generated by a line passing through a fixed point and making constant angle with fixed line through vertex is called a right circular cone. The fixed point is called as the vertex, fixed line is called axis of the cone and constant angle is called semi vertical angle of the cone (as shown in fig.)




**Theorem:** To find the eqn. of the right circular con with vertex V$(\alpha, \beta, \gamma)$ , eqn. of the axis as $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and semi vertical angle θ

**Proof:**




**Let P$(x_1, y_1, z_1)$ be any point on the surface of the cone (i.e anywhere on the surface) then VP is the generator of the cone which makes an angle θ with axis of the cone VA.**

D.R.'s of VP are $(x_1 - \alpha), (y_1 - \beta), (z_1 - \gamma)$ ,

D.R.'s of VA are $l$, m, n and angle between VP and VA is θ

By using the formula for angle between the lines,

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} = \frac{(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n}{\sqrt{(x_1-\alpha)^2+(y_1-\beta)^2+((z_1-\gamma)^2}\sqrt{l^2+m^2+n^2}}$$

$\therefore \cos\theta = \dfrac{(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n}{\sqrt{(x_1-\alpha)^2+(y_1-\beta)^2+((z_1-\gamma)^2}\sqrt{l^2+m^2+n^2}}$

Squaring both the sides we get

$$\cos^2\theta = \frac{[(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n]^2}{\left[(x_1-\alpha)^2+(y_1-\beta)^2+\left((z_1-\gamma)\right)^2\right][l^2+m^2+n^2]}$$

i.e $\cos^2\theta[(x_1-\alpha)^2+(y_1-\beta)^2+((z_1-\gamma)^2][l^2+m^2+n^2] = [(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n]^2$

**But P($x_1, y_1, z_1$) be any point on the surface of the cone and hence locus of the point P i.e replace ($x_1, y_1, z_1$) by (x, y, z) in the above eqn. we get**

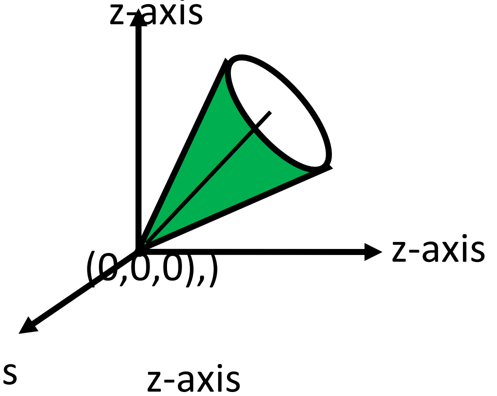
$\cos^2\theta[(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2][l^2+m^2+n^2] = [x-\alpha)l+(y-\beta)m+(z-\gamma)n]^2$

**Thus the eqn. of the right circular cone with vertex V($\alpha, \beta, \gamma$), eqn. of the axis as $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and semi vertical angle θ is**

$\boldsymbol{\cos^2\theta[(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2][l^2+m^2+n^2] = [x-\alpha)l+(y-\beta)m+(z-\gamma)n]^2}$ **--(I)**

**Corollary 1:** The eqn. of the right circular cone with vertex V(0, 0, 0), eqn. of the axis as $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and semi vertical angle θ is

$\cos^2\theta[x^2+y^2+z^2][l^2+m^2+n^2] = [xl+ym+zn]^2$




**Proof:** In (I) if we put ($\alpha, \beta, \gamma$) as (0,0,0) we get

$\cos^2\theta[x^2+y^2+z^2][l^2+m^2+n^2] = [xl+ym+zn]^2$

**Corollary 2:** The eqn. of the right circular cone with vertex V(0, 0, 0), the axis as $z-axis$ and semi vertical angle θ is $\mathbf{z^2\tan^2\theta = x^2 + y^2}$ **(as in fig.)**

**Proof:** If Z –axis is axis of cone , its D.R.'s 0,0,1,

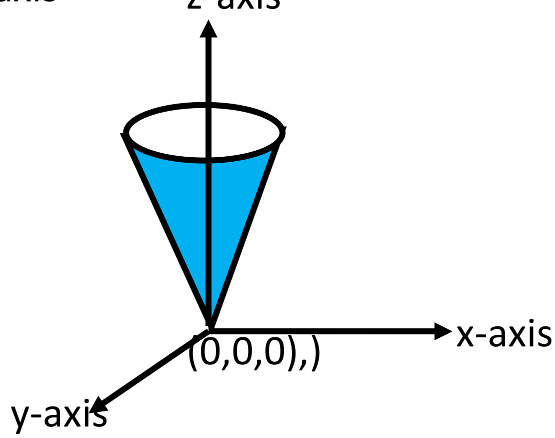
$\therefore$ In (I ) ($\alpha, \beta, \gamma$) as (0,0,0) and l, m, n, as 0,0,1

We get, $\cos^2\theta[x^2+y^2+z^2][0+0+1]=[0+0+z]^2$

i.e $\cos^2\theta[x^2+y^2] = z^2[1 - \cos^2\theta]$

i.e $\cos^2\theta[x^2+y^2] = z^2\sin^2\theta$

i.e $[x^2+y^2] = z^2\tan^2\theta$ or $z^2\tan^2\theta = x^2 + y^2$



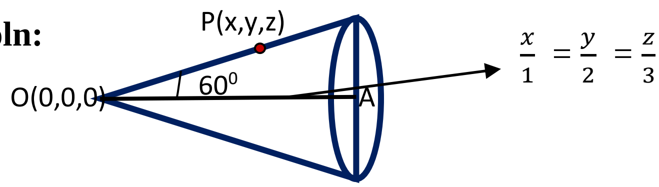
Similarly eqn. of the right circular cone with vertex V(0, 0, 0), the axis as $x-$ *axis* and semi vertical angle θ is $\mathbf{x^2\tan^2\theta = y^2 + z^2}$ **and** eqn. of the right circular cone with vertex V(0, 0, 0), the axis as $y-$ *axis* and semi vertical angle θ is $\mathbf{y^2\tan^2\theta = x^2 + z^2}$

**Note: These corollaries are important for 2 marks.**

**Examples:**

**1.** Find the eqn. of the right circular cone with vertex (0, 0, 0), eqn. of the axis as $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and semi vertical angle θ is $60^0$.

**Soln:**




Soln: Let P(x,y,z) be any point on the surface of the cone.

D.R.'s of OP are x-0, y-0, z-0,

D.R's of axis are 1, 2, 3 and θ $=60^0$

$\therefore \cos 60 = \frac{x1+y2+z3}{\sqrt{x^2+y^2+z^2}\sqrt{1+2^2+3^2}}$

i.e $\frac{1}{2} = \frac{x+2y+3z}{\sqrt{x^2+y^2+z^2}\sqrt{14}}$

Squaring both the sides we get ,

$14\,(x^2+y^2+z^2) = 4\,(x+2y+3z)^2$ ,

i.e $7\,(x^2+y^2+z^2) = 2\,(x+2y+3z)^2$ , on simplifying we get,

$\mathbf{5x^2 - y^2 - 11z^2 - 8xy - 24yz - 12zx = 0}$**, which is req. eqn. of rt. circular cone**

**2.** Find the eqn. of the right circular cone with vertex (0, 0, 0), eqn. of the axis as $\frac{x}{2} = \frac{y}{-4} = \frac{z}{3}$ and semi vertical angle θ is $30^0$.

**Soln:** D.R.'s of OP are x-0, y-0, z-0, and D.R's of axis are 2, -4, 3 and θ $=30^0$

$\therefore \cos 30 = \frac{2x+y(-4)+z(3)}{\sqrt{x^2+y^2+z^2}\sqrt{2^2+(-4^2)+3^2}}$

i.e $\frac{\sqrt{3}}{2} = \frac{2x-4y+3z}{\sqrt{x^2+y^2+z^2}\sqrt{29}}$

$\sqrt{3}\sqrt{x^2+y^2+z^2}\sqrt{29} = 2(2-4y+3z)$

Squaring both the sides , we get

$\mathbf{87(x^2+y^2+z^2) = 4\,(2-4y+3z)^2}$ **which is req. eqn. of rt. circular cone**

**3.** Find the eqn. of the right circular cone with

(i) vertex (0, 0, 0), eqn. of the axis as $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ and semi vertical angle θ is $45^0$.

(ii) vertex (0, 0, 0), eqn. of the axis as $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and semi vertical angle θ is 30.

(iii) vertex (0, 0, 0), eqn. of the axis as $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and semi vertical angle θ is $60^0$.

**Try above three examples as Home work**

**4.** Find the eqn. of the right circular cone with vertex as (0,0,0) , axis as z-axis and $\theta=30^0$.

**Soln.: We know that** eqn. of the right circular cone with vertex as (0,0,0) , axis as z-axis and smi vertical angle as θ is $z^2\tan^2\theta = x^2 + y^2$

**But $\theta = 30^0$, ∴ we have** $z^2\tan^2(30) = x^2 + y^2$

i.e $z^2\frac{1}{3} = x^2 + y^2$ => $3(x^2 + y^2) - z^2 = 0$, req. eqn. of cone.

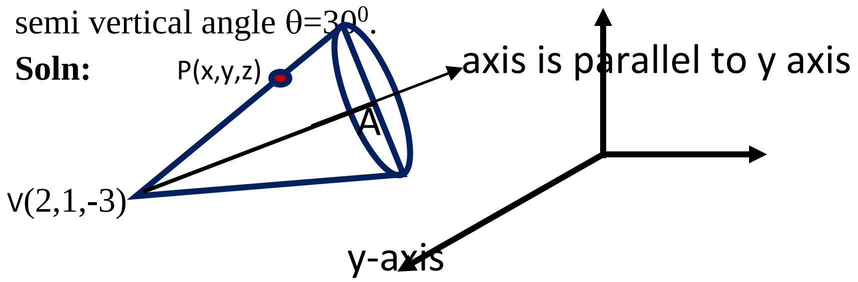
**5.** Find the eqn. of the right circular cone with vertex as (0,0,0) , axis as x-axis and $\theta=60^0$.

**6.** Find the eqn. of the right circular cone with vertex as (0,0,0) , axis as y-axis and $\theta=50^0$.

**Try above two examples as Home work**

**7.** Find the eqn. of the right circular cone with vertex as (2,1,-3) , axis is parallel to y-axis and semi vertical angle $\theta=30^0$.

**Soln:**




Let P(x,y,z) be any point on the surface of the cone and V(2,1,-3)be vertex of the cone

D.R.'s of VP are x-2, y-1, z+3,

Since axis of the cone is parallel to y-axis and hence DR's of axis are same as DR's of y-axis, they are 0,,1 0

∴ D.R's of axis are 0, 1, 0 and $\theta = 60^0$

∴ $\cos 30 = \frac{(x-2)0+(y-1)1+(z+3)0}{\sqrt{(x-2)^2+(y-1)^2+(z+3)^2}\sqrt{0+1+0^2}}$

i.e $\frac{\sqrt{3}}{2} = \frac{(y-1)}{\sqrt{(x-2)^2+(y-1)^2+(z+3)^2}\ 1}$ i.e $\sqrt{3}\,[\sqrt{(x-2)^2+(y-1)^2+(z+3)^2}] = 2(y-1)$

Squaring both the sides we get

$3[(x-2)^2+(y-1)^2+(z+3)^2] = 4(y-1)^2$

**i.e $3(x-2)^2-(y-1)^2+3(z+3)^2 = 0$ which is req. eqn. of rt. circular cone.**

**8.** Find the eqn. of the right circular cone with vertex as (2,-3, 5) , axis making equal angle with Coordinate axis and semi vertical angle $\theta=30^0$.

**Soln.**:Let P(x,y,z) be any point on the surface of the cone and V(2,-3, 5)be vertex of the cone

D.R.'s of VP are x-2, y+3, z-5,

Since axis of the cone makes equal angle with coordinate axis and hence

D.R's of axis are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ & $\theta = 30^0$

$$\therefore \cos 30 = \frac{(x-2)\frac{1}{\sqrt{3}}+(y+3)\frac{1}{\sqrt{3}}+(z-3)\frac{1}{\sqrt{3}}}{\sqrt{(x-2)^2+(y+3)^2+(z-5)^2}\sqrt{\frac{1}{\sqrt{3}}^2+\frac{1}{\sqrt{3}}^2+\frac{1}{\sqrt{3}}^2}}$$

i.e $\frac{\sqrt{3}}{2} = \frac{(x-2)+(y+3)+(z-3)}{\sqrt{3}\sqrt{(x-2)^2+(y+3)^2+(z-5)^2}\sqrt{1}}$

i.e $3\,[\sqrt{(x-2)^2+(y+3)^2+(z-5)^2}] = 2[(x-2)+(y+3)+(z-5)]$

Squaring both the sides we get,

$9[(x-2)^2+(y+3)^2+(z-5)^2] = 4\,[(x-2)+(y+3)+(z-5)]^2$

**i.e 9[$x^2+y^2+z^2$ -4x + 6y – 10z +38] =4[x + y + z -4]$^2$ , which is req. eqn. of cone.**

**9**. Find the eqn. of the right circular cone with vertex $(1, -2, 1)$, eqn. of the axis as

$\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z+1}{5}$ and semi vertical angle θ is $60^0$.

**Soln**: Let P(x,y,z) be any point on the surface of the cone and V(1,-2, -1 )be vertex of the cone

D.R.'s of VP are x-1, y+2, z+1,

D.R's of axis are 3, -4, 5 & $\theta = 60^0$

$$\therefore \cos 60 = \frac{(x-1)3+(y+2)(-4)+(z+1)5}{\sqrt{(x-1)^2+(y+2)^2+(z-1)^2}\sqrt{3^2+(-4)^2+5^2}}$$

i.e $\frac{1}{2} = \frac{3(x-1)-4(y+2)+5(z+1)}{\sqrt{(x-2)^2+(y+3)^2+(z+1)^2}\sqrt{50}}$

i.e $\sqrt{50}\,[\sqrt{(x-2)^2+(y+3)^2+(z+1)^2}] = 2[3(x-1)-4(y+2)+5(z+1)]$

Squaring both the sides we get,

$50[(x-2)^2+(y+3)^2+(z+1)^2] = 4\,[3x-4y+5z-6]^2$

**i.e 25[$x^2+y^2+z^2$ -4x + 6y +2z +14] =2[x + y + z -4]$^2$ , which is req. eqn. of cone.**

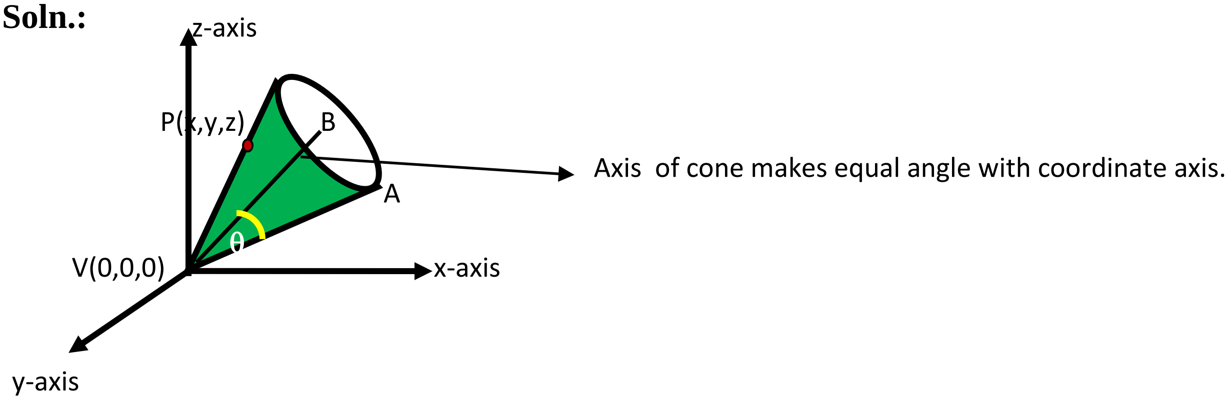
**10.** Find the eqn. of the right circular cone with vertex $(3, 2, 1)$, eqn. of the axis as

$\frac{x-3}{4} = \frac{y-2}{1} = \frac{z-1}{3}$ and semi vertical angle θ is $30^0$.

**HOME work**

**11.** Find the eqn. of the right circular cone with vertex origin, axis making equal angles with coordinate axis and whose generator has DR's 1, -2, 2.

**Soln.:**




In this example, θ is not given.

Given that , axis makes equal angles with coordinate axis and hence DR's of axis are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

And DR's of generator VA are 1, -2, 2.

We know that axis makes equal angle with all generators in rt. circular cone, ∴ θ is same throughout .

From the fig. angle between VA and VB is θ

$$\therefore \cos\theta = \frac{1\left(\frac{1}{\sqrt{3}}\right)+(-2)\frac{1}{\sqrt{3}}+2\left(\frac{1}{\sqrt{3}}\right)}{\sqrt{1^2+(-2)^2+2^2}\ \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2}} = \frac{1\left(\frac{1}{\sqrt{3}}\right)}{\sqrt{9}\ \sqrt{1}} = \frac{1}{3\sqrt{3}} \text{------------------------------(1)}$$

Next, let P(x, y, z) be any point on the surface of the cone, again from the fig. angle between VP and VB is also θ

We have $\cos\theta = \frac{x\left(\frac{1}{\sqrt{3}}\right)+y\frac{1}{\sqrt{3}}+z\left(\frac{1}{\sqrt{3}}\right)}{\sqrt{x^2+y^2+z^2}\ \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2+\left(\frac{1}{\sqrt{3}}\right)^2}}$ (b'cz, DR's of VP are x, y, z)

But from (1) $\cos\theta = \frac{1}{3\sqrt{3}}$

∴ we have $\frac{1}{3\sqrt{3}} = \frac{x+y+z}{\sqrt{3}\sqrt{x^2+y^2+z^2}\ \sqrt{1}}$

i.e $\sqrt{x^2+y^2+z^2} = 3(x+y+z)$

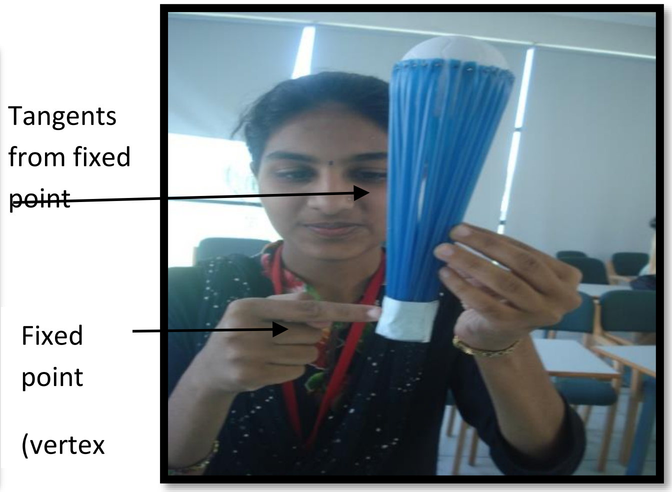
Squaring both the sides we get, $[x^2+y^2+z^2] = 9(x+y+z)^2$

i.e.$8(x^2 + y^2 + z^2)$ +18(xy+yz+zx) = 0

i.e **$4(x^2 + y^2 + z^2)$ +9(xy+yz+zx) = 0 which is req. eqn. of rt. circular cone.**

**Enveloping cone of a sphere:**

We know that from the external point to the surface of the sphere if we draw tangents throughout


the sphere i.e go on drawing tangent lines as much as possible to sphere from the fixed point, we get one surface which is in the form of cone, that is called enveloping cone of sphere.

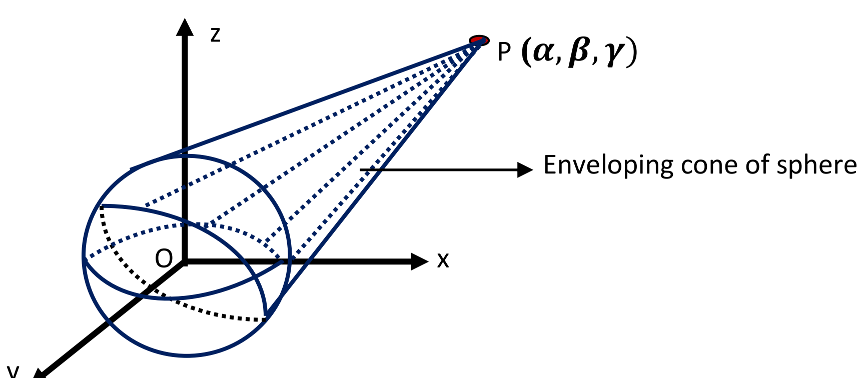
∴ we define enveloping cone of sphere as follows

**Defn:** The locus of tangent lines drawn from a given point to a given sphere is called enveloping cone of the sphere. The given point is vertex of the cone.

**Note:** Defn. is important for two marks**.**

**Theorem: Find the eqn. of enveloping cone of sphere $x^2 + y^2+z^2 =a^2$, from the vertex $(\alpha, \beta, \gamma)$.**

**Proof:**



Give sphere is $\mathbf{x^2 + y^2+z^2 = a^2}$---------------------------------------------------------------- **(1)**

Any line through P$(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ **with DR's l,m,n is** $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ =r (say)---------------(2)

$\therefore$ any point on the line (2) is $(\alpha + lr, \beta + mr, \gamma + nr)$

It lies on the sphere (1) if it satisfies eqn. (1).

i.e $(\alpha + lr)^2 + (\beta + mr)^2 + (\gamma + nr)^2 = \mathbf{a^2}$

i.e $r^2(l^2+m^2+n^2) + r(2\alpha l + 2\beta m + 2\gamma n) + (\alpha^2 + \beta^2 + \gamma^2 - a^2) = 0$---------------------------**(3)**

**which is quadratic in r, has two roots for r. For two different values of r we get two points at which a line will intersect sphere, i.e any line will intersect sphere at two points, but if it is a tangent then it will touch the sphere at only one point, hence both values of r same. Condition for equal roots of r discriminant $b^2 - 4ac = 0$ in (3)**

**i.e** $(2\alpha l + 2\beta m + 2\gamma n)^2 - 4(l^2+m^2+n^2)(\alpha^2 + \beta^2 + \gamma^2 - a^2) = 0$

i.e $(\alpha l + \beta m + \gamma n)^2 - (l^2+m^2+n^2)(\alpha^2 + \beta^2 + \gamma^2 - a^2) = 0$------------------------------------(4)

From (2), $l = \frac{x-\alpha}{r}$, $m = \frac{y-\beta}{r}$, $n = \frac{z-\gamma}{r}$, substitute these values in (4) we get

$(\alpha \frac{x-\alpha}{r} + \beta \frac{y-\beta}{r} + \gamma,)^2 - ((\frac{x-\alpha}{r})^2 + (\frac{y-\beta}{r})^2 + (\frac{z-\gamma}{r})^2)(\alpha^2 + \beta^2 + \gamma^2 - a^2) = 0$

i.e $[\alpha(x-\alpha) + \beta(y-\beta) + \gamma(z-\gamma)]^2 - [(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2](\alpha^2 + \beta^2 + \gamma^2 - a^2) = 0$

By simplifying we get, $[\alpha x + \beta y + \gamma z - a^2]^2 - [x^2 + y^2 + z^2 - a^2][\alpha^2 + \beta^2 + \gamma^2 - a^2] = 0$

**i.e** $\boldsymbol{[\alpha x + \beta y + \gamma z - a^2]^2 = [x^2 + y^2 + z^2 - a^2][\alpha^2 + \beta^2 + \gamma^2 - a^2]}$ ---(5)

which is req. eqn. of enveloping cone of sphere.

Easy method to remember this eqn. is as follows:

Eqn. (5) can be written as, $\mathbf{T^2 = SS_1}$

where $T = \alpha x + \beta y + \gamma z - a^2$ i.e eqn. of tangent to the sphere at $(\alpha, \beta, \gamma)$

$S = x^2 + y^2 + z^2 - a^2$ i.e given sphere

$S_1 = \alpha^2 + \beta^2 + \gamma^2 - a^2$ i.e Sphere at $(\alpha, \beta, \gamma)$

**Note:** (1) Above formula $\mathbf{T^2 = SS_1}$ , u have to remember for examples.

(2) Sphere may be given not necessarily with centre at origin, it may given general eqn.

$\mathbf{x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0}$**, then also same formula** $\mathbf{T^2 = SS_1}$**, but**

**T** $= \alpha x + \beta y + \gamma z + u(x+\alpha) + v(y+\beta) + w(z+\gamma) + d$ i.e Eqn. of tgt. to this sphere

$S = x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d$ , i.e eqn. of given sphere

$S_1 = \alpha^2 + \beta^2 + \gamma^2 + 2u\alpha + 2v\beta + 2w\gamma + d$, i.e sphere at $(\alpha, \beta, \gamma)$

**Note: Enveloping cone of the sphere is also locus of all tangents drawn from point (($\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$) to sphere**.

**Examples:**

**1.** Find the enveloping cone of the sphere $x^2 + y^2+z^2 = 11$ which has the vertex ( 2,4,1). Also prove that the plane z=0 cuts this cone in rectangular hyperbola.

**Soln.**: Given sphere $S = x^2 + y^2+z^2 - 11$ and $(\alpha, \beta, \gamma) =(2,4,1)$

$\therefore \quad S_1= 2^2 + 4^2+1^2 - 11 = 10$

$T = 2x+4y+z-11$

Enveloping cone of sphere is $\mathbf{T^2 = SS_1}$

**i.e** $(2x+4y+z-11)^2 = (x^2 + y^2+z^2 - 11)(10)$

i.e $10(x^2 + y^2+z^2 - 11) = (2x+4y+z-11)^2$, req. eqn. of enveloping cone of sphere.

Next, If plane z=0 cuts this cone, put z=0 in above eqn. we get,

$10(x^2 + y^2 - 11) = (2x+4y -11)^2$

i.e $6x^2 + 6y^2 - 16xy +44x -88y - 231 =0$, which is rectangular hyperbola.

**2.** Find the enveloping cone of the sphere $x^2 + y^2+z^2 = 9$ which has the vertex ( 0,1,1).

**Soln.:** Given sphere $S = x^2 + y^2+z^2 - 9=0$ and $(\alpha, \beta, \gamma) =(0,1,1)$

$\therefore \quad S_1= 0^2 + 1^2+1^2 - 9 = -7$

$T = 0x+1y+1z-9 = y+z-9$

Enveloping cone of sphere is $\mathbf{T^2 = SS_1}$

**i.e** $(y+z-9)^2 = (x^2 + y^2+z^2 - 9)(-7)$

i.e $7(x^2 + y^2+z^2 - 9) + (y+z-9)^2$, req. eqn. of enveloping cone of sphere.

**3**. Find the enveloping cone of the sphere $x^2 + y^2+z^2 -2x +4z-1=0$ with the vertex ( 1,1,1).

**Soln.**: Given sphere $S = x^2 + y^2+z^2 - 2x+4z-1 =0$ and $(\alpha, \beta, \gamma) =(1,1,1)$

$\therefore \quad S_1= 1^2 + 1^2+1^2 - 2+4 -1 = 4$

$T = \alpha x + \beta y + \gamma z + u(x+\alpha) + v(y+\beta) + w(z+\gamma) + d$

$= x+y+z-1(x+1)+0(y+1)+2(z+1)-1 = y+3z$

Enveloping cone of sphere is $\mathbf{T^2 = SS_1}$

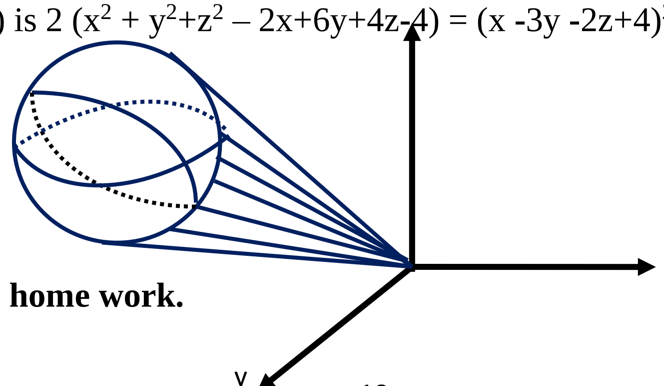
**i.e** $(y+3z)^2 = (x^2 + y^2+z^2 - 2x+4z-1)(4)$

i.e $4(x^2 + y^2+z^2 - 2x+4z-1) = (y+3z)^2$, req. eqn. of enveloping cone of sphere.

**4**. Find the enveloping cone of the sphere $x^2 + y^2+z^2 +2x -2y=2$ with the vertex ( 1,1,1).

**5**. Prove that the lines drawn from the origin so as to touch the sphere $x^2 + y^2+z^2 - 2x+6y+4z-4=0$ lie on the cone $-4 (x^2 + y^2+z^2 - 2x+6y+4z-4) = (x -3y -2z+4)^2$

**Hint:** In this example you have to prove enveloping cone of sphere $x^2 + y^2+z^2 - 2x+6y+4z-4=0$ From vertex (0,0,0) is $2 (x^2 + y^2+z^2 - 2x+6y+4z-4) = (x -3y -2z+4)^2$




**Try (4) and (5) as home work.**

Try following examples also:

1. Prove that equation (i) $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$ represents cone with vertex (-1, -2, -3).

K.L.E Society's

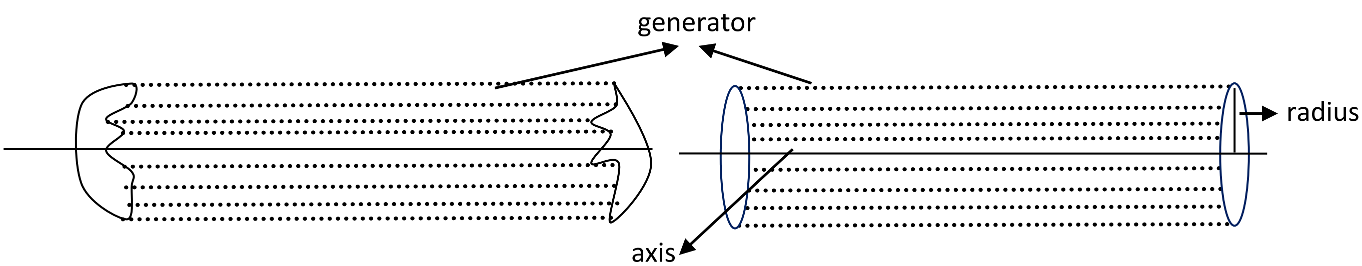
G.I. Bagewadi Arts, Science & Commerce College, NIPANI

**DEPARTMENT OF MATHEMATICS**

**B.Sc. II Sem. Paper II**

***Lecture on Cylinder April 2020***

***Definition:*** A surface generated by variable straight line moving in space parallel to a fixed line and satisfying the condition that interesting given curve is called cylinder. Variable line is called generator and intersecting curve is called guiding curve.




**Oblique cylinder** **Right circular cylinder**

**Right circular cylinder:** A surface generated by variable straight line moving in space parallel to a fixed line with constant distance from fixed line and satisfying the condition that interesting given curve is called cylinder. Fixed line is called axis of the cylinder, variable line is called generator, constant distance is called radius of cylinder and intersecting curve is called guiding curve.

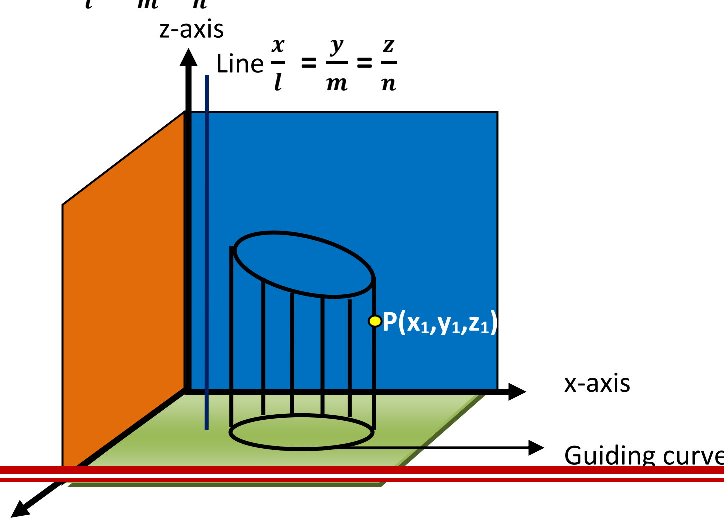
**Note:(1)** In oblique cylinder generators are not necessarily at constant distance from fixed line where as in rt. circular cylinder they are at constant distance.

(2) Most of the properties and theorems in cylinder are same as that of cone.

(3) It is not having vertex, only fixed line and conic

**Theorem: To find eqn. of cylinder with given conic $ax^2+2hxy+by^2+2gx+2fy+c=0$, $z=0$ and generators parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$**

**Proof:**



**Given conic $ax^2+2hxy+by^2+2gx+2fy+c=0$, z=0 -----------------------(1)**

**Let the equation of the fixed line be $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ --------------------(2)**

**Let P($x_1,y_1,z_1$) be any point on the cylinder, then equation to this line parallel to fixed line (2) is $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ ---------------------------------------(3)**

But this line intersect xy-plane at z=0 ( b'cz on xy plane z coordinate is zero)

∴ we have $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{0-z_1}{n}$---------------------------------------------**(4)**

⇨ $\frac{x-x_1}{l} = \frac{0-z_1}{n}$ and $\frac{y-y_1}{m} = \frac{0-z_1}{n}$

⇨ $x = x_1 - \frac{lz_1}{n}$, $y = y_1 - \frac{mz_1}{n}$, z = 0,

But line (4) intersect the given conic (1) if this point lies on conic (1), as shown in fig. ∴ substitute this point in (1) we get ,

$a(x_1 - \frac{lz_1}{n})^2 + 2h(x_1 - \frac{lz_1}{n})(y_1 - \frac{mz_1}{n}) + b(y_1 - \frac{mz_1}{n})^2 + 2g(x_1 - \frac{lz_1}{n}) + 2f(y_1 - \frac{mz_1}{n}) + c = 0$

(not necessary to remove l, m,n as they are given)

Simplifying this eqn. we get

$a(nx_1 - lz_1)^2 + 2h(nx_1 - lz_1)(ny_1 - mz_1) + b(nx_1 - lz_1)^2 + 2g(nx_1 - lz_1) + 2f(nx_1 - lz_1) + cn^2 = 0$

Then locus of P($x_1$, $y_1$, $z_1$) , i.e replace (x, y, z) we get,

$a(nx - lz)^2 + 2h(nx - lz)(ny - mz) + b(nx - lz)^2 + 2g(nx - lz) + 2f(nx_1 - lz_1) + cn^2 = 0.$

**Thus required eqn. of cylinder with generators parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and given conic $ax^2+2hxy+by^2+2gx+2fy+c=0$, z=0 (i.e guiding curve) is**

**$a(nx - lz)^2 + 2h(nx - lz)(ny - mz) + b(nx - lz)^2 + 2g(nx - lz) + 2f(nx_1 - lz_1) + cn^2 = 0$.----(5)**

**Corollary 1.** If the generator of the cylinder parallel to z-axis then l=0, m=0 and n=1 substitute these in (5) we get eqn. of cylinder is **$ax^2+2hxy+by^2+2gx+2fy+c=0$, z=0 i.e f(x,y) =0, z=0**

**similarly,** If the generator of the cylinder parallel to x-axis then l=1,m=0 and n=0 substitute these in (5) we get eqn. of cylinder is **f(z,y) =0, x=0.**

**And** If the generator of the cylinder parallel to y-axis then l=0,m=1 and n=0 substitute these in (5) we get eqn. of cylinder is **f(x,z) =0, y=0.**

**Corollary 2.** The eqn. of cylinder which is having guiding curve as intersection of two curves f(x,y,z) =0----(1) and g(x, y, z)=0-----(2) and whose generators are parallel to

(i) x –axis is obtained by eliminating x between (1) and (2)
(ii) y–axis is obtained by eliminating y between (1) and (2)
(iii) z–axis is obtained by eliminating z between (1) and (2)

Examples:

1. Find eqn. to the cylinder which passes through the curve of intersection of plane lx+my+nz=p and $ax^2+by^2+cz^2 = 1$ and generators parallel to z-axis.

**Soln.** Given guiding curve is intersection of lx+my+nz=p ---------------------(1)

$ax^2+by^2+cz^2 = 1$ ----------------------(2)

Since generators are parallel to z-axis, req. eqn. to the cylinder is obtained by eliminating z-coordinate between (1) and (2)

From (1) $z = \frac{p-lx-my}{n}$, substitute this in (1) we get

$ax^2+by^2+c\left(\frac{p-lx-my}{n}\right)^2 = 1$

i.e **$an^2x^2+bn^2y^2+c(p-lx-my)^2 = n^2$, which is the req. eqn. of the cylinder.**

**2.** Find eqn. to the cylinder which passes through the curve of intersection of plane lx+my+nz=p and $by^2+cz^2 = 2ax$ and generators parallel to x -axis.

**Hint:** get x from first eqn. and substitute in second as in above example.

3. Find eqn. to the cylinder which passes through the curve of intersection of plane lx+my+nz=p and $ax^2+cz^2 = 2y$ and generators parallel to y -axis.

**Hint:** get y from first eqn. and substitute in second as in above example.

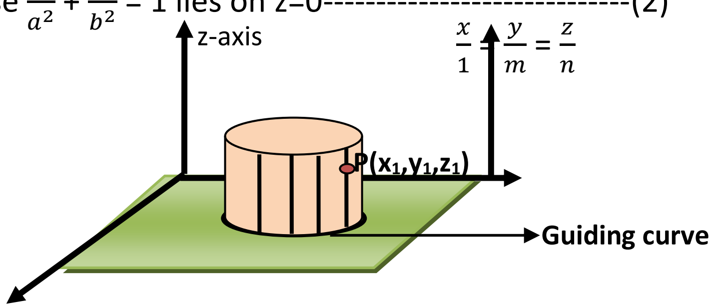
**Try (2)and (3) as home work.**

**4.** Find eqn. to the surface generated by the line which is parallel to y=mx and z=nx and intersecting the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, z=0

i.e nothing but , finding eqn. of cylinder whose generators are parallel to the line y=mx and z=nx and intersecting the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ lies on z=0 i.e xy plane

**Soln:** Given line y=mx and z=nx i.e $x = \frac{y}{m} = \frac{z}{n}$, i.e $\frac{x}{1} = \frac{y}{m} = \frac{z}{n}$ ----------------(1)

Guiding curve is ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ lies on z=0-----------------------------(2)




**Let $P(x_1,y_1,z_1)$ be any point on the cylinder, then equation generator parallel to**

**given line (1) passing through is** $\frac{x-x_1}{1} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ **----------------------------------------(3)**

But this line intersect xy-plane at z=0 ( b'cz on xy plane z coordinate is zero)

$\therefore$ we have $\frac{x-x_1}{1} = \frac{y-y_1}{m} = \frac{0-z_1}{n}$**------------------------------------------------------------------(4)**

⇨ $\frac{x-x_1}{1} = \frac{0-z_1}{n}$ and $\frac{y-y_1}{m} = \frac{0-z_1}{n}$

⇨ $x = x_1 - \frac{z_1}{n}$, $y = y_1 - \frac{mz_1}{n}$, z = 0,

But line (4) intersect the given conic (2) if this point lies on conic (2), as shown in fig. $\therefore$ substitute this point in (2) we get ,

$$\frac{(x_1 - \frac{z_1}{n})^2}{a^2} + \frac{(y_1 - \frac{mz_1}{n})^2}{b^2} = 1$$

**i.e** $b^2(x_1 - \frac{z_1}{n})^2 + a^2(y_1 - \frac{mz_1}{n})^2 = a^2b^2$

i.e $b^2(nx_1 - z_1)^2 + a^2(ny_1 - z_1)^2 = a^2b^2n^2$

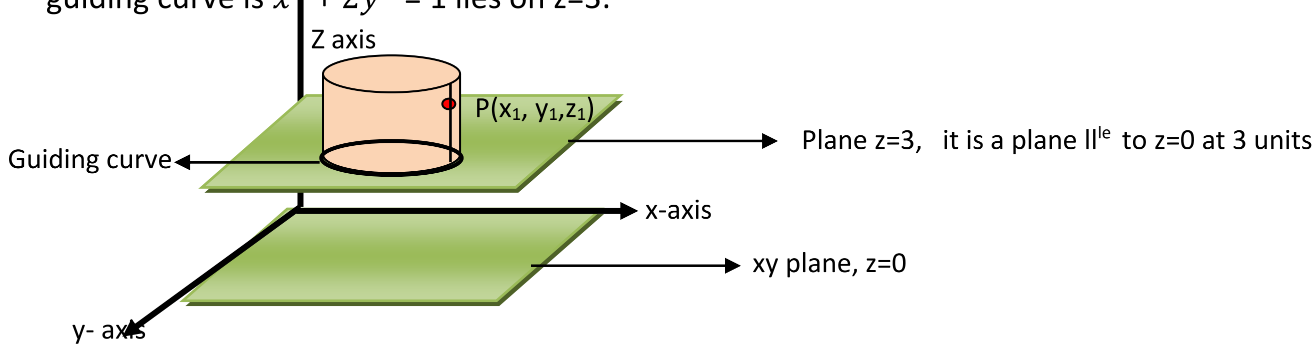
Locus of $P(x_1,y_1,z_1)$ i.e replace $(x_1,y_1,z_1)$ by (x, y, z) we get

$b^2(nx - z)^2 + a^2(ny - z)^2 = a^2b^2n^2$**, req. Eqn. of cylinder**.

**5.** Find eqn. of cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{2}$ and whose guiding curve is $x^2 + 2y^2 = 1$ lies on z=0.

**Try this example, similar to above example**.

**6.**Find eqn. of cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is $x^2 + 2y^2 = 1$ lies on z=3.




**Soln.:Gven** line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ ----------------------(1) guiding curve is $x^2 + 2y^2 = 1$, z=3-----(2)

**Let $P(x_1,y_1,z_1)$ be any point on the cylinder, then equation of generator parallel to given line (1) passing through is** $\frac{x-x_1}{1} = \frac{y-y_1}{-2} = \frac{z-z_1}{3}$ **----------------------------------------(3)**

But this line intersect plane z=3 ( plane II[le] to xy plane at distance z=3)

$\therefore$ we have $\frac{x-x_1}{1} = \frac{y-y_1}{-2} = \frac{3-z_1}{3}$-------------------------------------------------------------------------**(4)**

⇨ $\frac{x-x_1}{1} = \frac{3-z_1}{3}$ and $\frac{y-y_1}{-2} = \frac{3-z_1}{3}$

⇨ $(3x - 3x_1) = 3 - z_1$, $3(y - y_1) = -2(3 - z_1)$, z = 3

⇨ $x = \frac{3x_1+3-z_1}{3}$, $y = \frac{3y_1-6-2z_1}{3}$, z=3

If the line (4) intersect the given guiding curve, the above point satisfies eqn. (2)

$\therefore$ we have $(\frac{3x_1+3-z_1}{3})^2 + 2(\frac{3y_1-6-2z_1}{3})^2 = 1$

i.e $(3x_1 + 3 - z_1)^2 + 2(3y_1 - 6 - 2z_1)^2 = 9$

**$\therefore$ locus of this eqn. is $(3x - z + 3)^2 + 2(3y - 2z - 6)^2 = 9$, which is the req. eqn.of cylinder**.

**7**. Find eqn. of cylinder whose guiding curve is $x^2 + y^2 = 16$ lies on z=0 and generators are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

**Soln.:** Given line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ ------------------------------------------------------------(1)

Guiding curve is ellipse $x^2 + y^2 = 16$ lies on z=0----------------------------(2)

**Let P($x_1$,$y_1$,$z_1$) be any point on the cylinder, then equation generator parallel to given line (1) passing through is $\frac{x-x_1}{1} = \frac{y-y_1}{2} = \frac{z-z_1}{3}$ ---------------------------------------(3)**

But this line intersect xy-plane at z=0 ( b'cz on xy plane z coordinate is zero)

$\therefore$ we have $\frac{x-x_1}{1} = \frac{y-y_1}{2} = \frac{0-z_1}{3}$-------------------------------------------------------------------------**(4)**

⇨ $\frac{x-x_1}{1} = \frac{0-z_1}{3}$ and $\frac{y-y_1}{2} = \frac{0-z_1}{3}$

⇨ $x = x_1 - \frac{z_1}{3}$, $y = y_1 - \frac{2z_1}{3}$, z = 0,

But line (4) intersect the given conic (2) if this point lies on conic (2), as shown in fig. $\therefore$ substitute this point in (2) we get ,

$(x_1 - \frac{z_1}{3})^2 + (y_1 - \frac{2z_1}{3})^2 = 16$

i.e $(3x_1 - z_1)^2 + (3y_1 - 2z_1)^2 = 16$

$\therefore$ locus of this eqn. is $(3x - z)^2 + (3y - 2z)^2 = 16$

**i.e $9x^2 + 9y^2 + 10z^2 - 6xz - 12yz - 16 = 0$ req. eqn. of cylinder**.

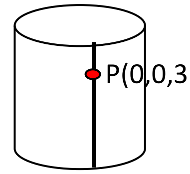
**8.** Find eqn. of cylinder whose generators are parallel to the line $x = \frac{y}{2} = -z$,

i.e $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and passing through the curve (i.e. guiding curve) $3x^2 + 2y^2 = 1$,z=0.

**Try this example as home work.**

**9. Show that the line** $\frac{x}{1} = \frac{y}{-1} = \frac{z-3}{1}$ is a generator of the cylinder **$x^2 + y^2 + z^2 + xy + yz - zx = 9$.**

**Soln.: Given line** $\frac{x}{1}=\frac{y}{-1}=\frac{z-3}{1}$ i.e $\frac{x-0}{1}=\frac{y-0}{-1}=\frac{z-3}{1}$-------------------(1)




i.e the line passing through point P(0,0,3)

And given cylinder $\mathbf{x^2 + y^2+z^2+xy+yz -zx = 9}$ **-------------------------(2),**

**To prove the line (1) is generator for the cylinder (2)we have to prove that(0,0,3) lies on (2), i.e this point satisfies (2).**

**i.e 0+0+9+0+0+0=9**

**9= 9**

⇨ **(1) is the generator of the cylinder (2).**

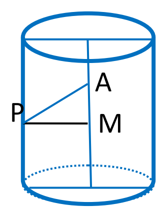
**10.** Find eqn. of cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is $z^2+y^2=1$ lies on x=2.

**Right circular cylinder:**

We know that **right circular cylinder** is a surface generated by variable straight line parallel to a fixed line with constant distance from fixed line and satisfying the condition that interesting given curve, that curve is circle.

**Finding equation of the right circular cylinder whose radius is r and axis is the line** $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ **(important for 5 marks)**

Slon:




Let $(x_1, y_1, z_1)$ be any point on the cylinder and let $A(\alpha,\beta,\gamma)$ be a fixed point on the axis and PM = r, radius of the cylinder.

Then from the fig. $AP^2 = PM^2 + AM^2$

⇨ $PM^2 = AP^2 - AM^2$ --------------------------(1)

By using distance formula, AP = $\sqrt{(x_1-\alpha)^2+(y_1-\beta)^2+(z_1-\gamma)^2}$

AM = Projection of AP on the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$

AM = $\frac{(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n}{\sqrt{l^2+m^2+n^2}}$ (By using dot product)

Squaring both the sides we get,

AM$^2$ = $\left(\frac{(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n}{\sqrt{l^2+m^2+n^2}}\right)^2 = \frac{[(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n]^2}{(l^2+m^2+n^2)}$ and PM =r

Then (1) becomes r$^2$ = $[(x_1-\alpha)^2+(y_1-\beta)^2+(z_1-\gamma)^2]$ -- $\frac{[(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n]^2}{(l^2+m^2+n^2)}$

$$r^2((l^2+m^2+n^2) = [(x_1-\alpha)^2+(y_1-\beta)^2+(z_1-\gamma)^2](l^2+m^2+n^2) - [(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n]^2$$

i.e $[(x_1-\alpha)l+(y_1-\beta)m+(z_1-\gamma)n]^2 = [(x_1-\alpha)^2+(y_1-\beta)^2+(z_1-\gamma)^2-r^2](l^2+m^2+n^2)$

**Locus of the above eqn. is**

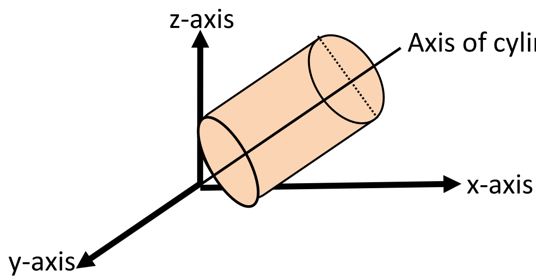
$$[(\mathbf{x}-\boldsymbol{\alpha})\mathbf{l}+(\mathbf{y}-\boldsymbol{\beta})\mathbf{m}+(\mathbf{z}-\boldsymbol{\gamma})\mathbf{n}]^2 = [(\boldsymbol{x}-\boldsymbol{\alpha})^2+(\boldsymbol{y}-\boldsymbol{\beta})^2+(\boldsymbol{z}-\boldsymbol{\gamma})^2]-\mathbf{r}^2](\mathbf{l}^2+\mathbf{m}^2+\mathbf{n}^2)$$

**Which is the required eqn. of the rt. circular cylinder with axis and radius r.**

**Corollary 1.** Eqn. of the rt. circular cylinder with axis $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ and radius r is

$[lx+my+nz]^2 = [x^2+y^2+z^2-r^2][l^2+m^2+n^2]$

**Proof:**




We know that eqn. of cylinder with axis $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ and radius as r is

$[(x-\alpha)l+(y-\beta)m+(z-\gamma)n]^2 = [(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2]-r^2](l^2+m^2+n^2)$

$\therefore$ eqn. of cylinder with axis $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ (i.e line passing through origin with DR's l, m, n) and radius as r is $[xl+ym+zn]^2 = [x^2+y^2+z^2-r^2](l^2+m^2+n^2)$

i.e $[lx+my+nz]^2 = [x^2+y^2+z^2-r^2][l^2+m^2+n^2]$

**Corollary 2.** Eqn. of the rt. circular cylinder with radius r and (i) z –axis as axis of cylinder is $x^2+y^2=r^2$ (ii) y –axis as axis of cylinder is $x^2+z^2=r^2$ (iii) x –axis as axis of cylinder is $y^2+z^2=r^2$

**Proof:**

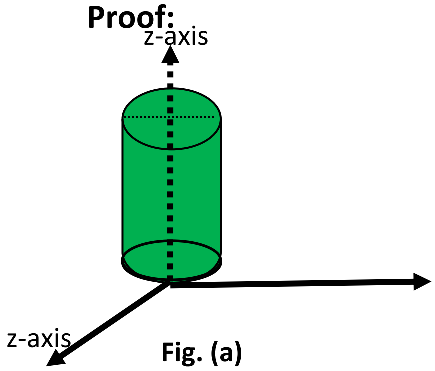



**Fig. (a)**

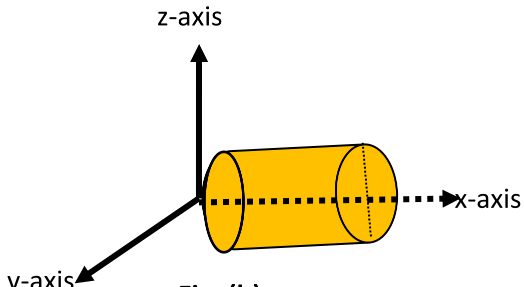



**Fig. (b)**

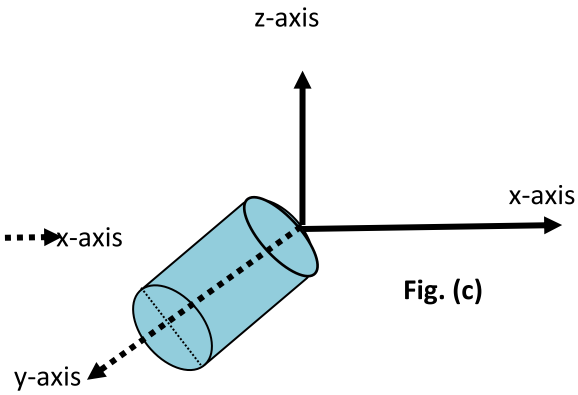



**Fig. (c)**

We know that eqn. of rt. circular cylinder with axis $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and radius as r is

$[lx + my + nz]^2 = [x^2 + y^2 + z^2 - r^2][l^2 + m^2 + n^2]$ by **cor. (1)--------------------(d)**

If z-axis is axis of cylinder as in fig. (a), then $l = 0$, m = 0, n = 1

$\therefore$ eqn. (d) becomes $[0 + 0 + z]^2 = [x^2 + y^2 + z^2 - r^2][0 + 0 + 1^2]$

i.e $z^2 = x^2 + y^2 + z^2 - r^2$ => $x^2 + y^2 = r^2$

$\therefore$ eqn. of rt. circular cylinder with $z - axis$ as axis and radius as r is $\mathbf{x^2 + y^2 = r^2}$

**Similarly,** eqn. of rt. circular cylinder with $x - axis$ as axis and radius as r is $\mathbf{z^2 + y^2 = r^2}$ **fig. (b)**

**And** eqn. of cylinder with $y - axis$ as axis and radius as r is $\mathbf{x^2 + z^2 = r^2}$ **fig. (c)**

**Examples:**

**1. Find the eqn. of right circular cylinder of radius 2 and axis is the line** $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$.

**Soln:** Axis of the cylinder $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1} = \frac{x-\alpha}{2} = \frac{y-\beta}{2} = \frac{z-\gamma}{-1}$ $\therefore (\alpha, \beta, \gamma) = (1,3,5)$,

$l$=2, m= 2 , n= -1, radius r =2

We know that eqn. of rt. circular cylinder with axis $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius as r is

$[(x-\alpha)l + (y-\beta)m + (z-\gamma)n]^2 = [(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 - r^2](l^2 + m^2 + n^2)$

i.e $[(x-1)2 + (y-3)2 + (z-5)(-1)]^2 = [(x-1)^2 + (y-3)^2 + (z-5)^2 - 2^2](4 + 2 + 1)$

i.e $[2x + 2y - z - 3]^2 = 9[(x-1)^2 + (y-3)^2 + (z-5)^2 - 2^2]$ req. . eqn of rt. circular cylinder.

**2. Find the eqn. of right circular cylinder of radius 2 and axis is the line x =2y = -z**

**Soln.:** eqn. of rt. circular cylinder with axis **x =2y = -z** **i.e x =2y = -z i.e** $\frac{x}{2}=\frac{y}{1}=\frac{z}{-2}$ and radius as r = 2, is

$$[(x-\alpha)l+(y-\beta)m+(z-\gamma)n]^2=[(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2-r^2](l^2+m^2+n^2)$$

$$(\alpha,\beta,\gamma)=(0,0,0)$$

i.e $[(x)2+(y)1+z(-2)]^2=[(x)^2+(y)^2+(z)^2-2^2](4+1+4)$

i.e $[2x+y-z]^2=9[x^2+y^2+z^2-4]$

i.e $5x^2+8y^2+8z^2+4xy-2yz-4zx=6x^2+6y^2+6z^2-36$

**i.e** $5x^2+8y^2+8z^2-4xy+2yz+4zx-36=0$

**3. Find the eqn. of right circular cylinder of radius 3 and axis passes through (2, 3, 4) and D.C's proportional to 2,1,-2.**

**Soln.: Given that axis passes through (2, 3, 4) with D.R.'s 2, 1, -2. ie e** $\frac{x-2}{2}=\frac{y-3}{1}=\frac{z-4}{-2}$

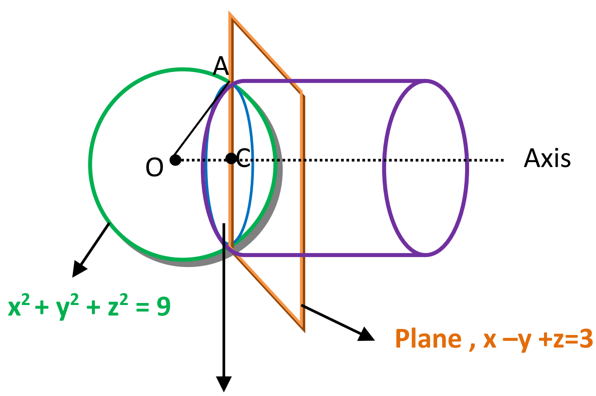
$\therefore(\alpha,\beta,\gamma)=(2,3,4)$ and r =3

**4. Find the eqn. of right circular cylinder of radius 2 and axis e** $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$

Try these two example (3) and (4).

**5. Find the eqn. of rt. circular cylinder whose guiding curve is circle** $x^2+y^2+z^2=9$: $x-y+z=3$.

**Soln.:**




Given that circle of intersection of sphere S: $x^2+y^2+z^2-9=0$-- (1) and plane P: $x-y+z=3$.------------------------------------------------(2) We studied in sphere that intersection of sphere by the plane is circle, that circle is the guiding curve for cylinder.

In the following fig. O( 0,0,0) is centre of the sphere (1) and C is the centre of the circle of intersection, radius AC f circle is radius of the cylinder. The line joining OC is the axis of the cylinder.

So we have to find O, C, OA, AC and D.R.'s of OC.

By using the procedure of finding centre and radius of circle of intersection of sphere (1) by the plane (2) we have to find.

Centre of the sphere (1) is O = (0,0,0) and radius of the sphere OA = 3

And OC is perpendicular drawn from O to the plane (2)

$\therefore$ OC = $\left|\frac{1(0)-1(0)+1(0)-3)}{\sqrt{1^1+(-1)^2+1^2}}\right| = \left|\frac{-3)}{\sqrt{3}}\right| = \sqrt{3}$

$\therefore$ from rt. angled triangle OAC, $AC^2 = OA^2 - OC^2 = 9 - 3 = 6$

Radius of the circle i.e radius of rt. circular cylinder , AC = r =$\sqrt{6}$

And D.R.'s of axis OC are 1, -1, 1 and axis passing through $(\alpha, \beta, \gamma) = (0,0,0)$

$\therefore$ eqn. of rt. circular cylinder with radius r=$\sqrt{6}$, $(\alpha, \beta, \gamma) = (0,0,0)$, and l, m, n as 1, -1, 1

is $[lx + my + nz]^2 = [x^2 + y^2 + z^2 - r^2][l^2 + m^2 + n^2]$

i.e $[(1)x + (-1)y + (1)z]^2 = [x^2 + y^2 + z^2 - 6][1 + 1 + 1]$

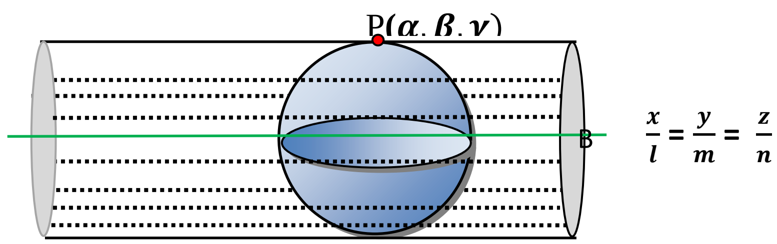
i.e $[x - y + z]^2 = 3[x^2 + y^2 + z^2 - 6]$

i.e $x^2 + y^2 + z^2 - 2xy - 2yz + 2xz = 3(x^2 + y^2 + z^2) - 18$

**i.e $2(x^2 + y^2 + z^2) + 2xy + 2yz - 2xz - 18 = 0$, req. eqn. of cylinder.**

## Enveloping cylinder of sphere:

If we go on drawing tangent lines to the sphere parallel to the line AB, those tangent lines envelope (cover)sphere and form a surface in the form of cylinder as show in the




above figure. That cylinder formed is called enveloping cylinder of sphere. So we define enveloping cylinder as follows:

**Definition:** The locus of tangent lines drawn to a given sphere parallel to given line is called the enveloping cylinder of sphere.

**Theorem: Enveloping cylinder of the sphere $x^2 + y^2 + z^2 = a^2$ with tangents drawn parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is $[lx + my + nz]^2 = [x^2 + y^2 + z^2 - a^2][l^2 + m^2 + n^2 - a^2]$**

**Proof:** Given sphere is $x^2 + y^2 + z^2 = a^2$ -----------------------------------------(1)

Given line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ --------------------------------------------(2)

Let $P(\alpha, \beta, \gamma)$ be any point on the locus, any line through $P(\alpha, \beta, \gamma)$ parellel to (2) is

$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ =r (say)------------------------------------------------------------------------------(3)

$\therefore$ any point on the line (3) is $(\alpha + lr, \beta + mr, \gamma + nr)$, It lies on the sphere (1) if it satisfies eqn. (1).

i.e $(\alpha + lr)^2 + (\beta + mr)^2 + (\gamma + nr)^2 = a^2$

i.e $r^2(l^2+m^2+n^2) + r(2\alpha l + 2\beta m + 2\gamma n) + (\alpha^2 + \beta^2 + \gamma^2 - a^2) = 0$---------------------------**(4)**

which is quadratic in r, has two roots for r. For two different values of r we get two points at which a line will intersect sphere, i.e any line will intersect sphere at two points, but if it is a tangent then it will touch the sphere at only one point, hence both values of r same.

**Condition for equal roots of r discriminant $b^2 - 4ac = 0$ in (3)**

**i.e** $(2\alpha l + 2\beta m + 2\gamma n)^2 - 4(l^2+m^2+n^2)(\alpha^2 + \beta^2 + \gamma^2 - a^2) = 0$

i.e $(\alpha l + \beta m + \gamma n)^2 - (l^2+m^2+n^2)(\alpha^2 + \beta^2 + \gamma^2 - a^2) = 0$---------------------------------(5)

Hence, locus of $(\alpha, \beta, \gamma)$ i.e replace $(\alpha, \beta, \gamma)$ by (x, y, z) in (5) we get

$\boldsymbol{(xl + ym + zn)^2 - (l^2+m^2+n^2)(x^2 + y^2 + z^2 - a^2) = 0}$

**i.e.** $\boldsymbol{(lx + my + nz)^2 - (l^2+m^2+n^2)(x^2 + y^2 + z^2 - a^2) = 0}$

which is req. eqn. of enveloping cylinder of sphere.

**Note:** (i)Procedure for enveloping cone and cylinder are same , only change is, in cone tangents are drawn from a fixed point to the sphere and in cylinder tangents are drawn parallel to the given line.

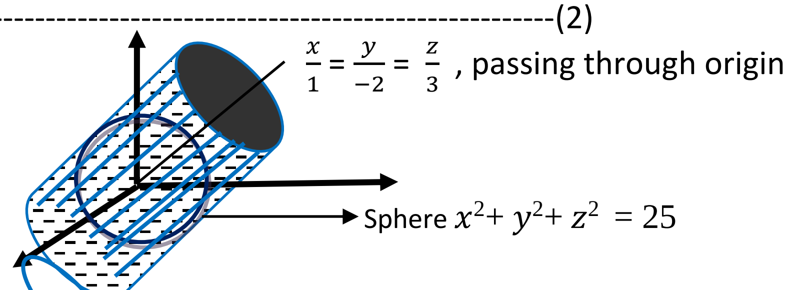
(ii) Enveloping cylinder of the sphere is also sometimes is locus of generators touch the sphere and parallel to the given line.

**Examples:**

**1.**Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 = 25$ whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$

**Soln.:** Given sphere $x^2 + y^2 + z^2 = 25$ ---------------------------------------------(1)

Given line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ ------------------------------------------------------------------(2)




Here a =5, $l$ =1, m = --2 , n =3

We know that enveloping cone of sphere $(lx + my + nz)^2 - (l^2+m^2+n^2)(x^2 + y^2 + z^2 - a^2) = 0$

i.e $((1)x + (-2)y + 3z)^2 - (1+4+9)(x^2 + y^2 + z^2 - 25)$

**i.e (x -2y +3z)$^2$ = 14($x^2$+ $y^2$+ $z^2$-25) which is req. eqn. of enveloping cone of sphere.**

**2.** Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2 = a^2$ whosegenerators are parallel to the line $x = y = z$

**Soln.:** Given sphere $x^2+y^2+z^2 = a^2$ ------------------------------------------------(1)

Given line $x = y = z$ i.e $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ ----------------------------------------------------(2)

Here a =a, $l$ =1, m = 1 , n =1

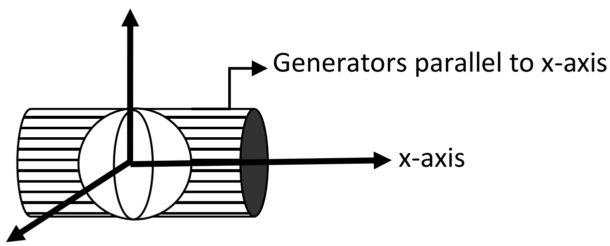
We know that enveloping cone of sphere $(lx + my + nz)^2 - (l^2+m^2+n^2)(x^2+y^2+z^2-a^2) = 0$

i.e $(x + y + z)^2 - (1+1+1)(x^2+y^2+z^2-a^2)$

**i.e (x +y +z)$^2$ = 3($x^2$+ $y^2$+ $z^2$-a$^2$) which is req. eqn. of enveloping cone of sphere.**

**3.** Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2 = a^2$ whose generators are parallel x-axis.

Soln.:




Generators parallel to x-axis => D.R.'s of generators are 1,0,0

$\therefore$ put l=1, m=0, n=0 in above eqn. we get $x^2 = (x^2+y^2+z^2-a^2)$

i.e $y^2+z^2 = a^2$

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**K. L. E. Society's**
**G.I.Bagewadi Arts, Science & Commerce College, Nipani**
Accredited at 'A' level by NAAC with CGPA 3.35
**DEPARTMENT OF MATHEMATICS**

**B. Sc. III Sem. 2020-21**

**Paper II**

# UNIT III

# APPLICATIONS OF DEFINITE INTEGRALS

# (VOLUME AND SURFACE AREA BY REVOLUTION)

## CONTENTS OF THE UNIT



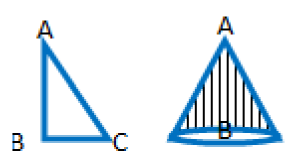
## INTRODUCTION:

We already studied formulas for area, surface area and volume of solid figures and examples in school level, but we don't know that why surface area of sphere is $4\pi r^2$ ? and volume is $\frac{4}{3}\pi r^3$ ?

These formulas we get by using definite integral. Similarly length of curve also we get by using definite integral.

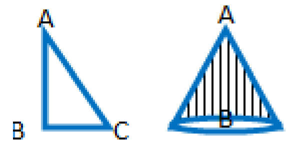
If any plane figure is rotated ( one full rotation) about x-axis or y-axis or any other line we get one solid figure that is called solid of revolution generated by plane curve.

**For example**: If A right angled triangle is rotated about its perpendicular line we get a 3-D figure, cone.




**Definition:** If a plane area is rotated about a fixed straight line lying in its own plane, then the body so generated by the plane area is called **solid of revolution** and surface generated by the boundary of the plane area is called **surface area of**

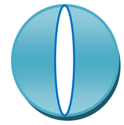
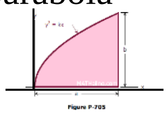
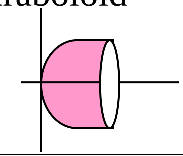
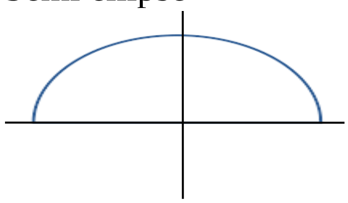
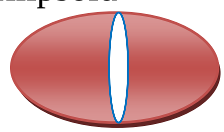
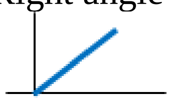
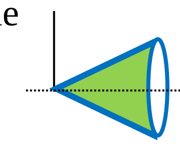
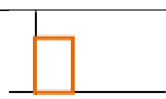
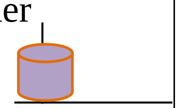
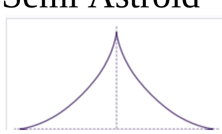
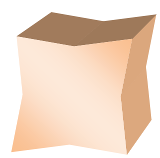
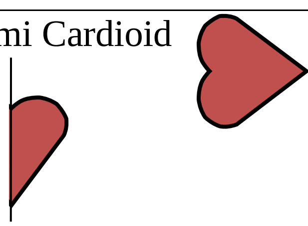
**revolution** and volume formed by surface is called **volume of revolution.** The fixed line about which the plane area is rotated is called **axis of axis of rotation.**



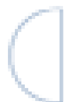
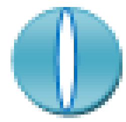
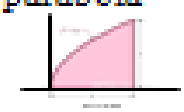
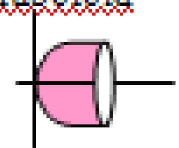
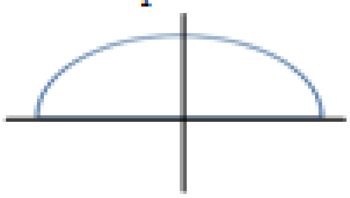
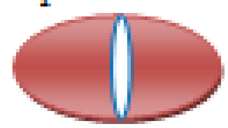
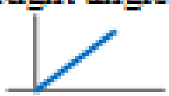
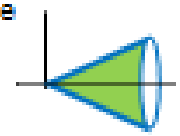
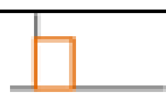
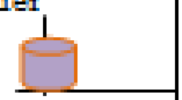
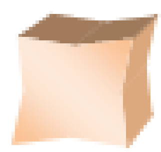
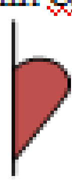
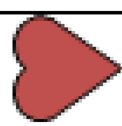

In the above figure , Plane area is **right angled triangle**, solid of revolution is **cone** and axis of cone is **AB**.

**Examples:**

1. If a **semi circle** is rotated about its diameter then we get solid figure **sphere**.
2. If a **semi ellipse** is rotated about its major axis then we get solid figure **called ellipsoid**.
3. If a **semi parabola** is rotated about x- axis then we get solid figure **called paraboliod**.
4. If a **semi hyperbola** is rotated about x- axis then we get solid figure **called hyperboloid**.
5. If a **rectangle** is rotated about its one of the side, we get solid figure **called cylinder.**

| S. No. | Plane figure | Equation | Solid Obtained | Axis |
|---|---|---|---|---|
| 1. | Semi Circle  | $x^2 + y^2 = a^2$ | Sphere  | Diameter |
| 2. | Semi parabola  | $y^2 = 4ax$ | Paraboloid  | x-axis or y-axis |
| 3. | Semi ellipse  | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | Ellipsoid  | Major axis or minor axis |
| 4. | Right angle triangle  | y = mx | Cone  | x-axis or y-axis |
| 5. | Rectangle  | x =k | Cylinder  | Any side |
| 6. | Semi Astroid  | $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ |  | x-axis or y-axis |
| 7. | Semi Cardioid  | $r = a(1+\cos\theta)$ | Cardiac  | Initial line or line $\theta = \frac{\pi}{2}$ |

## Solid figures obtained by plane figures ofter revolution

| S. No. | Plane figure | Equation | Solid Obtained | Axis |
|---|---|---|---|---|
| 1. | Semi Circle  | $x^2 + y^2 = a^2$ | Sphere  | Diameter |
| 2. | Semi parabola  | $y^2 = 4ax$ | Paraboloid  | x-axis or y-axis |
| 3. | Semi ellipse  | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | Ellipsoid  | Major axis or minor axis |
| 4. | Right angle triangle  | $y = mx$ | Cone  | x-axis or y-axis |
| 5. | Rectangle  | $x = k$ | Cylinder  | Any side |
| 6. | Semi Astroid  | $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ |  | x-axis or y-axis |
| 7. | Semi Cardioid   | $r = a(1+\cos\theta)$ | Heart shaped figure  | Initial line or line $\theta = \frac{\pi}{2}$ |

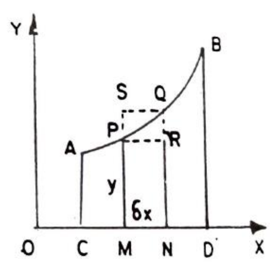
## Cartesian, Parametric and Polar equations:

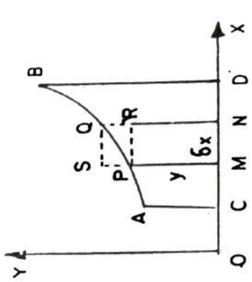
Equations in terms **of x and y** are called **Cartesian**, if **x and y** are in terms of **parameter t are $\theta$** are called **parametric** and equations in terms of **r and $\theta$** are called **polar**.

**For example**:

| S. No. | Curve | Equations | | |
|---|---|---|---|---|
| | | Cartesian | Parametric | Polar |
| 1. | Circle | $x^2+y^2 = a^2$ | $x = a\cos\theta,\ y = a\sin\theta$ | $r = a\cos(\pi+\theta)$ |
| 2. | Parabola | $y^2 = 4ax$ | $x = at^2,\ y = 2at$ | - |
| 3. | Ellipse | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ | $x = a\cos\theta,\ y = b\sin\theta$ | - |
| 4. | Hyperbola | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ | $x = a\sec\theta,\ y = b\tan\theta$ | - |
| 5. | Astroid | $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ | $x = a\cos^3\theta,\ y = b\sin^3\theta$ | - |
| 6. | Cardioid | - | - | $r = a(1+\cos\theta)$ |

# I. Volume and surface area formulas for Cartesian curve.




(i) The volume of a solid generated by the area ACDM i.e bounded by the curve y = f(x), x – axis and ordinates x=a and x=b by the revolution about x-axis is

$$V = \int_a^b \pi y^2 dx \quad \text{or} \quad V = \int_a^b \pi x^2 dy$$

(ii) The surface area of a solid generated by the area ACDM i.e bounded by the curve y = f(x), x –axis and ordinates x=a and x=b by the revolution about x-axis is

$$S = \int_a^b 2\pi y \sqrt{1+(\frac{dy}{dx})^2}\ dx$$

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**G.I.Bagewadi Arts, Science & Commerce College, Nipani**
Accredited at 'A' level by NAAC with CGPA 3.35
**DEPARTMENT OF MATHEMATICS**

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**Paper I**

**Mathematical Logic and Real Analysis**

# Unit III

# Real Analysis II

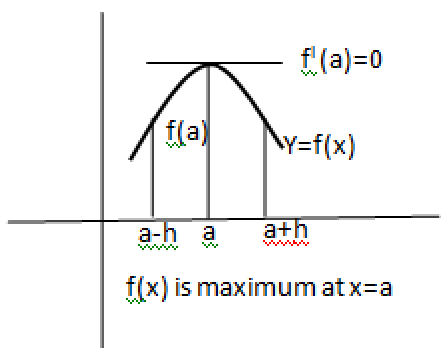
# Maxima, Minima of two variables and Lagrange's Undetermined Multipliers

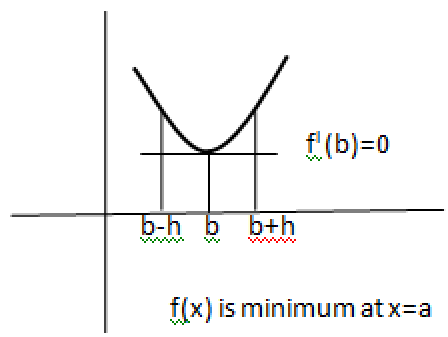
## Contents of this unit

- **Introduction**
- **Definitions**
- **Conditions for extreme values**
- **Theorem, Lagarnge's method of undetermined multipliers for optimization**
- **Examples.**

## Introduction:

**We already studied maxima and minima of a function f(x) of single variable x.**



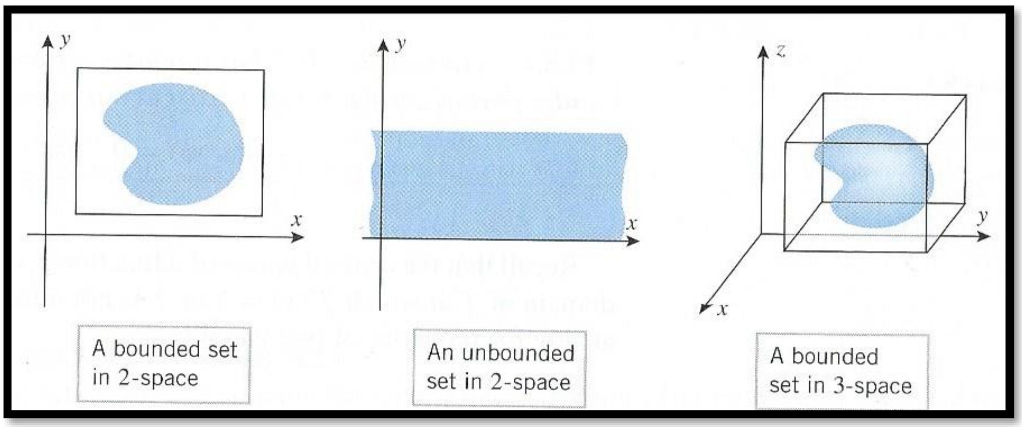

**At the point x=a in first fig. and x=b in second fig. tangents are parallel to x-axis**

**In the same way we have maxima and minima for function of two variables f(x,y). There are many practical situations in which it is necessary and desirable to know maximum and minimum of functions of two or more variables. In many applied problems of science and engineering a function of two or more variables has to be optimized subject to one or more conditions on variables. The so called Lagrnge's method of undetermined multipliers provides a recipe to handle these problems.**

**For example: It is given a metal sheet of fixed surface area *l* , We have to construct a recangular box with maximum volume. We use this Lagrnge's method of undetermined multipliers and calculate.**

## Maxima and Minima for functions of two variables:

**Continuous function of two or three variables assumes extreme values on closed and bounded regions.**




**We study in this section how to get extreme values using first and second order partial derivatives.**

**Definition (2015,17,19): Let f(x,y) be a continuous function of two variables in a neighbourhood of a point (a,b). Then f(a, b) said to be maximum of f(x,y) at (a,b) if f(a,b)>f(a+h, b+k) in a certain deleted neighbourhood of (a, b). And value f(a,b) is maximum value.**

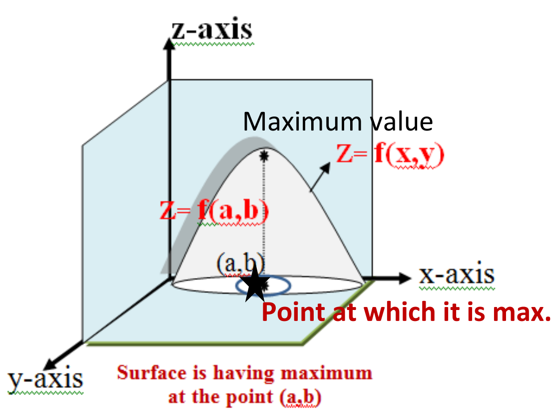



Fig. (1)

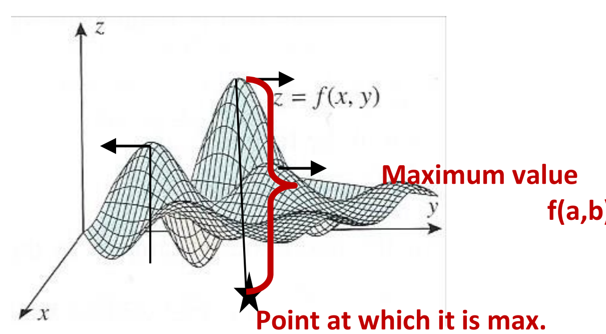



Fig. (2)

**Definition (2015,17,19):** Let f(x,y) be a continuous function of two variables in a neighbourhood of a point (a,b). Then f(a, b) said to be **minimum of f(x,y) at (a,b)** if f(a,b)<f(a+h, b+k) in a certain deleted neighbourhood of (a, b). And **value f(a,b)** is minimum value.

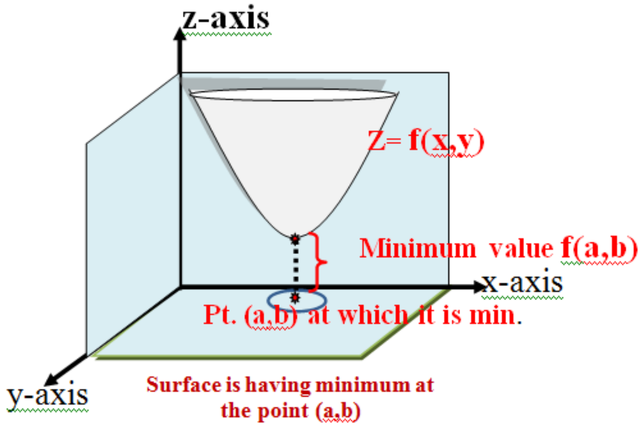



Fig. (3)

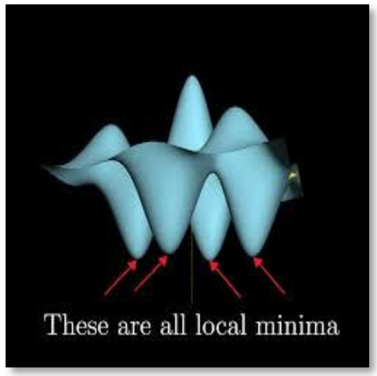


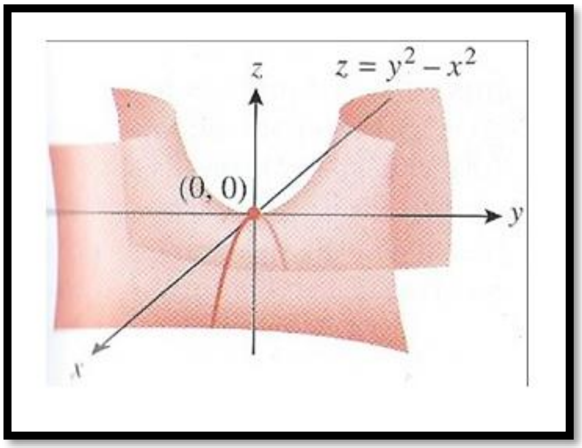

Fig. (4)

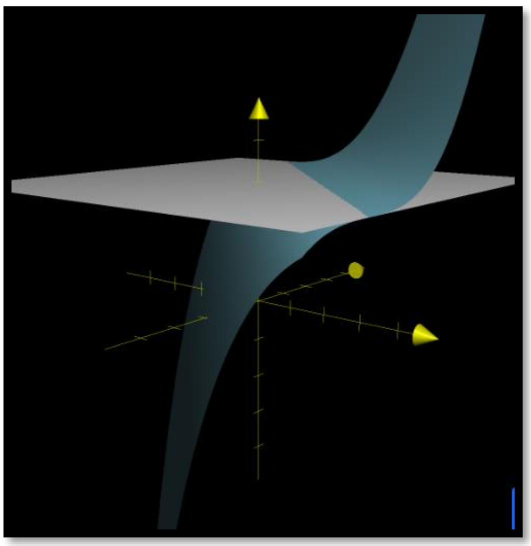
**Note:** In given heighbourhood only one point of minimum or maximum or minimum as shown in the fig.(1) and (3) and it is not necessary that surface has only one point as point of minimum, may be more than one point as points of minimum as shown in the fig. (2) and (4).

**Definition of Extreme Value** : For the function f(x, y), f(a,b) is said to be extreme value or extremum of f(x,y) if f(a,b) is maximum or minimum.

**Critical or stationary point**: A point (a, b) is called critical or stationary point if f(x, y) has extremum at (a,b).

## Saddle point:



**Saddle point: A point (a,b) is called saddle point if f(x,y) is neither maximu nor minimum at (a, b), as shown in fig.**

## Theorem (Necessary condition for extreme values):

**Necessary conditions for a function f(x, y) to have an extreme value (maxima or minima) at (a, b) are that**

$$f_x(a, b) = 0 \text{ and } f_y(a, b) = 0$$

**Above conditions are only necessary but not sufficient conditions.**

**i.e even though $f_x(a, b) = 0$ and $f_y(a, b) = 0$ but function is not having extreme values at (a, b).**

**Classification of critical or stationary points:**

**Finding the extreme values by definition is very difficult, so by using next theorem we will find very easily the extreme values.**

**Theorem(sufficient conditions for extreme values):**

**Let f(x, y) be defined in a certain neighbourhood of (a, b) and posses continuous partial derivatives upto third order.**

**Let $f_x(a, b) = 0$ and $f_y(a,b) = 0$.**

**Let $A = f_{xx}$ or $f_{x^2}(a, b)$, $B = f_{xy}(a, b)$, and $C = f_{yy}$ or $f_{y^2}(a, b)$**

**Then**

**(i)** **f has maximum at (a,b) if $AC - B^2 > 0$ & $A < 0$**
ie. in other words $f_{xx} f_{yy} - (f_{xy})^2 > 0$, $f_{xx} < 0$
and f(a,b) is maximum value.

**(ii)** **f has minimum at (a,b) if $AC - B^2 > 0$ & $A > 0$**
ie. in other words $f_{xx} f_{yy} - (f_{xy})^2 > 0$, $f_{xx} > 0$
and f(a,b) is minimum value.

**(iii)** **f is neither maximum nor minimum at (a, b) or it has a saddle point at (a, b) if $AC - B^2 < 0$.**

**(iv)** **If $AC - B^2 = 0$,** then anything is possible.

## *Procedure for calculating maxima , minima and saddle points:*

**I Step:** **Calculate $f_x$, $f_y$, $f_{xy}$, $f_{xx}$, $f_{yy}$**

**II Step:** **For maximum or minimum we have $f_x = 0$ and $f_y=0$.**

**III Step:** **Solving above two equations $f_x= 0$ and $f_y=0$ get the values of a, b.**

**IV Step:** **Calculate $AC - B^2$ at that point and check it conditions of maxima and minima along with $A<0$, $A>0$ or $A=0$.**

### Examples:

**1. Find maximum or minimum values of $f(x,y) = x^2 + y^2$**

Lets work out the stationary points for the function

$$f(x, y) = x^2 + y^2$$

and classify them into maxima, minima and saddles.
We need all the first and second derivatives so lets work them out. we have

$$\begin{aligned} f_x &= 2x \\ f_y &= 2y \\ f_{xx} &= 2 \\ f_{yy} &= 2 \\ f_{xy} &= 0 \end{aligned}$$

For stationary points we need $f_x = f_y = 0$.

**This gives 2x = 0 and 2y = 0 so that there is just one stationary point, namely (x, y) = (0, 0). We now need to classify it.**

**$f_{xx} f_{yy} - (f_{xy})^2 = 2.2 - 0 = 4>0$ and $A= f_{xx} = 2>0$**

⇨ **Hence it is a minimum at (0,0) and minimum value is f(0,0)=0**

**2. Find stationary points of the function $f(x, y) = x^3 + y^3 - 3xy$ (2018)**

**Soln.:** Given function be $f(x, y) = x^3 + y^3 – 3xy$-----------------(1)

Then $f_x = 3x^2 – 3y$

$f_y = 3y^2 – 3x$

$f_{xx} = 6x = A$

$fxy = -3 = B$

$f_{yy} = 6y = C$

For extreme values we have $f_x = 0$ and $f_y = 0$

i.e $3x^2 – 3y = 0$ and $3y^2 – 3x = 0$ ------------(2)

i.e $y = x^2$ -----(3), then (2) becomes $x^4 – x = 0$

$x(x^3 – 1) = 0$

⇨ x =0 and x =1

⇨ If x = 0 then (from (3) ) we have y =0

And if x =1 then y =1

Points at which function is max. Or min. are (0, 0), (1, 1) qnd (1, -1)

(i) At (0,0)

$AC – B^2 = 0 – 9 < 0$ , f(0, 0) is not extreme value

(ii) At (1,1)

$AC – B^2 = -36 – 9 = - 45 < 0$

⇨ **f has minimum value at (1, 1) and that min. value is f(1,1) = -1**

3. Show that $f(x,y) = x^3 + y^3 - 3xy +1$ is minimum at (1, 1). (2018)

Same as example (2)

**4. Find stationary values of $f(x, y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$, (2019)**

**Soln.:** Given function be $f(x, y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$

Then $f_x = 2x - \frac{2}{x^2}$ => $f_{xx} = 2 - \frac{4}{x^3} = A$

$f_y = 2y - \frac{2}{y^2}$ => $f_{yy} = 2 - \frac{4}{y^3} = C$

$f_{xy} = 0 = B$

**For extreme values we have $f_x = 0$ and $f_y = 0$**

**i.e $2x - \frac{2}{x^2} = 0$ and $2y - \frac{2}{y^2} = 0$**

**=> $x = \frac{1}{x^2}$ and $y = \frac{1}{y^2}$**

**=> $x^3 = 1$ and $y^3 = 1$**

**=> x=1 and y= 1**

**At the point (1, 1) , A = -2, B = 0, C = -2**

**$AC - B^2 = (-2)(-2) - 0 = 4 > 0$ and A = -2**

**⇨ f is max. at (1, 1) and max. value is f(1, 1) = 6**

**5. Find extreme values of $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ (2018)**

**Soln.: Given function be $f(x, y) = y^2 + 4xy + 3x^2 + x^3$--------------(1)**

**Then $f_x = 4y+6x+3x^2$ , $f_y = 2y+4x$**

**$f_{xx} = 6 + 6x = A$**

**$f_{xy} = 4 = B$**

**$f_{yy} = 2 = C$**

**For extreme values we have $f_x = 0$ and $f_y = 0$**

**i.e $4y+6x+3x^2 = 0$ -----(2) and $2y+4x = 0$-----(3)**

**from (3) => y = -2x-----------------------------------(4)**

**Sub. in (2) we get $-8x + 6x + 3x^2 = 0$**

**$3x^2 - 2x = 0$**

**i.e $x(3x - 2) = 0$**

**=> x =0 and $x = \frac{2}{3}$**

**From(4) , if x =0 then y =0**

**& if $x = \frac{2}{3}$ then $y = -\frac{4}{3}$**

**Stationery points are (0,0) and $(\frac{2}{3}, -\frac{4}{3})$**

**(i) At (0,0)**

**$AC - B^2 = 12 - 16 = -4 < 0$**

**∴ (0, 0 ) is saddle point**

**(ii)** At$(\frac{2}{3}, -\frac{4}{3})$

$AC - B^2 = 20 - 16 = 4 > 0$ and $A = 10 > 0$

⇨ **f is minimum at $(\frac{2}{3}, -\frac{4}{3})$ and minimum value is $-\frac{4}{27}$**

**6. Find extreme values of $f(x, y) = x^3y^2 (1 - x - y)$** **(2019)**

**Soln.:** Given function be $f(x, y) = x^3y^2 (1 - x - y) = x^3y^2 - x^4y^2 - x^3y^3$--(1)

Then $f_x = 3x^2y^2 - 4x^3y^2 - 3x^2y^3$

$f_y = 2x^3y - 2x^4y - 3x^3y^2$

$f_{xx} = 6xy^2 - 12x^2y^2 - 6xy^3 = A$

$fxy = 6x^2y - 8x^3y - 9x^2y^2 = B$

$f_{yy} = 2x^3 - 2x^4 - 6x^3y = C$

For extreme values we have $f_x = 0$ and $f_y = 0$

i.e $3x^2y^2 - 4x^3y^2 - 3x^2y^3 = 0$ and $2x^3y - 2x^4y - 3x^3y^2 = 0$

i.e $x^2y^2 ( 3 - 4x - 3y) = 0$ and $x^3y( 2 - 2x - 3y) = 0$

=> $x = 0$ and $y = 0$

And $3 - 4x - 3y = 0$ -----(a) & $2 - 2x - 3y = 0$ ---------(b)

(a) – (b) gives $3 - 4x - 3y - 2 + 2x + 3y = 0$

i.e $1 - 2x = 0$

=> $x = \frac{1}{2}$

Then from (b) $2 - 2(\frac{1}{2}) - 3y = 0$

i.e $2 - 1 - 3y = 0$

i.e $1 - 3y = 0$

=> $y = \frac{1}{3}$

Therefore stationary points are (0, 0) and $(\frac{1}{2}, \frac{1}{3})$.

$f_{xx} = 6xy^2 - 12x^2y^2 - 6xy^3 = A$
$f_{xy} = 6x^2y - 8x^3y - 9x^2y^2 = B$
$f_{yy} = 2x^3 - 2x^4 - 6x^3y = C$

**(i) At (0, 0)**

$A = f_{xx} = 0$, $B = f_{xy} = 0$, $C = f_{yy} = 0$

⇨ $AC - B^2 = 0$

**We cannot tell anything about f at (0, 0).**

**(ii) At $(\frac{1}{2}, \frac{1}{3})$**

$A = f_{xx} = 6(\frac{1}{2})(\frac{1}{3})^2 - 12(\frac{1}{2})^2(\frac{1}{3})^2 - 6(\frac{1}{2})(\frac{1}{3})^3 = (\frac{1}{3}) - (\frac{1}{3}) - (\frac{1}{9})$

$= -\frac{1}{9}$

$B = f_{xy} = 6(\frac{1}{2})^2(\frac{1}{3}) - 8(\frac{1}{2})^3\frac{1}{3} - 9(\frac{1}{2})^2(\frac{1}{3})^3 = (\frac{1}{2}) - (\frac{1}{3}) - (\frac{1}{12}) = \frac{1}{12}$

$C = f_{yy} = 2(\frac{1}{2})^3 - 2(\frac{1}{2})^4 - 6(\frac{1}{2})^3(\frac{1}{3}) = \frac{1}{4} - \frac{1}{8} - \frac{1}{4} = -\frac{1}{8}$

Then $AC - B^2 = (-\frac{1}{9})(-\frac{1}{8}) - \frac{1}{144} = \frac{1}{72} - \frac{1}{144} > 0$

And $A = -\frac{1}{9} < 0$

Thus $AC - B^2 > 0$ and $A < 0$

**and hence function f has maximum at $(\frac{1}{2}, \frac{1}{3})$**

**Maximum value is $f(\frac{1}{2}, \frac{1}{3}) = (\frac{1}{2})^3(\frac{1}{3})^2(1 - \frac{1}{2} - \frac{1}{3}) = \frac{1}{72}(\frac{1}{6}) = \frac{1}{432}$**

∴ **Extreme value is $\frac{1}{432}$**

**7. Find extreme values of f(x, y) = xy (6 –x –y)** **(2016)**

**Soln.:** Given function be $f(x, y) = xy(6 - x - y) = 6xy - x^2y - xy^2$

$f_x = 6y - 2xy - y^2 = y(6 - 2x - y)$

$f_y = 6x - x^2 - 2xy = x(6 - x - 2y)$

$f_{xx} = -2y = A$

$f_{xy} = 6 - 2x - 2y = B$

$f_{yy} = -2x = C$

For extreme values we have $f_x = 0$ and $f_y = 0$

i.e $y(6 - 2x - y) = 0$ and $x(6 - x - 2y) = 0$

**=> y = 0, 6 -2x – y = 0 and x =0 , 6-x – 2y = 0**

**=> (0, 0) is first extreme point**

**Next consider, 6 -2x – y = 0----------(1)**

**6 -x – 2y = 0---------(2)**

**Then (1) – 2(2) gives**

**6 -2x – y- 12 +2x +4y = 0**

**i.e 3y – 6= 0 => y = 2**

**From (1) 6 – 2x -2 =0 => -2x = -4 => x = 2**

$f_{xx} = -2y = A$
$f_{xy} = 6 - 2x - 2y = B$
$f_{yy} = -2x = C$

**Then another point is (2, 2)**

**(i) At (0, 0)**

**A=0, B= 6, C=0 => $AC - B^2 = -36 < 0$**

**⇨ (0, 0) is saddle point**

**(ii) At (2, 2)**

**A = -4, B = 6 -4 – 4 = -2, C = -4**

**∴ $AC - B^2 = 16 - 4 = 8 > 0$ and A = -4 < 0**

**i.e $AC - B^2 < 0$ and A<0**

**=> Function f has maximum at (2, 2) and maximum value is f(2,2).**

**i.e f(2,2) = 4(6 – 2 - 2) = 8**

**∴ Extreme value is 8.**

**8. Find extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ (2015)**

**Soln.: Given function is $f(x, y) = x^3 + y^3 - 3x - 12y + 20$----(1)**

**Then $f_x = 3x^2 - 3$**

**$f_y = 3y^2 - 12$**
**$f_{xx} = 6x \quad = A$**
**$f_{xy} = 0 \quad = B$**
**$f_{yy} = 6y \quad = C$**

**For extreme values we have $f_x = 0$ and $f_y = 0$**
**i.e $3x^2 - 3=0$ and $3y^2 - 12 =0$**
**i.e $x^2 - 1=0$ and $y^2 - 4 =0$**
**=> $x = \pm 1$ and $y =\pm 2$**

**∴ Stationary points are ( 1, 2), (-1, 2) , (1, -2) and (-1, -2)**

**(i) At (1, 2)**
**$A = f_{xx} = 6 \quad B = f_{xy} = 0, \quad C = f_{yy} = 12$**
**∴ $AC - B^2 = 72>0$ and $A =6>0$**
**=> f is minimum at (1, 2) and minimum value is f(1,2)= 2**

**(ii) At (-1, 2)**
**$A = f_{xx} = -6 \quad B = f_{xy} = 0, \quad C = f_{yy} = 12$**
**∴ $AC - B^2 = -72<0$ => (-1, 2) is saddle point.**

**(iii) At (1, -2)**
**$A = f_{xx} = 6 \quad B = f_{xy} = 0, \quad C = f_{yy} = -12$**
**∴ $AC - B^2 = -72<0$ => (1, -2) is saddle point.**

**(iv) At (-1, -2)**
**$A = f_{xx} = -6 \quad B = f_{xy} = 0, \quad C = f_{yy} = -12$**
**∴ $AC - B^2 = 72>0$ and $A = -6 <0$**
**=> f has maximum at (-1, -2) and maximum value is f(-1,-2) = 38**

**Thus extreme points are (1, 2) and (-1, -2) and extreme values are 2 and 38.**

**9. Examine the maxima and minima for the function**

**$f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$ (2017)**

**Soln.:** Given function is $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$

Then $f_x = 3x^2 + y^2 + 10x$

$f_y = 2xy + 2y$

$f_{xx} = 6x+10 \quad = A$

$f_{xy} = 2y \quad = B$

$f_{yy} = 2x+2 \quad = C$

For extreme values we have $f_x = 0$ and $f_y = 0$

- ⇨ $3x^2 + y^2 + 10x = 0$ ---(1) and $2xy + 2y = 0 \Rightarrow y(x+1)=0$
- ⇨ $\Rightarrow y = 0$ and $x = -1$
- ⇨ From (1) , for y=0, we get $3x^2 + 10x = 0 \Rightarrow x = 0$ and $x = -\frac{10}{3}$
- ⇨ First two points are (0, 0) and $(-\frac{10}{3}, 0)$

Next for x =-1, eqn. (1) becomes.

$3 + y^2 - 10 = 0 \Rightarrow y^2 = 7 \Rightarrow y = \pm\sqrt{7}$

∴ Other two stationary points are= $(-1, \sqrt{7})$ and $(-1, -\sqrt{7})$

$f_x = 3x^2 + y^2 + 10x$

$f_y = 2xy + 2y$

$f_{xx} = 6x+10 \quad = A$

$f_{xy} = 2y \quad = B$

$f_{yy} = 2x+2 \quad = C$

i) At (0, 0)

$A = f_{xx} = 10 \quad B = f_{xy} = 0, \quad C = f_{yy} = 2$

∴ $AC - B^2 = 20 > 0$ and $A = 10 > 0$

=> f is minimum at (0, 0) and minimum value is f(0,0)= 2

ii) At $(-\frac{10}{3}, 0)$

$A = f_{xx} = -10 \quad B = f_{xy} = 0, \quad C = f_{yy} = -\frac{14}{3}$

∴ $AC - B^2 = \frac{140}{3} > 0$ and $A = -10 < 0$

=> f is maximum at $(-\frac{10}{3}, 0)$ and maximum value is

**$f(-\frac{10}{3}, 0) = -\frac{500}{27}$**

**iii) At (-1, $\sqrt{7}$ ) and (-1,$-\sqrt{7}$), AC- $B^2$ < 0 and hence these are saddle points.**

**10. Find maximum or minimum of the function**

$$f(x,y) = e^{-(x^2+y^2)}$$

**Soln.: Given function be f(x, y) = $e^{-(x^2+y^2)}$**

The first and second order partial derivatives of this function are:

$$f_x = -2xe^{-(x^2+y^2)}$$
$$f_y = -2ye^{-(x^2+y^2)}$$
$$f_{xx} = -2e^{-(x^2+y^2)}(1-2x^2) \quad \text{by the product rule}$$
$$f_{yy} = -2e^{-(x^2+y^2)}(1-2y^2)$$
$$f_{xy} = 4xye^{-(x^2+y^2)}$$

For Stationary points we have fx = 0 and fy = 0

i.e **-2x $e^{-(x^2+y^2)}$ = 0 => x = 0 and -2y $e^{-(x^2+y^2)}$ = 0 => y = 0**

So there is only one stationary point, at (x, y) = (0, 0).

Substituting (x, y) = (0, 0) into the expressions for fxx, fyy and fxy gives

fxx = −2, fyy = −2, fxy = 0

Therefore

AC – $B^2$ = fxx fyy – $(fxy)^2$ = (−2)(−2) − 0 = 4 > 0

so that (0, 0) is either a min or a max.

Since A = fxx = -2 < 0 .

**Thus AC – $B^2$ >0, A<0**

**Hence f is maximum at (0, 0) and maximum value is f(0,0) = 1.**

**11. Find maximum or minimum of the function**

**Soln: Given function be $f(x, y) = 2 - x^2 - xy - y^2$**

For this function

$$f_x = -2x - y$$
$$f_y = -x - 2y$$
$$f_{xx} = -2$$
$$f_{yy} = -2$$
$$f_{xy} = -1$$

For Stationary points we have fx = 0 and fy = 0

⇨ -2x-2y=0 and –x-2y=0

i.e x+y = 0 and x+2y =0

Solving these two equations we get

x+y = 0

-(x+2y)=0

=> -y =0 i.e y =0 => x =0

So there is only one stationary point, at (x, y) = (0, 0).

Substituting (x, y) = (0, 0) into the expressions for fxx, fyy and fxy gives

$$f_{xx} = -2,\ f_{yy} = -2,\ f_{xy} = -1$$

Therefore

$AC - B^2 = f_{xx} f_{yy} - (f_{xy})^2 = (-2)(-2) - (-1)^2 = 3 > 0$

so that (0, 0) is either a min or a max.

Since $A = f_{xx} = -2 < 0$ .

**Thus $AC - B^2 > 0$, $A < 0$**

**Hence f is maximum at (0, 0) and maximum value is f(0,0) = 2.**

**Example 4: Find the stationary values of**

**$f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$**

**Soln: Given function is $f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$**

The first and second order partial derivatives of this function are

$$f_x = 6x^2 + 6y^2 - 150$$
$$f_y = 12xy - 9y^2$$
$$f_{xx} = 12x$$
$$f_{yy} = 12x - 18y$$
$$f_{xy} = 12y$$

For Stationary points we have fx = 0 and fy = 0

⇨ -2x-2y=0 and –x-2y=0

$$6x^2 + 6y^2 - 150 = 0 \text{ and } 12xy - 9y^2 = 0$$

i.e. $x^2 + y^2 = 25$ --------(1) and $y(4x - 3y) = 0$--------(2)

From (2) we get

$y = 0$ or $4x = 3y$

If $y = 0$ then from (1) we get

$x^2 = 25$ so that $x = \pm 5$

gives (5, 0) and (−5, 0) as stationary points.

Next, If $4x = 3y$ then $x = \frac{3}{4}y$ ----------------(3)

From (1) we get

$$\left(\frac{3}{4}y\right)^2 + y^2 = 25$$

$$\frac{25}{16}y^2 = 25 \implies y^2 = 16$$

Therefore $y = \pm 4 \implies x = \pm 3$

Now $y = 4$ gives $x = 3$ and $y = -4$ gives $x = -3$,

so we have two further stationary points (3, 4) and (−3, −4).

Thus in total there are four stationary points

(5, 0), (−5, 0), (3, 4) and (−3, −4).

Each of these must now be classified into max, min or saddle.

• At (5, 0).

For this stationary point,

$f_{xx} f_{yy} – (f_{xy})^2 = 60^2 - 0 > 0$ so it is either a max or a min.

But $f_{xx} = 60 > 0$ and $f_{yy} = 60 > 0$. **Hence at (5, 0) is a minimum**

**Minimum Value is f(5,0)= - 500**

• Next (−5, 0).

For this stationary point,

$f_{xx}f_{yy} − (f_{xy})^2 = (−60)^2 > 0$ so it is either a max or a min.

But $f_{xx} = −60 < 0$ and $f_{yy} = −60 < 0$. **Hence at (−5, 0) is a maximum.**

**Maximum Value is f(-5,0) = 500**

• Next(3, 4).

For this stationary point,

$f_{xx}f_{yy} − (f_{xy})^2 = −3600 < 0$ so **(3, 4) is a saddle.**

• Next (−3, −4).

For this stationary point,

$f_{xx}f_{yy} − (f_{xy})^2 = −3600 < 0$ so **(−3, −4) is a saddle.**

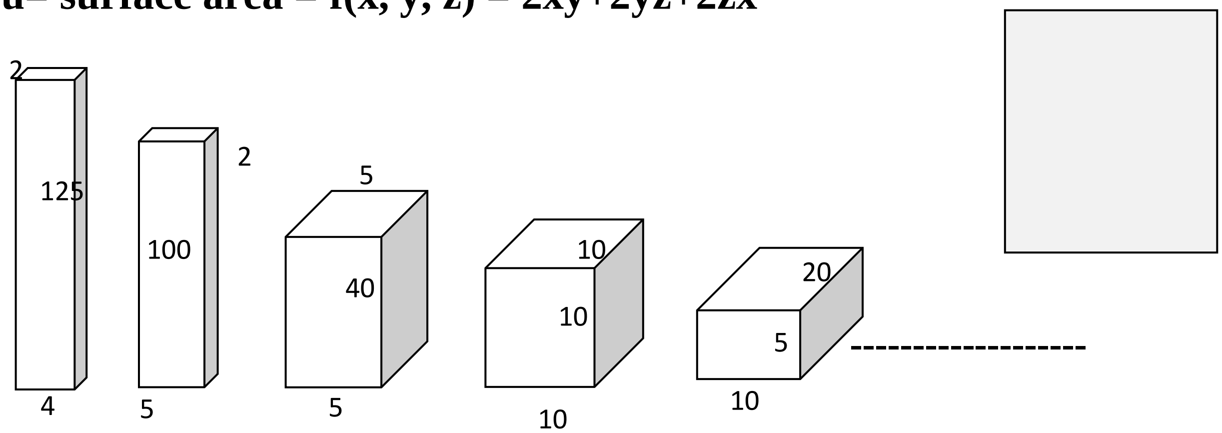
## Optimization by Lagrange's Method:

## I will explain first what is optimization?

1. **Consider an example, what is the minimum metal sheet required to construct a tin (cubide in shape) of capacity 1000 cc. ?**

**Here, given function is metal sheet required**

**u= surface area = f(x, y, z) = 2xy+2yz+2zx**




| | | | | |
|---|---|---|---|---|
| V=1000cc<br>S =1516 sqcm | V=1000cc<br>S =1420sqcm | V=1000cc<br>S =850 sqcm | V=1000cc<br>S = 600 sqcm | V=1000cc<br>S =700 sqcm |

**Among these boxes, the box with minimum surface area is 600 sq.cms. i.e minimum metal sheet required is of 600 sq.cms.**

**This minimizing surface area is optimization.**

2. **Consider another example**

**We have to find three numbers x, y, z so that their sum is 6 and (i) product is maximum (ii) Sum of their squares is minimum**

| Numbers | Product | Sum of their squares |
|---|---|---|
| 1,2,3 | 6 | 14 |
| 1,1, 4 | 4 | 18 |
| 2, 2, 2 | 8 | 12 |

**But for each examples of these type, this trial and error method is very difficult. So in such cases we can use Lagrange's Undermined Multipliers.**

**(i) Lagrange's Method of undetermined Multipliers when variables x, y, z are connected by one equation.**

**Theorem:** Explain the method of Lagrange's method of undetermined multipliers to find optimum value of f(x, y, z) where variables are connected by the relation $\emptyset(x, y, z)$ =0 . **[2016, 2018]**

**Explanation:** Let u = f(x, y, z) be a function of three variables x, y, z.

These variables x, y, z are connected by the equation $\emptyset(x, y, z)$ =0------(1)

For maximum or minimum value of u we have du =0

I.e $\frac{\partial u}{\partial x}$ dx + $\frac{\partial u}{\partial y}$ dy+ $\frac{\partial u}{\partial z}$dz =0 ---------------------(2)

Also from equation (1)

$d\emptyset = \frac{\partial \emptyset}{\partial x} dx + \frac{\partial \emptyset}{\partial y} dy + \frac{\partial \emptyset}{\partial z} dz = 0$-------------------(3)

Multiplying equation (2) and (3) by 1 and $\mu$ and adding them, where $\mu$ is constant, we get

$\left(\frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz\right) + \mu \left(\frac{\partial \emptyset}{\partial x} dx + \frac{\partial \emptyset}{\partial y} dy + \frac{\partial \emptyset}{\partial z} dz\right) = 0$

$\left(\frac{\partial u}{\partial x} + \mu \frac{\partial \emptyset}{\partial x}\right) dx + \left(\frac{\partial u}{\partial y} + \mu \frac{\partial \emptyset}{\partial y}\right) dy + \left(\frac{\partial u}{\partial z} + \mu \frac{\partial \emptyset}{\partial z}\right) dz = 0$---------------------(4)

Since $\mu$ is arbitrary constant, i.e for all values of $\mu$, (4) is true only when

$$\left.\begin{aligned} \frac{\partial u}{\partial x} + \mu \frac{\partial \emptyset}{\partial x} = 0 \\ \frac{\partial u}{\partial y} + \mu \frac{\partial \emptyset}{\partial y} = 0 \end{aligned}\right\} \quad \text{-----------------(5)}$$

$$\frac{\partial u}{\partial z} + \mu \frac{\partial \emptyset}{\partial z} = 0$$

Using (1) and (5), we can determine the values $\mu$, x, y and z for which u is maximum or minimum.

Hence the proof.

**Examples:**

**1. Find the minimum value of $x^2+y^2+z^2$ having given that ax+by +cz =p. [2016]**

**[in other words find the three numbers x, y, z so that sum of their squares is minimum and linear combination of them is p.]**

**Soln.**: We have to find minimum value of

$u = x^2+y^2+z^2$ --------------------------(1)

With condition ax+by +cz =p

i.e $\emptyset = ax+by+cz-p = 0$----------------------(2)

Consider $du+\mu\, d\emptyset = 0$, where $\mu$ is constant

i.e $2x\,dx + 2y\,dy + 2z\,dz + \mu(a\,dx + b\,dy + c\,dz) = 0$

$(2x + \mu a)\,dx + (2y + \mu b)\,dy + (2z + \mu c)\,dz = 0$ -----(3)

Since $\mu$ is arbitrary, (3) is true only when

$2x + \mu a = 0$ ---------------------(4)

$2y + \mu b = 0$----------------------(5)

$2z + \mu c = 0$--------------------- (6)

Multiplying eqn. (4) by x , (5) by y (6) by Z and adding we get

$2x^2 + \mu ax + 2y^2 + \mu by + 2z^2 + \mu cz = 0$

i.e $2(x^2+y^2+z^2) + \mu(ax+by+cz) = 0$

$2u + \mu p = 0$

⇨ $\boldsymbol{\mu = -\frac{2u}{p}}$ **-------------------------------------------(7)**

Next , multiplying equations (4), (5) and (6) by a, b, c resply. And adding we get

$2(ax+by+cz) + \mu(a^2 + b^2 + c^2) = 0$

i.e $2p - \frac{2u}{p}(a^2 + b^2 + c^2) = 0$

⇨ $\frac{2u}{p}(a^2 + b^2 + c^2) = 2p$

⇨ $\boldsymbol{u = \frac{p^2}{a^2+b^2+c^2}}$ **which is the minimum value of u.**

Next from (4)& (7) $2x = -a\mu = -a\left(-\frac{2u}{p}\right) = 2a\frac{1}{p}\frac{p^2}{a^2+b^2+c^2} = \frac{2ap}{a^2+b^2+c^2}$

$\therefore x = \frac{ap}{a^2+b^2+c^2}$

similarly

(5), (6) and (7) we get

$\boldsymbol{y = \frac{bp}{a^2+b^2+c^2}, \quad z = \frac{cp}{a^2+b^2+c^2}}$

**Thus u is minimum for**

**$x = \frac{ap}{a^2+b^2+c^2}$ , $y = \frac{bp}{a^2+b^2+c^2}$ , $z = \frac{cp}{a^2+b^2+c^2}$ and minimum value is $\frac{p^2}{a^2+b^2+c^2}$ .**

**2. Find three numbers so that their sum is 24 and sum of their squares is minimum.**

**Soln.**: Let the numbers be x, y, z so that x + y + z = 24

**Let u =** $x^2+y^2+z^2$ --------------------------(1)

And condition is x + y + z = 24

i.e $\emptyset = x + y + z - 24 = 0$---------------------(2)

Consider du + $\mu$ d$\emptyset$ =0, where $\mu$ is constant

i.e 2x dx + 2y dy + 2z dz + $\mu$( dx + dy + dz) =0

$( 2x + \mu ) dx + ( 2y + \mu ) dy + ( 2z + \mu ) dz =0$ -----(3)

This is true only when

$2x + \mu = 0$ --------------------(4)

$2y + \mu = 0$---------------------(5)

$2z + \mu = 0$-------------------- (6)

Multiplying eqn. (4) by x , (5) by y (6) by Z and adding we get

$$2x^2 + \mu x + 2y^2 + \mu y + 2z^2 + \mu z = 0$$

i.e $2(x^2+y^2+z^2) + \mu (x+y+z) =0$

$$2u + \mu 24 = 0$$

⇨ $\boldsymbol{\mu} = -\frac{2u}{24}$ --------------------------------------------(7)

Again adding (4), (5) and (6) we get

$$2(x+y+z) + 3\mu = 0$$

$$48 = -3\mu = -3\left(-\frac{2u}{24}\right)$$

$$\text{i.e}\quad 48 = \frac{u}{4}$$

⇨ u = 192, **which is the minimum value of u.**

And from (4), (5), (6) and (7) we get

$$x = \frac{ap}{a^2+b^2+c^2},\quad y = \frac{bp}{a^2+b^2+c^2},\quad z = \frac{cp}{a^2+b^2+c^2}$$

$$x = \frac{24}{3} = 8,\ y = 8,\ z = 8$$

**3. Find the maximum or minimum value of $x^2y^2z^2$ subject to the condition $x^2+y^2+z^2 = a^2$ [ 2016].**

**Soln.: Let u = $x^2y^2z^2$**

**And given condition ∅ :** $x^2+y^2+z^2=a^2$ i.e $x^2+y^2+z^2-a^2= 0$ ----------(1)

Consider

$du+\mu\, d\emptyset = 0$, where $\mu$ is constant

i.e $2x\,y^2z^2\,dx + 2x^2yz^2\,dy + 2x^2y^2z\,dz + \mu\,(2x\,dx + 2y\,dy + 2z\,dz) = 0$

$2x(y^2z^2 + \mu)dx + 2y(x^2z^2 + \mu)\,dy + 2z(x^2y^2 + \mu)\,dz = 0$

**This is true for all values o** $\mu$, only when

$y^2z^2 + \mu = 0$-------------------------(2)

$x^2z^2 + \mu = 0$-------------------------(3)

$x^2y^2 + \mu = 0$-------------------------(4)

**Multiplying (2), (3), and (4) resply. by $x^2$, $y^2$, $z^2$ and adding them we get**

$x^2 y^2z^2 + x^2 y^2z^2 + x^2 y^2z^2 + \mu (x^2+y^2+z^2) = 0$

i.e $3 x^2 y^2z^2 + \mu a^2 = 0$ from (1)

$3u + a^2\mu = 0$

$\Rightarrow \mu = -\frac{3u}{a^2}$ -----------------(5)

Again from (2), (3) and (4)

We get $\frac{\mu}{x^2y^2} = -1 \Rightarrow \mu = -x^2y^2$

**Similarly** $\mu = -z^2y^2$ **and** $\mu = -x^2z^2$

$x^2y^2 = z^2y^2 = z^2x^2$

$\Rightarrow x^2 = y^2 = z^2$

From (1) we get $3x^2 = a^2$

$\Rightarrow x = \frac{a}{\sqrt{3}}$

$\Rightarrow y = \frac{a}{\sqrt{3}}$ and $z = \frac{a}{\sqrt{3}}$

Hence minimum or maximum value of is $= x^2y^2z^2 = \frac{a^6}{27}$

[ for example: If a = 3 then $a^2 = 9$, we have to find three numbers x,y,z so that $x^2 + y^2 + z^2 = 9$ **and $x^2y^2z^2$ is extrem. Then three numbers are $\sqrt{3}, \sqrt{3}, \sqrt{3}$, so that $x^2 + y^2 + z^2 = 9$ and $x^2y^2z^2 = 27$**

**Suppose, if numbers are $\sqrt{2}$, $\sqrt{3}$, and 2 so that $x^2 + y^2 + z^2 = 9$ and $x^2y^2z^2 = 24$ which is less than 27, in this way 27 is maximum value]**

**4. If u = $a^3x^2 + b^3y^2 + c^3z^2$ where $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ =1 then show that the stationary points of u are given by ax = by = cz = (a+b+c)**

Soln.: Let u = $a^3x^2 + b^3y^2 + c^3z^2$ ------------------------(1)

And let g = $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ -1 = 0 --------------------------(2)

For stationary values of u we have **du +k dg = 0** where k is constant.

i.e $2x a^3dx + 3y b^3dy + 2zc^3dz + k ( - \frac{1}{x^2} dx + ( - \frac{1}{y^2} )dy + ( - \frac{1}{z^2} dz) = 0$

$(2x a^3 - \frac{k}{x^2})dx + (2y b^3 - \frac{k}{y^2} )dy + (2zc^3 - \frac{k}{z^2} )dz = 0$

It is true for all values of k, if

$2x a^3 - \frac{k}{x^2} = 0$ ------------------------------------------(3)

$2y b^3 - \frac{k}{y^2} = 0$ ------------------------------------------(4)

$2zc^3 - \frac{k}{z^2} = 0$------------------------------------------(5)

From (3) $\frac{k}{x^2} = 2x a^3$ ∴ $k = 2 x^3 a^3$ and from (4) and (5) $k =2 y^3 b^3 = 2 z^3 c^3$

⇨ $x^3 a^3 = y^3 b^3 = z^3 c^3 = \frac{k}{2}$

⇨ ax = by = cz------------------------(6)

⇨ i. e $z = \frac{a}{c}x$ and $y =\frac{a}{b}x$

Then (2) becomes, $\frac{1}{x}+\frac{b}{ax}+\frac{c}{ax} = 1$

i.e $\frac{a+b+c}{ax} = 1$

=> ax = a+b+c = $\sum a$

∴ From (6) ax= by = cz = $\sum a$

**Thus the stationary points of u are given by ax = by = cz = (a+b+c) and stationary value is u = $a^3\frac{\sum a}{a} + b^3\frac{\sum a}{b} + a^3\frac{\sum a}{c} = (a^2+ b^2+c^2)\sum a = \sum a^2 \sum a$**

**5. Find the maximum value product of three positive numbers whose sum is 24 [2019]**

**Soln:** Let the three positive numbers be x, y, z so that u = xyz-----------------(1)

and x+y+z =24 i.e ∅ = x+y+z-24=0--------------------------------------------------(2)

For maximum or minimum value we have

$$du+\mu d\emptyset = 0$$

i.e $(yz+\mu)dx + (zx+\mu)dy + (xy+\mu)\,dz=0$

=> $yz + \mu = 0$--------------------------------------------------------------(3)

$zx + \mu = 0$--------------------------------------------------------------(4)

$xy + \mu = 0$------------------------------------------------------------(5)

From (3), (4), (5) we get $\mu = -yz = -zx = -xy$

i.e $xy = yz = zx$

=> $x = y = z$

From (2) we get $3x = 24$ => $x = 8$

$\therefore y = z = 8$

=> $x = y = z = 8$

Thus maximum value is $u = xyz = 512$

And numbers are 8, 8, 8

**6. Find the extreme values of xyz subject to the condition $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$**

**Soln.:** Let $u = xyz$--------------------------(1)

And $g = \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} - 1 = 0$------------------(2)

Then $du = yz\,dx + xz\,dy + xy\,dz$

And $dg = \frac{2x}{a^2}dx + \frac{2y}{b^2}dy + \frac{2z}{c^2}dz = 0$

For extreme values we have $du + k\,dg = 0$ for arbitrary constant k.

i.e $yz\,dx + xz\,dy + xy\,dz + k(\frac{2x}{a^2}dx + \frac{2y}{b^2}dy + \frac{2z}{c^2}dz) = 0$

i.e $(yz + k\frac{2x}{a^2})dx + (xz + K\frac{2y}{b^2})dy + (xy + k\frac{2z}{c^2})dz = 0$---------------(3)

This is true for all k if

$(yz + k\frac{2x}{a^2}) = 0$-----------------------------------------------------(4)

$(xz + K\frac{2y}{b^2}) = 0$---------------------------------------------------(5)

$(xy + k\frac{2z}{c^2}) = 0$-----------------------------------------------------(6)

From (4) $k\frac{2x}{a^2} = -yz$ => $k = -\frac{a^2}{2x}yz$

From (5) and (6) also we get $K = -\frac{b^2}{2y}zx$ and $k = -\frac{c^2}{2z}xy$

These implies $k = -\frac{a^2}{2x}yz = -\frac{b^2}{2y}zx = -\frac{c^2}{2z}xy$

i.e $\frac{a^2}{x}yz = \frac{b^2}{y}zx = \frac{c^2}{z}xy$

i.e $\frac{x}{a^2yz} = \frac{y}{b^2xz} = \frac{z}{c^2xy}$ Bc'z $P = Q = R$ => $\frac{1}{P} = \frac{1}{Q} = \frac{1}{R}$

Multiplying throughout by xyz we get

$\frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2}$

⇨ $\frac{z^2}{c^2} = \frac{x^2}{a^2}$ and $\frac{y^2}{b^2} = \frac{x^2}{a^2}$ --------------------(7)

*Substitute in (2)we get*

$3\frac{x^2}{a^2} = 1$

⇨ $x = \frac{a}{\sqrt{3}}$

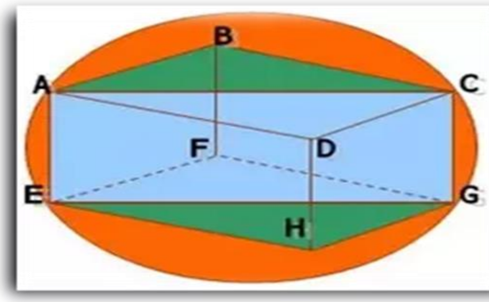
By using (7) we get $y = \frac{b}{\sqrt{3}}$ and $z = \frac{c}{\sqrt{3}}$

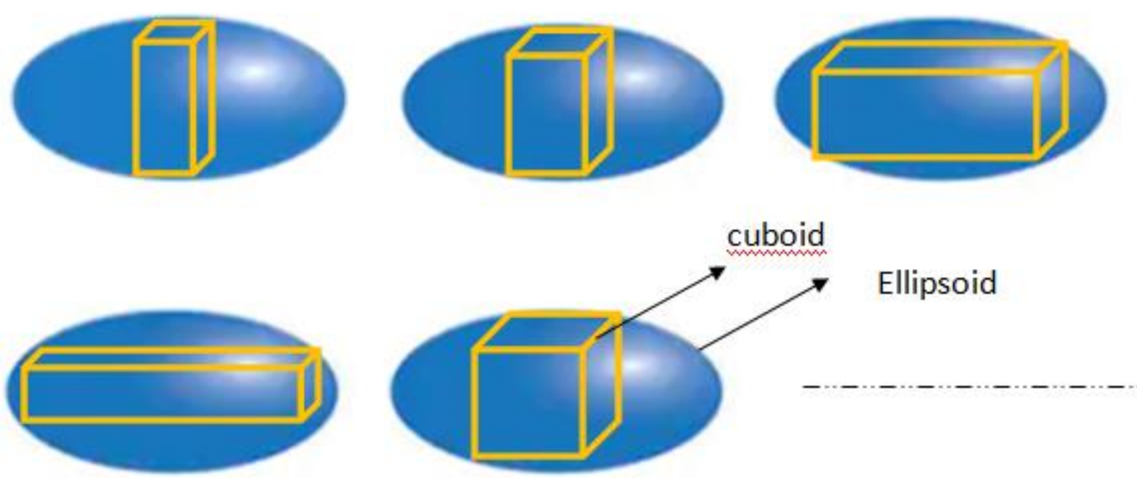
Therefore extreme value of u = xyz = $\frac{abc}{3\sqrt{3}}$ and extrmemum point is $(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}})$

**Same example is asked in other way as below**

**7. Prove that the volume of largest cubid that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$.**

**Proof:** See in this example ellipsoid is fixed, surface area of ellipsoid is fixed, inside that ellipsoid we have to construct cubide with maximum volume.




Among these cuboids inscribed in ellipsoid, one cuboid is having maximum volume, what are its length, breadth and width, that we have to find by LMUDM.

Let u = xyz (Volume of cuboide)----------------------------------(1)

And x, y, z are connected by **ellipsoid** $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0$.

**i.e** $\emptyset = \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1=0$**----------------------------------------(2)**

**Proceed as in above example**.

## Home work examples

8. **Find the maximum or minimum value of $x^2y^2z^2$ subject to the condition $x^2+y^2+z^2 = 1$**
9. **Find the maximum or minimum value of u $=x^2+y^2+z^2$ subject to the condition x+y+z=3a [ 2018].**
10. **Find the extreme value of u $=x^2+y^2+z^2$ subject to the condition x+3y-2z=4.**
11. **Find the maximum value of u = (x+1)(y+1)(z+1) subject to the condition $a^x b^y c^z = A$.**
12. **Prove that cuboide of max. valome that can be inscribed in a sphere is cube.**

**(ii) Lagrange's Method of undetermined Multipliers when variables x, y, z are connected by two equations.**

**Theorem:** Explain the method of Lagrange's method of undetermined multipliers to find optimum value of f(x, y, z) where variables are connected by the relation $g(x,y,z)$ =0 and h(x, y, z) =0 **[2015, 2017, 2019]**

**Explanation:** Let u = f(x, y, z) ----------(1) be a function of three variables x, y, z.

These variables x, y, z are connected by the equation

$g(x,y,z)$ =0-------------------------------------------(2)

h(x, y, z) = 0-----------------------------------------(3)

For maximum or minimum value of u we have du =0

I.e $\frac{\partial u}{\partial x}$ dx + $\frac{\partial u}{\partial y}$ dy+ $\frac{\partial u}{\partial z}$dz =0 ---------------------(2)

Also from equation (1)

d$g$ = $\frac{\partial g}{\partial x}$ dx + $\frac{\partial g}{\partial y}$ dy+ $\frac{\partial g}{\partial z}$dz =0-------------------(3)

d$h$ = $\frac{\partial h}{\partial x}$ dx + $\frac{\partial h}{\partial y}$ dy+ $\frac{\partial h}{\partial z}$dz =0-------------------(4)

Multiplying equation (2), (3) and (4) by 1 ,$\mu$ and $\lambda$ adding them, where $\lambda, \mu$ are constants , we get

$(\frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z}dz) + \mu (\frac{\partial g}{\partial x} dx + \frac{\partial g}{\partial y} dy + \frac{\partial g}{\partial z}dz) + \lambda( \frac{\partial h}{\partial x} dx + \frac{\partial h}{\partial y} dy + \frac{\partial h}{\partial z}dz ) = 0$

$(\frac{\partial u}{\partial x} + \mu \frac{\partial g}{\partial x} + \lambda \frac{\partial h}{\partial x}) dx + (\frac{\partial u}{\partial y} + \mu \frac{\partial g}{\partial y} + \lambda \frac{\partial h}{\partial y})dy + (\frac{\partial u}{\partial z} + \mu \frac{\partial g}{\partial z} + \lambda \frac{\partial h}{\partial z})dz = 0$--**(5)**

Since $\lambda$ *and* $\mu$ is arbitrary constants, i.e for all values of $\lambda$ *and* $\mu$, (4) is true only when

$$\left.\begin{aligned} \frac{\partial u}{\partial x} + \mu \frac{\partial g}{\partial x} + \lambda \frac{\partial h}{\partial x} = 0 \\ \frac{\partial u}{\partial y} + \mu \frac{\partial g}{\partial y} + \lambda \frac{\partial h}{\partial y} = 0 \\ \frac{\partial u}{\partial z} + \mu \frac{\partial g}{\partial z} + \lambda \frac{\partial h}{\partial z} = 0 \end{aligned}\right\} \text{ - -----------------(5)}$$

Using (2) , (3) and (5), we can determine the values $\mu$, $\lambda$, x, y and z for which u is maximum or minimum.

Hence the explanation.

**Examples:**

1. **Find the extreme values of $x^2+y^2+z^2$ subject to the conditions $ax+by+cz=1$ and $a^l x + b^1 y + c^l z = 1$.**

**Soln.:** Let u = $x^2+y^2+z^2$ -------------------------------(1)

Conditions

$g = ax+by+cz-1 = 0$------------------------------(2)

& $h = a^l x + b^1 y + c^l z - 1 = 0$----------------------(3)

For extreme values we have

$du + k_1 dg + k_2 dh = 0$

i.e $2x\,dx + 2y\,dy + 2z\,dz + k_1(a\,dx + b\,dy + c\,dz) + k_2(a^l dx + b^l dy + c^l dz) = 0$

**i.e** $(2x + k_1 a + k_2 a^l)dx + (2y + k_1 b + k_2 b^l)dy + (2z + k_1 c + k_2 c^l)dz = 0$

=> $2x + k_1 a + k_2 a^l = 0$---------------------------------(4)

$2y + k_1 b + k_2 b^l = 0$ -------------------------------(5)

$2z + k_1 c + k_2 c^l = 0$--------------------------------(6)

**x(4) + y(5) + z(6) gives**

$2(x^2+y^2+z^2) + k_1(ax+by+cz) + k_2(a^l x + b^1 y + c^l z) = 0$

i.e $2u + k_1(1) + k_1(1) = 0$ **from (1), (2), (3)**

**i.e** $2u + k_1 + k_1 = 0$ ----------------------------(7)

**Next,** **a(4) + b(5) + c(6) gives**

$2(ax+by+cz) + k_1(a^2+b^2+c^2) + k_2(aa^l + bb^1 + cc^l) = 0$

i.e $2 + k_1 \sum a^2 + k_2 \sum aa^l = 0$ -----------------(8)

Similarly **$a^l$(4) + $b^1$(5) + $c^1$(6) gives**

$2 + k_1 \sum aa^l + k_2 \sum (a^l)^2 = 0$ --------------(9)

Eliminating $k_1$, $k_2$ from (7), (8) and (9) we get (using determinant)

$$\begin{vmatrix} 2u & 1 & 1 \\ 2 & \sum a^2 & \sum aa^l \\ 2 & \sum aa^l & \sum (a^l)^2 \end{vmatrix} = 0$$

i.e $\begin{vmatrix} u & 1 & 1 \\ 1 & \sum a^2 & \sum aa^l \\ 1 & \sum aa^l & \sum (a^l)^2 \end{vmatrix} = 0$

which gives extreme value of u.

1. **Find the max. or min. values of $x^2+y^2+z^2$ subject to the conditions $ax^2+by^2+cz^2=1$ and $lx + my + nz = 0$.**

**Soln.:** Let $u = x^2+y^2+z^2$ --------------------------------(1)

Conditions

$g = ax^2+by^2+cz^2 - 1 = 0$--------------------------(2)

& $h = lx + my + nz = 0$---------------------------(3)

For max. or min. values we have

$du + k_1 dg + k_2 dh = 0$

i.e $2x\,dx + 2y\,dy + 2z\,dz + k_1(2ax dx + 2by dy + 2cz dz) + k_2(l dx + m dy + n dz) = 0$

i.e $(2x + 2k_1 ax + k_2 l)dx + (2y + 2k_1 by + k_2 m)dy + (2z + 2k_1 cz + k_2 n)dz = 0$

=> $2x + 2k_1 ax + k_2 l = 0$---------------------------------(4)

$2y + 2k_1 by + k_2 m = 0$ --------------------------------(5)

$2z + 2k_1 cz + k_2 n = 0$--------------------------------(6)

x(4) + y(5) + z(6) gives

$2(x^2+y^2+z^2) + 2k_1(ax+by+cz) + k_2(l x + my + n z) = 0$

i.e $2u + 2k_1 (1) + k_1 (0) = 0$ from (1), (2), (3)

i.e $2u + 2k_1 = 0$ ----------------------------(7)

=> $k_1 = -u$

**Substitute $k_1 = -u$ in (4) we get**

**$2x(1+(-u)a) + k_2 l = 0$**

**i.e $2x\,(1-au) = -\,k_2 l$**

**=> $x = -\dfrac{k_2 l}{1-au} = \dfrac{k_2 l}{au-1}$**

**Similarly $y = \dfrac{k_2 m}{bu-1}$ and $z = \dfrac{k_2 n}{cu-1}$**

**Substitute these values in (3) we get**

**$l\left(\dfrac{k_2 l}{au-1}\right) + m\left(\dfrac{k_2 m}{bu-1}\right) + n\left(\dfrac{k_2 n}{cu-1}\right) = 0$**

**⇨ $\left(\dfrac{l^2}{au-1}\right) + \left(\dfrac{m^2}{bu-1}\right) + \left(\dfrac{n^2}{cu-1}\right) = 0$**

**Which gives max. or min. value of u.**

## Excersice on Extreme Values and Lagrange's Undetermined Multipliers

1. Define maxima and minima of function of two variables (2015, 2017, 2019).
2. Find necessary condition for a function to have an extreme values. (2017, 201 9)
3. State sufficient condition for extreme values for functions of two variables. (2018)
4. Show that $f(x,y) = x^3 + y^3 - 3xy + 1$ is minimum at (1, 1). (2018)
   (Same as example solved example (2))
5. Find max. and min. values of $f(x, y) = x^3 + 3x^2y - 3x^2 - 3y^2 + 7$.
6. Find max. and min. values of $f(x, y) = 2x^4 + y^4 - 2x^2 - 2y^2$ .
7. Find max. and min. values of $f(x, y) = x^3 - y^3 + (x^2 + y^2) - 9x$.
8. Find the maximum or minimum value of $x^2y^2z^2$ subject to the condition $x^2+y^2+z^2 = 1$

9. Find the maximum or minimum value of u $=x^2+y^2+z^2$ subject to the condition $x+y+z=3a$ [ 2018].
10. Find the extreme value of u $=x^2+y^2+z^2$ subject to the condition $x+3y-2z=4$.
11. Find the maximum value of $u = (x+1)(y+1)(z+1)$ subject to the condition $a^x b^y c^z = A$.
12. Prove that cuboide of max. valome that can be inscribed in a sphere is cube.

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## Department of Mathematics

## B.Sc. IV Sem.

## Paper I: Vector Calculus and Infinite Series

### Unit I : Vector Calculus I

**EXAMPLES:**

**1. If $\vec{S}(t)$ = (t+1) $\hat{\imath}$ + ($t^2$+t+1) $\hat{\jmath}$ +($t^3$+ $t^2$+t+1) $\hat{k}$ then find $\left|\frac{d\vec{S}}{dt}\right|$ and $\left|\frac{d^2\vec{S}}{dt^2}\right|$.**

**Soln.:** Now $\vec{S}(t)$ = (t+1)$\hat{\imath}$ + ($t^2$+t+1) $\hat{\jmath}$ +($t^3$+ $t^2$+t+1) $\hat{k}$ is vector fn. of t.

$\therefore \frac{d\vec{S}}{dt} = \hat{\imath} + (2t+1)\,\hat{\jmath} + (3t^2+2t+1)\,\hat{k}$

And $\frac{d^2\vec{S}}{dt^2} = 2\,\hat{\jmath} + (6t+2)\,\hat{k}$

$\therefore \left|\frac{d\vec{S}}{dt}\right| = \sqrt{1+(2t+1)^2+(3t^2+2t+1)^2}$

$= \sqrt{1+4t^2+4t+1+9t^4+4t^2+1+12t^3+4t+6t^2}$

$= \sqrt{9t^4+12t^3+14t^2+8t+3}$

And $\left|\frac{d^2\vec{S}}{dt^2}\right| = \sqrt{4+(6t+2)^2} = \sqrt{4+4(3t+1)^2} = 2\sqrt{1+(3t+1)^2}$

**2. If $\vec{r}$ (t)= $t^2\hat{\imath}$ + sint $\hat{\jmath}$ +( 1-t) $\hat{k}$ then find $\left|\frac{d\vec{r}}{dt}\right|$ and $\left|\frac{d^2\vec{r}}{dt^2}\right|$.**

**Soln.** Now $\vec{r}$(t) = $t^2\hat{\imath}$ + sint $\hat{\jmath}$ +( 1-t) $\hat{k}$

$\therefore \frac{d\vec{r}}{dt} = 2t\,\hat{\imath} + \cos t\,\hat{\jmath} - \hat{k} \Rightarrow \left|\frac{d\vec{r}}{dt}\right| = \sqrt{4t^2+\cos^2 t+1}$

And $\frac{d^2\vec{r}}{dt^2} = 2\,\hat{\imath} - \sin t\,\hat{\jmath} \Rightarrow \left|\frac{d^2\vec{r}}{dt^2}\right| = \sqrt{4+\sin^2 t}$

3. **If $\vec{r}$ (t)= $e^{nt}\vec{a}$ + $e^{-nt}\vec{b}$ where $\vec{a}$ $and$ $\vec{b}$ are constant vectors then prove that $\frac{d^2\vec{r}}{dt^2}$ - $n^2$ $\vec{r}$ = 0.**

**Proof:** Now $\vec{r} = e^{nt}\vec{a} + e^{-nt}\vec{b}$

$\therefore \frac{d\vec{r}}{dt} = n\, e^{nt}\vec{a} - n\, e^{-nt}\vec{b} = n\,(e^{nt}\vec{a} - e^{-nt}\vec{b})$

And $\frac{d^2\vec{r}}{dt^2} = n\,(n\, e^{nt}\vec{a} + n\, e^{-nt}\vec{b})$

$= n^2(e^{nt}\vec{a} + e^{-nt}\vec{b})$

$= n^2\vec{r}$

⇨ $\frac{d^2\vec{r}}{dt^2} - n^2\,\vec{r} = 0.$

4. **If $\vec{r}$ (t)= (1-cost)$\hat{\imath}$ + (t-sint) $\hat{\jmath}$ then find $\vec{r} \cdot \frac{d\vec{r}}{dt}$.**

**Soln.:** Now $\vec{r}$ (t)= (1-cost)$\hat{\imath}$ + (t-sint) $\hat{\jmath}$

$\therefore \frac{d\vec{r}}{dt}$ = sint $\hat{\imath}$ + (1-cost) $\hat{\jmath}$

Therefore $\vec{r} \cdot \frac{d\vec{r}}{dt}$ = (1-cost) sint + (t-sint) (1-cost)

= (1-cost)[sint + t –sint]

$\vec{r} \cdot \frac{d\vec{r}}{dt}$ = t (1 –cost).

5. **If $\vec{r}$ (t)= cosnt $\hat{\imath}$ + sin nt $\hat{\jmath}$ , n is constant then prove that $\vec{r} \times \frac{d\vec{r}}{dt} = n\,\hat{k}$.**

**Soln.:** Now $\vec{r}$ (t)= cosnt $\hat{\imath}$ + sin nt $\hat{\jmath}$

Then $\frac{d\vec{r}}{dt}$ = - nsinnt $\hat{\imath}$ + ncosnt $\hat{\jmath}$

We have

$$\vec{r} \times \frac{d\vec{r}}{dt} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ cosnt & sinnt & 0 \\ -nsinnt & ncosnt & 0 \end{vmatrix}$$

$= \hat{\imath}(0) - \hat{\jmath}(0) + \hat{k}$ (n $\cos^2$nt+ n $\sin^2$nt)

$= \hat{k}$ n = n$\hat{k}$

**Hence** $\vec{r} \times \frac{d\vec{r}}{dt} = n\hat{k}$.

6. **If $\vec{s}$ (t)= cost $\hat{\imath}$ + sint $\hat{\jmath}$ +t tan$\alpha\hat{k}$ then find $\frac{d\vec{S}}{dt} \times \frac{d^2\vec{r}}{dt^2}$.**

**Soln.:** Now $\vec{s}$ (t)= cost $\hat{\imath}$ + sint $\hat{\jmath}$ +t tan$\alpha\hat{k}$

Then $\frac{d\vec{S}}{dt}$ = -sint $\hat{\imath}$ + cost $\hat{\jmath}$ + tan$\alpha\hat{k}$

And $\frac{d^2\vec{S}}{dt^2}$ = -cost $\hat{\imath}$ - sint $\hat{\jmath}$ + 0$\hat{k}$

= -cost $\hat{\imath}$ - sint $\hat{\jmath}$

$$\frac{d\vec{S}}{dt} \times \frac{d^2\vec{S}}{dt^2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -sint & cost & tan\alpha \\ -cost & sint & 0 \end{vmatrix}$$

= $\hat{\imath}$ (0 + $tan\alpha$ sint) - $\hat{\jmath}$(0 + $tan\alpha$ cost)+ $\hat{k}$(- $\sin^2$t + $\cos^2$t)

= $tan\alpha$($\hat{\imath}$ sint - $\hat{\jmath}$ cost)+ $\hat{k}$ cos2t.

**Thus** $\frac{d\vec{S}}{dt} \times \frac{d^2\vec{S}}{dt^2} = tan\alpha(\hat{\imath}$ **sint -** $\hat{\jmath}$ **cost)+** $\hat{k}$ **cos2t.**

7. **If $\vec{r}$ (t)= $cos\omega t\, \vec{a} + sin\omega t\, \vec{b}$, where $\vec{a}$ and $\vec{b}$ are constant vectors and $\omega$ is scalar then prove that** $\frac{d^2\vec{r}}{dt^2} + \omega^2\vec{r}$= 0 and

$\vec{r} \times \frac{d\vec{r}}{dt} = \omega(\vec{a} \times \vec{b})$.

**Soln.**: Now $\vec{r}$ (t)= $cos wt\ \vec{a} + sin wt\ \vec{b}$ , where $\vec{a}$ and $\vec{b}$ are constant vectors and $w$ is scalar.

Then $\frac{d\vec{r}}{dt} = -w\ sin wt\ \vec{a} + w\ cos wt\ \vec{b}$

(i) $\frac{d^2\vec{r}}{dt^2} = -w^2\ cos wt\ \vec{a} - w^2 sin wt\ \vec{b}$

$= -w^2\ (cos wt\ \vec{a} + sin wt\ \vec{b})$

$\frac{d^2\vec{r}}{dt^2} = -w^2 \vec{r}\ (t)$

$\therefore \frac{d^2\vec{r}}{dt^2} + w^2 \vec{r}\ (t) = 0$

(ii) **Next,** $\vec{r} \times \frac{d\vec{r}}{dt} = (cos wt\ \vec{a} + sin wt\ \vec{b}) \times (-w\ sin wt\ \vec{a} + w\ cos wt\ \vec{b})$

$= -w\ sin wt\ cos wt\ (\vec{a} \times \vec{a}) + w\ cos w^2 t\ (\vec{a} \times \vec{b})$

$-w\ sin w^2 t\ (\vec{b} \times \vec{a}) + w\ sin wt\ cos wt\ (\vec{b} \times \vec{b})$

$= 0 + w\ [cos w^2 t\ (\vec{a} \times \vec{b}) + sin w^2 t\ (\vec{a} \times \vec{b})] + 0$

$= w\ (\vec{a} \times \vec{b})$

Thus $\vec{r} \times \frac{d\vec{r}}{dt} = w\ (\vec{a} \times \vec{b})$

**8. If $\vec{A}(t) = t^2\hat{i} - t\,\hat{j} + 2t\,\hat{k}$ and $\vec{B}(t) = (2t-3)\hat{i} + \hat{j} + t\,\hat{k}$ then find (i) $\frac{d}{dt}(\vec{A} + \vec{B})$ (ii) $\frac{d}{dt}(\vec{A} \times \frac{d\vec{A}}{dt})$.**

Soln.: Now $\vec{A}(t) = t^2\hat{i} - t\,\hat{j} + 2t\,\hat{k}$ and $\vec{B}(t) = (2t-3)\hat{i} + \hat{j} + t\,\hat{k}$

Then (i) $\frac{d}{dt}(\vec{A} + \vec{B}) = \frac{d}{dt}[(t^2 + 2t - 3)\hat{i} + (-t+1)\,\hat{j} + 3t\,\hat{k}]$

$= (2t+2)\hat{i} - \hat{j} + 3\,\hat{k}$

(ii) $\frac{d\vec{A}}{dt} = 2t\,\hat{i} - \hat{j} + 2\,\hat{k}$

$$\vec{A} \times \frac{d\vec{A}}{dt} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ t^2 & -t & 2t \\ 2t & -1 & 2 \end{vmatrix}$$

$= \hat{\imath}(-2t+2t) - \hat{\jmath}(2t^2 - 4t^2) + \hat{k}(-t^2 + 2t^2)$

$= 2t^2\hat{\jmath} + t^2\hat{k}$

$= t^2(2\hat{\jmath} + \hat{k})$

**9. If $\vec{r}$ (t)= a cost $\hat{\imath}$ + a sint $\hat{\jmath}$ +t $\hat{k}$ then find $[\vec{r}\ \frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}]$.**

**Soln.:** Now $\vec{r}$ (t)= a cost $\hat{\imath}$ + a sint $\hat{\jmath}$ +t $\hat{k}$

Then $\frac{d\vec{r}}{dt}$ = - a sint $\hat{\imath}$ + a cost $\hat{\jmath}$ + $\hat{k}$

And $\frac{d^2\vec{r}}{dt^2}$ = - a cost $\hat{\imath}$ - a sint $\hat{\jmath}$

We have $[\vec{r}\ \frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}] = \vec{r}\cdot \frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}$

$$= \begin{vmatrix} a\cos t & a\sin t & t \\ -a\sin t & a\cos t & 1 \\ -a\cos t & -a\sin t & 0 \end{vmatrix}$$

$= a^2[\cos t(0+\sin t) - \sin t(0+\cos t) + t(\sin^2 t+\cos^2 t)]$

Thus $[\vec{r}\ \frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}] = a^2 t$

**10.If $\frac{d\vec{u}}{dt} = \vec{w}\times\vec{u}$ and $\frac{d\vec{v}}{dt} = \vec{w}\times\vec{v}$ then show that $\frac{d}{dt}(\vec{u}\times\vec{v}) = \vec{w} \times \overrightarrow{(u \times v)}$.**

**Proof**: Now $\frac{d\vec{u}}{dt} = \vec{w}\times\vec{u}$ and $\frac{d\vec{v}}{dt} = \vec{w}\times\vec{v}$

$\therefore \frac{d}{dt}(\vec{u}\times\vec{v}) = \vec{u}\times\frac{d\vec{v}}{dt} + \frac{d\vec{u}}{dt}\times\vec{v} = \vec{u}\times(\vec{w}\times\vec{v}) + (\vec{w}\times\vec{u})\times\vec{v}$

i.e $\frac{d}{dt}(\vec{u}\times\vec{v}) = 0 + (\vec{w}\times\vec{u})\times\vec{v} = \vec{w}\times\overrightarrow{(u\times v)}$.

11. **If $\vec{r}$ (t)= $sinh\mu t\,\vec{a} +\ cosh\mu t\,\vec{b}$, where $\vec{a}$ and $\vec{b}$ are constant vectors and $\mu$ is scalar then prove that** $\frac{d^2\vec{r}}{dt^2}$ - $\mu^2\vec{r}$= 0.

Soln.: ***Soln.***: Now $\vec{r}$ (t)= $sinh\mu t\ \vec{a}$ + $cosh\mu t\ \vec{b}$ , where $\vec{a}$ and $\vec{b}$ are constant vectors and $\mu$ is scalar.

Then $\frac{d\vec{r}}{dt}$ =$\mu cosh\mu t\ \vec{a}$ +$\mu\ sinh\mu t\ \ \vec{b}$

$\frac{d^2\vec{r}}{dt^2}$ = $\mu^2\ sinh\mu t\,\overrightarrow{a}$ + $\mu^2 cosh\mu\ \vec{b}$

= $\mu^2\ (sinh\mu t\ \vec{a} +\ cosh\mu t\ \vec{b})$

$\frac{d^2\vec{r}}{dt^2}$ = $\mu^2\vec{r}$

$\therefore \frac{d^2\vec{r}}{dt^2}$ -$\mu^2\ \vec{r}$ = 0.

**Home work Examples:**

1. If $\vec{a}$ (t)= 5t$^2$ $\hat{\imath}$ + t $\hat{\jmath}$ +t$^3$ $\hat{k}$ and $\vec{b}$ (t)= sint $\hat{\imath}$ - cost $\hat{\jmath}$ then find (i) $\frac{d}{dt}(\vec{a}.\vec{b})$ and (ii) $\frac{d}{dt}(\vec{a}$ x $\vec{b})$ .
2. If $\vec{A}(t)$ = t$^2\hat{\imath}$ –t $\hat{\jmath}$ + (2t+1) $\hat{k}$ and $\vec{B}(t)$ = (2t -3)$\hat{\imath}$ + $\hat{\jmath}$ + t $\hat{k}$ then find (i) $\frac{d}{dt}(\vec{A}\ .\ \overrightarrow{B})$ (ii) $\frac{d}{dt}(\vec{A}$ x $\frac{d\vec{B}}{dt})$ at t =1.
3. If Ø(t) = 3t$^2$- 5t+1 and $\vec{f}(t)$= (2t -3)$\hat{\imath}$ + $t\ \hat{\jmath}$ + 2t $\hat{k}$ then find

   $\frac{d}{dt}$(Ø(t) $\vec{f}(t)$).
4. If $\vec{a}$ (t)= t$^2$ $\hat{\imath}$ + 2t $\hat{\jmath}$ + $\hat{k}$ and $\vec{b}$ (t)= $\hat{\imath}$ – t $\hat{\jmath}$ +$\hat{k}$ and $\vec{c}$ (t)= 2t$\hat{\imath}$ – t$\hat{\jmath}$ + 2$\hat{k}$ then find(i) $\frac{d}{dt}(\vec{a}.\overrightarrow{(b}$x $\vec{c}$ )) and (ii) $\frac{d}{dt}(\vec{a}$ x $(\vec{b}$x $\vec{c}))$ .

## Department of Mathematics

## B.Sc. VI Sem., Paper II: Complex Analysis and Ring Theory

### Introduction:

**Complex analysis**, traditionally known as the theory of functions of a complex variable, is the branch of mathematical analysis that investigates functions of complex numbers. This course provides an introduction to complex analysis which is the theory of complex functions of a complex variable. We will start by introducing the complex plane, along with the algebra and geometry of complex numbers, and then we will make our way via differentiation, integration, power series representation, Taylor's and Laurent series.

In previous classes we studied analysis w.r.t real variable x, in this paper we are studying the analysis i.e limits, continuity, differentiability , integration w.r.t complex variable z.

### Applications of Complex Analysis:

It is useful in many branches of mathematics, including algebraic geometry, number theory, analytic combinatorics, applied mathematics; as well as in Physics, including the branches of hydrodynamics, thermodynamics, and particularly quantum mechanics. By extension, use of complex analysis also has applications in engineering fields such as nuclear, aerospace, mechanical and electrical engineering specially in signal processing.

### Bridge Course:

Basics to study Complex analysis:

### Complex number:

As we know the equation $x^2$- 4 =0 has solution as x= 2 or -2 i.e in R but the equation $x^2$+4 =0 is not having solution in R we come across new number i.e x= $\sqrt{-4}$ = $2\sqrt{-1}$ = 2i which is called imaginary or complex number.

Thus any number in the form of x+iy where x and y are real is called complex number and is denoted by z. And set of complex numbers is denoted by C.

**Modulus of a complex no**.: If z= x+iy then $\sqrt{x^2+y^2}$ is called modulus of a complex number and is denoted by lzl.

$\therefore$ $|z| = \sqrt{x^2 + y^2}$

And $\bar{z}$ = x-iy is called conjugate of the complex no. z so that $z\bar{z} = (x+iy)(x-iy) = |z|^2$

Geometrically every point (x,y) in the plane can be represented by complex number x+iy.

i.e (x, y) = x+iy

## Algebra of Complex numbers:

**Addition, multiplication and division of z:**

**Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$ then we have**

$$z_1 \pm z_2 = (x_1+iy_1) \pm (x_2+iy_2) = (x_1 \pm x_2) + i(y_1 \pm y_2) .$$

$$z_1 z_2 = (x_1x_2 - y_1y_2) + i\,(x_1y_2 + y_1x_2)$$

$$\frac{z_1}{z_2} = \frac{x_1+iy_1}{x_2+iy_2} = \frac{x_1+iy_1}{x_2+iy_2}\,\frac{x_2-iy_2}{x_2-iy_2} = \frac{(x_1x_2+y_1y_2)+i(y_1x_2-x_1y_2)}{x_2^2+y_2^2}$$

$$= \frac{(x_1x_2+y_1y_2)}{x_2^2+y_2^2} + i\frac{(y_1x_2-x_1y_2)}{x_2^2+y_2^2}$$

$$= x+iy$$

**i.e addition (subtraction ) 0r multiplication or division of two complex nos. is again complex no.**

**Also if z= x = (x, 0) then it is called purely real and if z = iy = (0,y) then it is called purely imaginary.**

**0 = 0+i0 and 1= 1+i0 and hence every real no. is complex.**

**Example:** 1.If $z_1 = x_1+iy_1$ and $z_2 = x_2+iy_2$ then find $|z_1 - z_2|$

Soln.: Now $z_1 = x_1+iy_1$ and $z_2 = x_2+iy_2$ then $z_1 - z_2 = (x_1+iy_1) - (x_2+iy_2)$

$$= (x_1 - x_2) + i\,(y_1 - y_2)$$

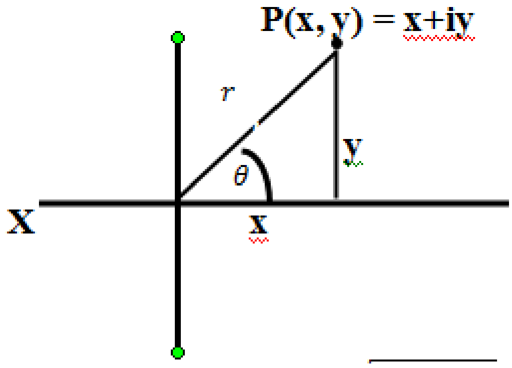
$$\therefore |z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

2. Modulus of $2x + 3y - 4i = |2x + 3y - 4i| = \sqrt{(2x + 3y)^2 + (-4)^2}$

$= \sqrt{4x^2 + 9y^2 + 12\,xy + 16}$

**Geometrical Representation of complex no.:**

**Let P(x,y) be any pt. in the plane, draw PM perpendicular to x-axis.**




**Then $x = r\cos\theta$, $y = r\sin\theta$**

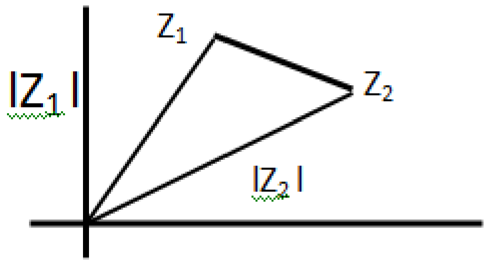
**$\therefore z = x+iy = r(\cos\theta + i\sin\theta)$ where**

**$r = \sqrt{x^2+y^2}$ and $\theta = \tan^{-1}(\frac{y}{x})$**

**Here $\theta$ is called argument or amplitude of z.**

**Thus $z = r(\cos\theta + i\sin\theta) = r\,e^{i\theta}$ is called polar form of complex number.**

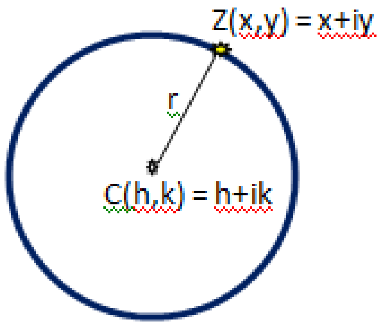
**Distance between points $z_1$ and $z_2$ : If $z_1 = x_1+iy_1$ and $z_2 = x_2+iy_2$ are two points then $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$**




**i.e $|z_1 - z_2| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$, which is distance between to pts $(x_1, y_1)$ and $(x_2, y_2)$.**

**Thus $|z_1 - z_2|$ represent distance between the points $z_1$ and $z_2$**

## EQUATION OF A CIRCLE IN TERMS OF COMPLEX NUMBERS (VIP)




We all know that equation of circle with centre at C(h.K) and radius as r is **$(x-h)^2 + (y-k)^2 = r^2$**

Taking the root on both the sides we get

$\sqrt{(x-h)^2+(y-k)^2}$ = r

$|(x-h)+i(y-k)|$ = r  b'cz $|a+ib| = \sqrt{a^2+b^2}$

i.e $|(x+iy)-(y+ik))|$ = r

i.e $|z-c|$ = r

**Thus equation of a circle with centre at a(h,k) and radius r is $|z-a|$ = r**

**Note: And equation of circle with centre at (0,0) and radius 1 is $|z-(0,0)|$ = 1**

**i.e $|z|$ = 1 which is called unit circle**

**Thus equation of a circle with centre at (0, 0) and radius 1 is $|z|$ = 1**

**Or Equation of unit circle is $|z|$ = 1**

**Examples:**

**1. Equation of circle with centre at a(2,3) and radius 4 is** $|z-a|$ = 4

i.e $|z-(2,3)|$ = 4

or $|z-(2+3i)|$ = 4

or $|x+iy-(2+3i)|$ = 4

2. **Equation of circle with centre at (2,0) and radius 1 is** $|z-2|$ = 1

3. **Equation of circle with centre at (0,1) and radius 5 is** $|z-(0+1i|$ = 5

i.e $|z - i| = 5$

4. **Equation of circle with centre at (-2,1) and radius 5 is** $|z - (-2 + 1i| = 5$

i.e $|z + 2 - i| = 5$

5. **Equation of circle with centre at a(-2,-3) and radius 3/2 is** $|z - (-2 - 3i| = 3/2$

i.e $|z + 2 + 3i| = 3/2$

6. **Equation of circle with centre at a(0,-5) and radius 1 is** -----------------

7. **Equation of circle with centre at 4 and radius 5 is** -----------------

8. **Equation of circle with centre at 3i and radius 3 is** -----------------

Examples on finding centre and radius of a circle if equation is given

Example:- 1) Find the centre and radius of the circle $|z+3|=1$

ie $|z-(-3)|=1$

$\therefore$ Centre $(-3,0)$ & radius 1.

2) Find the centre and radius of the circle $|x-2i+iy|=3$

ie $|(x+iy)-2i| = 3$

ie $|z-2i| = 3$

∴ Centre is at $(0, 2)$ and radius 3.

3) Centre and radius of the circle $|z| = 3$ are

$(0, 0)$ and 3 resply.

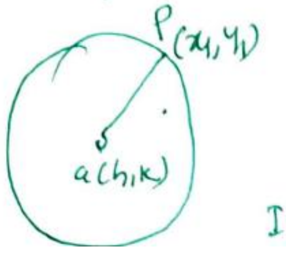
5. Centre and radius of the circle

$|z+3-2i| = 5$ are

$(-3, 2)$ and 5 resply.

Finding whether given point lies inside or out side the circle $|z-a| = r$.

To find the given point $(x_1, y_1)$ lies inside or out side or on the boundary we should find distance between centre $a(h, k)$ and given pt $(x_1, y_1)$. ie distance between $(x_1, y_1)$ and centre $(h, k)$ is




$$AP = \sqrt{(x_1-h)^2 + (y_1-k)^2}$$

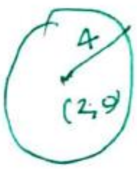
If $AP < r$ then point P lies inside the

If $AP < r$ then point P lies inside the circle

If $AP > r$ then " " outside the circle

If $AP = r$ then " " is on the circle or on the boundary of the circle

Examples:- Find the position of the pt (1,3) w.r.t the circle $|z-2| = 4$.




Distance between centre (2,0) & (1,3)

$$= \sqrt{(2-1)^2 + (0-3)^2}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10} < \sqrt{16} = 4$$

ie distance is $< 4$

$\therefore$ Point lies inside the circle.

2. Find the position of the point $Z = 3$ w.r.t circle $|z - 5| = 3/2$

Distance between the pt $(3,0)$ and centre $(5,0)$ is

$$= \sqrt{4+0} = 2$$
$$> 3/2$$

∴ pt lies outside the circle.

3. Find the position of the point $z = \frac{\pi}{2}i$ w.r.t the circle $|z + 2i| = 3$.

Centre $(0,-2)$ and point is $(0, \frac{\pi}{2})$

Distance between them is $\sqrt{0 + (-2 - \frac{\pi}{2})^2}$

$$= \sqrt{\left(\frac{-4-\pi}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{\left(4 + \frac{22}{7}\right)^2}$$

$$= \frac{1}{2 \times 7}\sqrt{(28+22)^2}$$

$$= \frac{50}{14} = 3.5 > 3$$

∴ pt $(0, \frac{\pi}{2})$ lies outside the circle.

4. Find the position of the point $z = (3 - i)$ w.r.t the circle $|z| = 2$.

Distance between the pt $(3, -1)$ and centre $(0,0)$ is $\sqrt{3^2 + 1^2} = \sqrt{10} > 2$

$\Rightarrow$ point $(3, -1)$ lies outside the circle.

5. Find the position of the pt $z = 3i$ $(0,3)$ w.r.t the circle $|z + 2 - i| = 5$. $(-2, 1)$

Distance $= \sqrt{4 + 4}$

$= \sqrt{8} < \sqrt{25} = 5$

$\therefore$ pt $3i$ lies inside the circle.

Eg. Find whether the point (1, 3) lies on the circle or inside or outside the circle

(i) $|z-1-3i| = 2$

(ii) $|z| = \sqrt{10}$.

Soln :- (i) $|z-1-3i| = 2$ Centre (1, 3)


Clearly given pt (1,3) itself is a centre and hence it lies inside the circle.

(ii) $|z| = \sqrt{10}$

Distance betn (1,3) and centre (0,0) is

$$\sqrt{1^2+3^2}$$

$$= \sqrt{10} = \text{radius } \sqrt{10}$$

$\therefore$ pt. lies on the circle

Eg. Find the position of the point $(0,\pi)$ w.r.t the circle $|z-\sqrt{3}+i|=4$.

Now centre is $(\sqrt{3},-1)$

$\therefore$ distance betn $(0,\pi)$ & $(\sqrt{3},-1)$

$$= \sqrt{(-\sqrt{3})^2+(\pi+1)^2}$$

$$= \sqrt{3+\left(\frac{22}{7}+1\right)^2} = \sqrt{3+\left(\frac{29}{7}\right)^2}$$

$$= \sqrt{\frac{52+801}{49}}$$

$$= \frac{1}{7}\sqrt{853} \quad ?$$

H.W. Find the position of

(i) $(1,5)$ w.r.t circle $|z+3|=4$

(ii) $(0,7)$ " " $|z|=1$

(iii) $z=-i$ " " $|z|=1$

(iv) $z=3i$ " " $|z-2i+3|=5$.

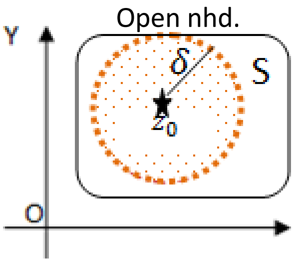
## Unit I: Analytic Function

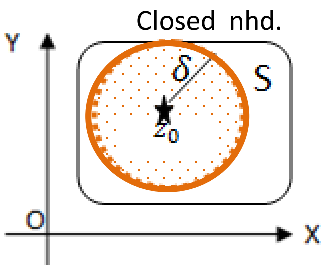
### Definitions:

**(i) Neighbourhood(nhd.) of a point $z_0$:**

Let S be a subset of complex Plane C and $z_0$ be a point in S then collection of all points z in S which are very close to $z_0$ i.e $|z - z_0| < \delta$ , is called nhd. of a point $z_0$ for a smallest positive real no. $\delta$. And is also called $\delta$ nhd. of $z_0$ or open disc or open sphere, denoted by $N_\delta(z_0)$.

Thus $N_\delta(z_0) = \{ z \in C / |z - z_0| < \delta\}$

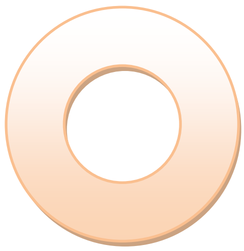



And collection of all the elements z whose distance from $z_0$ is less than or equal to $\delta$ is called closed nhd. denoted by $\overline{N_\delta(z_0)} = \{ z \in C / |z - z_0| \leq \delta\}$

**Deleted nhd.:** Deleted nhd. of $z_0$ is $N_\delta(z_0)$ - $z_0$, i.e collection of all points except $z_0$ is called deleted nhd. of $z_0$.

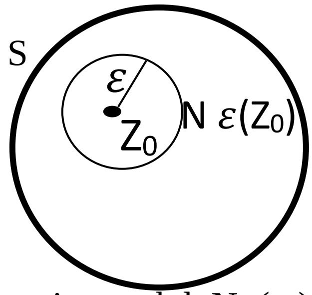
**For example**: Coins, plane paper, etc. are nhd. where as CD, discus throw disc tennicoit ring, punctured ball are examples of deleted nhds.



**For example**: If Niapni is the centre place i.e like (a, b) and $\delta$ be distance 5 kms. Then set of all villages which are within 5 kms is called nhd. of Nipani i.e they are neighbours of Nipani. i.e Lakanapur, Yamagarni, Chikli, Stavanidhi etc. are in nhd. of Nipani where as Galataga is not.

## Interior , exterior and boundary points:

A point $z_0$ is said to be an interior point of a set S if the neighbourhood of $z_0$ is within S or completely contained in S




**OR**

Let $S \subset \mathbb{C}$ and a point $z_0 \in S$ is called **interior point** of S if there exists a nhd. $N\varepsilon(z_0)$ such that $N\varepsilon(z_0) \subset S$(as shown in fig.) And set of interior points of S is called interior of S.

A point $z_0 \in S$ is called an **exterior point** of S if any nhd. of $z_0$ contains no points of S.

OR

**Let $S \subset \mathbb{C}$ and a point $z_0 \in S$ is called an exterior point of S if there exists a nhd. $N\varepsilon(z_0)$ such that $N\varepsilon(z_0) \subset S^l$**

Points of S are called **boundary points** if they are neither interior nor exterior.

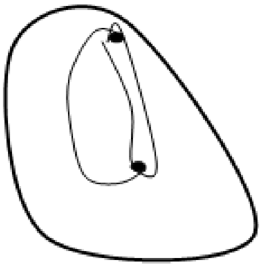
- **Open and closed subsets of complex plane:**

**Definition:** A set S of complex plane is said to be **open set** if it contains all its interior points and set of all interior and boundary points is called **closed set**.

**For example**: A set S ={ z / $|z| < 1$ } is an open set where as $\bar{S}$ ={ z / $|z| \leq 1$ } is closed set.

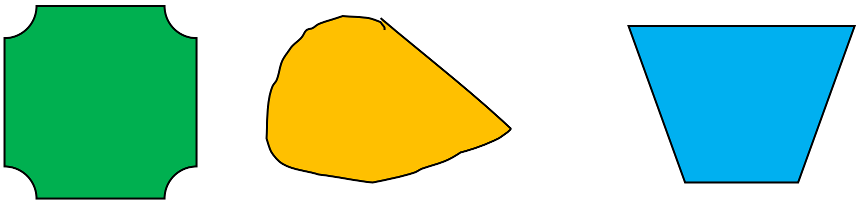
## Connected Set:

Definition: An open set is said to be connected if any two of its points can be joined by finite number of line segments then all of them lies in that set only.



### Domain or region:

**Definition:** A set S in the complex plane which is open and connected is called region or domain in the complex plane i.e. part of complex plane.



**Continuous arc:** An arc z(t) = x(t) +iy(t) for real no. t∈(a, b) is called continuous arc if x(t) and y(t) are continuous for all t.

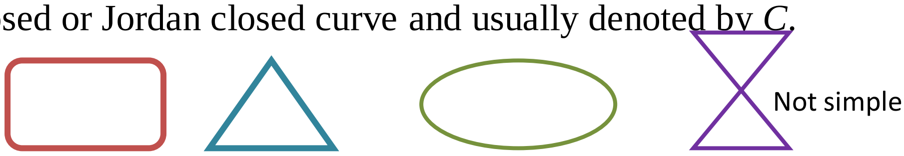
**Simple or Jordan curve :** A curve without multiple points ( point at which curve crosses itself once more than once) is called simple curve**.**

**For example:** Parabola, ellipse, circle etc. are examples for simple curves

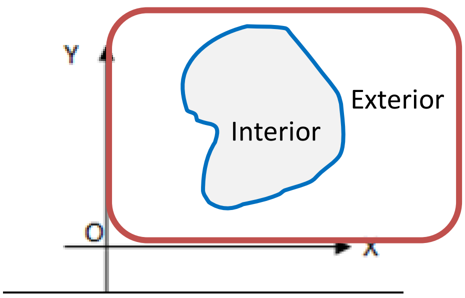
**Simple closed curve or Jordan closed curve:**

If the initial and terminating points of simple curve are coinciding then that curve is called simple closed or Jordan closed curve and usually denoted by *C*.

**For example**:




**Jordan Curve theorem**: A simple closed curve divides whole complex plane into two parts one is interior and another is exterior.



## FUNCTION OF COMPLEX VARIABLE Z:

Let S be a set of complex numbers then to each complex number Z of S there corresponds a unique complex number w then w is called function of complex variable z and is denoted by w = f(z).

i.e $f : S \rightarrow Z$ is defined f(z) = w

i.e w = f(z) is called a function of complex variable z.

For example: **1. $w = z^2$ 2. w = sinz 3. $w = e^z$**

The function w = f(z) is said to be **single valued** if for given value of z there corresponds only one value of w otherwise it is called multi valued function.

For example: **$w = z^2$, $e^z$, cosz, ----are single valued functions where as $\sqrt{z}$. , $z^{1/3}$ are multi valued functions.**

## Complex function in the form of real and imaginary parts

**Let w = f(z) = f(x+iy)**

**= u(x,y) + iv(x,y)--- Cartesian form**

**where u(x,y) and v(x,y) are real valued functions of x and y.**

For example: **1. Let $w = f(z) = z^2 = (x+iy)^2 = (x^2-y^2) + i2xy$**

**= u(x,y) +iv(x,y)**

**2. $w = e^z = e^{(x+iy)} = e^x e^{iy} = e^x(\cos y + i\sin y)$**

**$= e^x\cos y + i\, e^x\sin y$**

**= u(x,y) +iv(x,y)**

**3. f(z) =sinz = sin(x+iy) = sinx cosiy + cosx siniy**

**= sinx coshy + cosx (isinhy)**

**= sinx coshy + i cosx sinhy**

**= u(x,y) +iv(x,y)**

**4. f(z) = cosz (HW)**

**5. f(z) = logz = log(x+iy) = log(r $e^{i\theta}$) where r = $\sqrt{x^2+y^2}$ and θ = $\tan^{-1}(\frac{y}{x})$**

**= logr + log $e^{i\theta}$ = logr + iθ log e**

**= log $\sqrt{x^2+y^2}$ + iθ**

**logz = $\frac{1}{2}\log(x^2+y^2)$ + i $\tan^{-1}(\frac{y}{x})$**

**= u(x,y) +iv(x,y)**

**6. Find real and imaginary parts of $ze^z$**

**Definition of limit of f(z):**

A function w = f(z) is said to tends to a limit l as $z \to z_0$ if for every $\epsilon>0$ however small there exists a positive real no. $\delta$ such that $|f(z) - l| < \epsilon$ for

$|z-z_0| < \delta$ .

And is written as $\lim_{z\to z_0} f(z) = l$

Here whatever the path we choose from z to $z_0$ but limit should be unique.

* **Definition of continuity of f(z):**

A function w=f(z) is said to be continuous at z = $z_0$

if for every $\varepsilon>0$ however small there exists a positive real no. $\delta$ such that $|f(z) - f(z_0)| < \varepsilon$ for $|z-z_0| < \delta$ .

And is written as $\lim_{z\to z_0} f(z) = f(z_0)$

* **Definition of differentiability of f(z):**

A function w=f(z) is said to be differentiable at z = $z_0$ if $\lim_{z\to z_0}\frac{f(z)-f(z_0)}{z-z_0}$ exists and unique and is denoted by $f^{l}(z_0)$ or$\{\frac{d}{dz}(f(z)\}_{z=z_0}$

If $z-z_0=\delta z$ then we can also write

$$f^{l}(z) = \lim_{z\to z_0} \frac{f(z+\delta z)-f(z)}{\delta z}$$

i.e for given $\varepsilon>0$ however small there exists a positive real no. $\delta$ such that $\left|\frac{f(z)-f(z_0)}{z-z_0} - f'(z_0\right| < \varepsilon$

along the path whatever we choose.

**Note: A function w =f(z) is continuous or differentiable in the domain or region D if f(z) is continuous or differentiable at each point of the region D.**

**Example:** Prove that differentiability of f(z) gives continuity.

**Proof:** Let f(z) be differentiable at $z_0$ then by the definition of differentiability

$$f^{l}(z_0) = \lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$$ exists and unique.

We shall write $f(z) - f(z_0) = \frac{f(z)-f(z_0)}{z-z_0}(z-z_0)$

$$\therefore \lim_{z\to z_0} f(z) - f(z_0) = \lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}(z-z_0)$$

$$= f^{l}(z_0)\, 0 = 0$$

i.e $\lim_{z\to z_0} f(z) = f(z_0)$

=> f(z) is continuous.

**Note: Converse is not true.**

i.e continuous function is not necessarily differentiable.

**For example**: $f(z) = \bar{z} = x - iy$ is continuous but not differentiable

It is continuous at every point z of C but not differentiable

Bcz, $$\lim_{\delta z\to 0} \frac{\delta w}{\delta z} = \lim_{\delta z\to 0} \frac{f(z+\delta z)-f(z)}{\delta z}$$

$$= \lim_{\delta z\to 0} \frac{\overline{z+\delta z}-\bar{z}}{\delta z}$$

$$= \lim_{\delta z\to 0} \frac{[(x+\delta x)-i(y+\delta y)]-(x-iy)}{\delta x+i\delta y}$$

$$= \lim_{\delta x \to 0} \frac{[(\delta x) - i(\delta y)]}{\delta x + i\delta y}$$

Let $\delta z \to 0$ along real axis then $\delta z = \delta x$ and $\delta y$=0

$\therefore \lim_{\delta z \to 0} \frac{\delta w}{\delta z} = = \lim_{\delta x \to 0} \frac{(\delta x)}{\delta x}$ = 1 ---------(1)

Let $\delta z \to 0$ along y- axis then $\delta z = i\delta y$ and $\delta x$=0

And $\therefore \lim_{\delta z \to 0} \frac{\delta w}{\delta z} = = \lim_{\delta y \to 0} \frac{(-i\delta y)}{i\delta y}$ = -1 -------(2)

From (1) and (2) f(z) = $\overline{z}$ is not differentiable.

**Analytic function:**

**Defn.:** A function f(z) is said to be analytic at $z_0$ if it is single valued and differentiable in the nhd. of $z_0$.

Analytic function is also called **regular** or **holomarphic** function.

The points in the domain at which the function f(z) is not analytic are called **singular points or singularities**.

**For example:**

1. Every polynomial function f(z)=$a_0z^n$+ $a_1z^{n-1}$+

$a_2z^{n-2}$+----- $a_n$, exponential $e^z$ , sinz are analytic for all z in the entire plane such functions are called entire functions.

2. If f(z) = $\frac{1}{z-1}$ then f(z) is not analytic at z=1 and hence z=1 is singularity of f(z).

**Cauchy's Reimann equations:** Let f(z) = u(x,y) +iv(x,y) be defined and continuous and differentiable in some nhd. of point z then at that point the first order partial derivatives exist and satisfy two conditions $u_x = v_y$ and $u_y = -v_x$ which are called Cauchy's Reimann (or shortly C-R) equations.

**Note: (i)** If the function f(z) = u + iv is differentiable in the region D means it is differentiable at all the points z of the region D.

i.e $f^l(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$ exists along whatever the path we choose.

**(ii)** Let f(z) = u+iv

i.e f(x+iy)= u(x,y) +iv(x,y), *then* $\boldsymbol{f}(\boldsymbol{z} + \Delta \boldsymbol{z})$= u(x+$\Delta\boldsymbol{x}$,y+$\Delta\boldsymbol{y}$) +iv(x+$\Delta\boldsymbol{x}$,y+$\Delta\boldsymbol{y}$),

(iii) $\lim_{\Delta x \to 0} \frac{u(x+\Delta x,y) - u(x,\ y)}{\Delta x} = \frac{\partial u}{\partial x} = u_x$ , by definition of partial differentiation.

**Necessary and sufficient conditions for f(z) to be analytic:**

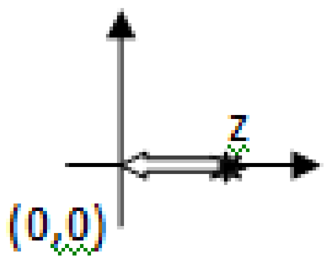
**Necessary conditions:** If the function w = f(z) = u+iv be analytic in the given region D then Cauchy's Reimann equations are satisfied i.e $u_x = v_y$ and $u_y = -v_x$ **(VIP)**

**Proof:** Let the function w = f(z) = u+iv be analytic in the given region D, then it is differentiable at all the points z of region D.

i.e $\frac{dw}{dz} = f^{l}(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$ exists along whatever the path we choose.

Where $\Delta z = \Delta x + i\Delta y$

(i) We will choose first $\Delta z \to 0$ along horizontal line, that is along x-axis.




Along x axis $\Delta y = 0$. $\therefore \Delta z = \Delta x$

$$\frac{dw}{dz} = f^{l}(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{u(x+\Delta x,\ y+\Delta y) + i\,v(x+\Delta x,\ y+\Delta y) - u(x,y) - iv(x,y)}{\Delta z} \quad \text{becomes}$$

$$= \lim_{\Delta x \to 0} \frac{u(x+\Delta x,\ y) + i\,v(x+\Delta x,\ y) - u(x,y) - iv(x,y)}{\Delta x}$$

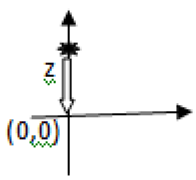
$$= \lim_{\Delta x \to 0} \frac{[u(x+\Delta x,\ y) - u(x,y)] + i\,[v(x+\Delta x,\ y) - v(x,y)]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left\{ \frac{[u(x+\Delta x,\ y) - u(x,y)]}{\Delta x} + i\,\frac{[v(x+\Delta x,\ y) - v(x,y)]}{\Delta x} \right\}$$

$$= \lim_{\Delta x \to 0} \frac{[u(x+\Delta x,\ y) - u(x,y)]}{\Delta x} + i \lim_{\Delta x \to 0} \frac{[v(x+\Delta x,\ y) - v(x,y)]}{\Delta x}$$

$$\frac{dw}{dz} = f^{l}(z) = \frac{\partial u}{\partial x} + i\,\frac{\partial v}{\partial x} \quad \text{-----------------(i)}$$

(ii) We will choose next $\Delta z \to 0$ along vertical line, that is along y-axis.




Along vertical line axis $\Delta x = 0$. $\therefore \Delta z = i\, \Delta y$

$$\frac{dw}{dz} = f^{I}(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$= \lim_{\Delta z \to 0} \frac{u(x+\Delta x,\ y+\Delta y)+i\,v(x+\Delta x,\ y+\Delta y)-u(x,y)-iv(x,y)}{\Delta z} \quad \text{becomes}$$

$$= \lim_{i\Delta y \to 0} \frac{u(x,\ y+\Delta y)+i\,v(x,\ y+\Delta y)-u(x,y)-iv(x,y)}{i\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{[u(x,\ y+\Delta y)-u(x,y)]+i\,[v(x,\ y+\Delta y)-v(x,y)]}{i\Delta y}$$

$$= \lim_{\Delta y \to 0} \left\{\frac{[u(x,\ y+\Delta y)-u(x,y)]}{i\Delta y} + i\,\frac{[v(x,\ y+\Delta y)-v(x,y)]}{i\Delta y}\right\}$$

$$= \frac{1}{i} \lim_{\Delta y \to 0} \frac{[u(x,\ y+\Delta y)-u(x,y)]}{\Delta y} + \lim_{\Delta x \to 0} \frac{[v(x,\ y+\Delta y)-v(x,y)]}{\Delta y}$$

$$\frac{dw}{dz} = f^{I}(z) = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} + i\left(-\frac{\partial u}{\partial y}\right)$$

i.e $\frac{dw}{dz} = f^{I}(z) = \frac{\partial v}{\partial y} + i\left(-\frac{\partial u}{\partial y}\right)$-----------------(ii)

From (i) and (ii), we have $\frac{\partial u}{\partial x} + i\,\frac{\partial v}{\partial x} = = \frac{\partial v}{\partial y} + i\left(-\frac{\partial u}{\partial y}\right)$

Equating real and imaginary parts we get, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = - \frac{\partial u}{\partial y}$

These are necessary conditions i.e C-R equations

**Thus if f(z) is analytic then C-R equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = - \frac{\partial v}{\partial x}$**

**i.e $u_x = v_y$ and $u_y = -v_x$ are satisfied.**

**Sufficient conditions**: ***If all the four partial derivatives $u_x$, $u_y$, $v_x$ and $v_y$ exist and continuous and satisfy the C-R equations then f(z) = u+iv is analytic.***

***i.e if we want to show f(z) is analytic or differentiable we have to show four partial derivatives $u_x$, $u_y$, $v_x$ and $v_y$ exist and continuous and C-R equations*** **$u_x = v_y$ and $u_y = -v_y$ are satisfied.**

***Procedure to show given function f(z)= u+iv is analytic***

(i) Given function be f(z) =u(x,y)+iv(x,y) , write in terms of real and imaginary Parts

(ii) Get ***$u_x$, $u_y$, $v_x$ and $v_y$ , if they are poly. Or trigonometric or rational functions with non zero D^r then they are all continuous.***

**(iii)** ***We have get C-R equations*** **$u_x = v_y$ and $u_y = -v_x$ are satisfied.**
Then we say f(z) is analytic means f(z) is differentiable.

And $f^{I}(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ **or** $f^{I}(z) = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial x}$

**But most of the time we prefer $f^{I}(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$**

**Examples:**

**1. Prove that $f(z) = e^z$ is analytic for all z**

**Soln.:** Now $f(z) = e^z = e^{x+iy} = e^x e^{iy} = e^x(\cos y + i\sin y)$

$= e^x\cos y + i\, e^x\sin y$

$= u(x,y) + iv(x,y)$

Where $u(x,y) = e^x\cos y$ and $v(x,y) = e^x\sin y$

∴ $u_x = e^x\cos y$, $u_y = -e^x\sin y$, $v_x = e^x\sin y$, $v_y = e^x\cos y$

Clearly these are rational functions with non zero denominator and hence they exists and continuous.

Also C-R equations **$u_x = v_y$ and $u_y = -v_x$ are satisfied.**

**Hence $f(z) = e^z$ is analytic and $f^I(z) = e^z$**

**2. Prove that f(z) = sinz is analytic for all z and find $f^I(z)$**

**Soln.:** Now f(z) = sinz = sin(x+iy) = sinx coshy + icosx sinhy

$= u(x,y) + iv(x,y)$

Where u(x,y) = sinx coshy and v(x,y) = cosx sinhy

∴ $u_x$ = cosx coshy, $u_y$ = sinx sinhy, $v_x$ = - sinx sinhy, $v_y$ = cosx coshy

Clearly these are rational functions with non zero denominator and hence they exists and continuous.

Also C-R equations **$u_x = v_y$ and $u_y = -v_x$ are satisfied.**

**Hence f(z) = sinz is analytic and $f^I(z)$ = cosz**

**3. Prove that f(z) = cosz is analytic for all z and find $f^I(z)$ (HW)**

**4. Prove that f(z) = logz is analytic for all z and find $f^I(z)$**

**Proof:** Now f(z) = logz $= \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}(\frac{y}{x})$

$= u(x,y) + iv(x,y)$

Where $u(x,y) = \frac{1}{2}\log(x^2+y^2)$ and $v(x,y) = \tan^{-1}(\frac{y}{x})$

∴ $u_x = \frac{1}{2}\frac{2x}{x^2+y^2} = \frac{x}{x^2+y^2}$ , $u_y = \frac{y}{x^2+y^2}$,

$v_x = \frac{1}{1+(\frac{y}{x})^2}\left(\frac{y}{-x^2}\right) = \frac{x^2}{x^2+y^2}\left(\frac{y}{-x^2}\right) = \frac{-y}{x^2+y^2}$ and $v_y = \frac{1}{1+(\frac{y}{x})^2}\left(\frac{1}{x}\right) = \frac{x^2}{x^2+y^2}\left(\frac{1}{x}\right) = \frac{x}{x^2+y^2}$

Clearly these are rational functions with non zero denominator and hence they exists and continuous.

Also C-R equations $u_x = v_y$ **and** $u_y = -v_x$ **are satisfied.**

**Hence f(z) =logz is analytic and $f^l(z) = 1/z$**

**5. Show that a function f(z) =xy+iy is everywhere continuous but not analytic.**

Soln: Given function be f(z) =xy+iy = u+iv where u =xy and v =y

Both u and v being polynomials in x and y are continuous for all x & y. Hence f(z) is continuous every where.

Here $\frac{\partial u}{\partial x}$ = y, $\frac{\partial u}{\partial y}$ =x, $\frac{\partial v}{\partial x}$ = 0 and $\frac{\partial v}{\partial y}$ = 1

So we have $\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} \neq -\frac{\partial v}{\partial x}$

i.e C-R equations are not satisfied and hence f(z) is not analytic.

**6. Show that $f(z)= y^3-3x^2y+i(x^3- 3xy^2)$ is analytic and find its derivative.**

**Harmonic function**:

**Definition:** A function u of x and y is said to be harmonic if partial derivatives of u of order first and second exist and satisfy **Laplace equation** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is called **harmonic function**.

In Physics, the descriptor "harmonic" in the name harmonic function originates from a point on a stretched tight string which is undergoing harmonic motion. The solution to the differential equation for this type of motion can be written in terms of sines and cosines, functions which are thus referred to as *harmonics*.

**Theorem:** If the function f(z) = u+iv be analytic in the given region D then u and v are harmonic functions.

**Proof:** Consider f(z) = u(x,y) +iv(x,y) be analytic in the given region D. Then all the four partial derivatives exit and C-R equations are satisfied.

i.e $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ -------(1) and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$------(2)

Differentiate (1) partially w.r.t x and (2) w.r.t y and adding we get,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial x\,\partial y} - \frac{\partial^2 v}{\partial y\,\partial x} = \frac{\partial^2 v}{\partial x\,\partial y} - \frac{\partial^2 v}{\partial x\,\partial y} = 0$$

i.e $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ i.e $\nabla^2 u = 0$ -------(3)

Similarly Differentiate (1) partially w.r.t y and (2) w.r.t x and adding we get,

$\frac{\partial^2 u}{\partial y\,\partial x}$ - $\frac{\partial^2 u}{\partial x\,\partial y}$ = $\frac{\partial^2 v}{\partial y^2}$ + $\frac{\partial^2 v}{\partial x^2}$

$0 = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$

i.e $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ i.e $\nabla^2 v = 0$ --------(4)

From (3) and (4) we say that u and v satisfy Laplace eqn. hence are harmonic functions.

**Thus if the function f(z) = u+iv be analytic in the given region D then u and v are harmonic functions.**

**Harmonic conjugates:** The real and imaginary parts u and v of analytic function f(z) are harmonic functions and they are called harmonic conjugates i.e u is harmonic conjugate of v and v is harmonic conjugate of u. Also Harmonic functions satisfy the linear equations in the Cartesian form. Theory of such harmonic functions is called Potential Theory.

**For example:**

**Verify that u = $x^2$- $y^2$-y is harmonic in the whole complex plane. **

**Soln.: Now u = $x^2$- $y^2$-y**

**Then** $\frac{\partial u}{\partial x} = 2x$, $\frac{\partial^2 u}{\partial x^2} = 2$, $\frac{\partial u}{\partial y} = -2y-1$, $\frac{\partial^2 u}{\partial y^2} = -2$

All these partial order derivatives exists as they are polynomial functions and satisfy Laplace eqn. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} = 2-2 = 0$

Thus u is harmonic function.

**Determination of the harmonic conjugate:**

**If f(z) = u+iv be analytic function and being given u then determine its harmonic conjugate v.**

**Procedure**: Let f(z) =u+iv be analytic and u(x,y) will be given, we have to find its conjugate v(x,y).

By partial differentiation we have

$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$

$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ ---------by C-R eqns as f(z) is analytic.

Which is in the form of Mdx +Ndy where M $= -\frac{\partial u}{\partial y}$ and N = $\frac{\partial u}{\partial x}$

So that $\frac{\partial M}{\partial y} = -\frac{\partial^2 u}{\partial y^2}$ and $\frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial x^2}$

But $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ b'cz u is harmonic.

=> $\frac{\partial^2 u}{\partial y^2} = -\frac{\partial^2 u}{\partial x^2}$ or => $-\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial x^2}$

i.e $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

=> Hence eqn (1) is exact and integrable by the method of exact differential eqn.

i.e $v = \int M dx + \int (\textbf{terms in N not containing x}) dy$

**Examples:**

**1. Prove that u = $y^3 - 3x^2y$ is harmonic function. Determine its harmonic conjugate and find the corresponding analytic function f(z) in terms of z.**

**Sol$^n$:** Given u = $y^3 - 3x^2y$

$\therefore \frac{\partial u}{\partial x} = u_x = -6xy$, $\frac{\partial^2 u}{\partial x^2} = u_{xx} = -6y$,

& $\frac{\partial u}{\partial y} = u_y = 3y^2 - 3x^2$, $\frac{\partial^2 u}{\partial y^2} = u_{yy} = 6y$

Clearly all these derivatives exist & continuous and Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is satisfied.

Hence u is harmonic.

**Let v be harmonic conjugate of u so that f(z) = u+iv is analytic.**

We have $dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$

$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ C-R eq$^{ns}$

dv $= (3x^2 - 3y^2)\,dx + -6xy\,dy$ which is exact.

Integrating both the sides, we get

$v = \int (3x^2 - 3y^2)dx + \int (\text{terms in N not containing x}) dy$

v = $x^3 - 3y^2x + c$

**which is harmonic conjugate of u.**

**And f(z) = u +iv =** $(y^3-3x^2y) + i(x^3 - 3y^2x) + ic$

$= i(iy)^3 + i\,3x^2(iy) + ix^3 + i\,3(iy)^2x + ic$

$= i[x^3 + 3x^2(iy) + 3x(iy)^2 + (iy)^3] + ic$

$= i(x+iy)^3 + ic = iz^3 + ic$

**2. Prove that u = $x^3 - 3xy^2$ is harmonic function. Determine its harmonic conjugate and find the corresponding analytic function f(z) in terms of z.**

**Sol^n:** Given u = $x^3 - 3xy^2$

$\therefore \frac{\partial u}{\partial x} = u_x = 3x^2 - 3y^2$, $\quad \frac{\partial^2 u}{\partial x^2} = u_{xx} = 6x$,

& $\frac{\partial u}{\partial y} = u_y = -6xy$, $\quad \frac{\partial^2 u}{\partial y^2} = u_{yy} = -6x$

Clearly all these derivatives exist & continuous and Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is satisfied.

Hence u is harmonic.

**Let v be harmonic conjugate of u so that f(z) = u+iv is analytic.**

We have dv = $\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$

$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ C-R eq^ns

dv $= (6xy)\,dx + (3x^2-3y^2)dy$ which is exact

and hence its soln. is v $= \int (6xy)\,dx + \int (-3y^2)dy$

i.e $v = 6y\frac{x^2}{2} - y^3 + c$

**v = $3x^2y - y^3 + c$, is harmonic conjugate of u.**

**And f(z) = u +iv =** $(x^3 - 3xy^2) + i(3x^2y - y^3) + ic$

$= x^3 + 3x(iy)^2 + 3x^2(iy) + (iy)^3 + ic$

$= (x+iy)^3 + ic = z^3 + ic$

**3. Prove that $u = (x-1)^3-3xy^2+3y^2$ is harmonic function. Determine its harmonic conjugate so that the corresponding analytic function f(z) = u+iv is regular.**

**Sol$^n$:** Given $u = (x-1)^3-3xy^2+3y^2$

$\therefore \frac{\partial u}{\partial x} = u_x = 3(x-1)^2-3y^2, \quad \frac{\partial^2 u}{\partial x^2} = u_{xx} = 6(x-1),$

$\& \frac{\partial u}{\partial y} = u_y = -6xy+6y, \quad \frac{\partial^2 u}{\partial y^2} = u_{yy} = -6x+6$

Clearly all these derivatives exist & continuous and Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is satisfied.

Hence u is harmonic.

**Let v be harmonic conjugate of u so that f(z) = u+iv is analytic.**

We have $dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$

$= -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$ --by C-R eq$^{ns}$

$dv = (6xy-6y)dx + (3(x-1)^2-3y^2)dy$

$dv = (6xy-6y)dx + (3x^2-3x + 3 -3y^2)dy$ which is exact

Hence its soln. is

$v = \int (6xy - 6y)dx\,] + \int (3 - 3y^2)dy$

$v = \quad 6\frac{x^2}{2}y - 6xy + 3y - y^3 + c$

$= 3x^2y-6xy+3y-y^3+c = 3(x^2-2x+1)y-y^3+c$

**$v = 3(x-1)^2y - y^3 + c$, is harmonic conjugate of u.**

**4. If $u = x^2-y^2$ and $v = \frac{-y}{x^2+y^2}$ then prove that, both u and v satisfy Laplace's equation, but u+iv is not an analytic function of z.**

**Soln.:** Now $u = x^2-y^2$

$\therefore u_x = 2x, u_{xx} = 2, u_y = -2y$ and $u_{yy} = -2$

Clearly u satisfied Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

Similarly v = $\frac{-y}{x^2+y^2}$

$\therefore v_x = \frac{y\,(2x)}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2},$

$v_{xx} = y\,\frac{(x^2+y^2)^2 2 - 2x[2(x^2+y^2)2x]}{(x^2+y^2)^4} = y\,\frac{(x^2+y^2)[x^2+y^2)^1 2 - 2x\,.4x]}{(x^2+y^2)^4}$

$= y\,\frac{[2y^2-6x^2]}{(x^2+y^2)^3} = \frac{\mathbf{2y(y^2-3x^2)}}{\mathbf{(x^2+y^2)^3}}$

Next $v_y = -\,\frac{(x^2+y^2)1 - y\,(2y)}{(x^2+y^2)^2} = -\,\frac{(x^2-y^2)}{(x^2+y^2)^2} = \frac{(y^2-x^2)}{(x^2+y^2)^2}$

$V_{yy} = \frac{(x^2+y^2)^2 2y - (y^2-x^2)[2(x^2+y^2)2y]}{(x^2+y^2)^4} = \frac{(x^2+y^2)[x^2+y^2)^1 2y - 4y\;(y^2-x^2)]}{(x^2+y^2)^4}$

$= 2y\,\frac{[x^2+y^2-2y^2+2x^2]}{(x^2+y^2)^3} = 2y\,\frac{[3x^2-y^2]}{(x^2+y^2)^3} = -\,\frac{\mathbf{2y(y^2-3x^2)}}{\mathbf{(x^2+y^2)^3}}$

Clearly v also satisfied Laplace equation $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0.$

Thus u and v are satisfying Laplace equation but u+iv is not analytic

as $u_x \neq v_y$ and $u_y \neq -v_x$ i.e C-R equations are not satisfied

**5. Prove that the function u = $e^x$(x cosy –ysiny) satisfies Laplace's equation, find its harmonic conjugate and the corresponding analytic function f(z).**

**Soln:** Let u= $e^x$(x cosy –ysiny)

Then $u_x = e^x$( cosy )+ $e^x$(x cosy –ysiny)

= $e^x$((1+x) cosy –ysiny)

$U_{xx} = e^x$( cosy) + $e^x$((1+x) cosy –ysiny)

= $e^x$((2+x) cosy –ysiny) ----(1)

$U_y$ = $e^x$[x (-siny) –ycosy-siny]

= $e^x$[- (1+x) siny –ycosy]

$U_{yy} = e^x$[- (1+x) cosy + ysiny-cosy]

$= e^x[-(2+x)\cos y + y\sin y]$----(2)

From (1) and (2) $U_{xx} + U_{yy} = 0$

- ⇨ function u satisfies Laplace equation and is continuous also.
- ⇨ U is harmonic

**Let v be harmonic conjugate of u so that f(z) = u+iv is analytic.**

We have $dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$

$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ --by C-R eq$^{ns}$

$= - e^x[-(1+x)\sin y - y\cos y]\,dx + e^x((1+x)\cos y - y\sin y)dy$

$= e^x[(1+x)\sin y + y\cos y]\,dx + e^x((1+x)\cos y - y\sin y)dy$ which is exact

Hence Solution is $v = \int e^x[(1+x)\sin y + y\cos y]\,dx + \int(\text{terms of N not containing x})dy$

i.e $v = \int e^x(1+x)\sin y\,dx + \int e^x y\cos y\,dx + 0$

$= \sin y[e^x(1+x) - e^x] + e^x y\cos y$

$= x e^x \sin y + e^x y\cos y = e^x[x\sin y + y\cos y]$

∴ f(z) = u+iv

$= e^x(x\cos y - y\sin y) + ie^x[x\sin y + y\cos y]$

$= e^x[x(\cos y + i\sin y) + iy(\cos y + i\sin y)]$

$= e^x[(x+iy)(\cos y + i\sin y)] = ze^z$

**6. Show that the function $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic and find its harmonic conjugate.**

**Soln.: Now $u = \frac{1}{2}\log(x^2+y^2)$**

$\therefore u_x = \frac{1}{2}\frac{2x}{x^2+y^2} = \frac{x}{x^2+y^2},\quad u_{xx} = \frac{(x^2+y^2)1 - x(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

$u_y = \frac{1}{2}\frac{2y}{x^2+y^2} = \frac{y}{x^2+y^2},\quad u_{yy} = \frac{(x^2+y^2)1 - y(2y)}{(x^2+y^2)^2} = \frac{-y^2+x^2}{(x^2+y^2)^2}$

Clearly all these four partial derivatives exist as they are rational functions with non zero denominators and satisfying Laplace equation and hence u is harmonic.

**Let v be harmonic conjugate of u so that f(z) = u+iv is analytic.**

We have dv = $\frac{\partial v}{\partial x}$ dx+$\frac{\partial v}{\partial y}$ dy

= - $\frac{\partial u}{\partial y}$ dx + $\frac{\partial u}{\partial x}$ dy --by C-R eq$^{ns}$

= - $\frac{y}{x^2+y^2}$ dx + $\frac{x}{x^2+y^2}$ dy which is exact

Hence its soln. is

V = $\int -\frac{y}{x^2+y^2}$dx + $\int$(terms of N not containing x)dy

V = - y$\int \frac{1}{x^2+y^2}$dx + $\int 0$ dy = -y$\frac{1}{y}$ tan$^{-1}$($\frac{x}{y}$) + c

V = tan$^{-1}$($\frac{x}{y}$) + c

**7. Show that the function u = $cosx$ coshy is harmonic and find its harmonic conjugate and f$^{l}$ (z) in terms of z.**

**Soln.:** Now u = $cosx$ coshy

∴ $u_x$ = -sinx cushy, $u_{xx}$ = - cosx coshy, $u_y$ = cosx sinhy, $u_{yy}$ = cosxsinhy

Clearly all these four partial derivatives exist as they are rational functions with non zero denominators and satisfying Laplace equation and hence u is harmonic.

**Let v be harmonic conjugate of u so that f(z) = u+iv is analytic.**

We have dv = $\frac{\partial v}{\partial x}$ dx+$\frac{\partial v}{\partial y}$ dy

= - $\frac{\partial u}{\partial y}$ dx + $\frac{\partial u}{\partial x}$ dy --by C-R eq$^{ns}$

= - (cosx sinhy)dx + (-sinx cushy) dy which is exact.

Hence its soln. is

V = $\int$ − (cosx sinhy)dx + $\int$(terms of N not containing x)dy

= - sinhy $\int$ cosx dx + 0 = - sinhy sinx + c

V = -sinhy sinx +c

∴ f(z) = u + iv = cosx coshy + i ( - sinx sinhy)= cosx coshy - i sinx sinhy +ic

= cos(x+iy) +ic = cosz + ic

Hence $f^{l}(z) = \frac{\partial u}{\partial x} + i\frac{\partial x}{\partial x}$ = - sinx coshy – i cosx sinhy

= - (sinx coshy + i cosx sinhy) = - sinz

## Milne's method of finding f(z) directly if u or v will be given

**Let u will be given, find f(z) = u+iv without finding v.**

**Let** $\frac{\partial u}{\partial x} = \emptyset_1(x,y)$ and $\frac{\partial u}{\partial y} = \emptyset_2(x,y)$

By Milne's method $f^{l}(z) = \emptyset_1(z,0) - i\emptyset_2(z,0)$

Integrating we get $f(z) = \int \emptyset_1(z,o)dz - i\int \emptyset_2(z,o)dz$

**If v will be given then** $f(z) = \int \varphi_1(z,o)\, dz + i\int \varphi_2(z,o)dz$ where $\frac{\partial v}{\partial y} = \varphi_1(x,y)$ and $\frac{\partial v}{\partial x} = \varphi_2(x,y)$

**Examples:**

**1.Prove that the function $u = e^x(x \cos y - y\sin y)$**

**satisfies Laplace's equation and find the corresponding analytic function f(z).**

**Sol$^n$:** Now $u = e^x(x \cos y - y\sin y)$

$\therefore \frac{\partial u}{\partial x} = e^x(x\cos y - y\sin y) + e^x(\cos y) = e^x[(x+1)\cos y - y\sin y]$

i.e $\frac{\partial u}{\partial x} = e^x[(x+1)\cos y - y\sin y] = \emptyset_1(x,y)$

& $\frac{\partial^2 u}{\partial x^2} = e^x[(x+1)\cos y - y\sin y] + e^x\cos y = e^x[(x+2)\cos y - y\sin y]$

Next $\frac{\partial u}{\partial y} = e^x(x(-\sin y) - y\cos y - \sin y) = e^x[-(x+1)\sin y - y\cos y] = \emptyset_2(x,y)$

& $\frac{\partial^2 u}{\partial y^2} = e^x[-(x+1)\cos y + y\sin y - \cos y] = e^x[-(x+2)\cos y + y\sin y]$

Clearly $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Therefore u satisfies Laplace's equation.

**By Milne's method** $f^{I}(z) = \emptyset_1(z,0) - i\emptyset_2(z,0)$

i.e $f^{I}(z) = e^z(z+1) - ie^z 0 = ze^z + e^z$

i.e $f^{I}(z) = ze^z + e^z$

Integrating, $f(z) = \int (ze^z + e^z)dz$

$= ze^z - e^z + e^z + c$

$= ze^z + c$

**2.If $u = \frac{sin2x}{cosh2y + cos2x}$ find the corresponding analytic function f(z).**

**Sol$^n$:** Now $u = \frac{sin2x}{cosh2y + cos2x}$

$\therefore \frac{\partial u}{\partial x} = \frac{(cosh2y + cos2x)2cos2x - sin2x(-2sin2x)}{(cosh2y + cos2x)^2}$

$= \frac{2[1 + cos2xcosh2y]}{(cosh2y + cos2x)^2} = \emptyset_1(x,y)$

And $\frac{\partial u}{\partial y} = \frac{(cosh2y + cos2x).0 - sin2x(2sinh2y)}{(cosh2y + cos2x)^2}$

$= \frac{-sin2x(2sinh2y)}{(cosh2y + cos2x)^2} = \frac{-2sin2x\, sinh2y}{(cosh2y + cos2x)^2} = \emptyset_2(x,y)$

**By Milne's method** $f^{I}(z) = \emptyset_1(z,0) - i\emptyset_2(z,0)$

$= \frac{2[1 + cos2z]}{(1 + cos2z)^2} - i\,0$

$f^{I}(z) = \frac{2}{1 + cos2z} = \frac{2}{2cos^2 z} = sec^2 z$

Integrating we get $f(z) = tanz + c$

**3.Find the regular function w = u+iv where $u = e^{-x}\{(x^2 - y^2) cosy + 2xy\, siny\}$**

**Sol$^n$:** Now $u = e^{-x}\{(x^2 - y^2) cosy + 2xy\, siny\}$

$\therefore \frac{\partial u}{\partial x} = -e^{-x}\{(x^2 - y^2) cosy + 2xy\, siny\} + e^{-x}(2xcosy + 2ysiny)$

$= -e^{-x}\{(x^2 - y^2 - 2x) cosy + 2y(x-1) siny\} = \emptyset_1(x,y)$

& $\frac{\partial u}{\partial y}$ = $e^{-x}\{(x^2-y^2)(-\sin y)-2y\cos y+2x\sin y+2xy\cos y\}$

= $e^{-x}\{-(x^2-y^2-2x)(\sin y)-2y\cos y(1-x)\}$ = **$\emptyset_2$(x,y)**

**By Milne's method** $f^I(z)= \emptyset_1(z,0)-i\emptyset_2(z,0)$

$f^I(z) = -e^{-z}(z^2-2z) - i.\ 0$

$= e^{-z}(2z - z^2)$

Integrating we get $f(z)=- e^{-z}(2z - z^2)- e^{-z}(2 - 2z)+ - e^{-z}(-2)$

$= -e^{-z}(2z-z^2+2-2z-2)= e^{-z}(z^2)+c$

$f(z) = z^2 e^{-z}+c$

**4. Prove that the function u = sinx coshy+2cosx sinhy +$x^2$-$y^2$+4xy**

**satisfies Laplace's equation and find the corresponding halomarphic function f(z).**

**Soln.: Now u = sinx coshy+2cosx sinhy +$x^2$-$y^2$+4xy**

$\frac{\partial u}{\partial x}$ = cosx coshy – 2sinx sinhy +2x +4y = **$\emptyset_1$(x,y)**

& $\frac{\partial^2 u}{\partial x^2}$ = - sinx coshy - 2cosx sinhy+2

$\frac{\partial u}{\partial y}$ = sinx sinhy + 2cosx coshy - 2y +4x = **$\emptyset_2$(x,y)**

$\frac{\partial^2 u}{\partial y^2}$ = sinx coshy + 2cosx sinhy – 2

Clearly $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$

∴ u **satisfies Laplace's equation**

**By Milne's method** $f^I(z)= \emptyset_1(z,0)-i\emptyset_2(z,0)$

i.e $f^I(z)$= cosz +2z –i (2 cosz +4z)

= (1-2i) cosz + 2z(1-2i)

$f^I(z)$ = (1 -2i)(cosz + 2z)

Integrating, we get

f(z) = (1 – 2i) [sinz + $z^2$] +c

**5. Prove that the function u = $x^3$-3x$y^2$+3$x^2$ – 3$y^2$+1**

**satisfies Laplace's equation and find the corresponding halomarphic function f(z).**

## Finding f(z)= u+iv if either u-v or u+v will be given

Let U = u-v and V = u+v so that

F(z) = U+iV = (u-v)+i(u+v)= (u+iu+iv-v)

= u(1+i)+iv(1+i)

= (1+i)(u+iv)

F(z) = (1+i)f(z)

$\therefore$ f(z) = $\frac{1}{1+i}$ F(z)

**Examples:**

1. **If f(z) = u+iv is an analytic function of z and u-v = $e^x$(cosy-siny), find f(z) in terms of z.**

**Sol$^n$:** Now f(z) =u+iv be analytic function.

Let U= u-v = $e^x$(cosy-siny) and V=u+v (1)

So that F(z) = U+iV = (1+i)f(z)

Differentiate (1) p.w.r.t x and y

$\frac{\partial U}{\partial x}$ = $e^x$(cosy-siny) = $\emptyset_1(x,y)$

$\frac{\partial U}{\partial y}$ = $e^x$(-siny-cosy)= $\emptyset_2(x,y)$

By Milne's method $F^I(z)$= $\emptyset_1(z,0)$-i$\emptyset_2(z,0)$

= $e^z$(1-0) –i$e^z$(0-1)

$F^I(z)$ = $e^z$(1+i)

Integrating on both the sides we get

F(z) = $e^z$(1+i) + c

$(1+i)f(z) = e^z(1+i) + c$

⇨ $f(z)= e^z + c$

2. **If f(z) = u+iv is an analytic function of z and u-v = $(x-y)(x^2+4xy+y^2)$, find f(z) in terms of z.**

**Sol$^n$:** Now f(z) =u+iv be analytic function.

Let $U=u-v=(x-y)(x^2+4xy+y^2)$,and $V=u+v$ (1)

So that $F(z) = U+iV = (1+i)f(z)$

Differentiate (1) p.w.r.t x and y

$\frac{\partial U}{\partial x} = (x-y)(2x+4y)+ (x^2+4xy+y^2)$

$=2x^2+4xy-2xy-4y^2+x^2+4xy+y^2$

$= 3x^2+6xy-3y^2 = \emptyset_1(x,y)$

$\frac{\partial U}{\partial y} =(x-y)(4x+2y)+ (x^2+4xy+y^2)(-1)$

$= 4x^2+2xy-4xy-2y^2-x^2-4xy-y^2$

$= 3x^2-6xy-3y^2 = \emptyset_2(x,y)$

By Milne's method $F^l(z)= \emptyset_1(z,0)-i\emptyset_2(z,0)$

$F^l(z) = 3z^2-i3z^2= 3z^2(1-i)$

$\therefore F(z)=z^3(1-i)+c$

$(1+i)f(z) = z^3(1-i)+c$

⇨ $f(z) = \frac{1-i}{1+i} z^3+c = -i z^3+c$

**Theoretical Examples:**

**1. Prove that an analytic function with onstant modulus is constant.**

**Proof:** Let f(z) = u+iv be analytic function so that $|f(z)| = \sqrt{u^2 + v^2}$ = constant = k

⇨ $u^2 + v^2 = k^2$ --------------(1)

Diff. (1) w.r.t x and then y we get

$2u\frac{\partial U}{\partial x} + 2v\frac{\partial v}{\partial x} = 0$

$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} = 0$ -----------------(2)

& $u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} = 0$

i.e $u(-\frac{\partial v}{\partial x}) + v\frac{\partial u}{\partial x} = 0$ ------By C-R eqns.

i.e $-u\frac{\partial v}{\partial x} + v\frac{\partial u}{\partial x} = 0$ ---------(3)

Squaring and adding (2) and (3) we get

$[u^2(\frac{\partial u}{\partial x})^2 + v^2(\frac{\partial v}{\partial x})^2 + 2uv\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}] + [u^2(\frac{\partial v}{\partial x})^2 + v^2(\frac{\partial u}{\partial x})^2 - 2uv\frac{\partial v}{\partial x}\frac{\partial u}{\partial x}] = 0$

i.e $(u^2+v^2)[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial x})^2] = 0$

=> $(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial x})^2 = 0$

i.e $\left|\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right|^2 = 0$

i.e $|f^l(z)|^2 = 0$ since $f^l(z) = u_x + iv_x$

=> $f^l(z) = 0$

=> f(z) is constant

**2. Prove that an analytic function with constant real (imaginary ) part is constant.**

**Proof:** Let f(z) = u+iv be analytic function so that u be constant.

i.e u = k

=> $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial u}{\partial y} = 0$

But $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = 0$ & $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = 0$ by C-R equns.

i.e $\frac{\partial v}{\partial y} = 0$ & $\frac{\partial v}{\partial x} = 0$

=> v is constant

=> u+iv is constant

i.e f(z) is constant.

Thus analytic function with constant real part is constant.

Similarly we can prove for v.

**3. If f(z) is analytic function of z, prove that $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})|R\, f(z)|^2 = 2\,|f^l(z)|^2$**

**i.e $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})u^2 = 2\,|f^l(z)|^2$**

**Proof:** Let f(z) = u+iv be analytic so that

R f(z) = u (i.e RP of f(z) = u)

Now $\frac{\partial}{\partial x}(u^2) = 2u\frac{\partial u}{\partial x}$

$\therefore \frac{\partial^2}{\partial x^2}(u^2) = 2[\, u\frac{\partial^2 u}{\partial x^2} + (\frac{\partial u}{\partial x})^2]$

Similarly $\frac{\partial^2}{\partial y^2}(u^2) = 2[\, u\frac{\partial^2 u}{\partial y^2} + (\frac{\partial u}{\partial y})^2]$

Adding these two we get,

$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})u^2 = 2[\, u\frac{\partial^2 u}{\partial x^2} + (\frac{\partial u}{\partial x})^2 + u\frac{\partial^2 u}{\partial y^2} + (\frac{\partial u}{\partial y})^2]$

$= 2[(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 + u(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})]$

$= 2[(\frac{\partial u}{\partial x})^2 + (-\frac{\partial v}{\partial x})^2 + 0]$ (since u is harmonic)

$= 2|f^l(z)|^2$

**4. If f(z) is analytic function of z, prove that $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})|f(z)|^2 = 4|f^l(z)|^2$**

**Proof:** Let f(z) = u+iv be analytic so that

$|f(z)|^2 = u^2+v^2$

From previous result we

$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})u^2 = 2|f^l(z)|^2$ ------(1)

Similarly

$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})v^2 = 2|f^l(z)|^2$ ------(2)

Adding (1) and (2) , we get

$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})(u^2+v^2) = 4|f^l(z)|^2$

**5.Prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z\,\partial \bar{z}}$**

**Proof:** We have z =x+iy so that $\bar{z}$ = x-iy

$\therefore x = \frac{1}{2}(z+\bar{z})$ and $y = \frac{1}{2i}(z-\bar{z})$

⇨ $\frac{\partial x}{\partial z} = \frac{1}{2}, \frac{\partial x}{\partial \bar{z}} = \frac{1}{2}, \frac{\partial y}{\partial z} = \frac{1}{2i} = -\frac{i}{2}, \frac{\partial y}{\partial \bar{z}} = \frac{i}{2}$

Now $\frac{\partial}{\partial z} = \frac{\partial}{\partial x}\frac{\partial x}{\partial z} + \frac{\partial}{\partial y}\frac{\partial y}{\partial z} = \frac{\partial}{\partial x}\frac{1}{2} + \frac{\partial}{\partial y}\frac{-i}{2} = \frac{1}{2}(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y})$

and $\frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial x}\frac{\partial x}{\partial \bar{z}} + \frac{\partial}{\partial y}\frac{\partial y}{\partial \bar{z}} = \frac{\partial}{\partial x}\frac{1}{2} + \frac{\partial}{\partial y}\frac{i}{2} = \frac{1}{2}(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y})$

Multiplying above two equations we get

$\frac{\partial^2}{\partial z\,\partial \bar{z}} = \frac{1}{2}(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y})\frac{1}{2}(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}) = \frac{1}{4}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})$

i.e $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z\,\partial \bar{z}}$

**6. If w = f(z) is regular function of z prove that ( $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ ) log$|f^l(z)|$= 0**

**Proof:** We have $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z\,\partial \bar{z}}$

$\therefore (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \log|f^l(z)| = 4\frac{\partial^2}{\partial z\,\partial \bar{z}} \log|f^l(z)|$

$= 4\frac{\partial^2}{\partial z\,\partial \bar{z}}\frac{1}{2}\log|f^l(z)|^2$

$= 4\frac{\partial^2}{\partial z\,\partial \bar{z}}\frac{1}{2}\log\{f^l(z)\,\overline{f^l(z)}\}$

$= 2\frac{\partial^2}{\partial z\,\partial \bar{z}}\log\{f^l(z)\,f^l(\bar{z})\}$

$=2\frac{\partial^2}{\partial z\,\partial \bar{z}}\{\log f^l(z) + \log f^l(\bar{z})\}$

$= 2\frac{\partial}{\partial z}(\frac{\partial}{\partial \bar{z}}\{\log f^l(z) + \log f^l(\bar{z})\})$

$= 2\frac{\partial}{\partial z}(0 + \frac{1}{f^l(\bar{z})}f^{ll}(\bar{z}))$

$= 0$

Thus $(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) \log|f^l(z)| = 0$

**7. If f(z) = u+iv is analytic then prove that $\frac{\partial(u,v)}{\partial(x,y)} = (\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2$**

**OR**

**J(u,v) = $(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 = (u_x)^2 + (u_y)^2$**

**Proof:** Now f(z) = u+iv be analytic

$\therefore \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ -\frac{\partial u}{\partial y} & \frac{\partial u}{\partial x} \end{vmatrix}$ ----by C-R equns.

$= (\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2$

## Examples of functions where C-R equations are satisfied at origin but $f^1(0)$ does not exist

**1. Prove that the function f(z) = u+iv where f(z) = $\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ (z$\neq 0$)**

**f(0) = 0 is continuous and that Cauchy-Riemann equations are satisfied at the origin, yet $f^{I}(z)$ does not exist there.**

**Sol$^n$:** Let the function be f(z) = u+iv

$= \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ (z$\neq 0$)

f(0) = 0 = u(0,0)+i v(0,0)

where u(x,y) = $\frac{x^3-y^3}{x^2+y^2}$, v (x,y)= $\frac{x^3+y^3}{x^2+y^2}$

Clearly u and v are rational functions with non zero D$^r$ and hence they exist and continuous.

⇨ f(z) = u+iv is continuous.

Next C-R equations at origin,

$\frac{\partial u}{\partial x} = \lim_{x\to 0} \frac{u(x,0)-u(0,0)}{x} = \lim_{x\to 0} \frac{x-0}{x} = 1$

$\frac{\partial u}{\partial y} = \lim_{y\to 0} \frac{u(0,y)-u(0,0)}{y} = \lim_{y\to 0} \frac{-y-0}{y} = -1$

$\frac{\partial v}{\partial x} = \lim_{x\to 0} \frac{v(x,0)-v(0,0)}{x} = \lim_{x\to 0} \frac{x-0}{x} = 1$

$\frac{\partial v}{\partial y} = \lim_{y\to 0} \frac{v(0,y)-v(0,0)}{y} = \lim_{y\to 0} \frac{y-0}{y} = 1$

Hence C-R equations are satisfied at the origin.

Again, $f^{I}(0) = \lim_{z\to 0} \frac{f(z)-f(0)}{z}$

$= \lim_{z\to 0} \frac{x^3(1+i)-y^3(1-i)}{(x^2+y^2)(x+iy)}$

Let z→0 along y= mx ,

$= \lim_{x \to 0} \frac{x^3(1+i)-m^3x^3(1-i)}{(x^2+m^2x^2)(x+imx)} = \lim_{x \to 0} \frac{x^3[(1+i)-m^3 \ \ (1-i)]}{x^3(1+m^2)(1+im)}$

$= \frac{[(1+i)-m^3 \ \ (1-i)]}{(1+m^2)(1+im)}$ which depends on m and hence it is not unique.

⇨ $f^{I}(z)$ does not exist at zero.

**2. Prove that the function f(z) = u+iv where f(z) = $\frac{x^2y^5(x+iy)}{x^4+y^{10}}$ (z≠ 0) f(0) = 0 is continuous and that Cauchy-Reimann equations are satisfied at the origin, yet $f^{I}(z)$ does not exist there. OR examine the nature of the function f(z).**

**Sol$^n$:** Let the function be f(z) = u+iv

$= \frac{x^2y^5(x+iy)}{x^4+y^{10}}$ (z≠ 0)

f(0) = 0 = u(0,0)+i v(0,0)

where u(x,y) = $\frac{x^3y^5}{x^4+y^{10}}$ , v (x,y)= $\frac{x^2y^6}{x^4+y^{10}}$

Clearly u and v are rational functions with non zero $D^r$ and hence they exist and continuous.

⇨ f(z) = u+iv is continuous.

Next C-R equations at origin,

$\frac{\partial u}{\partial x} = \lim_{x \to 0} \frac{u(x,0)-u(0,0)}{x} = \lim_{x \to 0} \frac{0-0}{x} = 0$

$\frac{\partial u}{\partial y} = \lim_{y \to 0} \frac{u(0,y)-u(0,0)}{y} = \lim_{y \to 0} \frac{0-0}{y} = 0$

$\frac{\partial v}{\partial x} = \lim_{x \to 0} \frac{v(x,0)-v(0,0)}{x} = \lim_{x \to 0} \frac{0-0}{x} = 0$

$\frac{\partial v}{\partial y} = \lim_{y \to 0} \frac{v(0,y)-v(0,0)}{y} = \lim_{y \to 0} \frac{0-0}{y} = 0$

Hence C-R equations are satisfied at the origin.

Again, $f^{I}(0) = \lim_{z \to 0} \frac{f(z)-f(0)}{z} = \lim_{z \to 0} \frac{x^2y^5(x+iy)-0}{(x^4+y^{10})(x+iy)}$

$= \lim_{z \to 0} \frac{x^2y^5}{(x^4+y^{10})}$

**Let z→0 along y= mx ,**

= $\lim_{x\to 0}\frac{x^2m^5x^5}{(x^4+m^{10}x^{10})}$ = $\lim_{x\to 0}\frac{x^7m^5}{x^4(1+m^{10}x^6)}$ = 0

**Now Let z→0 along $y^5$= $x^2$ , then**

$f^{I}(z) = \lim_{x\to 0}\frac{x^2x^2}{(x^4+x^4)}$ = ½

Thus limits along y=mx and along the curve $y^5$= $x^2$ are not same.

⇨ $f^{I}(z)$ does not exist at zero.

**3. Prove that the function f(z) = $\sqrt{|xy|}$ is not regular at the origin, although Cauchy-Reimann equations are satisfied at the origin.**

**Sol$^n$:** Let the function be f(z) = u+iv where u = $\sqrt{|xy|}$ and v = 0 .

At the origin,

$\frac{\partial u}{\partial x} = \lim_{x\to 0}\frac{u(x,0)-u(0,0)}{x} = \lim_{x\to 0}\frac{0-0}{x}=0$

$\frac{\partial u}{\partial y} = \lim_{y\to 0}\frac{u(0,y)-u(0,0)}{y} = \lim_{y\to 0}\frac{0-0}{y}=0$

$\frac{\partial v}{\partial x} = \lim_{x\to 0}\frac{v(x,0)-v(0,0)}{x} = \lim_{x\to 0}\frac{0-0}{x}= 0$

$\frac{\partial v}{\partial y} = \lim_{y\to 0}\frac{v(0,y)-v(0,0)}{y} = \lim_{y\to 0}\frac{0-0}{y}=0$

Hence C-R equations are satisfied at the origin.

Again, $f^{I}(0)=\lim_{z\to 0}\frac{f(z)-f(0)}{z}= \lim_{z\to 0}\frac{\sqrt{|xy|}}{x+iy}$

**Let z→0 along y= mx ,**

∴ $f^{I}(0) = \lim_{z\to 0}\frac{\sqrt{|mx^2|}}{x+imx} = \lim_{z\to 0}\frac{\sqrt{|m|}}{1+im}$

which depends on m, hence $f^{I}(0)$ is not unique.

∴ f(z) is not analytic at origin even though C-R equations are satisfied at the origin.

**4. Prove that $\lim_{z\to 0}\frac{\bar{z}}{z}$ does not exist.**

**Soln.:** Now $\lim_{z\to 0}\frac{\bar{z}}{z} = \lim_{(x,y)\to(0,0)}\frac{x-iy}{x+iy}$

Let $(x, y) \to (0,0)$ along x axis then y=0, we get

$\lim_{(x,y)\to(0,0)} \frac{x-iy}{x+iy} = \lim_{x\to 0)} \frac{x}{x} = 1$

Let $(x, y) \to (0,0)$ along y axis then x=0, we get

$\lim_{(x,y)\to(0,0)} \frac{x-iy}{x+iy} = \lim_{y\to 0)} \frac{-iy}{iy} = -1$

**Thus limit is not unique and hence $\lim_{z\to 0} \frac{\bar{z}}{z}$ does not exist.**

**5. If f(z) = $\frac{x^2y(y-ix)}{x^6+y^2}$ (z$\neq 0$)**

**f(0) = 0**

**show that $f^{I}(z)$ does not exist at origin.**

## ASSIGNMENT QUESTIONS ON UNIT I

1. Use the C-R equations to show that the following functions are differentiable(analytic) and find $f^{I}(z)$

   a) f(z) =iz+4i (b) f(z) = $z^2$ (c) f(z) =-2(xy+x) +i($x^2$ – 2y – $y^2$)

[Hint: First write f(z) in terms of u+iv , get u and v and then proceed.]

2. Show that f(z) = $\frac{y+ix}{x^2+y^2}$ , is differentiable for all z≠0, ie to prove analytic means satisfying C-R equations.

3. Show that f(z) = $e^{2xy}[\cos(y^2 - x^2) + I \sin(y^2 - x^2)$ is differentiable for all z, ie to prove analytic ( satisfying C-R equations.)
4. Show that following functions are nowhere differentiable i.e not analytic.

   a) $f(z) = \bar{z}$ b) $g(z) = z + \bar{z}$ c) $h(z) = e^y \cos x + I e^y \sin x$
5. If (z) = $x^3 + 3xy^2 + i(y^3 + 3xy^2)$, is f analytic? Why?
6. If (z) = $8x - x^3 - xy^2 + i(y^3 + xy^2 - 8y)$, is f analytic? Why?
7. Find the analytic function f(z) = u+iv given tha following

   a) $U = y^3 - 3xy^2$, b) u(x,y) = sinx sinhy c) v(x,y)= $e^x$ siny d) v(x,y)= sinx sinhy
8. Find the constants a and b such tha function f(z) =(2x-y) +I (ax+by) is differentiable for all z, analytic **.(Hint: Apply C-R equations and find a and b)**
9. Let $u_1(x, y) = x^2 - y^2$ and $u_2(x, y) = x^3 - 3xy^2$. Show that $u_1$ and $u_2$ are harmonic functions and that their product $u_1(x, y)u_2(x, y)$ is not a harmonic function.

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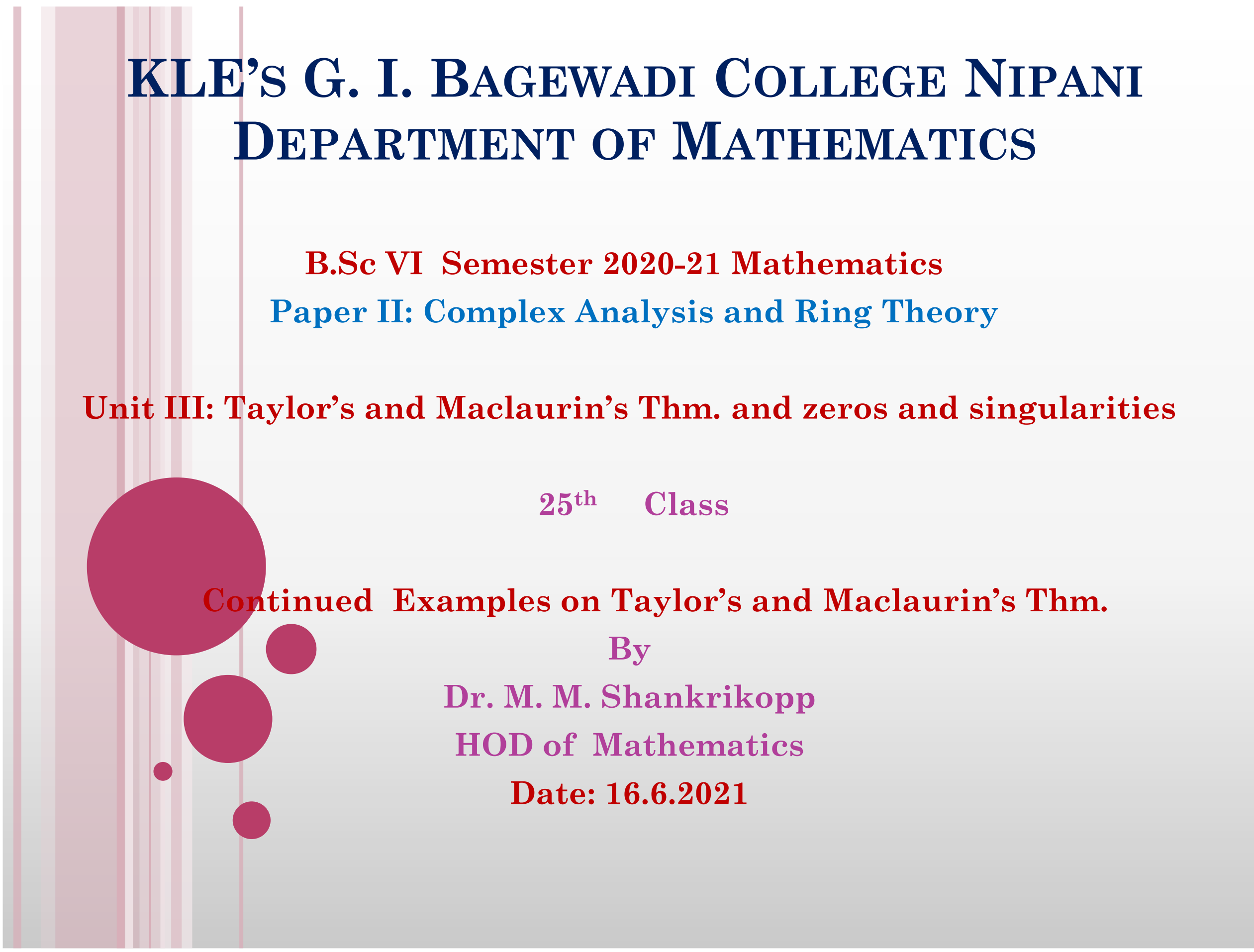
## Department of Mathematics

## B.Sc. VI Sem., Paper II: Complex Analysis and Ring Theory

# UNIT III: TAYLOR'S AND LAURENT'S SERIES , ZEROS POLES AND RESIDUES OF ANALYTIC FUNCTION

### CONTINUED EXAMPLES.




**4. Obtain Laurent's series for $f(z) = \frac{4z+3}{z(z+2)(z-3)}$ (i) within $|z| = 1$ (ii) $2<|z|<3$ (iii) $|z|>3$**

**Soln.**: Given function $f(z) = \frac{4z+3}{z(z+2)(z-3)} = \frac{A}{z} + \frac{B}{(z+2)} + \frac{C}{(z-3)}$

$$= \frac{-\frac{1}{2}}{z} + \frac{-\frac{1}{2}}{(z+2)} + \frac{1}{(z-3)}$$

$$f(z) = \frac{-1}{2z} + \frac{-1}{2(z+2)} + \frac{1}{(z-3)}$$

**(i) When $|z| < 1$ => $|z| < 2$ and $|z| < 3$**

$$\therefore \quad \frac{|z|}{2} < 1 \text{ and } \frac{|z|}{3} < 1$$

Then $f(z) = \frac{-1}{2z} + \frac{-1}{2\,(2)(1+\frac{z}{2})} + \frac{1}{(-3)(1-\frac{z}{3})}$

$$= \frac{-1}{2z} - \frac{1}{4}\left(1+\frac{z}{2}\right)^{-1} - \frac{1}{3}\left(1-\frac{z}{3}\right)^{-1}$$

$$= \frac{-1}{2z} - \frac{1}{4}\left(1-\frac{z}{2}+\left(\frac{z}{2}\right)^2-\left(\frac{z}{2}\right)^3+--\right) - \frac{1}{3}\left(1+\frac{z}{3}+\left(\frac{z}{3}\right)^2+\left(\frac{z}{3}\right)^3+---\right)$$

$= \frac{-1}{2z} - \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{2}\right)^n - \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n$ Laurent's series

**(ii) When $2 < |z| < 3$ => $|z| > 2$ and $|z| < 3$**

$$\therefore \quad \frac{2}{|z|} < 1 \text{ and } \frac{|z|}{3} < 1$$

Then $f(z) = \frac{-1}{2z} + \frac{-1}{2\,(z)(1+\frac{2}{z})} + \frac{1}{(-3)(1-\frac{z}{3})}$

$$= \frac{-1}{2z} - \frac{1}{2z}\left(1+\frac{2}{z}\right)^{-1} - \frac{1}{3}\left(1-\frac{z}{3}\right)^{-1}$$

$$= \frac{-1}{2z} - \frac{1}{2z}\left(1-\frac{2}{z}+\left(\frac{2}{z}\right)^2-\left(\frac{2}{z}\right)^3+--\right) - \frac{1}{3}\left(1+\frac{z}{3}+\left(\frac{z}{3}\right)^2+\left(\frac{z}{3}\right)^3+--\right)$$

$$\mathbf{f(z)} = \frac{-1}{2z} - \frac{1}{2z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2}{z}\right)^n - \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{z}{3}\right)^n$$

**(iii) When $|z| > 3$ => $|z| > 2$**

$\therefore \quad \frac{3}{|z|} < 1$ **and** $\frac{2}{|z|} < 1$

Then $f(z) = \frac{-1}{2z} + \frac{-1}{2\,(z)(1+\frac{2}{z})} + \frac{1}{(z)(1-\frac{3}{z})}$

$= \frac{-1}{2z} - \frac{1}{2z}(1+\frac{2}{z})^{-1} + \frac{1}{z}\ (1-\frac{3}{z})^{-1}$

$= \frac{-1}{2z} - \frac{1}{2z}(1-\frac{2}{z} + (\frac{2}{z})^2 - (\frac{2}{z})^3 + --) + \frac{1}{z}\ (1+\frac{3}{z} + (\frac{3}{z})^2 + (\frac{3}{z})^3 + --)$

**f(z) =** $\frac{-1}{2z} - \frac{1}{2z}\sum_{n=0}^{\infty}(-1)^n(\frac{2}{z})^n + \frac{1}{z}\sum_{n=0}^{\infty}(\frac{3}{z})^n$

5. **Obtain Laurent's series for** $f(z) = \frac{4z+3}{(z+2)(z-3)}$ **(i) within |z| <2 (ii) 2<|z|<3 (iii) |z|>3 [2013]**

**Soln.**: Given function $f(z) = \frac{4z+3}{(z+2)(z-3)} = \frac{1}{(z+2)} + \frac{3}{(z-3)}$

$$f(z) = \frac{1}{(z+2)} + \frac{3}{(z-3)}$$

**(i) When |z| < 2 => |z| < 3**

$\therefore \quad \frac{|z|}{2} < 1$ **and** $\frac{|z|}{3} < 1$

Then $f(z) = \frac{1}{2\,(2)(1+\frac{z}{2})} + \frac{3}{(-3)(1-\frac{z}{3})}$

$= \frac{1}{4}(1+\frac{z}{2})^{-1} - (1-\frac{z}{3})^{-1}$

$= \frac{1}{4}(1-\frac{z}{2} + (\frac{z}{2})^2 - (\frac{z}{2})^3 + --) - (1+\frac{z}{3} + (\frac{z}{3})^2 + (\frac{z}{3})^3 + ---)$

$= \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n(\frac{z}{2})^n - \sum_{n=0}^{\infty}(\frac{z}{3})^n$ Taylor's series

**(ii) When 2< lzl < 3 => lzl > 2 and lzl < 3**

$$\therefore \quad \frac{2}{|z|} < 1 \text{ and } \frac{|z|}{3} < 1$$

Then $f(z) = \frac{1}{2\,(z)(1+\frac{2}{z})} + \frac{3}{(-3)(1-\frac{z}{3})}$

$= \frac{1}{2z}(1+\frac{2}{z})^{-1} - (1-\frac{z}{3})^{-1}$

$= \frac{1}{2z}(1-\frac{2}{z}+(\frac{2}{z})^2-(\frac{2}{z})^3+--) - (1+\frac{z}{3}+(\frac{z}{3})^2+(\frac{z}{3})^3+--)$

**f(z) =** $\frac{1}{2z}\sum_{n=0}^{\infty}(-1)^n(\frac{2}{z})^n - \sum_{n=0}^{\infty}(\frac{z}{3})^n$

**(iii) When lzl > 3 => lzl > 2**

$$\therefore \quad \frac{3}{|z|} < 1 \text{ and } \frac{2}{|z|} < 1$$

Then $f(z) = \frac{1}{2\,(z)(1+\frac{2}{z})} + \frac{3}{(z)(1-\frac{3}{z})}$

$= \frac{1}{2z}(1+\frac{2}{z})^{-1} + \frac{3}{z}(1-\frac{3}{z})^{-1}$

$= \frac{1}{2z}(1-\frac{2}{z}+(\frac{2}{z})^2-(\frac{2}{z})^3+--) + \frac{3}{z}(1+\frac{3}{z}+(\frac{3}{z})^2+(\frac{3}{z})^3+--)$

**f(z) =** $\frac{1}{2z}\sum_{n=0}^{\infty}(-1)^n(\frac{2}{z})^n + \frac{3}{z}\sum_{n=0}^{\infty}(\frac{3}{z})^n$

**6. Expand the function $f(z) = \frac{z+3}{z\,(z^2-z-2)}$ interms of powes z, when (i) lzl < 1 (ii) 1<lzl<2 (iii) lzl>2**

**Soln.**: Given function $f(z) = \frac{z+3}{z(z+1)(z-2)} = \frac{A}{z} + \frac{B}{(z+1)} + \frac{C}{(z-2)}$

$$f(z) = \frac{-\frac{3}{2}}{z} + \frac{\frac{2}{3}}{(z+1)} + \frac{\frac{5}{6}}{(z-2)}$$

$$f(z) = \frac{-3}{2z} + \frac{2}{3(z+1)} + \frac{5}{6(z-2)}$$

**(i) When lzl < 1 => lzl < 2**

**∴ lzl < 1 and $\frac{|z|}{2}$ < 1**

Then $f(z) = \frac{-3}{2z} + \frac{2}{3(1+z)} + \frac{5}{6(-2)\left(1-\frac{z}{2}\right)}$

$$= \frac{-3}{2z} + \frac{2}{3}(1+z)^{-1} - \frac{5}{12}\left(1-\frac{z}{2}\right)^{-1}$$

$$= \frac{-3}{2z} + \frac{2}{3}\left(1 - z + (z)^2 - (z)^3 + -\right) - \frac{5}{12}\left(1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + --\right)$$

$= \frac{-3}{2z} + \frac{2}{3}\sum_{n=0}^{\infty}(-1)^n (z)^n - \frac{5}{12}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$ Laurent's series

**(ii) When 1< lzl < 2 => lzl > 1 and lzl < 2**

**∴ $\frac{1}{|z|}$ < 1 and $\frac{|z|}{2}$ < 1**

Then $f(z) = \frac{-3}{2z} + \frac{2}{3\,(z)\left(1+\frac{1}{z}\right)} + \frac{5}{6(-2)\left(1-\frac{z}{2}\right)}$

$$= \frac{-3}{2z} + \frac{2}{3z}\left(1+\frac{1}{z}\right)^{-1} - \frac{5}{12}\left(1-\frac{z}{2}\right)^{-1}$$

$$= \frac{-3}{2z} + \frac{2}{3z}\left(1 - \frac{1}{z} + \left(\frac{1}{z}\right)^2 - \left(\frac{1}{z}\right)^3 + --\right) - \frac{5}{12}\left(1 + \frac{z}{2} + \left(\frac{z}{2}\right)^2 + \left(\frac{z}{2}\right)^3 + --\right)$$

**f(z) =** $\boldsymbol{\frac{-3}{2z} + \frac{2}{3z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{z}\right)^n - \frac{5}{12}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n}$

**(iii) When lzl > 2 => lzl > 1**

**∴ $\frac{2}{|z|}$ < 1 and $\frac{1}{|z|}$ < 1**

Then $f(z) = \frac{-3}{2z} + \frac{2}{3\,(z)(1+\frac{1}{z})} + \frac{5}{6(z)\left(1-\frac{2}{z}\right)}$

$$= \frac{-3}{2z} + \frac{2}{3z}\left(1+\frac{1}{z}\right)^{-1} + \frac{5}{6z}\left(1-\frac{2}{z}\right)^{-1}$$

$$= \frac{-3}{2z} + \frac{1}{2z}\left(1-\frac{1}{z} + \left(\frac{1}{z}\right)^2 - \left(\frac{1}{z}\right)^3 + --\right) + \frac{5}{6z}\left(1+\frac{2}{z} + \left(\frac{2}{z}\right)^2 + \left(\frac{2}{z}\right)^3 + --\right)$$

**f(z) =** $\frac{-3}{2z} + \frac{1}{2z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{z}\right)^n + \frac{5}{6z}\sum_{n=0}^{\infty}\left(\frac{2}{z}\right)^n$

# KLE'S G. I. BAGEWADI COLLEGE NIPANI
# DEPARTMENT OF MATHEMATICS

**B.Sc VI Semester 2020-21 Mathematics**

**Paper II: Complex Analysis and Ring Theory**

**Unit III: Taylor's and Maclaurin's Thm. and zeros and singularities**

**26th Class**

**Continued Examples on Taylor's and Maclaurin's Thm.**

**By**

**Dr. M. M. Shankrikopp**

**HOD of Mathematics**

**Date: 21.6.2021**

**7. Expand the function $f(z) = \frac{2z+1}{z(z-2)(2z-3)}$ interms of powers z, when (i) lzl < 3/2 (ii) 3/2<lzl<2 (iii) lzl>2** **[2017]**

**Soln.**: Given function $f(z) = \frac{2z+1}{z(z-2)(2z-3)} = \frac{A}{z} + \frac{B}{(z-2)} + \frac{C}{(2z-3)}$

$$= \frac{\frac{1}{6}}{z} + \frac{\frac{5}{2}}{(z-2)} + \frac{-\frac{16}{3}}{(2z-3)}$$

∴ $f(z) = \frac{1}{6z} - \frac{16}{3(2z-3)} + \frac{5}{2(z-2)}$

**(i) When lzl < 3/2 => lzl < 2**

∴ $\frac{2|z|}{3} < 1$ **and** $\frac{|z|}{2} < 1$

Then $f(z) = \frac{1}{6z} - \frac{16}{3(-3)(1-\frac{2z}{3})} + \frac{5}{2(-2)(1-\frac{z}{2})}$

$= \frac{1}{6z} + \frac{16}{9}\,(1-\frac{2z}{3})^{-1} - \frac{5}{4}\,(1-\frac{z}{2})^{-1}$

$= \frac{1}{6z} + \frac{16}{9}\,(1+\frac{2z}{3} + (\frac{2z}{3})^2 + (\frac{2z}{3})^3 + --) - \frac{5}{4}\,(1+\frac{z}{2} + (\frac{z}{2})^2 + (\frac{z}{2})^3 + --)$

$= \frac{1}{6z} + \frac{16}{9}\sum_{n=0}^{\infty}(\frac{2z}{3})^n - \frac{5}{4}\sum_{n=0}^{\infty}(\frac{z}{2})^n$ Laurent's series

**(ii) When 3/2< lzl < 2 => lzl > 3/2 and lzl < 2**

∴ $\frac{3}{2|z|} < 1$ **and** $\frac{|z|}{2} < 1$ $\left[ f(z) = \frac{1}{6z} - \frac{16}{3(2z-3)} + \frac{5}{2(z-2)} \right]$

Then $f(z) = \frac{1}{6z} - \frac{16}{3(2z)(1-\frac{3}{2z})} + \frac{5}{2(-2)(1-\frac{z}{2})}$

$= \frac{1}{6z} - \frac{8}{3z}\,(1-\frac{3}{2z})^{-1} - \frac{5}{4}\,(1-\frac{z}{2})^{-1}$

$= \frac{1}{6z} - \frac{8}{3z}\,(1+\frac{3}{2z} + (\frac{3}{2z})^2 + (\frac{3}{2z})^3 + --) - \frac{5}{4}\,(1+\frac{z}{2} + (\frac{z}{2})^2 + (\frac{z}{2})^3 + --)$

$\mathbf{f(z)} = \frac{1}{6z} + \frac{8}{3z}\sum_{n=0}^{\infty}(\frac{3}{2z})^n - \frac{5}{4}\sum_{n=0}^{\infty}(\frac{z}{2})^n$

**(iii) When lzl > 2 => lzl > 3/2**

∴ $\frac{2}{|z|} < 1$ **and** $\frac{3}{2|z|} < 1$ $\left[ f(z) = \frac{1}{6z} - \frac{16}{3(2z-3)} + \frac{5}{2(z-2)} \right]$

Then f(z) = $\frac{1}{6z}$ - $\frac{16}{3(2z)(1-\frac{3}{2z})}$ + $\frac{5}{2z(1-\frac{2}{z})}$

= $\frac{1}{6z}$ - $\frac{8}{3z}$ $(1-\frac{3}{2z})^{-1}$ + $\frac{5}{2z}$ $(1-\frac{2}{z})^{-1}$

= $\frac{1}{6z}$ - $\frac{8}{3z}$ $(1+\frac{3}{2z}+(\frac{3}{2z})^2+(\frac{3}{2z})^3+-)$ + $\frac{5}{2z}$ $(1+\frac{2}{z}+(\frac{2}{z})^2+(\frac{2}{z})^3+--)$

**f(z) =** $\frac{\mathbf{1}}{\mathbf{6z}}$ **-** $\frac{\mathbf{8}}{\mathbf{3z}}\sum_{n=0}^{\infty}(\frac{3}{2z})^n$ + $\frac{5}{2z}\sum_{n=0}^{\infty}(\frac{2}{z})^n$

**8. Expand the function f(z) = $\frac{1}{z(z^2+1)}$ interms of powes z, when**
**(i) 0<lzl < 1 (ii) (iii) lzl>1** **[2018]**

**Soln.:** Now f(z) = $\frac{1}{z(z^2+1)}$ = $\frac{A}{z}+\frac{Bz+C}{(z^2+1)}$

i.e $1 = 1(z^2+1) + (Bz+C)z$

Equating coeffts of $z^2$ and Z on both the sides we get, 1+B =0 & C=0
=>B= -1 and C =0

∴ f(z) = $\frac{1}{z(z^2+1)}$ = $\frac{1}{z}+\frac{(-1)z}{(z^2+1)}$

i.e f(z) = $\frac{1}{z}-\frac{z}{(z^2+1)}$

(i) If lzl<1 => $|z|^2<1$

$\therefore f(z) = \frac{1}{z}-\frac{z}{(z^2+1)}$

= $\frac{1}{z}-z(1+z^2)^{-1}$ = $\frac{1}{z}-z(1-z^2+z^4-+$-------------------)

= $\frac{1}{z}-(z-z^3+z^5-+$-------------------)

= $\frac{1}{z}-\sum_{n=0}^{\infty}(-1)^n(z)^{2n-1}$

(ii) If lzl>1 => $|z|^2>1$ => $\frac{1}{|z|^2}<1$

$\therefore f(z) = \frac{1}{z} - \frac{z}{z^2\,(1+\frac{1}{z^2})} = \frac{1}{z} - \frac{1}{z(1+\frac{1}{z^2})}$

$= \frac{1}{z} - \frac{1}{z}(1+\frac{1}{z^2})^{-1} = \frac{1}{z} - \frac{1}{z}(1 - \frac{1}{z^2} + \frac{1}{z^4}$--------------$)$

$= \sum_{n=1}^{\infty}(-1)^{n-1}(\frac{1}{z})^{2n+1}$

**9. Expand the function f(z) = $\frac{1}{z(z^2-1)}$ interms of powers z, when**
**(i) 0<lzl < 1 (ii) lzl>1** **(2017 Repeaters)**

**10. Expand the function f(z) = $\frac{1}{z(z-3)}$ interms of powers z-1, when**
**(i) 0<lz-1l < 1 (ii) 1<lz-1l < 2 (iii) lz-1l>2**

**Soln.:** Now f(z) = $\frac{1}{z(z-3)}$

Put z-1 =t => z = 1+t

$\therefore f(t) = \frac{1}{z(z-3)} = \frac{1}{(1+t)(t-2)} = \frac{A}{(t+1)} + \frac{B}{(t-2)}$

$= \frac{-\frac{1}{3}}{(t+1)} + \frac{\frac{1}{3}}{(t-2)} = \frac{-1}{3(t+1)} + \frac{1}{3(t-2)}$

(i) When 0<lz-1l < 1 i.e 0< ltl < 1 => ltl < 2

=> $\frac{|t|}{2} < 1$

Then $f(t) = \frac{-1}{3(1+t)} + \frac{1}{3(-2)\left(1-\frac{t}{2}\right)}$

$= \frac{-1}{3}(1+t)^{-1} - \frac{1}{6}(1-\frac{t}{2})^{-1}$

$= \frac{-1}{3}(1 - t + t^2 - t^3 +$ -----$) - \frac{1}{6}(1 + \frac{t}{2} + (\frac{t}{2})^2 +$---------$)$

$= \frac{-1}{3}\sum_{n=0}^{\infty}(-1)^n\,(t)^n - \frac{1}{6}\sum_{n=0}^{\infty}(\frac{t}{2})^n$

i.e f(z) = $\frac{-1}{3}\sum_{n=0}^{\infty}(-1)^n\,(z-1)^n - \frac{1}{6}\sum_{n=0}^{\infty}(\frac{z-1}{2})^n$

**(ii) 1<lz-1l < 2   i.e  1<ltl < 2**

**i.e   1<ltl   &   ltl < 2**

=> $\frac{1}{ltl}<1$   & $\frac{ltl}{2}<1$

[ $f(t) = \frac{-1}{3(t+1)} + \frac{1}{3(t-2)}$ ]

$\therefore\ f(t) = \frac{-1}{3t\left(1+\frac{1}{t}\right)} + \frac{1}{3(-2)\left(1-\frac{t}{2}\right)}$

$= \frac{-1}{3t}(1+\frac{1}{t})^{-1} - \frac{1}{6}(1-\frac{t}{2})^{-1}$

$= \frac{-1}{3t}\sum_{n=0}^{\infty}(-1)^n(\frac{1}{t})^n - \frac{1}{6}\sum_{n=0}^{\infty}(\frac{t}{2})^n$

i.e f(z) = $\frac{-1}{3(z-1)}\sum_{n=0}^{\infty}(-1)^n(\frac{1}{z-1})^n - \frac{1}{6}\sum_{n=0}^{\infty}(\frac{z-1}{2})^n$

**(iii) When lz-1l>2  i.e ltl>2  => ltl>1**

=> $\frac{2}{ltl}<1$ and $\frac{1}{ltl}<1$

[ $f(t) = \frac{-1}{3(t+1)} + \frac{1}{3(t-2)}$ ]

$\therefore\ f(t) = \frac{-1}{3t\left(1+\frac{1}{t}\right)} + \frac{1}{3(t)\left(1-\frac{2}{t}\right)}$

$= \frac{-1}{3t}(1+\frac{1}{t})^{-1} + \frac{1}{3t}(1-\frac{2}{t})^{-1}$

$= \frac{-1}{3t}\sum_{n=0}^{\infty}(-1)^n(\frac{1}{t})^n + \frac{1}{3t}\sum_{n=0}^{\infty}(\frac{2}{t})^n$

i.e f(z) = $\frac{-1}{3(z-1)}\sum_{n=0}^{\infty}(-1)^n(\frac{1}{z-1})^n + \frac{1}{3(z-1)}\sum_{n=0}^{\infty}(\frac{2}{(z-1)})^n$

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**G.I.Bagewadi Arts, Science & Commerce College, Nipani**
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**DEPARTMENT OF MATHEMATICS**

**B. Sc. III Sem. 2020-21**

# Paper I

# Mathematical Logic and Real Analysis

## Revision i.e Bridge Course

## Function of independent variable:

**If x is a variable choosing independently and u is a function of x i.e**

**u = f(x) then u is called function of independent variable x and in this case differentiation of u w.r.t x is $\frac{du}{dx}$ , ordinary differentiation.**

**Similarly if x and y are independent variables and u = f(x, y) then u is function of independent variables x and yand in this case differentiation of u w.r.t x is $\frac{\partial u}{\partial x}$ & differentiation of u w.r.t y is $\frac{\partial u}{\partial y}$ which are partial differentiations.**

**For example: If u = $x^2$ then u is function of single independent variable x and if u = $x^2 + y^2$ then u is function of two independent variables x and y.**

## Explicit and Implicit function:

**(i) Explicit function: If u and v are directly function of x and y then u is called explicit function of x and y .**
**i.e u= u(x,y) and v= v(x,y) are explicit functions of x and y.**

**For example: u = x+y and v=xy then u and v are explicit functions of x and y.**

**Partial differentiation in case of explicit function:**

**If u= u(x, y) and v = v(x, y) explicit functions of independent variables x and y then** $\frac{\partial u}{\partial x} = \frac{du}{dx}$ **(treating y constant) and** $\frac{\partial u}{\partial y} = \frac{du}{dy}$ **(treating x constant).**

**(ii) Implicit function :If u and v are not directly functions of x and y , i.eif u, v, x, y are all interrelated then it is called implicitfunction.i.e f(x,y,u,v)=0**

**For example: u +v –x-y+z=0 ,uv = y+z , uvw =z are examples of implicit function.**

**Partial differentiation in case of implicit function:**

**If x and y are two variables and u, v are implicit functions of x and y ,i.e F(u,v, x,y) =0 be implicit function then differentiation of F w.r.t x is**

$$\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial x}+\frac{\partial F}{\partial x}\cdot\frac{\partial x}{\partial x}+\frac{\partial F}{\partial y}\cdot\frac{\partial y}{\partial x}=0$$

**i.e** $\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial x}+\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}\cdot\frac{\partial y}{\partial x}=0$ **---------(1)**

**If x and y are independent then** $\frac{\partial y}{\partial x}=0$ **and hence (1) becomes**

$$\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial x}+\frac{\partial F}{\partial x}=0$$

**Similarlydifferentiation of F w.r.t y is**

$$\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial y}+\frac{\partial F}{\partial y}=0$$

**For example: If $u^2 + v^2 = x^2 + y^2$, then it is written as $F = u^2 + v^2 - x^2 - y^2 = 0$**

**Then differentiation of F w.r.t x is** $\frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial F}{\partial v} \cdot \frac{\partial v}{\partial x} + \frac{\partial F}{\partial x} \quad = 0$

**i.e** $2u \cdot \frac{\partial u}{\partial x} + 2v \cdot \frac{\partial v}{\partial x} + -2x \quad = 0$ **i.e** $u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} - x \quad = 0$

**and** $u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y} - y \quad = 0$

## Chain Rule for partial differentiation:

**If u is a function of x and y and x, y are functions of t then**

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}$$

**Or** $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$

**Similarly for three variables, If u is a function of x , y and z and x, y and z are functions of t then**

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t}$$

**Example: $x = r \sin\theta \cos\emptyset$, $y = r \sin\theta\, sin\emptyset$ and $z = r\cos\theta$**

**Then find** $\frac{\partial x}{\partial r}, \frac{\partial x}{\partial \theta}, \frac{\partial x}{\partial \emptyset}, \frac{\partial y}{\partial r}, \frac{\partial y}{\partial \theta}, \frac{\partial y}{\partial \emptyset}, \frac{\partial z}{\partial r}, \frac{\partial z}{\partial \theta}, \frac{\partial z}{\partial \emptyset}.$

# Unit II

# Real Analysis I

# Jacobian of order 2 and 3,

# Lagrange’s Mean Value Theorem and Tayler’s Theorem

## Contents of this unit

- **Introduction**
- **Definition**
- **Theorems on Jacobian (Properties) andExamples**
- **Lagrange's Mean Value Theorem, Taylor's and Maclaurin's Theorem for function of two variables andExamples**

**Introduction:**

**We already studied maxima, minima, Taylor's series, Maclurin's series and so on for function of one variable, in this unit we are studying these properties for function of more than one variables. An important application of partial differentiation is finding maxima and minima of curves and surface. In many applied problems of science and engineering a function of two or more variables has to be optimized, which can be handled by Lagrange's method of undetermined multipliers.**

# I. Jacobian:

**The Jacobian Marix was first developed by Jacobi (1804-1851)a German Mathematician. The Jacobian matrix is defined as a matrix whose entries are first order partial derivatives of a real n valued function with respect to other m variables where all these functions are continuous and differentiable. If m=n then matrix is square matrix and determinant of this matrix is called Jacobian determinant or simply Jacobian.**

**Jacobians are very much useful in multiple integrals in calculating area and volume, in solving system of linear equations, it is used for determining probability distribution of variable y.**

**Definition of Jacobian of order Two[2017]: If u and v are functions of two independent variables x and y with continuous partial derivatives**

$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ **then the determinant denoted by**

$J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$ **is called Jacobian or Jacobian determinant of u and v with respect to x and y and is denoted by** $\frac{\partial(u,v)}{\partial(x,y)} = J$

**Thus** $J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$ **is Jacobian of order 2.**

**Similarly we define Jacobian of order 3[2017]:**

**If u, v and w are functions of three independent variables x , y and z with continuous partial derivatives** $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial z}, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial z},$ **then the determinant denoted by**

$J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}, \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial y} \end{vmatrix}$ **is called Jacobian of u, v and w with respect to x , y and z is denoted by** $\frac{\partial(u,v,w)}{\partial(x,y,z)} = J$

**Similarly Jacobian of order n, if** $f_1, f_2,$ **-----------**$f_n$**are functions of independent variables** $x_1, x_2,$ **--------------**$x_n$**then Jacobian of** $f_1, f_2,$ **------**

**-----$f_n$ w.r.t $x_1$, $x_2$, --------------$x_n$ be** **J =**

$$\frac{\partial(f1,f2,-----------fn)}{\partial(x1,x2,--------------xn)} = \begin{vmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & ------ & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_1} & & \frac{\partial f_2}{\partial x_1} \frac{\partial f_2}{\partial x_1} & \\ & & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & & \frac{\partial f_n}{\partial x_n} & \end{vmatrix}$$

**Examples:**

**1. If u = xyand v = x+y then find $\frac{\partial(u,v)}{\partial(x,y)}$.**

**Soln: Now u = xyand v = x+y**

**Here** $\frac{\partial u}{\partial x} = y$, $\frac{\partial u}{\partial y} = x$, $\frac{\partial v}{\partial x} = 1$, $\frac{\partial v}{\partial y} = 1$

**Then Jacobian** $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} y & x \\ 1 & 1 \end{vmatrix} = y - x$

**2. If u = $x^2 + y^2$ and v = 2xy then find $\frac{\partial(u,v)}{\partial(x,y)}$.**

**Soln: Now u = $x^2 + y^2$ and v = 2xy**

**Here** $\frac{\partial u}{\partial x} = 2x$, $\frac{\partial u}{\partial y} = 2y$, $\frac{\partial v}{\partial x} = 2y$, $\frac{\partial v}{\partial y} = 2x$

**Then Jacobian** $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x & 2y \\ 2y & 2x \end{vmatrix} = 4(x^2 - y^2)$

**3. If u = x+y+z and v = xy+yz+zx, w = $x^2+y^2+z^2$ then P.T $\frac{\partial(u,v,w)}{\partial(x,y,z)}$=0**

**[2015]**

**Soln: Nowu = x+y+zand v = xy+yz+zx, w = $x^2+y^2+z^2$**

**Then** $\frac{\partial u}{\partial x} = 1$, $\frac{\partial u}{\partial y} = 1$, $\frac{\partial u}{\partial z} = 1$

$\frac{\partial v}{\partial x} = y+z$, $\frac{\partial v}{\partial y} = x+z$, $\frac{\partial v}{\partial z} = x+y$

$\frac{\partial w}{\partial x} = 2x$, $\frac{\partial w}{\partial y} = 2y$, $\frac{\partial w}{\partial z} = 2z$

**Then by definition,**

**Jacobian J =** $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}, \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ y+z & x+z & x+y \\ 2x & 2y & 2z \end{vmatrix}$

$=1[(x+z)2z-(x+y)2y]-1[(y+z)2z-(x+y)(2x)]+1[(y+z)2y-(x+z)2x]$

$= 2\{[zx + z^2 - xy - y^2] - [yz+z^2-x^2-xy]+[y^2+yz-x^2- xz]\}$

$= 2\{z^2 - y^2 - z^2 + x^2 + y^2 - x^2 + zx - xy - yz + xy + yz - xz\}$

**=0**

**Otherwise, use soln of determinant studied in B.Sc I sem.**

**i.e J =** $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}, \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ y+z & x+z & x+y \\ 2x & 2y & 2z \end{vmatrix}$

$$= 2\begin{vmatrix} 1 & 1 & 1 \\ y+z & x+z & x+y \\ x & y & z \end{vmatrix}$$

**$C_2=C_2 – C, C_3=C_3-C_1$**

$$= 2\begin{vmatrix} 1 & 0 & 0 \\ y+z & x-y & x-z \\ x & y-x & z-x \end{vmatrix}$$

$$= 2(x-y)(x-z)\begin{vmatrix} 1 & 0 & 0 \\ y+z & 1 & 1 \\ x & -1 & -1 \end{vmatrix}$$

**= 0 as 2$^{nd}$ and 3$^{rd}$ columns are identical.**

***4. If then find Jacobian***

$$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}$$

**Then find** $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)}$

**Soln.:Now x = r sin $\theta$ cos $\emptyset$ , y = r sin $\theta$ $sin\emptyset$ and z = r cos $\theta$**

**They all continuous functions**

**By definition**

$$J= \frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \emptyset} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \emptyset} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \emptyset} \end{vmatrix}$$

$$= \begin{vmatrix} \sin\theta\cos\emptyset & r\cos\theta\cos\emptyset & r\sin\theta\,(-\sin\emptyset) \\ \sin\theta\sin\emptyset & r\cos\theta\, sin\emptyset & r\sin\theta\cos\emptyset \\ \cos\theta & -r\sin\theta & 0 \end{vmatrix}$$

$= \sin\theta\cos\emptyset(0 + r^2\sin^2\theta\cos\emptyset) - r\cos\theta\cos\emptyset(0 - r\sin\theta\cos\emptyset\cos\theta)$

$+ r\sin\theta(-\sin\emptyset)[\sin\theta\sin\emptyset(-r\sin\theta) - r\cos\theta\, sin\emptyset(\cos\theta)]$

$= r^2\sin^3\theta\cos^2\emptyset + r^2\sin\theta\cos^2\emptyset\cos^2\theta + r^2\sin^3\theta\sin^2\emptyset + r^2\sin\theta\sin^2\emptyset\cos^2\theta$

$= r^2\sin\theta[\sin^2\theta\cos^2\emptyset + \cos^2\emptyset\cos^2\theta + \sin^2\theta\sin^2\emptyset + \sin^2\emptyset\cos^2\theta]$

$= r^2\sin\theta[\cos^2\emptyset(\sin^2\theta + \cos^2\theta) + \sin^2\emptyset(\sin^2\theta + \cos^2\theta)]$

$= r^2\sin\theta[\cos^2\emptyset(1) + \sin^2\emptyset(1)] =$ $\mathbf{r^2\sin\theta}$

It is transformation of Cartesian plane to sphere surface.

**5.If u = cosx ,v =sinxcosy, w= sinxsinycoszthen show that**

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3\sin^3x\sin^2y\sin z$$

**Soln.: Now u = cosx ,v =sinxcosy, w= sinxsinycosz**

**We have Jacobian** $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}, \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$

$$= \begin{vmatrix} -sinx & 0 & 0 \\ cosx\ cosy & -sinx\ siny & 0 \\ cosx\ siny\ cosz & sinx\ cosy\ cosz & -sinx\ siny\ sinz \end{vmatrix}$$

$$= sinx\ siny\ sinz \begin{vmatrix} -sinx & 0 & 0 \\ cosx\ cosy & -sinx\ siny & 0 \\ cosx\ siny\ cosz & sinx\ cosy\ cosz & -1 \end{vmatrix}$$

$$=\sin x \sin y \sin z \begin{vmatrix} -\sin x & 0 & 0 \\ \cos x \cos y & -\sin x \sin y & 0 \\ \cos x \sin y \cos z & \sin x \cos y \cos z & -1 \end{vmatrix}$$

$$= \sin^2 x \sin y \sin z \begin{vmatrix} -\sin x & 0 & 0 \\ \cos x \cos y & -\sin y & 0 \\ \cos x \sin y \sin z & \cos y \sin z & -1 \end{vmatrix}$$

$$= \sin^3 x \sin y \sin z \begin{vmatrix} -1 & 0 & 0 \\ \cos x \cos y & -\sin y & 0 \\ \cos x \sin y \sin z & \cos y \sin z & -1 \end{vmatrix}$$

$$= \sin^3 x \sin y \sin z\{ -1(\sin y)\} = (-1)^3 \sin^3 x \sin^2 y \sin z$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3 \sin^3 x \sin^2 y \sin z$$

6. If $u = x+y$ and $v = x^2+y^2$ , find $\frac{\partial(u,v)}{\partial(x,y)}$

**Soln:** **Now** $u = x+y$ **and** $v = x^2+y^2$

**Here** $\frac{\partial u}{\partial x} = 1, \quad \frac{\partial u}{\partial y} = 1, \quad \frac{\partial v}{\partial x} = 2x, \quad \frac{\partial v}{\partial y} = 2y$

**Then Jacobian** $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 2x & 2y \end{vmatrix} = 2(y - x)$

## Home work Examples:

1. **If u = $e^x$cosy and v = $e^x$siny then find Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$.**
2. **If u = sinxcoshyand v = cosxsinhy then find $\frac{\partial(u,v)}{\partial(x,y)}$.**
3. **If x = r sin∅**, y = **r cos∅**, z=z, then find $\frac{\partial(x,y,z)}{\partial(r,\emptyset,z)}$.
4. **If u = x+3$y^2$-$z^3$, v = 4$x^2$yz, w =2$z^2$- xy then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1,-1,0).**
5. **If x = r sin∅**, y = **r cos∅** then find $\frac{\partial(x,y)}{\partial(r,\emptyset)}$.**[2016, 2018, 2019]**
6. **If u = $\frac{y^2}{2x}$ , v = $\frac{x^2+y^2}{2x}$**then find $\frac{\partial(u,v)}{\partial(x,y)}$.
7. **If u =$\frac{yz}{x}$ , v =$\frac{zx}{y}$, w = $\frac{xy}{z}$**then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.
8. **If x =u(1+v) and y=v(1+u) then find the Jacobian of x, y w.r.t u and v.**
9. **If u =$\sqrt{x^2+y^2}$ and v =$\tan^{-1}\left(\frac{x}{y}\right)$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.**
10. **If u = x+yand v = $\frac{y}{x+y}$then find Jacobian$\frac{\partial(u,v)}{\partial(x,y)}$.[2016]**
11. **If u = x+$\frac{y^2}{x}$and v = $\frac{y^2}{x}$then find Jacobian$\frac{\partial(u,v)}{\partial(x,y)}$.[2017]**
12. **If u = xyzand v = $xy+yz+zx$ and $w = x+y+z$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ = (x-y)(y-z)(z-x)[2017 for 5 marks]**
13. **If u = xyzand v =$x^3$ - $y^3$+$z^2$ and $w = x^2+yz$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$[2018 for 5 marks]**
14. If u = x+y+z, v = x-y+z, w = $x^2$+$y^2$+$z^2$+2zx then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

# Properties of Jacobians

**Property1: If u and v are functions of x and y and x and y are functions of r and s then** $\frac{\partial(u,v)}{\partial(r,s)} = \frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(r,s)}$.

**Proof:** Since u and v are functions of x and y , x and y are functions of r and s, then by Chain Rule for two variable we have

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r}, \qquad \frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s}$$

**And** $$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r}, \qquad \frac{\partial v}{\partial s} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial s}$$

**By definition of Jacobian we have**

$$\frac{\partial(u,v)}{\partial(r,s)} = \begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial s} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial s} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} & \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} \\ \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r} & \frac{\partial v}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial s} \end{vmatrix} \text{ by chain Rule}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial s} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial s} \end{vmatrix} \because \begin{vmatrix} a_1 & a_2 \\ a_3 & a_4 \end{vmatrix} \begin{vmatrix} b_1 & b_2 \\ b_3 & b_4 \end{vmatrix} = \begin{vmatrix} a_1 b_1 + a_2 b_3 & a_1 b_2 + a_2 b_4 \\ a_3 b_1 + a_4 b_3 & a_3 b_2 + a_4 b_4 \end{vmatrix}$$

$$\boxed{\frac{\partial(u,v)}{\partial(r,s)} = \frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(r,s)}}$$

**Corollary: If u=r and v=s then by above theorem we have** $\frac{\partial(r,s)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(r,s)} = \frac{\partial(r,s)}{\partial(r,s)} = 1$

## That is next property.

**Note: We can generalize above theorem to three variables**

i.e $\frac{\partial(u,v,w)}{\partial(r,s,t)} = \frac{\partial(u,v,w)}{\partial(x,y,z)}\frac{\partial(x,y,z)}{\partial(r,s,t)}$

**Property 2: If u and v are functions of x, y then prove that** $\frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(u,v)} = 1.$

**OR  If J =** $\frac{\partial(u,v)}{\partial(x,y)}$ **and** $J^{l} = \frac{\partial(x,y)}{\partial(u,v)}$ **then** $J\,J^{l} = 1$

**Proof:** Now u and v are functions of x, y => x, y be functions of u,v

Let u = u(x,y)  and v = v(x, y)

Differentiate both partially w.r.t u andv on both sides we get

$$\left.\begin{aligned} 1 &= \frac{\partial u}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial u}, \qquad 0 = \frac{\partial v}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial u} \\ 0 &= \frac{\partial u}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial v}, \qquad 1 = \frac{\partial v}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial v} \end{aligned}\right\} \text{------(a)}$$

Then by  property-1 we have

$$\frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial u} & \frac{\partial u}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial v} \\ \frac{\partial v}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial u} & \frac{\partial v}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \text{from (a)}$$

$$= 1$$

**Thus** $\frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(u,v)} = 1$ **[it is similar to** $\frac{dy}{dx}.\frac{dx}{dy} = 1$**]**

**Property 3: Jacobian of implicit functions**

**Statement: If F(u,v, x, y) =0 and G(u,v, x, y) =0 be two implicit functions of independent variables x and y ( ie. if u and v are implicit functions of x and y) then prove that**

$$\frac{\partial(u,v)}{\partial(x,y)} = (-1)^2 \frac{\left[\frac{\partial(F,G)}{\partial(x,y)}\right]}{\left[\frac{\partial(F,G)}{\partial(u,v)}\right]}$$

**Proof: Given that u and v are implicit functions of x and y**

$$\left.\begin{aligned} &\text{i.e } F(u,v,x,y)=0 \\ &G(u,v,x,y)=0 \end{aligned}\right\} \quad \text{-------(1)}$$

Differentiating both expressions of (1) partially w.r.t. x and y, we get

$$\left.\begin{aligned}
\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial x}+\frac{\partial F}{\partial x} &= 0\\
\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial y}+\frac{\partial F}{\partial y} &= 0\\
\frac{\partial G}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial G}{\partial v}\cdot\frac{\partial v}{\partial x}+\frac{\partial G}{\partial x} &= 0\\
\frac{\partial G}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial G}{\partial v}\cdot\frac{\partial v}{\partial y}+\frac{\partial G}{\partial y} &= 0
\end{aligned}\right\} \quad \text{------(2)}$$

**Now consider**

$$\frac{\partial(F,G)}{\partial(u,v)}\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{vmatrix} \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

**By using product of determinants**

$$= \begin{vmatrix} \frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial x} & \frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial y} \\ \frac{\partial G}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial G}{\partial v}\cdot\frac{\partial v}{\partial x} & \frac{\partial G}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial G}{\partial v}\cdot\frac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} -\frac{\partial F}{\partial x} & -\frac{\partial F}{\partial y} \\ -\frac{\partial G}{\partial x} & -\frac{\partial G}{\partial y} \end{vmatrix} \quad \textbf{from (2)}$$

$$= (-1)^2 \begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix} = (-1)^2 \frac{\partial(F,G)}{\partial(x,y)}$$

**Thus** $\frac{\partial(F,G)}{\partial(u,v)} \frac{\partial(u,v)}{\partial(x,y)} = (-1)^2 \frac{\partial(F,G)}{\partial(x,y)}$

$$\Rightarrow \frac{\partial(u,v)}{\partial(x,y)} = (-1)^2 \frac{\left[\frac{\partial(F,G)}{\partial(x,y)}\right]}{\left[\frac{\partial(F,G)}{\partial(u,v)}\right]}$$

**Corollary:** Generalizing the above result for three variables, ie if F(u,v,w,x,y,z) = 0, G(u,v,w,x,y,z) = 0 and H(u,v,w,x,y,z) = 0 be implicit functions of independent variables x,y, z then by above property we have

$$\Rightarrow \frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3 \frac{\left[\frac{\partial(F,G,H)}{\partial(x,y,z)}\right]}{\left[\frac{\partial(F,G,H)}{\partial(u,v,w)}\right]}$$

**Similarly if x,y,z are implicit functions of u,v and w**

**then** $\frac{\partial(x,y,z)}{\partial(u,v,w)} = (-1)^3 \frac{[\frac{\partial(F,G,H)}{\partial(u,v,w)}]}{[\frac{\partial(F,G,H)}{\partial(x,y,z)}]}$

# EXAMPLES

**1. If x = u(1-v), y = uv then find J =** $\frac{\partial(u,v)}{\partial(x,y)}$ **and** $J^1=\frac{\partial(x,y)}{\partial(u,v)}$

**Soln:** **Now x = u(1-v), y = uv -------(1)**

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} 1-v & -u \\ v & u \end{vmatrix}$$

**= u(1-v) + uv = u**

**Next we have to get u and v in terms of x and y**

**Fromm (1) x = u –uv and y = uv**

⇨ **x = u – y ∴ u = x+y and v =** $\frac{y}{u} = \frac{y}{x+y}$

**i.e u = x+y and v =** $\frac{y}{x+y}$

Then $J^{l} = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$

$$= \begin{vmatrix} 1 & 1 \\ \frac{-y}{(x+y)^2} & \frac{(x+y)1-y}{(x+y)^2} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ \frac{-y}{(x+y)^2} & \frac{x}{(x+y)^2} \end{vmatrix}$$

$$= \frac{x}{(x+y)^2} + \frac{y}{(x+y)^2}$$

$$= \frac{x+y}{(x+y)^2} = \frac{1}{x+y}$$

$$= \frac{1}{u}$$

$\therefore J^{l} = \frac{\partial(u,v)}{\partial(x,y)} = \frac{1}{u}$

**Thus** $J\,J^{l} = u\,\frac{1}{u} = 1$

**2. If u =2xy , v =$x^2 - y^2$, x = r$\cos\theta$ and y =r$\sin\theta$ then find $\frac{\partial(u,v)}{\partial(r,\theta)}$**

**Soln.:** Now u =2xy , v =$x^2 - y^2$, x = r$\cos\theta$ and y =r$\sin\theta$

**i.e u and v are Composite functions of $r, \theta$.**

$$\therefore \frac{\partial(u,v)}{\partial(r,\theta)} = \frac{\partial(u,v)}{\partial(x,y)} \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix}$$

$$= \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$= -4\,(x^2+y^2)r(\cos^2\theta + \sin^2\theta) = -4r(x^2+y^2)$

**3. If $u^2 - v^2 + x^2 + y^2 = 0$ , $uv + xy = 0$ then prove that**

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{x^2 - y^2}{u^2 + v^2}$$

Soln.:Given $u^2 - v^2 + x^2 + y^2 = 0$, $uv + xy = 0$

Let $F = u^2 - v^2 + x^2 + y^2 = 0$

& $G = uv + xy = 0$

We have $\frac{\partial(u,v)}{\partial(x,y)} = (-1)^2 \frac{\left[\frac{\partial(F,G)}{\partial(x,y)}\right]}{\left[\frac{\partial(F,G)}{\partial(u,v)}\right]}$ ---------(1)

Now $\frac{\partial(F,G)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix} = \begin{vmatrix} 2x & 2y \\ y & x \end{vmatrix} = 2(x^2 - y^2)$

And $\frac{\partial(F,G)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{vmatrix} = \begin{vmatrix} 2u & -2v \\ v & u \end{vmatrix} = 2(u^2+v^2)$

$\therefore$ from (1) , $\frac{\partial(u,v)}{\partial(x,y)} = (-1)^2 \frac{2(x^2 - y^2)}{2(u^2 + v^2)} = \frac{x^2 - y^2}{u^2 + v^2}$

$i.e\ \frac{\partial(u,v)}{\partial(x,y)} = \frac{x^2 - y^2}{u^2 + v^2}$

**4. If $u^3+v^3+w^3 = x+y+z$, $u^2+v^2+w^2 = x^3+y^3+z^3$ and $u+v+w = x^2+y^2+z^2$ then prove that**

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$$

**Proof:** Now $u^3+v^3+w^3 = x+y+z$, $u^2+v^2+w^2 = x^3+y^3+z^3$ and $u+v+w = x^2+y^2+z^2$ which are implicit functions.

Let $F = u^3+v^3+w^3 - x - y - z = 0$

$G = u^2+v^2+w^2- x^3-y^3-z^3=0$ ----------(1)

$H = u+v+w- x^2-y^2-z^2 = 0$

By using one of the property we have

$$\Rightarrow \frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3 \frac{\left[\frac{\partial(F,G,H)}{\partial(x,y,z)}\right]}{\left[\frac{\partial(F,G,H)}{\partial(u,v,w)}\right]} \text{ ------------------(p)}$$

Now $\frac{\partial(F,G,H)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z}, \\ \frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} & \frac{\partial H}{\partial z} \end{vmatrix} = \begin{vmatrix} -1 & -1 & -1 \\ -3x^2 & -3y^2 & -3z^2 \\ -2x & -2y & -2z \end{vmatrix}$

$$= -(-3)(-2)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x & y & z \end{vmatrix}$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get

$$= -6\begin{vmatrix} 1 & 0 & 0 \\ x^2 & y^2-x^2 & z^2-x^2 \\ x & y-x & z-x \end{vmatrix}$$

$$= -6\,(y-x)(z-x)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & y+x & z+x \\ x & 1 & 1 \end{vmatrix}$$

$= -6\,(y-x)(z-x)[\,1\{y+x-z-x\}]$

$= -6(y-x)(z-x)\,(y-z)$

$= 6(x-y)\,)\,(y-z)\,(z-x)$---------------------------------(2)

Next, $\frac{\partial(F,G,H)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} & \frac{\partial F}{\partial w} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} & \frac{\partial G}{\partial w} \\ \frac{\partial H}{\partial u} & \frac{\partial H}{\partial v} & \frac{\partial H}{\partial w} \end{vmatrix} =$

$$\begin{vmatrix} 3u^2 & 3v^2 & 3w^2 \\ 2u & 2v & 2w \\ 1 & 1 & 1 \end{vmatrix}$$

$= 6\begin{vmatrix} u^2 & v^2 & w^2 \\ u & v & w \\ 1 & 1 & 1 \end{vmatrix} = 6\begin{vmatrix} u^2 & v^2-u^2 & w^2-u^2 \\ u & v-u & w-u \\ 1 & 0 & 0 \end{vmatrix}$

$= 6\,(v-u)(\,w-u)\begin{vmatrix} u^2 & v+u & w+u \\ u & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$

$= 6\,(v-u)(\,w-u)[u^2\,(0) - (v+u)(0-1) + (w+u)(0-1)]$

$= 6\,(v-u)(\,w-u)[\ (v+u) - w-u)]$

$= 6(v-u)(\,w-u)(v-w)=$

$= -\ 6(u-v)(v-w)(w-u)$-------------------------(3)

From (2) and (3) , Eqn.(p) becomes

$\therefore \frac{\partial(u,v,w)}{\partial(x,y,z)} = (-1)^3 \frac{6(x-y))\ (y-z)\ (z-x)}{-\ 6(u-v)(v-w)(w-u)}$

i.e $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y))\ (y-z)\ (z-x)}{(u-v)(v-w)(w-u)}$

3. If x+y+z = u, y+z=uv and z= uvwthen show that

$\frac{\partial(x,y,z)}{\partial(u,v,w)}$ =$u^2v$

Soln: Now x+y+z = u, y+z=uv and z= uvw

i.ex,y,z are implicit functions of u, v, w

Let F = x+y+z – u =0

G = y+z-uv=0, H = z-uvw =0

We have $\frac{\partial(F,G,H)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z}, \\ \frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} & \frac{\partial H}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1$

And $\frac{\partial(F,G,H)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} & \frac{\partial F}{\partial w} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} & \frac{\partial G}{\partial w} \\ \frac{\partial H}{\partial u} & \frac{\partial H}{\partial v} & \frac{\partial H}{\partial w} \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 \\ -v & -u & 0 \\ -vw & -uw & -uv \end{vmatrix}$

$= -1(u^2v) = -u^2v$

We have $\frac{\partial(x,y,z)}{\partial(u,v,w)} = (-1)^3 \frac{\left(\frac{\partial(F,G,H)}{\partial(u,v,w)}\right)}{\left(\frac{\partial(F,G,H)}{\partial(x,y,z)}\right)} = -1\frac{-u^2v}{1} = u^2v$

**4.If x =u+v+w, y = uv+vw+wu, z=uvw and F =f(x,y,z) is a differentiable function of x,y,z then prove that**

$u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v} + w\frac{\partial F}{\partial w} = x\frac{\partial F}{\partial x} + 2y\frac{\partial F}{\partial y} + 3z\frac{\partial F}{\partial z}$.

**Soln.:** Nowx =u+v+w, y = uv+vw+wu, z=uvw and F =f(x,y,z)

Then $\frac{\partial F}{\partial u} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial u} = \frac{\partial F}{\partial x}1 + \frac{\partial F}{\partial x}(v+w) + \frac{\partial F}{\partial z}vw$

i.e $\frac{\partial F}{\partial u} = \frac{\partial F}{\partial x} + (v+w)\frac{\partial F}{\partial y} + vw\frac{\partial F}{\partial z}$

Then $u\frac{\partial F}{\partial u} = u\frac{\partial F}{\partial x} + (uv+uw)\frac{\partial F}{\partial y} + uvw\frac{\partial F}{\partial z}$

Similarly $v\frac{\partial F}{\partial v} = v\frac{\partial F}{\partial x} + (vw+vu)\frac{\partial F}{\partial y} + uvw\frac{\partial F}{\partial z}$

And $w\frac{\partial F}{\partial w} = w\frac{\partial F}{\partial x} + (wv+wu)\frac{\partial F}{\partial y} + uvw\frac{\partial F}{\partial z}$

**Adding all these we get**

$$u\frac{\partial F}{\partial u} + v\frac{\partial F}{\partial v} + w\frac{\partial F}{\partial w} = (u+v+w)\frac{\partial F}{\partial x} + 2(uv+uw+vw)\frac{\partial F}{\partial y} + 3uvw\frac{\partial F}{\partial z}$$

$$= x\frac{\partial F}{\partial x} + 2y\frac{\partial F}{\partial y} + 3z\frac{\partial F}{\partial z}$$

# Home work Examples:

1. If u =2xy, v =$x^2$-$y^2$ and x =$r^2\cos2\theta$,y = $r^2\sin2\theta$ then valuate $\frac{\partial(u,v)}{\partial(r,\theta)}$
2. If x =a(u+v), y =b(u-v) and u =$r^2\cos2\theta$,v = $r^2\sin2\theta$ then valuate $\frac{\partial(x,y)}{\partial(r,\theta)}$
3. If u = x+y+z, vu =$x^2+y^2+z^2$, uvw = xy+yz+zx then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

# Dependent and Independent functions:

**Definition:** **Functions u=f(x,y) and v =g(x,y) are said to be dependent ( or functionally dependent) if u and v are related by the equation F(u v) = 0 , i.e x and y are removed in terms of u and v and get equation interms of u and v ,i.e F(u,v) =0.**

**For example:** **Let u = $\frac{x}{y}$ and v = $\frac{x+y}{x-y}$ then u and v are dependent.**

$$\text{Bec's } v = \frac{x+y}{x-y} = \frac{\frac{x}{y}+1}{\frac{x}{y}-1} = \frac{u+1}{u-1}$$

**i.e v = $\frac{u+1}{u-1}$**

**=> v(u-1) = u+1 or v(u-1) – (u+1) =0**

**i.e F(u, v) =0**

**=> u and v are dependent.**

**Note:** **Determining whether given functions dependent or independent by using above method as in example is tedious , so in such case we can apply Jacobian and easily determine.**

**Theorem:** **If functions u=f(x,y) and v=g(x,y) are dependent then**

$\frac{\partial(u,v)}{\partial(x,y)} = 0.$

**Proof:** Let **u=f(x,y) and v=g(x,y) be dependent then F(u,v) =0**

**Differentiate this partially w.r.t. x and y we get,**

$$\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial x}=0 \quad \text{--------------(1)}$$
$$\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial y}=0 \quad \text{------------(2)}$$

**This system of equation has non trivial solution $\frac{\partial F}{\partial u} \neq 0$ and $\frac{\partial F}{\partial v} \neq 0$ if the determinant of coefficient matrix is zero.**

In B.Sc. II sem. we studied system of equations $a_1x_1 + a_2x_2 = 0$ and $b_1x_1 + b_2x_2 = 0$ has non trivial solution ( i.e at least one $x_i \neq 0$) if determinant $\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = 0$

**i.e** $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$

Change rows to columns and columns to rows we get

i.e $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$

i.e $\frac{\partial(u,v)}{\partial(x,y)} = 0.$

**Thus given functions are dependent if $\frac{\partial(u,v)}{\partial(x,y)} = 0$ and independent if $\frac{\partial(u,v)}{\partial(x,y)} \neq 0$.**

**Similarly three functions u, v, w are dependent if $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$.**

## Examples:

**1. Prove that functions $u = \frac{x}{y}$ and $v = \frac{x+y}{x-y}$ are functionally dependent.**

Soln: Now $u = \frac{x}{y}$ and $v = \frac{x+y}{x-y}$

**Consider** $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} \frac{1}{y} & -\frac{x}{y^2} \\ \frac{(x-y)1-(x+y)(1)}{(x-y)^2} & \frac{(x-y)1-(x+y)(-1)}{(x-y)^2} \end{vmatrix}$

$$= \begin{vmatrix} \frac{1}{y} & -\frac{x}{y^2} \\ \frac{-2y}{(x-y)^2} & \frac{2x}{(x-y)^2} \end{vmatrix} = \frac{2x}{y(x-y)^2} - \frac{2xy}{y^2(x-y)^2}$$

$$= \frac{2x}{y(x-y)^2} - \frac{2x}{y(x-y)^2} = 0$$

**Thus** $\frac{\partial(u,v)}{\partial(x,y)} = 0$ **=> u and v are dependent**

**2. Determine whether or not the functions u =$e^x$siny and v = $e^x$cosy functionally dependent.**

**Soln.:** Given functions are u = **$e^x$sinyand v = $e^x$cosy**

Consider $\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} e^x \text{ siny} & e^x \text{ cosy} \\ e^x \text{ cosy} & -e^x \text{ siny} \end{vmatrix}$

$$= e^{2x}(-\sin^2 y - \cos^2 y)$$

$$= -e^{2x}(\sin^2 y + \cos^2 y)$$

$$= -e^{2x} \neq 0 \forall x$$

⇨ u and v are independent

**Home work**

**3.Prove that u = $\frac{x^2-y^2}{x^2+y^2}$ v= $\frac{2xy}{x^2+y^2}$ are functionally dependent.**

**4. Prove that u = $x^2e^{-y}$coshz, v = $x^2e^{-y}$sinhz and w=$3x^4 e^{-2y}$ are functionally dependent.**

**Soln.**Now u = $x^2e^{-y}$coshz, v = $x^2e^{-y}$sinhz and w=$3x^4 e^{-2y}$

Consider $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}, \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$ $u = x^2e^{-y}\cosh z$, $v = x^2e^{-y}\sinh z$ and $w=3x^4 e^{-2y}$

$$= \begin{vmatrix} 2xe^{-y}coshz & -x^2e^{-y}coshz & x^2e^{-y}sinhz \\ 2xe^{-y}sinhz & -x^2e^{-y}sinhz & x^2e^{-y}coshz \\ 12x^3e^{-2y} & -6x^4e^{-2y} & 0 \end{vmatrix}$$

$$= (2xe^{-y})(-x^2e^{-y})(x^2e^{-y}) \begin{vmatrix} coshz & coshz & sinhz \\ sinhz & sinhz & coshz \\ 6x^2e^{-y} & 6x^2e^{-y} & 0 \end{vmatrix}$$

$$= (-2x^5e^{-3y})(6x^2e^{-y}) \begin{vmatrix} coshz & coshz & sinhz \\ sinhz & sinhz & coshz \\ 1 & 1 & 0 \end{vmatrix}$$

$= -12x^7e^{-4y}(0)$ **bc'z first two columns are identical**

**= 0**

**i.e** $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$ **=> u, v, w are dependent.**

**And relation between them is** $u^2 - v^2 = x^4e^{-2y}(\cosh^2 z - \sinh^2 z)$

$= x^4e^{-2y} = \frac{w}{3}$

**i.e** $3(u^2 - v^2) = w$

**Home work**

**5. Prove that $u = x+2y+z$, $v = x-2y+3z$, $w=2xy - xz + 4yz - 2z^2$**

**are dpendent.**

# Lagrange's Mean Value Theorem for two variables

**Statement:** **If f(x, y) possess continuous first order partial derivatives in the neighborhood of (a,b) then there exists a number $\theta \in (0, 1)$ such that $f(a+h, b+k) - f(a,b) = h f_x(a+\theta h, b+\theta k) + k f_y(a+\theta h, b+\theta k)$ for every point (a+h, b+k) in a certain neighbourhood of (a, b).**

To prove above theorem we are using L.M.V theorem for one variable studied in B. Sc. I sem.

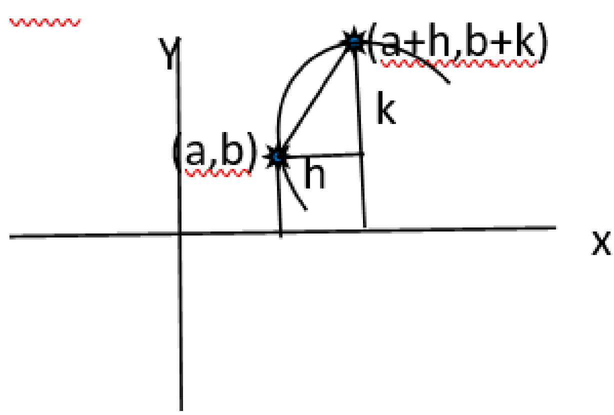
**i.e if g(x) is continuous and differentiable in (a, b) then there exists c in (a, b) such that $g^{l}(c) = \frac{g(b)-g(a)}{b-a}$**

Anther form of the above theorem is

**if g(x) is continuous and differentiable in (a, a+h) then there exists $\theta \in (0, 1)$ such that $g^{l}(a+\theta h) = \frac{g(a+h)-g(a)}{h}$ or $g(a+h) - g(a) = h\, g^{l}(a+\theta h)$.**

**Proof of above Theorem:**

**Let (a+h, b+k) be any neighbouring point of (a,b) in the plane so that $r = \sqrt{h^2+k^2}$.**




**Define a function $g(t) = f\left(a + t\frac{h}{r},\ b + t\frac{k}{r}\right)\ \forall r \in (0,1)$------------------(1)**

**Then $g(t+\delta t) = f\left(a + (t+\delta t)\frac{h}{r},\ b + (t+\delta t)\frac{k}{r}\right)$**

**$\therefore g(t+\delta t) - g(t) = f\left(a + (t+\delta t)\frac{h}{r},\ b + (t+\delta t)\frac{k}{r}\right) - f\left(a + t\frac{h}{r},\ b + t\frac{k}{r}\right)$**

$= f\left(\left(a + t\frac{h}{r}\right) + \delta t\frac{h}{r},\ \left(b + t\frac{k}{r}\right) + \delta t\frac{k}{r}\right) - f\left(a + t\frac{h}{r},\ b + t\frac{k}{r}\right)$

$= f\left(\in_1 + \delta t\frac{h}{r},\ \in_2 + \delta t\frac{k}{r}\right) - f(\in_1,\ \in_2)$ where $\in_1 = a + t\frac{h}{r},\ \in_2 = b + t\frac{k}{r}$

$= f(\in_1 + t_1,\ \in_2 + t_2) - f(\in_1,\ \in_2)$,

where $t_1 = \delta t\frac{h}{r}, t_2 = \delta t\frac{k}{r}$ & $t_1, t_2 \to 0$ as $\delta t \to 0$

**Thus** $g(t + \delta t) - g(t) = f(\in_1 + t_1,\ \in_2 + t_2) - f(\in_1,\ \in_2)$

**Dividing both the sides by $\delta t$ we get**

$$\frac{g(t+\delta t) - g(t)}{\delta t} = \frac{f(\in_1+t_1,\ \in_2+t_2) - f(\in_1,\ \in_2)}{\delta t}$$

$$= \frac{f(\in_1+t_1,\ \in_2+t_2) - f(\in_1,\ \in_2+t_2) + f(\in_1,\ \in_2+t_2) - f(\in_1,\ \in_2)}{\delta t}$$

$$= \frac{f(\in_1+t_1,\ \in_2+t_2) - f(\in_1,\ \in_2+t_2)}{\delta t} + \frac{f(\in_1,\ \in_2+t_2) - f(\in_1,\ \in_2)}{\delta t}$$

$$= \frac{h}{r}\left[\frac{f(\in_1+t_1,\ \in_2+t_2) - f(\in_1,\ \in_2+t_2)}{\delta t\frac{h}{r}}\right] + \frac{k}{r}\left[\frac{f(\in_1,\ \in_2+t_2) - f(\in_1,\ \in_2)}{\delta t\frac{k}{r}}\right]$$

**i.e** $$\frac{g(t+\delta t) - g(t)}{\delta t} = \frac{h}{r}\left[\frac{f(\in_1+t_1,\ \in_2+t_2) - f(\in_1,\ \in_2+t_2)}{t_1}\right] + \frac{k}{r}\left[\frac{f(\in_1,\ \in_2+t_2) - f(\in_1,\ \in_2)}{t_2}\right]$$

b'cz $t_1 = \delta t\frac{h}{r}, t_2 = \delta t\frac{k}{r}$

**Taking the limit as $\delta t \to 0$, on both sides, we get,**

$$\lim_{\delta t\to 0}\frac{g(t+\delta t) - g(t)}{\delta t} = \lim_{\delta t\to 0}\left\{\frac{h}{r}\left[\frac{f(\in_1+t_1,\ \in_2+t_2) - f(\in_1,\ \in_2+t_2)}{t_1}\right] + \frac{k}{r}\left[\frac{f(\in_1,\ \in_2+t_2) - f(\in_1,\ \in_2)}{t_2}\right]\right\}$$

**i.e** $$g^{l}(t) = \lim_{t_1\to 0}\frac{h}{r}\left[\frac{f(\in_1+t_1,\ \in_2+t_2) - f(\in_1,\ \in_2+t_2)}{t_1}\right] + \lim_{t_2\to 0}\frac{k}{r}\left[\frac{f(\in_1,\ \in_2+t_2) - f(\in_1,\ \in_2)}{t_2}\right] \text{-----------------}(P)$$

**We have** $\lim_{\delta x \to 0}\left[\frac{f(x+\delta x,\ y+\delta y) - f(x,\ y+\delta y)}{\delta x}\right] = f_x(x,y)$

**and** $\lim_{\delta y \to 0}\left[\frac{f(x,\ y+\delta y) - f(x,\ y)}{\delta y}\right] = f_y(x,y)$

**Using these in RHS of eqn. (P) we get**

**i.e** $g^l(t) = \frac{h}{r} f_x(\in_1,\ \in_2) + \frac{k}{r} f_y(\in_1,\ \in_2)$

$$g^l(t) = \frac{h}{r} f_x\left(a + t\frac{h}{r},\ b + t\frac{k}{r}\right) + \frac{k}{r} f_y\left(a + t\frac{h}{r},\ b + t\frac{k}{r}\right) \text{-----(P)}$$

Since by hypothesis, $f_x$, $f_y$ exist => $g^l(t)$ exists

i.e g(t) is differentiable in (0,r) & is continuous also and hence by LMV theorem for one variable

[ if g(x) is continuous and differentiable in (a, a+h) then there exists $\theta \in (0, 1)$ such that $g^l(a+\theta h) = \frac{g(a+h)-g(a)}{h}$ or $g(a+h) - g(a) = h\, g^l(a+\theta h)$. ]

$\exists\theta \in (0,1)$ such that $g(r) - g(0) = r g^l(\theta r)$ -----(Q)

a=0, a+h = r

⇨ a =0 and h =r

from (1) and (P), (Q) becomes

**f(a+h, b+k) – f(a, b)**

$g(t) = f\left(a + t\frac{h}{r},\ b + t\frac{k}{r}\right)\ \forall r \in (0,1)$-------------------(1)

$g^l(t) = \frac{h}{r} f_x\left(a + t\frac{h}{r},\ b + t\frac{k}{r}\right) + \frac{k}{r} f_y\left(a + t\frac{h}{r},\ b + t\frac{k}{r}\right)$-----(

$$= r\left\{\frac{h}{r} f_x\left(a + \theta r\frac{h}{r},\ b + \theta r\frac{k}{r}\right) + \frac{k}{r} f_y\left(a + \theta r\frac{h}{r},\ b + \theta r\frac{k}{r}\right)\right\}$$

$$= h f_x(a + \theta h,\ b + \theta k) + k f_y(a + \theta h,\ b + \theta k)$$

**i.e.** $f(a+h, b+k) - f(a, b) = h f_x(a + \theta h,\ b + \theta k) + k f_y(a + \theta h,\ b + \theta k)$

**Hence the proof**

## Other form of LMV theorem:

**If f(x, y) possess continuous first order partial derivatives in the neighborhood of (x,y) then there exists a number $\theta \in (0, 1)$ such that $f(x+\delta x, y+\delta y) - f(x,y) = \delta x\, f_x(x+\theta\delta x, y+\theta\delta y) + \delta y\, f_y(x+\theta\delta x, y+\theta\delta y)$ for every point $(x+\delta x, y+\delta y)$ in a certain neighbourhood of (x, y).**

## Taylore's Theorem for two variables:

In previous class we studied Taylore's Theorem for single variable i.e f(x) , Taylore's and Maclaurin's series also. In the same way we are studying here Taylore's and Maclaurin's series for two variables.

**In this theorem we use the notations**

$$hf_x + kf_y = h\frac{\partial f}{\partial x} + k\frac{\partial f}{\partial x} = \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial x}\right)f$$

$$h^2 f_{xx} + 2hk\, f_{xy} + k^2 f_{yy} = h^2\frac{\partial^2 f}{\partial x^2} + 2hk\frac{\partial^2 f}{\partial x \partial y} + k^2\frac{\partial^2 f}{\partial y^2} = \left(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial x}\right)^2 f$$

**and so on**

***Statement of Taylore's Theorem: (Only statement)***

**If a function f(x, y) possesses continuous partial derivatives of first, second, third and -----nth order in a neighbourhood of a point (a, b) then for any point (x, y) there exists a number $\theta \in (0, 1)$ such that**

$$f(x, y) = f(a, b) + \left((x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right)f(a, b) +$$

$$\frac{1}{2!}\left((x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right)^2 f(a, b)$$

$$+\frac{1}{3!}\left((x-a)\frac{\partial}{\partial x}+(y-b)\frac{\partial}{\partial y}\right)^3 f(a,b)+\text{------------------------}$$

$$\text{--}\qquad +\frac{1}{(n-1)!}\left((x-a)\frac{\partial}{\partial x}+(y-b)\frac{\partial}{\partial y}\right)^{n-1} f(a,b)+R_n$$

**Where** $R_n = \frac{1}{n!}\left((x-a)\frac{\partial}{\partial x}+(y-b)\frac{\partial}{\partial y}\right)^n f(a+\theta h, b+\theta k)$

## ***Taylor's Infinite series:***

**In the above theorem if number of terms tends to infinity then $R_n \to 0$, i.e**

**$\lim_{n\to\infty} Rn = 0$ then the series becomes**

$$f(x,y) = f(a,b) + \left((x-a)\frac{\partial}{\partial x}+(y-b)\frac{\partial}{\partial y}\right)f(a,b)$$

$$+\frac{1}{2!}\left((x-a)\frac{\partial}{\partial x}+(y-b)\frac{\partial}{\partial y}\right)^2 f(a,b)$$

$$+\frac{1}{3!}\left((x-a)\frac{\partial}{\partial x}+(y-b)\frac{\partial}{\partial y}\right)^3 f(a,b)+\text{------------------------}$$

$$+\frac{1}{n!}\left((x-a)\frac{\partial}{\partial x}+(y-b)\frac{\partial}{\partial y}\right)^n f(a,b)+\text{-----------infinite term}$$

**Thus Taylor's series about the point (a,b)**

$$f(x,y) = f(a,b) + [(x-a)\, f_x(a,b) + (y-b)\, f_y(a,b)]$$

$$+\frac{1}{2!}[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)] + \text{-------------}$$

**Sometimes it is also called series expansion in terms powers of (x-a) and (y-b).**

***Maclaurin's Theorem:(*** **Taylor's theorem at origin)**

**Statement: If a function f(x, y) possesses continuous partial derivatives of first, second, third and -----nth order in a neighbourhood of origin then for any point (x, y) we have**

$$f(x, y) = f(0, 0) + \left( x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right) f(0, 0) + \frac{1}{2!}\left( x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^2 f(0,0)$$

$$+ \frac{1}{3!}\left( x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^3 f(0,0) + \text{-----------------------}$$

$$+ \frac{1}{n!}\left( x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^n f(0,0) + \text{----------------}$$

$$\text{i.e } f(x, y) = f(0, 0) + \left( x\frac{\partial f(0,0)}{\partial x} + y\frac{\partial f(0,0)}{\partial x}\right) + \frac{1}{2!}\left(x^2 \frac{\partial^2 f(0,0)}{\partial x^2} + 2xy\frac{\partial^2 f(0,0)}{\partial x \partial y} + \right.$$

$$\left. y^2 \frac{\partial^2 f(0,0)}{\partial y^2} \right) + \frac{1}{3!}\left(x^3 \frac{\partial^3 f(0,0)}{\partial x^3} + + \text{------------}\right) + \text{--------------}$$

**Thus Maclaurin's series is**

$$f(x, y) = f(0, 0) + [x\, f_x(0,0) + y\, f_y(0,0)]$$

$$+ \frac{1}{2!}[x^2 f_{xx}(0,0) + 2xyf_{xy}(0,0) + y^2 f_{yy}(0,0)] + \text{------------}$$

**Applications of Taylor's and Maclaurin's series:**

**(i) To find the sum of many different infinite series**

**(ii) To find the limit of function**

**(iii) Taylor's polynomial is used to approximate polynomials**

**(iv) Used in solving differential equations to get series solution**

**(v) In Physics, if a system under conservative force is at stable equilibrium point $x_0$ then there are no net forces and the energy**

**function is concave upwards which can be derived using Taylor's series.**

**(vi)** **It is also used in special relativity to approximate Lorrentz factor $\gamma$**

**(vii)** **In software graphs of various functions uses Taylor's series.**

## EXAMPLES

**1. Expand sin(x+y) by Maclaurin's series (or Taylor's series at origin) upto 3rd**

**Soln.**: Let f(x, y) = sin(x+y) $\therefore$ f(0,0) = sin(0+0) = 0

Then $f_x = \cos(x+y)$ $\therefore$ $f_x(0,0) = 1$

$f_y = \cos(x+y)$ $\therefore$ $f_y(0,0) = 1$

$f_{xy} = -\sin(x+y)$ $\therefore$ $f_{xy}(0,0) = 0$

$f_{xx} = -\sin(x+y)$ $\therefore$ $f_{xx}(0,0) = 0$

$f_{yy} = -\sin(x+y)$ $\therefore$ $f_{yy}(0,0) = 0$

$f_{xxx} = f_{xxy} = f_{xyy} = f_{yyy} = -\cos(x+y)$ $\therefore$ $f_{xxx} = f_{xxy} = f_{xyy} = f_{yyy}(0,0) = -1$

and so on .

**By Maclaurin's series we have**

$$f(x, y) = f(0, 0) + [x f_x(0,0) + y f_y(0,0)]$$

$$+ \frac{1}{2!}[x^2 f_{xx}(0,0) + 2xyf_{xy}(0,0) + y^2 f_{yy}(0,0)] + \text{------------}$$

i.e $\sin(x+y) = 0 + (x(1)+y(1)) + \frac{1}{2!}(x^2(0)+2xy(0)+y^2(0)) + \frac{1}{3!}(x^3(-1) + 3x^2y(-1)$

$+3xy^2(-1)+ y^3(-1)) + \text{--------------------}$

$$= (x+y) + (0) - \frac{1}{3!}(x+y)^3 + \frac{1}{4!}(0) + \frac{1}{5!}(x+y)^5 + \text{------------------}$$

$$\therefore \sin(x+y) = (x+y) - \frac{1}{3!}(x+y)^3 + \frac{1}{5!}(x+y)^5 + \text{--------------------}$$

**Similarly you try for cos(x+y)**

**Ans: $\cos(x+y) = 1 - \frac{1}{2!}(x+y)^2 + \frac{1}{4!}(x+y)^4 +$ ---------------------**

**2. *Expand $e^{x+y}$ by Maclaurin's series up to third degree terms.(or* Taylor's series at origin)**

**Soln.**: Let $f(x, y) = \mathbf{e^{x+y}}$ $\therefore$ $f(0,0) = 1$

**Next** $f_x = f_y = \mathbf{e^{x+y}}$ $\therefore$ $f_x(0,0) = f_y(0,0) = 1$

**Similarly all derivatives at origin are 1**

**By Maclaurin's series we have**

$$f(x, y) = f(0, 0) + (x f_x(0,0) + y f_y(0,0)) + \frac{1}{2!}(x^2 fxx(0,0) + 2xy\, fxy(0,0) + y^2 fyy(0,0)) + \frac{1}{3!} \text{---------------------------------}$$

i.e $$e^{x+y} = 1 + (x.1 + y.1) + \frac{1}{2!}((x^2.1 + 2xy.1 + y^2.1) + \text{-------------------}$$

$$= 1 + (x+y) + \frac{1}{2!}(x+y)^2 + \frac{1}{3!}(x+y)^3 + \text{------------------------------}$$

**Thus**

$$e^{x+y} = 1 + (x+y) + \frac{1}{2!}(x+y)^2 + \frac{1}{3!}(x+y)^3 + \text{-------------}$$

**3. Expand $e^x$ siny by Maclaurin's series upto second degree term. (2018 and 2019)**

**Soln: Let $f(x, y) = e^x \sin y$** $\therefore$ $f(0,0) = e^0 \sin 0 = 0$

**Next**

| | | | |
|---|---|---|---|
| $f_x = e^x \sin y$ | $\therefore$ $f_x(0,0) = 0$ | $f_{xy} = e^x \cos y$ | $\therefore$ $f_{xy}(0,0) = 1$ |
| $f_y = e^x \cos y$ | $\therefore$ $f_y(0,0) = 1$ | $f_{yy} = -e^x \sin y$ | $\therefore$ $f_{yy}(0,0) = 0$ |
| $f_{xx} = e^x \sin y$ | $\therefore$ $f_{xx}(0,0) = 0$ | ----------------- | ------------------- |

**By Maclaurin's series we have**

$f(x, y) = f(0, 0) + (x\, f_x(0,0) + x f_y(0,0)) + \frac{1}{2!}(x^2 fxx(0,0) + 2xy\, fxy(0,0) + y^2 fyy(0,0)) + \frac{1}{3!}$ ----------------------------------

$= 0 + (x.0 + y.1) + \frac{1}{2!}(x^2 .0 + 2xy(1) + y^2 .0) + \frac{1}{3!}(x^3.0 + 3x^2y.1 + 3xy^2.0 + y^3.-1)$

\+ --------------------------------

**i.e** $e^x \sin y = y + xy + \frac{1}{3!} y(3x^2 - y^2) +$ --------------------

$$e^x \sin y = y + xy + \frac{1}{3!} y(3x^2 - y^2) + \text{--------------------}$$

**Home work**

**4. Expand $e^x \cos y$ by Maclaurin's series upto second degree term. (2016)**

**5. Obtain Taylor's series expansion of $\tan^{-1}(y/x)$ at (1, 1) upto second degree.**

Soln.: Let **Let f(x, y) = $\tan^{-1}(y/x)$** $\therefore$ $f(1,1) = \frac{\pi}{4}$

**Next**

$f_x = \frac{1}{1+\left(\frac{y}{x}\right)^2}\left(-\frac{y}{x^2}\right) = -\frac{y}{x^2+y^2}$ $\therefore$ $f_x(1,1) = -\frac{1}{2}$

$f_y = \frac{1}{1+\left(\frac{y}{x}\right)^2}\left(\frac{1}{x}\right) = \frac{x}{x^2+y^2}$ $\therefore$ $f_y(1,1) = \frac{1}{2}$

$f_{xx} = \frac{y(2x)}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2}$ $\therefore$ $f_{xx}(1,1) = \frac{1}{2}$

$f_{xy} = \frac{(x^2+y^2)\,1 - y(2y)}{(x^2+y^2)^2} = \frac{(x^2-y^2)}{(x^2+y^2)^2}$ $\therefore$ $f_{xy}(1,1) = 0$

$f_{yy} = -\frac{x\,(2y)}{(x^2+y^2)^2}$ $= -\frac{2xy}{(x^2+y^2)^2}$ $\therefore\ f_{yy}(1,1) = -\frac{1}{2}$

and so on-----------

**By Taylor's series expansion about the point (1, 1) we have**

$f(x, y) = f(1,1) + [(x-1)\, f_x(1,1) + (y-1)\, f_y(1,1)]$

$+ \frac{1}{2!}[(x-1)^2 f_{xx}(1,1) + 2(x-1)(y-1) f_{xy}(1,1) + (y-1)^2 f_{yy}(1,1)] +$ ------------

$= \frac{\pi}{4} + [(x-1)\frac{-1}{2} + (y-1)\frac{1}{2}] + \frac{1}{2!}[(x-1)^2\frac{1}{2} + 2(x-1)(y-1)(0) + (y-1)^2\frac{-1}{2}]$

$+$ -------------------------

**i.e $\tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{4} - \frac{1}{2}[x + y - 2] + \frac{1}{4}[(x-1)^2 - (y-1)^2] +$ --------------**

**6. Express $2x^2+xy-5y^2$ in powers of (x-1) and (y-1).**
**i.e Taylor's expansion about the point (1,1)**

**Soln.** **Let $f(x,y) = 2x^2+xy-5y^2$** $\therefore\ f(1,1) = -2$

**Next $f_x = 4x+y$** $\therefore\ f_x(1,1) = 5$

$f_y = x-10y$ $\therefore\ f_y(1,1) = -9$

$f_{xx} = 4$ $\therefore\ f_{xx}(1,1) = 4$

$f_{xy} = 1$ $\therefore\ f_{xy}(1,1) = 1$

$f_{yy} = -10$ $\therefore\ f_{yy}(1,1) = -10$

and next all higher order derivatives are zero

**By Taylor's series expansion about the point (1, 1) we have**

$f(x, y) = f(1,1) + [(x-1)\, f_x(1,1) + (y-1)\, f_y(1,1)]$

$+ \frac{1}{2!}[(x-1)^2 f_{xx}(1,1) + 2(x-1)(y-1) f_{xy}(1,1) + (y-1)^2 f_{yy}(1,1)] +$--------

$= -2 + [(x-1)(5) + (y-1)(-9)] + \frac{1}{2!}[[(x-1)^2(4) + 2(x-1)(y-1)(1) + (y-1)^2(-10)] + 0$

**i.e $2x^2+xy-5y^2 = -2 + [5(x-1) - 9(y-1)] + [2(x-1)^2 + (x-1)(y-1) - 5(y-1)^2]$**

**7. Obtain first three terms of the Maclaurin's expansion of the function $e^{x^2-y^2}$ in the neighbourhood of (0,0) upto 4th degree term.**

**Soln.: Soln: Let f(x, y) = $e^{x^2-y^2}$** $\therefore$ f(0,0) = $e^0$ = 1

**Next $f_x$ = 2x $e^{x^2-y^2}$** $\therefore$ $f_x(0,0) = 0$

$f_y = -2y\, e^{x^2-y^2}$ $\therefore$ $f_y(0,0) = 0$

$f_{xx} = 2[x\left(2x\, e^{x^2-y^2}\right) + e^{x^2-y^2}] = 2e^{x^2-y^2}[2x^2+1]$ $\therefore$ $f_{xx}(0,0) = 2$

$f_{xy} = 2x\left(-2y\, e^{x^2-y^2}\right) = -4xye^{x^2-y^2}$ $\therefore$ $f_{xx}(0,0) = 0$

$f_{yy} = -2[y\left(-2y\, e^{x^2-y^2}\right) + 1e^{x^2-y^2}] = 2e^{x^2-y^2}[2y^2-1]$ $\therefore$ $f_{yy}(0,0) = -2$

$f_{xxx} = 2e^{x^2-y^2}4x + [2x^2+1]2e^{x^2-y^2}(2x) = e^{x^2-y^2}[8x + 4x^3 + 4x]$

$= e^{x^2-y^2}[12x + 4x^3]$ $\therefore f_{xxx}(0,0) = 0$

Similarly $f_{xxy} = 2e^{x^2-y^2}[2x^2+1](-2y)\,]$ $\therefore$ $f_{xxy}(0,0) = 0$

$f_{xxxx} = e^{x^2-y^2}[12x + 4x^3](2x) + e^{x^2-y^2}(12 + 12x^2)$ $\therefore$ $f_{xxxx}(0,0) = 12$

$f_{xxxy} = e^{x^2-y^2}[12x + 4x^3](-2y)$ $\therefore$ $f_{xxxx}(0,0) = 0$

$f_{xxyy} = 2[2x^2+1]\{e^{x^2-y^2}(4y^2) + e^{x^2-y^2}(-2)\}$ $\therefore$ $f_{xxyy}(0,0) = -4$

and so on

----------------------------------------------------------------

**By Maclaurin's series we have**

**f(x, y ) = f(0, 0) +( x $f_x$ (0,0) + x$f_y$ (0,0)) +$\frac{1}{2!}$ ($x^2$fxx (0,0) + 2xy fxy (0,0) + $y^2$fyy (0,0) + $\frac{1}{3!}$ ($x^3$fxxx +3$x^2$y fxxy+ ----) + $\frac{1}{4!}$ ($x^4$fxxxx +4$x^3$y fxxxy + ------------------------------)+----------**

**i.e $e^{x^2-y^2}$= 1 + (0 +0) + $\frac{1}{2!}$ {$x^2$(2)+0+$y^2$(-2)}+ $\frac{1}{3!}$(0) + $\frac{1}{4!}$ { $x^4$(12)+ 4$x^3$y(0)+6$x^2y^2$(-4)+-----------------}+ ------------------------------**

$= 1+(x^2-y^2)+\frac{x4}{2} - x^2y^2$ -+-----------------)

**Thus $e^{x^2-y^2}$ = $1+(x^2-y^2)+ (\frac{x4}{2} - x^2y^2$ -+-----------------)+--------------**

**8. Expand $e^x$ log(1+y) by Maclaurin's series upto third degree term.**

**Soln.: Let** $f(x, y) = e^x \log(1+y)$ $\therefore f(0,0) = 0$

**Next** $f_x = e^x \log(1+y)$ $\therefore f_x(0,0) = 0$

$f_y = e^x \frac{1}{1+Y}$ $\therefore f_y(0,0) = 1$

$f_{xx} = e^x \log(1+y) = f_{xxx} = f_{xxxx}$--------=0

$f_{xy} = e^x \frac{1}{1+Y}$ $\therefore f_{xy}(0,0) = 1$

$f_{yy} = e^x \frac{-1}{(1+y)^2}$ $\therefore f_{yy}(0,0) = -1$

$f_{xxy} = e^x \frac{1}{1+Y}$ $\therefore f_{xxy}(0,0)=1$

$f_{xyy} = e^x \frac{-1}{(1+y)^2}$ $\therefore f_{xyy}(0,0) = -1$

$f_{yyy} = e^x \frac{2}{(1+y)^3}$ $\therefore f_{yyy}(0,0) = 2$

**and so on**

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**By Maclaurin's series we have**

$f(x, y) = f(0, 0) +( x f_x(0,0) + yf_y(0,0)) +\frac{1}{2!}(x^2 fxx(0,0) + 2xy\, fxy(0,0) +$

$y^2 fyy(0,0) + \frac{1}{3!}(x^3 fxxx + 3x^2y\, fxxy +$ ----) $+ \frac{1}{4!}(x^4 fxxxx + 4x^3y\, fxxxy +$

-----------------------------)+----------

$e^x \log(1+y) = 0+[x(0) + y(1)]+\frac{1}{2!}(x^2(0) + 2xy(1) + y^2(-1)] + \frac{1}{3!}[x^3(0)$

$+3x^2y(1)+3xy^2(-1)+y^3(2) +$ − − − − − − − − − − − − − −

**Thus $e^x \log(1+y) = y + xy - \frac{1}{2}y^2 + \frac{1}{3!}[3x^2y - 3xy^2 + 2y^3] +$ − − − − − − −**

## 9. Expand cosx cosy by Maclaurin's Theorem upto $4^{th}$ degree terms

**Soln.:** Let $f(x,y) = \cos x \cos y$ $\quad\therefore f(0,0) = 1$

Next $f_x = -\sin x \cos y$ $\quad\therefore f_x(0,0) = 0$

$f_y = -\cos x \sin y$ $\quad\therefore f_y(0,0) = 0$

$f_{xx} = -\cos x \cos y$ $\quad\therefore f_{xx}(0,0) = -1$

$f_{yy} = -\cos x \cos y$ $\quad\therefore f_{yy}(0,0) = -1$

$f_{xy} = \sin x \sin y$ $\quad\therefore f_{xy}(0,0) = 0$

$f_{xxx} = \sin x \sin y$ $\quad\therefore f_{xxx}(0,0) = 0$

$f_{xxy} = \cos x \sin y$ $\quad\therefore f_{xxy}(0,0) = 0$

$f_{xyy} = \sin x \cos y$ $\quad\therefore f_{xyy}(0,0) = 0$

$f_{yyy} = \cos x \sin y$ $\quad\therefore f_{yyy}(0,0) = 0$

$f_{xxxx} = \cos x \sin y$ $\quad\therefore f_{xxxx}(0,0) = 0$

$f_{xxxy} = \sin x \cos y$ $\quad\therefore f_{xxxy}(0,0) = 0$

$f_{xxyy} = \cos x \cos y$ $\quad\therefore f_{xxyy}(0,0) = 1$

$f_{xyyy} = \cos x \cos y$ $\quad\therefore f_{xyyy}(0,0) = 1$

$f_{yyyy} = -\sin x \sin y$ $\quad\therefore f_{yyyy}(0,0) = 0$

and so on ------------------------------------------------

**By Maclaurin's series we have**

$$f(x, y) = f(0, 0) + (x f_x(0,0) + y f_y(0,0)) + \frac{1}{2!}(x^2 fxx(0,0) + 2xy\, fxy(0,0) + y^2 fyy(0,0)) + \frac{1}{3!}(x^3 fxxx + 3x^2y\, fxxy + ----) + \frac{1}{4!}(x^4 fxxxx + 4x^3y\, fxxxy + ------------------------------) + ----------$$

$$\cos x \cos y = 1 + [x(0) + y(0)] + \frac{1}{2!}(x^2(-1) + 2xy(0) + y^2(-1)] + \frac{1}{3!}[x^3(0) + 3x^2y(0) + 3xy^2(0) + y^3(0)] + \frac{1}{4!}[x^4(0) + 4x^3y(0) + 6x^2y^2(1) + 4xy^3(0) + y^4(0)] + ----------------------------------------$$

**Thus** $\cos x \cos y = 1 - \frac{1}{2}(x^2 - y^2) + \frac{1}{4}x^2y^2 + ------------------------------$

## Home work example

**10. Obtain Taylor's series expansion of $e^x$ siny around the point $(0, \frac{\pi}{2})$**

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**K. L. E. Society's**

**G.I.Bagewadi Arts, Science & Commerce College, Nipani**

Accredited at 'A' level by NAAC with CGPA 3.35

**DEPARTMENT OF MATHEMATICS**

**B. Sc. V Sem. 2020-21**

**Paper I**

**Bridge Course**

**In this paper we come across intervals, bounded function, supremum and infimum of a function, properties of continuous functions etc. So we will have revision over them.**

**Open interval: Let a and b be two real numbers such that a<b then the set of real numbers x between a and b is called an open interval and denoted by (a, b).**

**Thus (a, b) = {x∈ $R/\ a < x < b$}**

**For example: 1. (1, 2) = { x∈ $R/\ 1 < x < 2$}**

**2. (2, ∞) = { x∈ $R/\ 2 < x$}**

**3. { x∈ $R/\ x < -5$} = (-∞, -5)**

**Closed interval: Let a and b be two real numbers such that a<b then the set of real numbers x from a to b is called an closed interval and denoted by [a, b].**

**Thus [a, b] = {x∈ $R/\ a \le x \le b$}**

**For example: 1. [-1, 3] = {x∈ $R/ -1 \le x \le 3$}**

**2. [2, ∞)= { x∈ $R/\ 2 \le x$}**

**3. { x∈ $R/\ x \ge -5$} = [-5, ∞)**

**Semi open and semi closed intervals: If one side it is open and another side it is closed, then it is called Semi open and semi closed intervals, denoted by [a, b) or (a, b].**

**i.e [a, b) = { x∈ $R/a \le x < b$}**

**(a, b] = { x∈ $R/a < x \le b$}**

**Examples: 1. [3, 5) = { x∈ $R/3 \le x < 5$}**

**2. (8, 100] = { x∈ $R/8 < x \le 100$}**

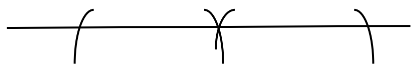
**3.[ -2,∞) = { x∈ $R/-2 \le x < \infty$}**

**Properties of intervals:**

1. Intervals are infinite sets
2. Intervals are subsets of set of real numbers

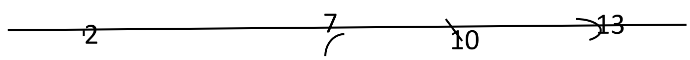
3. Interval contains infinitely many rationals and irrationals.
4. Union ( or intersection) of intervals need not be an interval unless it has a common point or common portion

   For example: $(1, 3)\cup(3, 4)$ is itself, not intermsof interval



   It is not an interval (1, 4) as 3 is not a point in the both intervals but $(1, 3)\cup[3, 4)=(1, 4)$ as 3 is included in second interval

   Similarly: $[2, 10]\cap(7, 13] =(7, 10]$




**Bounded function: A real valued function f: [a,b]→R, is called bounded in the interval [a,b] if there exist two real numbers $k_1$ and$k_2$ such that $k_1 \le f(x) \le k_2$ where $k_1$ is called lower bound and $k_2$ is called upper bound.**

**If any one bound does not exist then it is not bounded in that interval.**

**For example:**

**1. Let f: [1, 2] →R be defined by f(x) = 2x+5 then for all $x\in[1,2]$, $7\le f(x) \le 9$, ie. f(x) is bounded in [1,2] between 7 and 9.**

**All polynomial functions are bounded in [a,b]**

**2. If f(x) = $\frac{2}{x-2}$ in [1, 5] then at x = $2\in[1,5]$, f(2) = $\infty$ ,not bounded and hence this function is not bounded at this interval**

**3. If f(x) = sinx then is it bounded in the interval $[0, \pi]$, yes it is bounded between 0 and 1 because at x= 0 , f(x)= 0 and x = $\frac{\pi}{2}$,f(x)= 1 but even at $\pi$, $f(\pi) = 0$, so it is between 0 and 1**

**4. If f(x)= $|x|$ in the interval [-1 , 1] then f(x) is bonded between 0 and 1 because f(-1) = 1 , f(-.5) = .5, ---------f(0) =0, -----f(1)=1**

**Among all these 0 and 1 are min. and maximum**

**5. Every constant function is bounded.**

**For example: f(x) =2 then 2 ≤f(x) ≤ 2**

**Upper and lower bounds are same here.**

**Note:Some times bounded function is defined as 'A real valued function f: [a,b]→R, is called bounded in the interval [a,b] if there exist positive real number k such that** $|f(x)| \le k$, $\forall x \in [a,b]$

**Supremum and Infimum:**

If once the function f(x) is bounded it has an upper bound and all numbers more than that are also upper bounds for that. Among all these upper bounds, which is the least one is called the least upper bound or supremum.

**Defn.: A real number l is called the least upper bound of the function f(x) in [a, b] if (i) f(x) ≤ l** $\forall x \in [a,b]$**, ie l is theupper bound**

**(ii) it is the least one, that means if we take any number less than l i.e l -** $\varepsilon$ **then it is not an upper bound.**

i.e there exists some $x_i$ such that $f(x_i) > l-\varepsilon$.

Similarly we define greatest lower bound.

**Defn.: A real number k is called the greatest lower bound of the function f(x) in [a, b] if (i) f(x) ≥ k** $\forall x \in [a,b]$**, ie k is the lower bound**

**(ii) it is the greastest one, that means if we take any number greater than k i.e k +** $\varepsilon$ **then it is not a lower bound.**

i.e there exists some $x_i$ such that $f(x_i) < k+\varepsilon$ .i.e k it self is the greatest one, any other number greater than that is not a lower bound.

**For example: f(x) = x+3 in [3, 6]**

In this example if we put x=3, we get f(x)= 6, and f(x) values are increasing and at the end for x=6 we get f(x) = 9, it is the upper bound and the least upper bound.

$6 \le f(x) \le 9$.

2. If f(x) = sinx in [0, π] then f(x) will take values like 0,--- ½,------- √3/2, ---1, ---√3/2--, -----0 but among all these 0 is lower bound and 1 is the upper bound even though for x=0, f(x) =0 and x= π, f(x)= 0 but values are between 0 and 1.

3. If f(x) = $\frac{1}{3+cosx}$ in [0, $\frac{\pi}{2}$] then supremum, upper bound is 1/3 for x=$\frac{\pi}{2}$ and infimum, lower bound is ¼.

i.e $1/4 \le f(x) \le 1/3$.

# UNIT I

# Riemann Integration I

# Unit – I Riemann Integration-I

## Introduction:-

In the branch of Mathematics known as real Analysis the Riemann Integral created by German Mathematician Bernhard Riemann,(1854) was the first rigorous definition of the integral of a function on an interval.We all know that integration is an inverse process of differentiation. However the subject of integration was developed in connection with areas of planeregion, arc length, volume and surface area etc.

It is based on the concept the limit of a sum when the number of terms in the sum tends to infinity. So that division is very small. i.e. $\lim_{n\to\infty} \sum_{r=1}^{\infty} f(x_r)\delta_r$

For many functions and practical applications , the Riemann integral can be evaluated by Fundamental Theorem of Integral Calculus.

**Basic definitions**:

In this unit the function $f: [a,\ b] \to R$ is taken as bounded real valued function.

## 1. Partition of a closed interval $[a,\ b]$:-

Let I $=[a,\ b]$be a finite closed interval, divide it into 'n' number of sub divisions at the points $x_0 = a, x_1,\ x_2,\ x_3, \ldots .x_{n-1},\ x_n = b$[fig.1]. Then the finite ordered set [set of division points] $\{x_0 = a, x_1,\ x_2,\ \ldots .x_{n-1},\ x_n = b\}$is called partition of interval I $=[a,\ b]$. And is denoted by 'P' and the (n+1) points$x_0 = a, x_1,\ x_2,\ x_3, \ldots .x_{n-1}$, $x_n = b$

are called partition points of 'P'.

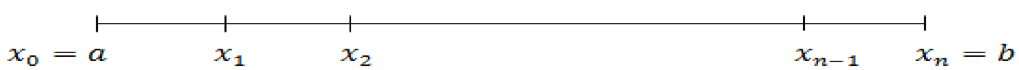



**Fig 1]**

And 'n' closed sub intervals

$I_1 = [x_0,\ x_1]$,

$I_2 = [x_1,\ x_2], \ldots \ldots, I_r = [x_{r-1},\quad x_r], \ldots . I_{n-1} = [x_{n-2},\quad x_{n-1}]$,

$I_n = [x_{n-1},\ x_n]$are called sub-divisions of I $=[a,\ b]$.

And length of sub-divisions$I_r$ is $x_r - x_{r-1}$ and which is denoted by $\delta_r$.

$\Rightarrow \delta_1 + \delta_2 + \delta_3 + \cdots + \delta_{n-1} + \delta_n = \sum_{r=1}^{n} \delta_r = b - a$

**Example:** Let I $=[0,\ 1]$

$P_1 = \left\{0,\ \frac{1}{2},\ 1\right\},\ \delta_r = \frac{1}{2}$

$P_2 = \left\{0,\ \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1\right\},\ \delta_r = \frac{1}{4}$

$P_3 = \{0,\ 0.1,\ 0.3,\ 0.6,\ 0.8,\ 0.9,\ 1\}$, in $P_3$, $\delta_1 = 0.1,\ \delta_2 = 0.2,\ \delta_3 = 0.3,\ \ \delta_4 = 0.2\ \delta_5 = 0.1,\ \delta_6 = 0.1$

**Note:** Partition of an interval $[a,\ b]$ is not necessarily unique. We can have (finite or infinite) many number of partitions on $[a,\ b]$. The collection of all these partitions of $[a,\ b]$ is denoted by P$[a,\ b]$.

If $P_1, P_2, P_3 \dots \dots$are partitions of $[a,\ b]$ then $P_1 \in$ P $[a,\ b]$, $P_2 \in$ P $[a,\ b]$, $P_3 \in$ P$[a,\ b]\dots$

P$[a, b]$=$\{P_1, P_2, P_3 \dots\}$

**Norm of a partition:**

It is not necessary that the length of sub division is same for all sub intervals of a partition. The maximum length of sub interval of a partition 'P' is called Norm of 'P'. And is denoted by $\|P\|$.

Therefore $\|P\|$ =max$\{\ \delta_r,\ \ r = 1, 2, \dots\dots n\}$

**Example:** Let I =$[a,\ b] =\ [1,\ 2]$

$$P_1 = \{1,\ \ 1.2,\ \ 1.4,\ \ 1.5,\ \ 1.7,\ \ 2\}$$

Where $\delta_1 = 0.2,\ \delta_2 = 0.2,\ \delta_3 = 0.1,\ \ \delta_4 = 0.2,\ \delta_5 = 0.3$

Therefore $\|P_1\|$ =max$\{\ \delta_r,\ \ r = 1, 2, \dots\dots n\}$= 0.3

$P_2 = \{1,\ \ 1.25,\ \ 1.5,\ \ 1.75,\ \ 2\}$

$\|P_2\|$ =max$\{\ \delta_r,\ \ r = 1, 2, \dots\dots n\}$= 0.25

Here length of sub division is uniform.

**Note:** If length $\delta_r$ of sub division is uniform then $\|P\|$= $\delta_r$

**Refinement of partition:**

If $P_1 and\ P_2$are two partitions of $[a,\ b]$ such that $P_1 \subseteq P_2$then $P_2$ is called refinement of $P_1[a,\ b]$.

**Example:**

1.Let us take $[a,\ b]$=$[0, \pi]$ and$P_1 = \left\{0,\ \frac{\pi}{2},\ \pi\right\}$, $P_2 = \left\{0,\ \frac{\pi}{4},\ \frac{\pi}{2},\ \frac{3\pi}{4},\ \pi\right\}$

$\Rightarrow P_1 \subseteq P_2 \therefore P_2$ is the refinement of$P_1$of$[0,\ \pi]$.

If $P_3$=$\left\{0,\ \frac{\pi}{6},\ \frac{\pi}{3},\ \pi\right\}$ then either $P_1, and\ P_3$or $P_2\ and\ P_3$are non comparable.

2. Let $[a,\ b]$= [1, 3]

$$P_1 = \{1,\ \ 1.4,\ \ 1.6,\ \ \ \ 2,\ \ 2.4,\ \ \ \ 2.6,\ \ \ \ 3\}$$

$$P_2 = \{1,\ \ \ \ 2,\ \ \ \ 3\}$$

$\|P_1\| = 0.4\ and\ \|P_2\| = 1$ and $P_2 \subseteq P_1$

$\Rightarrow P_1$is refinement of $P_2$of [1, 3].

**Lecture 3, 4.9.2020**

**Home work examples:**

1. Let [a, b] = [3, 6] and $P_1$ = {3, 3.3, 3.4, 3.9, 4.1, 4.7, 5, 5.6, 5.8, 6} Then find norm of partition $P_1$ and get at least one more partition $P_2$ on [3, 6] which is refinement of $P_1$.
2. [a, b] = [2, 4] and if $P_1$ = {2, 2.4, 2.6, 3.5, 3.7, 3.9, 4} and $P_2$ = {2, 2.6, 3.9, 4}then find $\|P_1\|$ and $\|P_2\|$ and which is refinement of which?
3. On [a, b] = [0, $\pi$], find two partitions $P_1$ and $P_2$ so that $P_2$ is refinement of $P_1$

## Upper and Lower Riemann sums or Darboux sums:

Let $f: [a,\ b] \to R$ is taken as bounded real valued function and

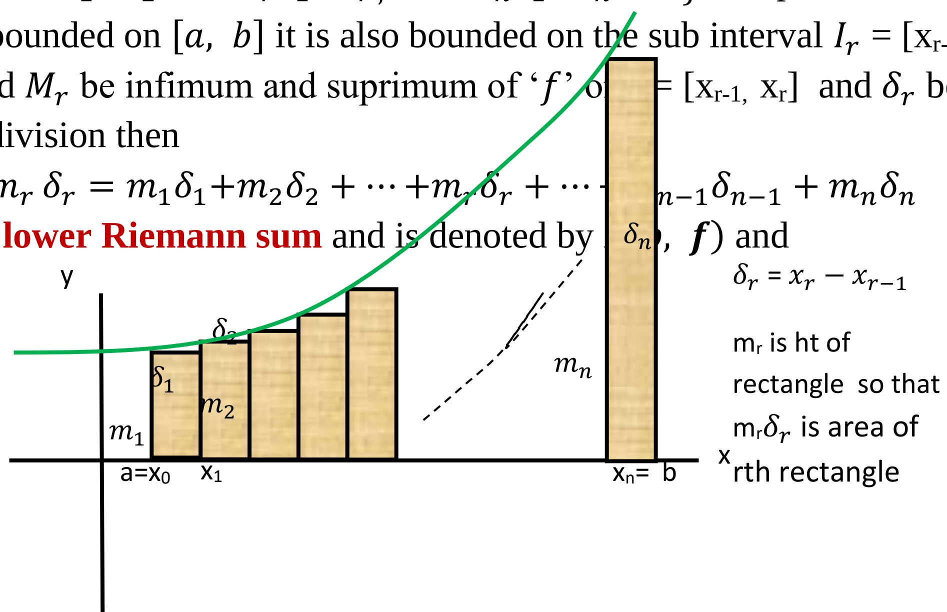
$P = \{x_0 = a, x_1,\ x_2, \ldots \ldots x_{r-1},\ x_{r\,,} \ldots \ldots . x_{n-1},\ x_n = b\}$ be a partition of $f$

As $f$ is bounded on $[a,\ b]$ it is also bounded on the sub interval $I_r$ = [$x_{r-1}$, $x_r$]

If $m_r$ and $M_r$ be infimum and suprimum of '$f$' on $I_r$ = [$x_{r-1}$, $x_r$] and $\delta_r$ be the length of the sub division then

***i***) $\sum_{r=1}^{n} m_r\,\delta_r = m_1\delta_1 + m_2\delta_2 + \cdots + m_r\delta_r + \cdots + m_{n-1}\delta_{n-1} + m_n\delta_n$

is called **lower Riemann sum** and is denoted by $\boldsymbol{L(p,\ f)}$ and

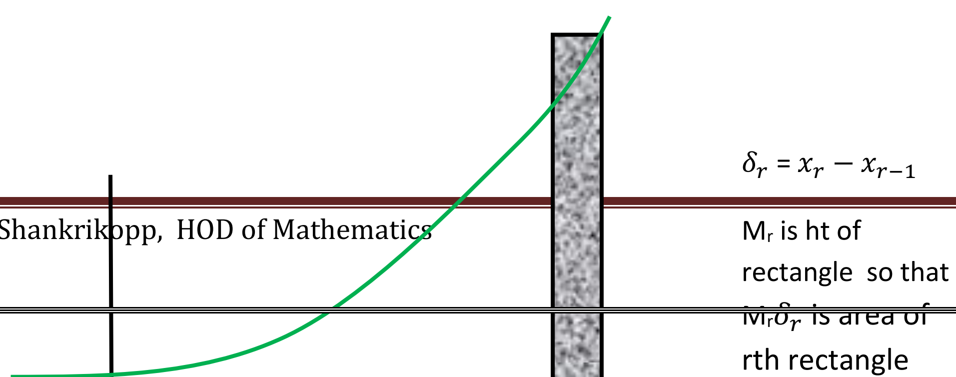


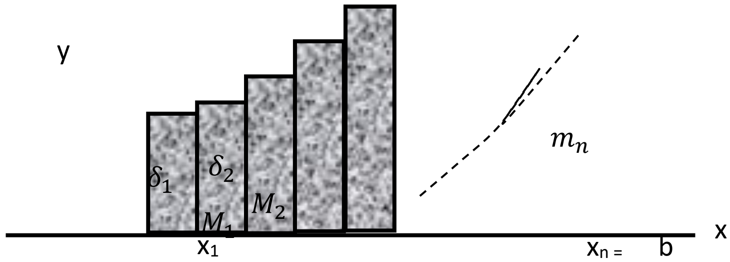

$\therefore\ \boldsymbol{L(p,\ f)} = \sum_{r=1}^{n} m_r\,\delta_r = m_1\delta_1 + m_2\delta_2 + \cdots + m_r\delta_r + \cdots + m_{n-1}\delta_{n-1} + m_n\delta_n$ is sum of

areas of all rectangles not exactly same as area between curve and x axis from a to b but less than that.

$$\boldsymbol{ii})\sum_{r=1}^{n} M_r\,\delta_r = M_1\delta_1 + M_2\delta_2 + \cdots + M_r\delta_r + \cdots + M_{n-1}\delta_{n-1} + M_n\delta_n$$

is called **upper Riemann sum** (upper sum) and is denoted by $\boldsymbol{U(p,\ f)}$






$\therefore$ $\boldsymbol{U(p,\ f)} = \sum_{r=1}^{n} M_r\,\delta_r = M_1\delta_1 + M_2\delta_2 + \cdots + M_r\delta_r + \cdots + M_{n-1}\delta_{n-1} + M_n\delta_n$ is sum of areas of all rectangles not exactly same as area between curve and x axis from a to b but more than that.

Thus lower and Upper sums are defined as

If $m_r$ and $M_r$ be Inf. and sup. of f on $I_r$ then sums

$L(p,\ f) = \sum_{r=1}^{n} m_r\,\delta_r$ *and* $U(p,\ f) = \sum_{r=1}^{n} M_r\,\delta_r$ are resply. called Riemann lower

and upper sums of f(x).

**Examples:**

**1.Let $f: [0,\ 1] \to R$ defined by $f(x) = 2x + 1$ and P $= \left\{0,\ \frac{1}{2},\ 1\right\}$ then find $L(p,\ f)$ and $U(p,\ f)$.**

**Solution:**

Let given function $f(x) = 2x + 1$ which is bounded on $[0,\ 1]$ and P $= \left\{0,\ \frac{1}{2},\ 1\right\}$

$$I_1 = \left[0,\quad \frac{1}{2}\right],\qquad I_2 = \left[\frac{1}{2}, 1\right]$$

Next, $m_1 = f(0) = 1$ ; $M_1 = f\left(\frac{1}{2}\right) = 2$ and $m_2 = f\left(\frac{1}{2}\right) = 2$ ; $M_2 = f(1) = 3$

And $\delta_1 = \frac{1}{2}$ and $\delta_2 = \frac{1}{2}$

Then $L(p,\ f) = m_1\delta_1 + m_2\delta_2 = 1 \times \frac{1}{2} + 2 \times \frac{1}{2} = \frac{3}{2} = 1.5$

$U(p,\ f) = M_1\delta_1 + M_2\delta_2 = 2 \times \frac{1}{2} + 3 \times \frac{1}{2} = \frac{5}{2} = 2.5$

Clearly $U(p,\ f) > L(p,\ f)$

**2. Let $f: [0,\ 1] \to R$ defined by $f(x) = 2x + 1$ and P $= \{0,\ 0.2, 0.5, 0.7\ \ 1\}$ then find $L(p,\ f)$ and $U(p,\ f)$.**

**Solution:** Let the given function is $f(x) = 2x + 1$ which is bounded on $[0,\ 1]$

P $= \{0,\ 0.2, 0.5, 0.7,\ 1\}$

$I_1 = [0,\quad 0.2],\quad I_2 = [0.2,\quad 0.5], I_3 = [0.5,\quad 0.7],\quad I_4 = [0.7,\quad 1]$

Next $m_1 = f(0) = 1\ M_1 = f(0.2) = 1.4$

$$m_2 = f(0.2) = 1.4 M_2 = f(0.5) = 2$$
$$m_3 = f(0.5) = 2 M_3 = f(0.7) = 2.4$$
$$m_4 = f(0.7) = 2.4\ M_4 = f(1) = 3$$

And $\delta_1 = 0.2\ \delta_2 = 0.3\ \delta_3 = 0.2$ and $\delta_4 = 0.3$

Then $L(p,\ f) = m_1\delta_1 + m_2\delta_2 + m_3\delta_3 + m_4\delta_4$

$L(p,\ f) = 1 \times 0.2 + 1.4 \times 0.3 + 2 \times 0.2 + 2.4 \times 0.3 = 1.74$

$U(p,\ f) = M_1\delta_1 + M_2\delta_2 + M_3\delta_3 + M_4\delta_4$

$U(p,\ f) = 1.4 \times 0.2 + 2 \times 0.3 + 2.4 \times 0.2 + 3 \times 0.3 = 2.26$

Clearly $U(p,\ f) > L(p,\ f)$

**3.Let $f: [0,\ 1] \to R$ defined by $f(x) = x$ and P $=\left\{0,\ \frac{1}{3},\ \frac{2}{3},\ 1\right\}$then find $L(p,\ f)$ and $U(p,\ f)$.*Dec-2014 &2016***

**Solution:** Now $f(x) = x$ and P $=\left\{0,\ \frac{1}{3},\ \frac{2}{3},\ 1\right\}$clearly $\delta_r = \frac{1}{3}$

$$I_1 = \left[0,\quad \frac{1}{3}\right],\qquad I_2 = \left[\frac{1}{3},\quad \frac{2}{3}\right], I_3 = \left[\frac{2}{3},\quad 1\right]$$

Next $m_1 = f(0) = 0,\ M_1 = f\left(\frac{1}{3}\right) = \frac{1}{3}$

$m_2 = f\left(\frac{1}{3}\right) = \frac{1}{3}, M_2 = f\left(\frac{2}{3}\right) = \frac{2}{3}$

$$m_3 = f\left(\frac{2}{3}\right) = \frac{2}{3}, M_3 = f(1) = 1$$

$$L(p,\ f) = m_1\delta_1 + m_2\delta_2 + m_3\delta_3 = \left(0 + \frac{1}{3} + \frac{2}{3}\right)\frac{1}{3} = 0.33$$

$$U(p,\ f) = M_1\delta_1 + M_2\delta_2 + M_3\delta_3 = \left(\frac{1}{3} + \frac{2}{3} + 1\right)\frac{1}{3} = \frac{2}{3} = 0.66$$

Clearly $U(p,\ f) > L(p,\ f)$

**Note:**$U(p,\ f)$& $L(p,\ f)$ are also ddenoted by S and s respectively.

**4. Compute S and s for $f(x) = x^2$ on $[0,\ 1]$ where P $=\left\{0,\ \frac{1}{4},\ \frac{2}{4},\ \frac{3}{4},\ 1\right\}$*Dec-2014***

**Solution:Now** $f(x) = x^2$, I $= [0,\ 1]$ where P $=\left\{0,\ \frac{1}{4},\ \frac{2}{4},\ \frac{3}{4},\ 1\right\}$ clearly $\delta_r = \frac{1}{4}$

$$I_1 = \left[0,\quad \frac{1}{4}\right],\qquad I_2 = \left[\frac{1}{4},\quad \frac{2}{4}\right], I_3 = \left[\frac{2}{4},\quad \frac{3}{4}\right],\qquad I_4 = \left[\frac{3}{4},\quad 1\right]$$

Next $m_1 = f(0) = 0\ M_1 = f\left(\frac{1}{4}\right) = \frac{1}{4^2} = \frac{1}{16}$

$m_2 = f\left(\frac{1}{4}\right) = \frac{1}{4^2} = \frac{1}{16}$ $M_2 = f\left(\frac{2}{4}\right) = \frac{4}{4^2} = \frac{1}{4}$ $m_3 = f\left(\frac{2}{4}\right) = \frac{4}{4^2} = \frac{1}{4}$ $M_3 = f\left(\frac{3}{4}\right) = \frac{9}{4^2} = \frac{9}{16}$

$$m_4 = f\left(\frac{3}{4}\right) = \frac{9}{4^2} = \frac{9}{16} \quad M_4 = f(1) = 1^2 = 1$$

Then $L(p,\ f) = s = m_1\delta_1 + m_2\delta_2 + m_3\delta_3 + m_4\delta_4$

$L(p,\ f) = s = \frac{1}{4}\left(0 + \frac{1}{4^2} + \frac{4}{4^2} + \frac{9}{4^2}\right) = \frac{7}{32}$

$U(p,\ f) = S = M_1\delta_1 + M_2\delta_2 + M_3\delta_3 + M_4\delta_4$

$U(p,\ f) = S = \frac{1}{4}\left(\frac{1}{4^2} + \frac{4}{4^2} + \frac{9}{4^2} + \frac{4^2}{4^2}\right) = \frac{1}{4} \times \frac{30}{4^2} = \frac{15}{32}$

Clearly $U(p,\ f) > L(p,\ f)$ i.e. $S > s$

**Home work examples:**

1. **If f(x) = 3 and I = [5, 7] then find s and S of f in I for the partition P = {5, 6, 7}**
2. **If f(x) = Sinx and I = [0, $\pi$] then find s and S of f in I for the partition P = {0, $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\pi$}**
3. **If f(x) = $|x|$, I = [-1, 1] and P ={ -1, 0, 0.5, 1} then L(P, f) and U(P, f).**

**Theorem 1. : Let $f: [a,\ b] \to R$ is taken as bounded real valued function $p \in$ P$[a,\ b]$ then $i)$ $L(p,\ f) \le U(p,\ f)$**

**$ii)$ $m(b-a) \le L(p,f) \le U(p,f) \le M(b-a)$ Or**

**$m(b-a) \le s \le S \le M(b-a)$ Where $m$ and $M$ be infimum and supremum of $f$ on $[a,\ b]$**

**Proof:** Let $p = \{x_0 = a, x_1,\ x_2, \dots\dots x_{r-1},\ x_r, \dots\dots x_{n-1},\ x_n = b\}$ be any partition of $[a,\ b]$ and $f$ be bounded on $[a,\ b]$

$i)$ Let $m_r,\ M_r$ be the infimum and supremum of $f$ on $I_r$ then

$$m_r \le M_r \forall r = 1, 2, \dots\dots n$$

$m_r\delta_r \le M_r\delta_r \forall r = 1, 2, \dots\dots n$ $\because \delta_r \ne 0$ and length is non-negative.

Taking summation both sides from r =1,2,---n we get, $\sum_{r=1}^{n} m_r\delta_r \le \sum_{r=1}^{n} M_r\delta_r$

$i.e$ $L(p,\ f) \le U(p,\ f)$

ii) If '$m$', '$M$' be infimum and supremum of $f$ on $[a,\ b]$ and $m_r,\ M_r$ be the infimum and supremum of f on $I_r$ then $m \le m_r \le M_r \le M$

Multiplying throughout by $\delta_r > 0$ we get,

$\Rightarrow m\delta_r \leq m_r\delta_r \leq M_r\delta_r \leq M\delta_r$

$$\Rightarrow \sum_{r=1}^{n} m\delta_r \leq \sum_{r=1}^{n} m_r\delta_r \leq \sum_{r=1}^{n} M_r\delta_r \leq \sum_{r=1}^{n} M\delta_r$$

$$\Rightarrow m\sum_{r=1}^{n} \delta_r \leq \sum_{r=1}^{n} m_r\delta_r \leq \sum_{r=1}^{n} M_r\delta_r \leq M\sum_{r=1}^{n} \delta_r$$

$$\Rightarrow m\sum_{r=1}^{n} \delta_r \leq L(p,f) \leq U(p,f) \leq M\sum_{r=1}^{n} \delta_r$$

$$\Rightarrow m(b-a) \leq L(p,f) \leq U(p,f) \leq M(b-a)$$

$\{\because$ If $[a,\ b]$ is interval then $\sum_{r=1}^{n} \delta_r = (b-a)\}$

**Theorem : If $f: [a,\ b] \rightarrow R$ be bounded function and $P_1, P_2 \in P[a,\ b]$ such that $P_1 \leq P_2$ ($P_2$ is refinement of $P_1$) then**

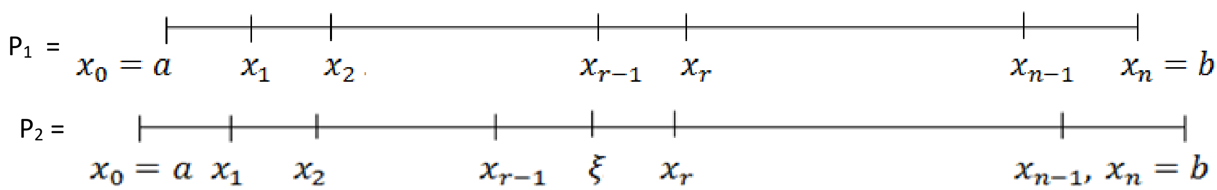
***i)*** $\boldsymbol{L(P_1,\ f) \leq L(P_2,\ f)}$ ***ii)*** $\boldsymbol{U(P_1,\ f) \geq U(P_2,\ f)}$ **and**

***iii)*** $\boldsymbol{L(P_1,\ f) \leq L(P_2,\ f) \leq U(P_2,\ f) \leq U(P_1,\ f)}$

**Proof:** Let $f: [a,\ b] \rightarrow R$ be bounded function and $P_1, P_2 \in P[a,\ b]$ such that $P_1 \subseteq P_2$ ($P_2$ is refinement of $P_1$),

$\therefore$ If $P_1 = \{x_0 = a,\ x_1,\ x_2, \ldots\ldots x_{r-1},\ x_r, \ldots\ldots x_{n-1},\ x_n = b\}$ and

$P_2 = \{x_0 = a,\ x_1,\ x_2, \ldots\ldots x_{r-1},\ \xi,\ x_r, \ldots\ldots x_{n-1},\ x_n = b\}$ be partitions of [a, b]




Clearly $P_1 \subseteq P_2$ i.e $P_2$ is refinement of $P_1$ as $P_2$ contains just one point($\xi$) more than $P_1$.

Let $m'_r m''_r$ and $m_r$ be infimum of $f$ on $[x_{r-1},\ \xi][\xi,\ x_r]$ and $[x_{r-1},\ x_r]$ respectively and $M'_r$, $M''_r$ and $M_r$ be supremum of $f$ on $[x_{r-1},\ \xi][\xi,\ x_r]$ and $[x_{r-1},\ x_r]$ respectively then

$m_r \leq m'_r m_r \leq m''_r$

$$M_r \geq M'_r M_r \geq M''_r \quad eqn\ (1)$$

***i)* To prove $\boldsymbol{L(P_1,\ f) \leq L(P_2,\ f)}$**

Consider, LHS = $L(P_2,\ f) - L(P_1,\ f)$

LHS = $\{(m_1\delta_1+m_2\delta_2+\cdots+m_{r-1}\delta_{r-1})+[m'_r(\xi-x_{r-1})+m''_r(x_r-\xi)]+$ $+m_{r+1}\delta_{r+1}+\cdots+m_n\delta_n\}-\{m_1\delta_1+m_2\delta_2+\cdots+m_{r-1}\delta_{r-1}+m_r\delta_r+m_{r+1}\delta_{r+1}+$ $\cdots+m_n\delta_n\}$

LHS = $m'_r(\xi-x_{r-1})+m''_r(x_r-\xi)-m_r\delta_r$

$\Rightarrow$LHS $=m'_r(\xi-x_{r-1})+m''_r(x_r-\xi)-m_r(x_r-\xi+\xi-x_{r-1})$

$\Rightarrow$ LHS $=(m''_r-m_r)(x_r-\xi)+(m'_r-m_r)(\xi-x_{r-1})$

But $(m''_r-m_r)(x_r-\xi)+(m'_r-m_r)(\xi-x_{r-1})\geq 0$ from eqn. (1)

$\Rightarrow$ LHS $\geq 0$ i.e. $L(P_2,\ f)-L(P_1,\ f)\geq 0$

$\therefore L(P_2,\ f)\geq L(P_1,\ f)$or $L(P_1,\ f)\leq L(P_2,\ f)$

***ii*) To prove $U(P_1,\ f)\geq U(P_2,\ f)$**

Consider LHS = $U(P_2,\ f)-\ U(P_1,\ f)$

$=\{(M_1\delta_1+M_2\delta_2+\cdots+M_{r-1}\delta_{r-1})+[M'_r(\xi-x_{r-1})+$ $M''_r(x_r-\xi)]++M_{r+1}\delta_{r+1}+\cdots+M_n\delta_n\}-\{M_1\delta_1+M_2\delta_2+\cdots+M_{r-1}\delta_{r-1}+$ $M_r\delta_r+M_{r+1}\delta_{r+1}+\cdots+M_n\delta_n\}$

$=\ M'_r(\xi-x_{r-1})+M''_r(x_r-\xi)-M_r\delta_r$

$\Rightarrow$LHS $=M'_r(\xi-x_{r-1})+M''_r(x_r-\xi)-M_r(x_r-\xi+\xi-x_{r-1})$

$\Rightarrow$ LHS $=(M''_r-M_r)(x_r-\xi)+(M'_r-M_r)(\xi-x_{r-1})$

But $(M''_r-M_r)(x_r-\xi)+(M'_r-M_r)(\xi-x_{r-1})\leq 0$ from eqn. (1)

$\Rightarrow$ LHS $\leq 0$ i.e. $U(P_2,\ f)-\ U(P_1,\ f)\leq 0$

$\therefore U(P_2,\ f)\leq\ U(P_1,\ f)$ Or $U(P_1,\ f)\geq U(P_2,\ f)$

***iii*) To prove $L(P_1,\ f)\leq L(P_2,\ f)\leq U(P_2,\ f)\leq U(P_1,\ f)$**

Thus if $P_1\subseteq P_2$ then

$L(P_2,\ f)\geq L(P_1,\ f)$or $L(P_1,\ f)\leq L(P_2,\ f)\rightarrow(a)$

$U(P_2,\ f)\leq\ U(P_1,\ f)$ Or $U(P_1,\ f)\geq U(P_2,\ f)\rightarrow(b)$

From $(a)$&$(b)$and as we know that lower sum is $\leq$ upper sum then

$\Rightarrow\ L(P_1,\ \ f)\leq L(P_2,\ \ f)\leq U(P_2,\ \ f)\leq U(P_1,\ \ f)$

**Corollary:** If $P_1,P_2\in P[a,\ b]$ then (i) $L(P_1,\ f)\leq U(P_2,\ f)$

(ii) $L(P_2,\ f)\leq U(P_1,\ f)$. No lower sum exceeds upper sum.

**Upper and Lower Riemann Integrals:**

If $f:[a,\ b]\rightarrow R$ be bounded function then we have

$m(b-a)\leq L(P,f)\leq U(P,f)\leq M(b-a)\rightarrow(1)\forall$ P$\in P[a,\ b]$ and where $m,M$ be infimum and supremum of $f$on$[a,\ b]$.

From (1) we have

$L(P,f)\leq M(b-a)\rightarrow(\mathbf{1})\ \forall P\in P[a,\ b]$

$$U(P,f) \geq m(b-a) \rightarrow (\mathbf{2}) \forall P \in P[a, \quad b]$$

From$(\mathbf{1})$ we have $L(P,f) \leq M(b-a) \forall P \in P[a,\ b]$

that means in the place of P if we go on substitute $P_1$, $P_2$, --------------we get

$L(P_1, f) \leq M(b-a)$, $L(P_2, f) \leq M(b-a)$, $L(P_3, f) \leq M(b-a)$, -----------

Thus the set { $L(P_i, f)$: $P_i \in P[a,\ b]$} is bounded above by M(b-a)

$\Rightarrow$ Set$\{L(P,\ f),\ P \in P[a,\ b]\}$ of lower sum is bounded above by $M(b-a)$,
and hence the set has the least upper bound(supremum) and is called lower Riemann integral denoted by $\int_{\underline{a}}^{b} f(x).dx$

**i.e. $\int_{\underline{a}}^{b} f(x).dx$ = Supremum of $\{L(P,\ f) : P \in P[a,\ b]\}$ = lub of $\{L(P,\ f) : P \in P[a,\ b]\}$ is called lower Riemann integral**

Similarly from $(\mathbf{2})$we have

$U(P,f) \geq m(b-a) \forall P \in P[a,\ b]$

$\Rightarrow$ set$\{U(P,\ f) :\ P \in P[a,\ b]\}$ of upper sum is bounded below by $m(b-a)$,
and hence the set has thegreatest lower bound(infimum) and is called upper Riemann integral denoted by $\int_{a}^{\bar{b}} f(x).dx$.

**i.e. $\int_{a}^{\bar{b}} f(x).dx$ = Infimum of $\{U(P,\ f) :\ P \in P[a,\ b]\}$ = glb of $\{U(P,\ f) :\ P \in P[a,\ b]\}$ is called upper Riemann integral.**

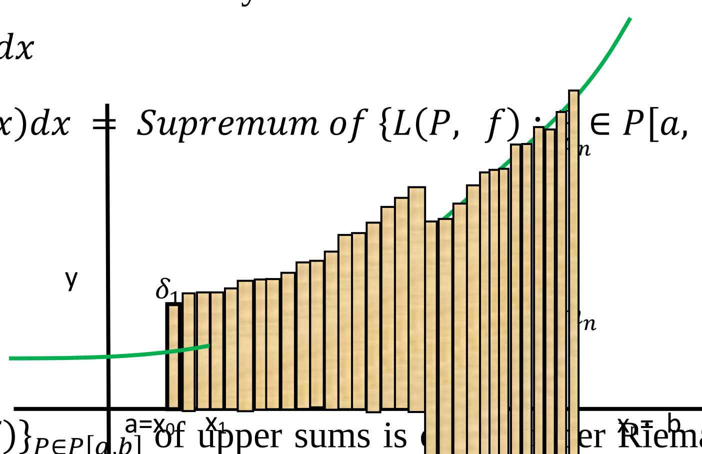
Thus the upper and lower Riemann integrals are defined as follows.

**Lower Riemann Integral:**

Supremum $(lub)$ of set $\{L(P,\ f)\}_{P\in P[a,b]}$ of lower sums is called lower Riemann integral and is denoted by

$\int_{\underline{a}}^{b} f(x)dx$

i.e$\int_{\underline{a}}^{b} f(x)dx$ = $Supremum\ of\ \{L(P,\ f) : P \in P[a,\ b]\}$ = $\lim_{n\to\infty} L(P,f)$or $|P| \rightarrow 0$




As n→ ∞ means no. of divisions tends to ∞, length $\delta_r$ is very small i.e tends to zero, ie $|P| \rightarrow 0$, in that case area is exactly bounded by curve and x-axis between a and b

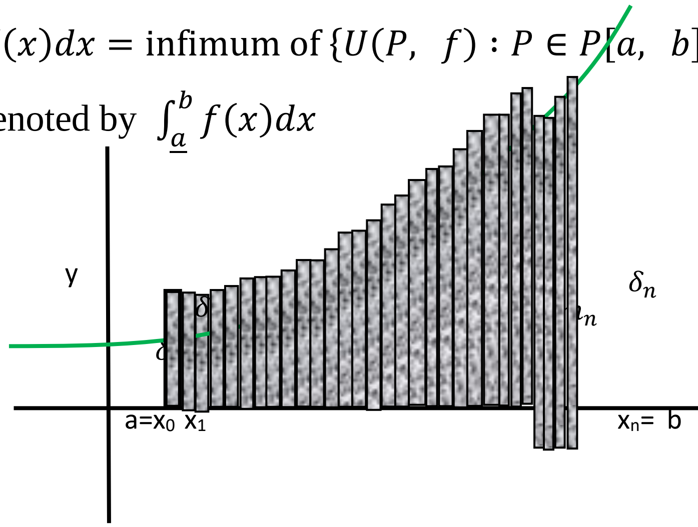
$\{U(P,\ f)\}_{P\in P[a,b]}$ of upper sums is ... er Riema... ed by

$\int_{a}^{\bar{b}} f(x)dx$**Upper Riemann Integral:**

Infimum $(glb)$ of set

$i.e\ \int_a^{\bar{b}} f(x)dx =$ infimum of $\{U(P,\ f) : P \in P[a,\ b]\} = \lim_{n\to\infty} U(P,f)$ or $|P| \to 0$

and is denoted by $\int_{\underline{a}}^{b} f(x)dx$



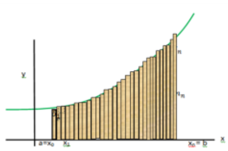
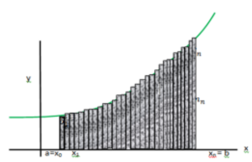

As n→ ∞ means no. of divisions tends to ∞, length $\delta_r$ is very small i.e tends to zero, ie $|P| \to 0$, in that case area is exactly bounded by curve and x-axis between a and b

**Riemann Integrable Function:**

A bounded function $f: [a,\ b] \to R$ is said to be Riemann Integrable (R-Integrable) or simply integrable, if $\int_a^{\bar{b}} f(x)dx = \int_{\underline{a}}^{b} f(x)dx$ and is denoted by $\int_a^b f(x)\ dx$.

Thus a function f(x) is R-integrable if f is bounded in [a, b] and Lower R I = Upper R. I.

i.e$\int_{\underline{a}}^{b} f(x)dx$ = $\int_a^{\bar{\bar{b}}} f(x)dx = \int_a^b f(x)\ dx$

i.e  = 

**Set of all Riemann Integrable function on $[a,\ b]$ are denoted by $R[a,\ b]$.**

**Note**: (**i**)**If** $f \in R[a,\ b]$ then we say that f is R − integrable.

$i.e\ f$ *is bounded and* $\int_a^{\bar{b}} f(x)dx = \int_{\underline{a}}^{b} f(x)dx$and that common value is value of integral$\int_a^b f(x)dx$

(ii) The interval$[a,\ b]$ is called range of integration where 'a' is lower limit and 'b' is an upper limit of integration.

(iii) If f and g are R-integrable functions means we write f, g$\in R[a,\ b]$

(iv) L(P,f) $\leq \int_{\underline{a}}^{b} f(x)dx$ and U(P, f)$\geq \int_a^{\bar{b}} f(x)dx, \forall$P$\in P[a,\ b]$

**Properties of Upper and Lower Riemann Integrals:**

**1. If $f:[a,\ b]\to R$ be a bounded function then $\int_{\underline{a}}^{b} f(x)dx \le \int_{a}^{\overline{b}} f(x)dx$**

**Proof:** Let $P_1, P_2 \in P[a,\ b]$ then we have $L(P_1,\ f) \le U(P_2,\ f) \to (1)$

This is true $\forall$ $P_1 \in P[a,\ b]$ , here by keeping $P_2$ fixedgo on changing $P_1$ as $P_3, P_4$, -----------we get $L(P_3,\ f) \le U(P_2,\ f)$,

$L(P_4, f) \le U(P_2,\ f)$,

$L(P_5, f) \le U(P_2,\ f)$

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Thus we get set $\{L(P_1,\ f)\}_{P_1 \neq P_2 \in P[a,b]}$ set of lower sums is bounded above by $U(P_2,\ f)$, but we have supremum of set $\{L(P_1,\ f)\}_{P_1 \in P[a,b]} = \int_{\underline{a}}^{b} f(x)dx$ and supremum $\le$ upper bound

$$\int_{\underline{a}}^{b} f(x)dx \le U(P_2,\ f), \forall P_2 \in P[a,b]$$

i.e $U(P_2,\ f) \ge \int_{\underline{a}}^{b} f(x)dx, \forall P_2 \in P[a,b] \to (2)$

In this put $P_2 = P_3, P_4$, ------------------- we get

$U(P_3,\ f) \ge \int_{\underline{a}}^{b} f(x)dx$,

$$U(P_4,\ f) \ge \int_{\underline{a}}^{b} f(x)dx$$

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Thus we get the set $\{U(P_2,\ f)\}_{P_2 \in P[a,b]}$ of upper sums is bounded below by $\int_{\underline{a}}^{b} f(x)dx$ but we have infimum (glb) of $\{U(P_2,\ f)\}_{P_2 \in P[a,b]} = \int_{a}^{\overline{b}} f(x)dx$

And GLB$\ge$ lower bound , i.e Infimum $\ge$ lower bound

$\int_{a}^{\overline{b}} f(x)dx \ge \int_{\underline{a}}^{b} f(x)dx$ i.e $\int_{\underline{a}}^{b} f(x)dx \le \int_{a}^{\overline{b}} f(x)dx$

$$\therefore \int_{\underline{a}}^{b} f(x)dx \le \int_{a}^{\overline{b}} f(x)dx$$

**II. If $f : [a, b] \to R$ be a bounded function then**

$$m(b-a) \leq \int_{\underline{a}}^{b} f(x)dx \leq \int_{a}^{\bar{b}} f(x)dx \leq M(b-a)$$

**Proof:** Now $f : [a, b] \to R$ be a bounded function and hence we have

$(i)\ m(b-a) \leq L(P,f) \leq U(P,f) \leq M(b-a) \to (1)$

$$(ii) \int_{\underline{a}}^{b} f(x)dx$$

$$\leq \int_{a}^{\bar{b}} f(x)dx ----------------- (2) \text{for any partition P}$$

We know that

$$\int_{\underline{a}}^{b} f(x)dx = \text{Supremum of } \{L(P, f) : P \in P[a, b]\}$$

i.e. $\int_{\underline{a}}^{b} f(x)dx$ is lub of set of lower sums

$$\Rightarrow \int_{\underline{a}}^{b} f(x)dx \text{ is an upper bound first}$$

$L(P,f) \leq \int_{\underline{a}}^{b} f(x)dx$ ---------------------- (3) for all L(P, f)

Similarly

$$U(P,f) \geq \int_{a}^{\bar{b}} f(x)dx$$

i.e $\int_{a}^{\bar{b}} f(x)dx \leq$ U(P,f)-------------------------------------------------(4)

from (1), (2), (3) & (4) we have

$$m(b-a) \leq L(P,f) \leq \int_{\underline{a}}^{b} f(x)dx \leq \int_{a}^{\bar{b}} f(x)dx \leq U(P,f) \leq M(b-a)$$

$$m(b-a) \le \int_{\underline{a}}^{b} f(x)dx \le \int_{a}^{\bar{b}} f(x)dx \le M(b-a)$$

**III. If $f \in \mathrm{R}[a,\ b]$, then $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$**

**Proof:** Let $f \in \mathrm{R}[a,\ b]$ then $f$ is bounded and

$$\int_{a}^{\bar{\bar{b}}} f(x)dx = \int_{\underline{a}}^{b} f(x)dx = \int_{a}^{b} f(x)dx ------------------(Q)$$

as $f$ is bounded $\Rightarrow m(b-a) \le L(P,f) \le U(P,f) \le M(b-a) ----- (1)$

We have supremum $\{L(P,\ f)\}_{P\in P[a,b]} = \int_{\underline{a}}^{b} f(x)dx$

$$\Rightarrow L(P,f) \le \int_{\underline{a}}^{b} f(x)dx = \int_{a}^{b} f(x)dx$$

$$\therefore L(P,f) \le \int_{a}^{b} f(x)dx ---------------------- (2)$$

Similarly,

We have infimum $\{U(p,\ f)\}_{p\in P[a,b]} = \int_{a}^{\bar{b}} f(x)dx$

$$U(p,f) \ge \int_{a}^{\bar{b}} f(x)dx = \int_{a}^{b} f(x)dx$$

$$U(p,f) \ge \int_{a}^{b} f(x)dx \textbf{ or } \int_{a}^{b} f(x)dx \le U(p,f) \to (3)$$

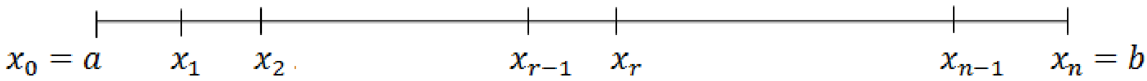
From (1)(2)&(3) we have

$$m(b-a) \le L(p,f) \le \int_{a}^{b} f(x)dx \le U(p,f) \le M(b-a)$$

$$m(b-a) \le \int_{a}^{b} f(x)dx \le M(b-a)$$

**Method of CalculatingLower Riemann and Upper Riemann Integrals for examples.**

**Step I.** Divide the given interval $[a,\ b]$into 'n' number of sub-divisions(uniform) such that $\delta_r = \frac{(b-a)}{n}$




Then $x_r = a + \frac{(b-a)r}{n}$ for $r = 1, 2, \dots . n$

**For Example:** If f(x) = x+1 and interval I = [0, 1] then $x_r = 0 + \frac{(1-0)r}{n} = \frac{r}{n}$

$\therefore x_r = \frac{r}{n}$for $r = 1, 2, \dots . n$

If $[a,\ b] = [1,\ 2]$ then $x_r = 1 + \frac{(2-1)r}{n} = 1 + \frac{r}{n}$

$\therefore x_r = 1 + \frac{r}{n}$for $r = 1, 2, \dots . n$

**Step II**: Calculate $m_r$ *and* $M_r$ in $I_r = [x_{r-1}, x_r]$ by $m_r = f(x_{r-1})$ *and* $M_r = f(x_r)$

For the above example $m_r = f(x_{r-1})= f(\frac{r-1}{n}) = \frac{r-1}{n} +1 =\frac{r+n-1}{n}$ ,

and $M_r = f(x_r)= f(\frac{r}{n}) = \frac{r}{n} +1 =\frac{r+n}{n}$

**Step III**. Calculate lower and upper sums for the above partition.

$$L(p,\ f) = \sum_{r=1}^{n} m_r\delta_r \ \& \quad U(p,\ f) = \sum_{r=1}^{n} M_r\delta_r$$

**Step IV**. Calculate lower and upper Riemann integral as follows.

$$\underline{\int_a^b} f(x)dx = \lim_{n\to\infty} \sum_{r=1}^{n} m_r\delta_r$$

$$\overline{\int_a^b} f(x)dx = \lim_{n\to\infty} \sum_{r=1}^{n} M_r\delta_r$$

To prove $f$ is R-integrable we have to prove that

$i)$ $f$ is bounded and

$$ii) \overline{\int_a^b} f(x)dx = \underline{\int_a^b} f(x)dx$$

**Examples on Riemann-Integral:**

**1. Prove that every constant function is R-integrable or if $f: [a,\ b] \to R$ defined by $f(x) = k$ then $f$ is R-Integrable.**

**Solution:** Let $f: [a,\ b] \to R$ defined by $f(x) = k\ \forall\ x \in [a,\ b]$

Clearly $k \leq f(x) \leq k\ \forall\ x \in [a,\ b]$ then $\Rightarrow f$ is bounded.

If P= $\{x_0 = a, x_1,\ \dots\dots x_{r-1},\ x_r, \dots\dots . x_{n-1},\ x_n = b\}$ be the partition of $[a,\ b]$then $L(P,\ f) = \sum_{r=1}^{n} m_r \delta_r$ where $m_r$ & $M_r$ be infimum & supremum of $f$on$I_r = [x_{r-1},\ x_r]$

$$L(P,\ f) = \sum_{r=1}^{n} f(x_{r-1})\delta_r = \sum_{r=1}^{n} k\delta_r = k\sum_{r=1}^{n} \delta_r = k(b-a)$$

$$\text{Similarly}, U(P,\ f) = \sum_{r=1}^{n} M_r\delta_r = \sum_{r=1}^{n} k\delta_r = k(b-a)$$

$$\underline{\int_{a}^{b}} f(x)dx = \lim_{n\to\infty} L(P,\ f) = \lim_{n\to\infty} k(b-a) = k(b-a)$$

$$\text{and} \overline{\int_{a}^{b}} f(x)dx = \lim_{n\to\infty} U(p,\ f) = \lim_{n\to\infty} k(b-a) = k(b-a)$$

$$\therefore \overline{\int_{a}^{b}} f(x)dx = \underline{\int_{a}^{b}} f(x)dx \Rightarrow f \text{is} R - \text{Integrable or} f \in R[a,\ b] \text{and}$$

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} kdx = k(b-a)$$

Thus every constant function is $R -$ Integrable

**2.If $f: [0,\ 1] \to R$ defined by $f(x) = 2x + 1$ then prove that $f$ is R-Integrable or $f \in R[a,\ b]$ andalso find $\int_0^1 f(x)dx$.**

**Solution:** Let$f: [0,\ 1] \to R$ defined by $f(x) = 2x + 1\forall\ x \in [0,\ 1]$

Clearly $1 \leq f(x) \leq 3\ \forall\ x \in [0,\ 1] \Rightarrow f$ is bounded.

P= $\left\{x_0 = 0, \frac{1}{n}, \frac{2}{n}, \dots . \frac{r-1}{n}, \frac{r}{n}, \dots\dots \frac{n}{n} = 1 = x_n\right\}$ be the partition of $[0,\ 1]$

If $m_r$ and $M_r$ be infimum and supremum of $f$on$I_r = \left[\frac{r-1}{n},\ \frac{r}{n}\right]$

Then $m_r = f(x_{r-1}) = f\left(\frac{r-1}{n}\right) = 2\left(\frac{r-1}{n}\right) + 1 = \frac{2r}{n} - \frac{2}{n} + 1$

$M_r = f(x_r) = f\left(\frac{r}{n}\right) = 2\left(\frac{r}{n}\right) + 1 = \frac{2r}{n} + 1$

And $L(P,\ f) = \sum_{r=1}^{n} m_r\delta_r = \sum_{r=1}^{n} f\left(\frac{r-1}{n}\right)\delta_r = \sum_{r=1}^{n}\left(\frac{2(r-1)}{n} + 1\right)\frac{1}{n}$ Where $\delta_r = \frac{1}{n}$

$$= \frac{1}{n}\left[\frac{2}{n}\sum_{r=1}^{n}(r-1) - \sum_{r=1}^{n} 1\right]$$

$= \frac{1}{n}\left[\frac{2}{n}\frac{(n-1)n}{2} + n\right] = \frac{1}{n}[(n-1) + n]$

$\sum_{r=1}^{n}(r-1)$= 0+1+2+--------(n-1)

Or $\sum_{r=1}^{n}(n-1)$= $\frac{(n-1)n}{2}$

And $\sum_{r=1}^{n}(n)$ = $\frac{n(n+1)}{2}$

$= \frac{1}{n}[2n-1]$

$$\therefore \underline{\int_0^1} f(x)dx = \lim_{n\to\infty} L(P,\ f) = \lim_{n\to\infty}\frac{1}{n}[2n-1] = 2$$

And $U(p,\ f) = \sum_{r=1}^{n} M_r\delta_r = \sum_{r=1}^{n} f(\frac{2r}{n})\delta_r = \sum_{r=1}^{n}\left(\frac{2r}{n} + 1\right)\frac{1}{n}$ where $\delta_r = \frac{1}{n}$

$$= \frac{1}{n}\left[\frac{2}{n}\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1\right] = \frac{1}{n}\left[\frac{2}{n}\frac{n(n+1)}{2} + n\right]$$

$$= \frac{1}{n}[(n+1) + n]$$

i.e $U(P,\ f) = \frac{1}{n}[2n+1]$

$$\therefore \overline{\int_0^1} f(x)dx = \lim_{n\to\infty} U(P,\ f) = \lim_{n\to\infty}\frac{1}{n}[2n+1] = 2$$

Clearly $\underline{\int_0^1} f(x)dx = \overline{\int_0^1} f(x)dx = 2 \Rightarrow f \in R[a,\ b]$

And $\int_0^1 f(x)dx = \int_0^1 (2x+1)dx = 2$ thus given function is R – integrable.

**3. If $f: [0,\ a] \to R$ defined by $f(x) = x^3\ \forall\ x \in [0,\ a]$ then prove that $f$ is R-Integrable or $f \in R[a,\ b]$ andalso $\int_0^1 f(x)dx = \frac{a^4}{4}$.**

**Solution**: Let $f: [0,\ a] \to R$ defined by $f(x) = x^3\ \forall\ x \in [0,\ a]$

Clearly $\forall\ x \in [0,\ a]\ 0 \le f(x) \le a^3 \Rightarrow f$ is bounded.

P= $\left\{0, \frac{a}{n}, \frac{2a}{n}, \ldots . \frac{(r-1)a}{n}, \frac{ra}{n}, \ldots .. a\right\}$ be the partition of $[0, a]$ where length of sub-interval $\delta_r = \frac{a}{n}$.

If $m_r$ and $M_r$ be infimum and supremum of $f$ on $I_r = \left[\frac{(r-1)a}{n}, \frac{ra}{n}\right]$

Then $L(p, f) = \sum_{r=1}^{n} m_r\delta_r = \sum_{r=1}^{n} f\left(\frac{(r-1)a}{n}\right)\frac{a}{n}$ Where $\delta_r = \frac{a}{n}$

$$L(p, f) = \sum_{r=1}^{n} m_r\delta_r = \sum_{r=1}^{n} \frac{(r-1)^3 a^3}{n^3} \times \frac{a}{n}$$

$\sum_{r=1}^{n}(r-1)^3$= 0+1³+2³+------(n-1)³

Or $\sum_{r=1}^{n}(n-1)^3 = \frac{[(n-1)n]^2}{4}$

Bcz $\sum_{r=1}^{n}(n)^3 = [\frac{n(n+1)}{2}]^2$

$$= \frac{a^4}{n^4}\sum_{r=1}^{n}(r-1)^3$$

$$= \frac{a^4}{n^4}\left[\frac{(n-1)n}{2}\right]^2 = \frac{a^4}{4n^4}(n-1)^2 n^2 = \frac{a^4}{4n^2}(n-1)^2$$

$$\underline{\int_0^a} f(x)dx = \lim_{n\to\infty} L(P, f) = \lim_{n\to\infty}\frac{a^4}{4n^2}(n-1)^2 = \lim_{n\to\infty}\frac{a^4 n^2}{4n^2}\left(1-\frac{1}{n}\right)^2 = \frac{a^4}{4}.$$

$U(P, f) = \sum_{r=1}^{n} M_r\delta_r = \sum_{r=1}^{n} f\left(\frac{ra}{n}\right)\frac{a}{n}$ Where $\delta_r = \frac{a}{n}$

$$= \sum_{r=1}^{n}\frac{r^3a^3}{n^3}\times\frac{a}{n} = \frac{a^4}{n^4}\sum_{r=1}^{n}(r)^3 = \frac{a^4}{n^4}\frac{n^2(n+1)^2}{4}$$

$$= \frac{a^4(n+1)^2}{4n^2}$$

$$\therefore \int_0^{\bar{a}} f(x)dx = \lim_{n\to\infty} U(p, f) = \lim_{n\to\infty}\frac{a^4(n+1)^2}{4n^2} = \frac{a^4}{4}\left[\frac{n^2\left(1+\frac{1}{n}\right)^2}{n^2}\right] = \frac{a^4}{4}$$

Clearly $\underline{\int_0^a} f(x)dx = \int_0^{\bar{a}} f(x)dx = \frac{a^4}{4} \Rightarrow f \in R[a, b]$

And $\int_0^a f(x)dx = \int_0^a x^3dx = \frac{a^4}{4}$ thus given function $f$ is R – integrable.

**4.If $f: [0, 1] \to R$ defined by $f(x) = x^2 + 2 \; \forall \; x \in [0, 1]$ then prove that $f$ is R-Integrable or $f \in R[a, b]$ andalso find $\int_0^1 f(x)dx$.**

**Solution:** Given that $f:[0,\ 1] \to R$ defined by $f(x) = x^2 + 2\ \forall\ x \in [0,\ 1]$

Clearly $x \in [0,\ 1]$ , $2 \le f(x) \le 3\ \forall \Rightarrow f$ is bounded.

P= $\left\{x_0 = 0, \frac{1}{n}, \frac{2}{n}, \ldots . \frac{r-1}{n}, \frac{r}{n}, \ldots .. \frac{n}{n} = 1 = x_n\right\}$ be the partition of $[0,\ 1]$ Where $\delta_r = \frac{1}{n}$ (length of sub-intervals)

If $m_r$ and $M_r$ be infimum and supremum of $f$ on $I_r = \left[\frac{r-1}{n}, \frac{r}{n}\right]$

Then $m_r = f(x_{r-1}) = f\left(\frac{r-1}{n}\right) = \left(\frac{r-1}{n}\right)^2 + 2$

$M_r = f(x_r) = f\left(\frac{r}{n}\right) = \left(\frac{r}{n}\right)^2 + 2$

And $L(P,\ f) = \sum_{r=1}^{n} m_r\delta_r = \sum_{r=1}^{n} f\left(\frac{r-1}{n}\right)\frac{1}{n} = \sum_{r=1}^{n}\left[\left(\frac{r-1}{n}\right)^2 + 2\right]\frac{1}{n}$ Where $\delta_r = \frac{1}{n}$

$= \frac{1}{n}\left[\sum_{r=1}^{n}\frac{(r-1)^2}{(n)^2} + \sum_{r=1}^{n} 2\right] = \frac{1}{n^3}\sum_{r=1}^{n}(r-1)^2 + \frac{1}{n}\sum_{r=1}^{n} 2 = \frac{1}{n^3}\frac{(n-1)n(2[n-1]+1)}{6} + \frac{1}{n}2n$

$\sum_{r=1}^{n}(r-1)^2$ = 0+1²+2²+-----(n-1)²

Or $\sum_{r=1}^{n}(r-1)^2 = \frac{(n-1)n(2n-1)}{6}$

And $\sum_{r=1}^{n}(n)^2 = \frac{n(n+1)(2n+1)}{6}$

$$L(p,\ f) = \sum_{r=1}^{n} m_r\delta_r = \frac{(n-1)(2n-1)}{6n^2} + 2$$

$$\therefore \int_0^1 f(x)dx = \lim_{n\to\infty} L(P,\ f) = \lim_{n\to\infty}\frac{(n-1)(2n-1)}{6n^2} + 2 = \frac{2}{6} + 2 = \frac{7}{3}$$

And $U(p,\ f) = \sum_{r=1}^{n} M_r\delta_r = \sum_{r=1}^{n} f\left(\frac{r}{n}\right)\frac{1}{n} = \sum_{r=1}^{n}\left[\left(\frac{r}{n}\right)^2 + 2\right]\frac{1}{n}$ Where $\delta_r = \frac{1}{n}$

$$= \frac{1}{n}\left[\sum_{r=1}^{n}\frac{r^2}{n^2} + \sum_{r=1}^{n} 2\right] = \frac{1}{n^3}\sum_{r=1}^{n} r^2 + \frac{1}{n}\sum_{r=1}^{n} 2$$

$$= \frac{n(n+1)(2n+1)}{6n^3} + \frac{1}{n}2n = \frac{(n+1)(2n+1)}{6n^2} + 2$$

$$\therefore \int_0^{\bar{1}} f(x)dx = \lim_{n\to\infty} U(p,\ f) = \lim_{n\to\infty}\frac{(n+1)(2n+1)}{6n^2} + 2 = \frac{2}{6} + 2 = \frac{7}{3}$$

$$\text{Clearly} \int_{\underline{0}}^{1} f(x)dx = \int_0^{\bar{1}} f(x).dx = \frac{7}{3} \Rightarrow f \in R[a,\ b]$$

And $\int_0^1 f(x)dx = \int_0^1 (x^2+2)dx = \frac{7}{3}$ thus given function is R − integrable.

**5.Prove that every bounded function need not be integrable. Or give an example to show that bounded function need not be R-Integrable.**

**Solution:** Let $f:[0,\ 1] \to R$ defined by $f(x) = \begin{cases} -1 & \forall\, rational\ x \in [0,\ 1] \\ 1 & \forall\, irrational\ x \in [0,\ 1] \end{cases}$

Clearly $\forall x \in [0,\ 1]\ -1 \le f(x) \le 1 \Rightarrow f$ is bounded.

If P be the partition of $[0,\ 1]$ and if $m_r$ and $M_r$ be infimum and supremum of $f$ on $I_r = \left[\frac{r-1}{n},\ \frac{r}{n}\right]$ then $m_r = -1$ and $M_r = 1$

$$\therefore L(P,\ f) = \sum_{r=1}^{n} m_r\delta_r = \sum_{r=1}^{n} (-1)\delta_r = (-1)(b-a) = -1$$

$$U(P,\ f) = \sum_{r=1}^{n} M_r\delta_r = \sum_{r=1}^{n} (1)\delta_r = (1)(b-a) = 1$$

And $\underline{\int_0^1} f(x)dx = \lim_{n\to\infty} L(P,\ f) = \lim_{n\to\infty} -1 = -1$

$\therefore \overline{\int_0^1} f(x)dx = \lim_{n\to\infty} U(P,\ f) = \lim_{n\to\infty} 1 = 1$

Thus $\underline{\int_0^1} f(x)dx \ne \overline{\int_0^1} f(x)dx \Rightarrow f$ is not R − integrable. i.e. $f \notin R[0,\ 1]$.

**6.Prove that function $f:[1,\ 2] \to R$ defined by $f(x) = 2x+1$ is R-Integrable.**

**Solution:** Let given that $f:[1,\ 2] \to R$ defined by $f(x) = 2x+1$

Clearly $\forall x \in [1,\ 2],\ 3 \le f(x) \le 5 \Rightarrow f$ is bounded.

If P be the partition of $[1,\ 2]$&if $m_r$&$M_r$ be infimum& supremum of $f$ on

$$I_r = \left[1 + \frac{r-1}{n},\qquad 1 + \frac{r}{n}\right]$$

then $m_r = f\left(1 + \frac{r-1}{n}\right) = 2\left(1 + \frac{r-1}{n}\right) + 1$ & $M_r = f\left(1 + \frac{r}{n}\right) = 2\left(1 + \frac{r}{n}\right) + 1$

$$\therefore L(P,\ f) = \sum_{r=1}^{n} m_r\delta_r = \sum_{r=1}^{n} \left[\left(2 + \frac{2(r-1)}{n}\right) + 1\right]\frac{1}{n}$$

$$= n + \frac{2n(n-1)}{2n^2}n + \frac{1}{n}n = 3 + \frac{(n-1)}{n}$$

$$\therefore \underline{\int_1^2} f(x)dx = \lim_{n\to\infty} L(P,\ f) = \lim_{n\to\infty}\left[3+\frac{(n-1)}{n}\right] = 4$$

and $U(P,\ f) = \sum_{r=1}^{n} M_r\delta_r = \sum_{r=1}^{n}\left[\left(2+\frac{2r}{n}\right)+1\right]\frac{1}{n}$

$$= \frac{2}{n}n + \frac{2n(n+1)}{2n^2}n + \frac{1}{n}n = \left[3+\frac{(n+1)}{n}\right]$$

$$\therefore \overline{\int_1^2} f(x)dx = \lim_{n\to\infty} U(P,\ f) = \lim_{n\to\infty}\left[3+\frac{(n+1)}{n}\right] = 3+1 = 4$$

Clearly $\underline{\int_1^2} f(x)dx = \overline{\int_1^2} f(x)dx = 4 \Rightarrow$ f isR − Integrable $\therefore\ f \in R[1,\ 2]$.

**6. Calculate upper and lower Riemann Integrals and hence prove that function $f: [2,\ 3] \to R$ defined by $f(x) = x^3 + 1$ is R-Integrable.**

**Darboux's Theorem:**

**Statement:** If function $f: [a,\ b] \to R$ is bounded function then for each $\varepsilon > 0, \exists\delta > 0$ such that

$$i)\ L(P,\ f) > \underline{\int_a^b} f(x)dx - \varepsilon$$

$ii)\ U(P,\ f) < \overline{\int_a^b} f(x).dx + \varepsilon P \in P[a,\ b]$ Suchthat $\|P\| < \delta$.

**Proof**: Let $f: [a,\ b] \to R$ is bounded function

$i)$ By the definition we have

$$\underline{\int_a^b} f(x)dx = \text{supremum}\ \{L(P_1,\ f)\}_{P_1\in P[a,b]}$$

i.e. $\underline{\int_a^b} f(x)dx$ is least upper bound of set$\{L(P_1,\ f)\}$ of lower sums.

By the definition of Supremum

$$(a) L(P_1, f) \leq \underline{\int_a^b} f(x)dx \; \forall \; P_1 \in P[a, \quad b]$$

$(b)$ For every $\varepsilon > 0$, $\exists P$ with $\|P\| < \delta$ such that $L(P, f) \nleq \underline{\int_a^b} f(x)dx - \varepsilon$

$$\Rightarrow L(P, f) > \underline{\int_a^b} f(x)dx - \varepsilon$$

**Similarly**

$ii)$ **By the definition we have** $\overline{\int_a^b} f(x)dx =$ **infimum** $\{U(P_1, f)\}_{P_1 \in P[a,b]}$

$i.e.$ $\overline{\int_a^b} f(x)dx$ **is greatest lower bound of set** $\{U(p_1, f)\}$ **of upper sums**

**∴by definition of Infimum**

$$(a) U(P_1, f) \geq \overline{\int_a^b} f(x).dx \; \forall P_1 \in P[a, \quad b]$$

$(b)$ For every $\varepsilon > 0, \exists P$ with $\|P\| < \delta$ such that $U(P, f) \ngeq \overline{\int_a^b} f(x)dx + \varepsilon$

$$\Rightarrow U(P, f) < \overline{\int_a^b} f(x)dx + \varepsilon$$

**Thus for every** $\varepsilon > 0$, $\exists P$ **with** $\|P\| < \delta$ **such that**

$i)$ $L(P, f) > \underline{\int_a^b} f(x)dx - \varepsilon$ **and**

$ii)$ $U(p, f) < \overline{\int_a^b} f(x)dx + \varepsilon$

**Necessary and sufficient condition for R-Integrability.**

**Theorem: A bounded function** $f: [a, b] \to R$ **is R-Integrable iff for** $\varepsilon > 0 \exists$ **P such that** $U(P, f) - L(P, f) < \varepsilon$.

**OR "A necessary and sufficient condition for bounded function to be R-Integrable is for** $\varepsilon > 0 \exists$ **P∈** $P[a, b]$ **such that** $U(P, f) - L(P, f) < \varepsilon$.**"**

**Proof:**

**Necessary Condition:**

Let we suppose that a bounded function $f$ be R-Integrable, now we need to prove that $U(P, f) - L(P, f) < \varepsilon$.

Now the bounded function $f$ is R-Integrable.

$$\overline{\int_a^b} f(x)dx = \underline{\int_a^b} f(x)dx = \int_a^b f(x)dx$$

But $\underline{\int_a^b} f(x)dx =$ supremum $\{L(P, f)\}_{P\in P[a,b]}$ and

$$\overline{\int_a^b} f(x)dx = \text{infimum } \{U(P, f)\}_{P\in P[a,b]}$$

∴ By the definition of supremum and infimum for $\varepsilon > 0 \exists$ partition $P_1, P_2$ of $[a, b]$ such that $U(P_1, f) < \overline{\int_a^b} f(x)dx + \frac{\varepsilon}{2}$

$$U(P_1, f) < \int_a^b f(x)dx + \frac{\varepsilon}{2} \to (1)$$

Similarly, $L(P_2, f) > \underline{\int_a^b} f(x)dx - \frac{\varepsilon}{2}$

$$L(P_2, f) > \int_a^b f(x)dx - \frac{\varepsilon}{2}$$

$$\int_a^b f(x)dx < L(P_2, f) + \frac{\varepsilon}{2} \to (2)$$

If P=$P_1 \cup P_2$ then P is refinement of $P_1$&$P_2$ both.

Then from (1)& (2) we have

$$U(P, f) \le U(P_1, f) < \int_a^b f(x)dx + \frac{\varepsilon}{2} \to (3)$$

$$\int_a^b f(x)dx < L(P_2, f) + \frac{\varepsilon}{2} < L(P, f) + \frac{\varepsilon}{2} \to (4)$$

i.e. from (3)& (4)

$$U(P,\ f) < \int_a^b f(x)dx + \frac{\varepsilon}{2} < \left\{L(P,\ f) + \frac{\varepsilon}{2}\right\} + \frac{\varepsilon}{2}$$

$\therefore U(P,\ f) < L(P,\ f) + \varepsilon$ for $P \in P[a,\ b]$ with $\|P\| < \delta$

$\Rightarrow U(P,\ f) - L(P,\ f) < \varepsilon$ for $P \in P[a,\ b]$ with $\|P\| < \delta$.

Conversely :( Sufficient condition)

If for $\varepsilon > 0 \exists$ partition $P \in P[a,\ b]$ such that $U(P,\ f) - L(P,\ f) < \varepsilon$

Then we need to show that $f$ is R-Integrable.

If $f$ is bounded, we have

$$L(P,f) \leq \int_{\underline{a}}^{b} f(x)dx \leq \int_a^{\bar{b}} f(x)dx \leq U(P,f)$$

$$\Rightarrow \int_a^{\bar{b}} f(x)dx - \int_{\underline{a}}^{b} f(x)dx \leq U(P,f) - L(P,f) < \varepsilon \rightarrow (5)$$

($\because$ Using properties of real numbers: if $a \leq b \leq c \leq d$ then $c - b \leq d - a$)

$\therefore$ from eqn. (5) $\int_a^{\bar{b}} f(x)dx - \int_{\underline{a}}^{b} f(x)dx < \varepsilon$

$$\Rightarrow \int_a^{\bar{b}} f(x)dx = \int_{\underline{a}}^{b} f(x)dx \text{ as } \varepsilon \rightarrow 0$$

**$\Rightarrow f$ is R-Integrable.**

## Algebra of Riemann Integrable Functions:

**I. Theorem: If $f: [a,\ b] \rightarrow R$ is R-Integrable and k is any non zero real then**

**k$f$ is also R-Integrable and $\int_a^b kf(x).dx = k\int_a^b f(x).dx$.**

**Proof:** Let $f$ be R- Integrable and k$\neq 0$ be any real number.

$\Rightarrow f$ is bounded and k$f$ is also bounded.

Let P= $\{x_0 = a,\ x_1,\ x_2, \dots\dots x_{r-1},\ x_r, \dots\dots . x_{n-1},\ x_n = b\}$ be any partition of $[a,\ b]$

and $m_r$ and $M_r$ be infimum and supremum of $f$ on $I_r$.

As f is R-integrable for $\varepsilon_1 > 0 \exists$ P$\in P[a,\ b]$ such that $U(P,\ f) - L(P,\ f) < \varepsilon_1$ ----(1)

If $m_r'$ & $M_r'$ be infimum and supremum of k$f$ on $I_r$ then

$m_r' = km_r$ and $M_r' = kM_r$ if k$> 0$

$m_r' = kM_r$ and $M_r' = km_r$ if k$< 0$

$\Rightarrow L(P,\ \mathrm{k}f) = \begin{cases} \sum_{r=1}^{n} m_r{}'\delta_r = \sum_{r=1}^{n} \mathrm{k}m_r\delta_r = \mathrm{k}\sum_{r=1}^{n} m_r\delta_r = \mathrm{k}L(P,\ f) \text{ if } \mathrm{k} > 0 \\ \sum_{r=1}^{n} M_r{}'\delta_r = \sum_{r=1}^{n} \mathrm{k}M_r\delta_r = \mathrm{k}\sum_{r=1}^{n} M_r\delta_r = \mathrm{k}U(P,\ f) \text{ if } \mathrm{k} < 0 \end{cases}$

Similarly $U(P,\ \mathrm{k}f) = \begin{cases} \mathrm{k}U(P,\ f) \text{ if } \mathrm{k} > 0 \\ \mathrm{k}L(P,\ f) \text{ if } \mathrm{k} < 0 \end{cases}$

$\therefore U(P,\ \mathrm{k}f) - L(P,\ \mathrm{k}f) = \mathrm{k}U(P,\ f) - \mathrm{k}L(P,\ f) = \mathrm{k}[U(P,\ f) - L(P,\ f)] < k\varepsilon_1$

$<\mathrm{k}\frac{\varepsilon}{k}$ if $\varepsilon_1 = \frac{\varepsilon}{k} > 0$

$<\varepsilon$

i.e $U(P,\ \mathrm{k}f) - L(P,\ \mathrm{k}f) < \varepsilon$ if k>0

And , if k<0 then $U(P,\ \mathrm{k}f) - L(P,\ \mathrm{k}f) = \mathrm{k}[L(P,\ f) - U(P,\ f)]$

$= -k[U(P,\ f) - L(P,\ f)]$

$< -k\varepsilon_1$

$<-\mathrm{k}\frac{\varepsilon}{-k}$ if $\varepsilon_1 = \frac{\varepsilon}{-k} > 0$,as k is negative, -k>0

$U(P,\ \mathrm{k}f) - L(P,\ \mathrm{k}f) < \varepsilon$ if k<0

Thus in both cases, $U(P,\ \mathrm{k}f) - L(P,\ \mathrm{k}f) < \varepsilon$ , $\forall k \neq 0$

=>kf is R-integrable

And $\underline{\int_a^b} \mathrm{k}f(x)dx =$ supremum $\{L(P,\ \mathrm{k}f)\}_{P\in P[a,b]}$ = sup $\{\mathrm{k}\, L(P,\ f)\}_{P\in P[a,b]}$ if $\mathrm{k} > 0$

$$= \mathrm{k}\sup\{L(p,\ f)\} = \mathrm{k}\underline{\int_a^b} f(x)dx \ = \text{ if } \mathrm{k} > 0$$

$$\therefore \underline{\int_a^b} \mathrm{k}f(x)dx = \mathrm{k}\underline{\int_a^b} f(x)dx = \mathrm{k}\int_a^b f(x)dx \text{ if } \mathrm{k} > 0 \to (p) \because f \in R[a,\ b]$$

Similarly $\overline{\int_a^b} \mathrm{k}f(x)dx = \mathrm{k}\overline{\int_a^b} f(x)dx = \mathrm{k}\int_a^b f(x)dx$

Thus $\int_a^b \mathrm{k}f(x)dx = \mathrm{k}\int_a^b f(x)dx$

**Cor:** $\int_a^b -f(x)dx = -\int_a^b f(x)dx$ **if k=-1**

**II. Theorem: If $f$ and $g$ are R-Integrable then $f+g$ is also R-Integrable.**

**i.e $f, g \in R[a, \ b] \Rightarrow f+g \in R[a, \ b]$ and hence prove that**

$$\int_a^b (f+g)(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

**Proof:** Let $f, g \in R[a, \ b] \Rightarrow f, g$ are bounded. {A function $f$ is bounded if $\exists$ positive real number 'k' such that $|f(x)| \leq k \forall x \in [a, \ b]$}.

$\therefore \exists$ positive real numbers $k_1$ and $k_2$ such that

$|f(x)| \leq k_1$ and $|g(x)| \leq k_2 \forall x \in [a, \ b]$ ---------- (1)

Consider, $|(f+g)(x)| = |f(x) + g(x)|$

$|(f+g)(x)| \leq |f(x)| + |g(x)| \leq k_1 + k_2$ from (1)

$\Rightarrow f+g$ is bounded.

And as $f$ & $g$ are R-Integrable then for $\varepsilon > 0 \exists$ a partition P such that

$U(P, \ f) - L(P, \ f) < \frac{\varepsilon}{2}$ and $U(P, \ g) - L(P, \ g) < \frac{\varepsilon}{2}$ -------(2)

Let P= $\{x_0 = a, \ x_1, \ x_2, \dots \dots x_{r-1}, \ x_r, \dots \dots . x_{n-1}, \ x_n = b\}$ be any partition of $[a, \ b]$ and let $m_r' \ m_r''$ and $m_r$ be infimum of $f, \ g$ &$(f+g)$ respectively on $I_r$ and $M_r'$, $M_r''$&$M_r$ be supremum of $f, \ g$ &$(f+g)$ respectively on $I_r$.

$m_r' \leq f(x) \leq M_r'$ and $m_r'' \leq g(x) \leq M_r'' \forall x \in I_r$.

Similarly, $m_r \leq (f+g)(x) \leq M_r \forall x \in I_r$.

And $f(x) + g(x) \leq M_r' + M_r'' \forall x \in I_r$. $\Rightarrow M_r' + M_r''$ is upper bound of $(f+g)$.

But $M_r$ is the least upper bound (i.e. supremum) of $(f+g)$.

$$\Rightarrow M_r \leq M_r' + M_r'' \Rightarrow$$

$$\sum_{r=1}^{n} M_r \delta_r \leq \sum_{r=1}^{n} M_r' \delta_r + \sum_{r=1}^{n} M_r'' \delta_r$$

$U(P, \ f+g) \leq U(P, \ f) + U(P, \ g)$ ------------ (3)

$L(P,\ f+g) \geq L(P,\ f) + L(P,\ g)$

$\Rightarrow -L(P,\ f+g) \leq -L(P,\ f) - L(P,\ g)$ − − − − − − − − − − − − − −(4)

$U(P,\ f+g) - L(P,\ f+g) \leq [U(P,\ f) - L(P,\ f)] + [U(P,\ g) - L(P,\ g)]$

$U(P,\ f+g) - L(P,\ f+g) \leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$ for $P \in P[a,\ b]$ from (1)

$$\boldsymbol{U(P, f+g) - L(P,\ f+g) < \varepsilon}$$

**$\Rightarrow \boldsymbol{f+g}$ is R-Integrable.**

Next we need to prove that $\int_a^b (\boldsymbol{f}+\boldsymbol{g})(\boldsymbol{x})\boldsymbol{dx} = \int_a^b \boldsymbol{f}(\boldsymbol{x})\boldsymbol{dx} + \int_a^b \boldsymbol{g}(\boldsymbol{x})\boldsymbol{dx}$

As $f$ & $g$ are R-Integrable, for $\varepsilon > 0 \exists$ partition P$\in P[a,\ b]$ such that

$$\left.\begin{array}{l} \int_a^b f(x)dx - \frac{\varepsilon}{2} = \underline{\int_a^b} f(x)dx - \frac{\varepsilon}{2} < L(P,\ f) \,\&\\ \int_a^b g(x)dx - \frac{\varepsilon}{2} = \underline{\int_a^b} g(x)dx - \frac{\varepsilon}{2} < L(P,\ g) \because \text{by Dourbox Theorem.} \end{array}\right\}$$

On adding we get,

$\int_a^b f(x)dx + \int_a^b g(x)dx - \varepsilon < L(P,\ f) + L(P,\ g) < L(P,\ f+g)$---from (4)

$\int_a^b f(x)dx + \int_a^b g(x)dx - \varepsilon < L(P,\ f+g) \leq \underline{\int_a^b}(f+g)(x)dx$

$\left\{\because \underline{\int_a^b}(f+g)(x)dx = \sup\{L(p,\ f+g)\}_{p\in P[a,b]}\right\}$

$\therefore \int_a^b f(x)dx + \int_a^b g(x)dx - \varepsilon \leq \int_a^b (f+g)(x)dx \ \because (f+g)$is R − Integrable.

$\therefore \int_a^b f(x)dx + \int_a^b g(x)dx \leq \int_a^b (f+g)(x)dx$ as $\varepsilon \to 0$ **eqn.** $\to (\boldsymbol{A})$

similarly $\int_a^b -f(x)dx + \int_a^b -g(x)dx \leq \int_a^b -(f+g)(x)dx$

$$\Rightarrow -\left[\int_a^b f(x)dx + \int_a^b g(x)dx\right] \le -\int_a^b (f+g)(x)dx$$

$$\Rightarrow \int_a^b (f+g)(x)dx \le \int_a^b f(x)dx + \int_a^b g(x)dx \;\; \boldsymbol{eqn.} \to (\boldsymbol{B})$$

From ***eqn.→(A) & eqn.*** $\to (\boldsymbol{B})$ ***we have***

$$\int_a^b f(x)dx + \int_a^b g(x)dx \le \int_a^b (f+g)(x).dx \le \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\therefore \int_a^b (\boldsymbol{f}+\boldsymbol{g})(\boldsymbol{x})\boldsymbol{dx} = \int_a^b \boldsymbol{f}(\boldsymbol{x})\boldsymbol{dx} + \int_a^b \boldsymbol{g}(\boldsymbol{x})\boldsymbol{dx}$$

**Corollary: If** $\boldsymbol{f}$ & $g \in \boldsymbol{R}[\boldsymbol{a}, \boldsymbol{b}]$ **then** $\boldsymbol{f} - \boldsymbol{g} \in \boldsymbol{R}[\boldsymbol{a}, \boldsymbol{b}]$.

**Proof:** As $g \in R[a, b]$and k=-1 $\Rightarrow (-1)g \in R[a, b] \Rightarrow -g \in R[a, b]$

$\therefore iff \in R[a, b]$and $-g \in R[a, b]$ then we have $f + (-g) \in R[a, b]$

$\Rightarrow f - g \in R[a, b]$

**III.If $\boldsymbol{f}$ and $\boldsymbol{g}$ are R-Integrable then $\boldsymbol{f}.\boldsymbol{g}$ is also R-Integrable.**

**i.e.** $\boldsymbol{f}, \boldsymbol{g} \in \boldsymbol{R}[\boldsymbol{a}, \boldsymbol{b}] \Rightarrow \boldsymbol{f}.\boldsymbol{g} \in \boldsymbol{R}[\boldsymbol{a}, \boldsymbol{b}]$.

**Proof:** Let$f, g \in R[a, b] \Rightarrow f, g$ are bounded

$\therefore \exists$Positive real numbers k such that

$|f(x)| \le$ k and $|g(x)| \le$ k $\forall x \in [a, b] \to (1)$and also$\exists \boldsymbol{\varepsilon}_1$ *and* $\boldsymbol{\varepsilon}_2$ such that

$\boldsymbol{U}(\boldsymbol{P}, \boldsymbol{f}) - \boldsymbol{L}(\boldsymbol{P}, \boldsymbol{f}) < \boldsymbol{\varepsilon}_1$ **and** $\boldsymbol{U}(\boldsymbol{P}, \boldsymbol{g}) - \boldsymbol{L}(\boldsymbol{P}, \boldsymbol{g}) < \boldsymbol{\varepsilon}_2 \to (2)$

WhereP= $\{x_0 = a, x_1, x_2, \dots\dots x_{r-1}, x_r, \dots\dots. x_{n-1}, x_n = b\}$ be any partition of $[a, b]$

let $m_r{}'\, m_r{}''$ and $m_r$ be infimum of $f, g$ &$(f.g)$ respectively on$I_r$ and $M_r{}'$, $M_r{}''$&$M_r$ be supremum of $f, g$ &$(f.g)$ respectively on$I_r$.

Let for $\alpha, \beta \in I_r$, Consider,

$|(f.g)(\beta)-(f.g)(\alpha)| = |f(\beta).g(\beta)-f(\alpha)g(\alpha)|$

$|M_r - m_r| = |f(\beta).g(\beta)-f(\beta)g(\alpha)+f(\beta)g(\alpha)-f(\alpha).g(\alpha)|$

$|M_r - m_r| = |f(\beta)[g(\beta)-g(\alpha)]+g(\alpha)[f(\beta)-f(\alpha)]|$

By using the properties of absolute values

$|M_r - m_r| \le |f(\beta)||g(\beta)-g(\alpha)|+|g(\alpha)||f(\beta)-f(\alpha)|$

$|M_r - m_r| \le k|g(\beta)-g(\alpha)|+k\,|f(\beta)-f(\alpha)|$

$|M_r - m_r| \le k|M_r'' - m_r''|+k\,|M_r' - m_r'|$

$(M_r - m_r) \le k(M_r'' - m_r'')+k\,(M_r' - m_r')$

$$\Rightarrow \sum_{r=1}^{n}(M_r - m_r)\delta_r \le k\sum_{r=1}^{n}(M_r'' - m_r'')\delta_r + k\sum_{r=1}^{n}(M_r' - m_r')\delta_r$$

$U(P,\ f.g)-L(P,\ f.g) \le k[U(P,\ g)-L(P,\ g)]+k[U(P,\ f)-L(P,\ f)]$

$U(P,\ f.g)-L(P,\ f.g) < k\,\boldsymbol{\varepsilon}_1 + k\boldsymbol{\varepsilon}_2$ $\because$ from (2)

$U(P,\ f.g)-L(P,\ f.g) < k\,\frac{\boldsymbol{\varepsilon}}{2k}+k\frac{\boldsymbol{\varepsilon}}{2k} = \varepsilon$

$\therefore U(P,\ f.g)-L(P,\ f.g) < \varepsilon \Rightarrow f.g$ is R-Integrable.

**Corollary: If $f$ is R-Integrable then $f^2$ is also R-Integrable i.e $f \in R[a,\ b]$**

$$\Rightarrow f^2 \in R[a,\ b]$$

**Proof:** From Result III, put $g = f$ we get $f^2 \in R[a,\ b]$.

**IV. Theorem: If $f$ is R-Integrable and f$\neq$ 0,then $f^{-1} = \frac{1}{f}$ is also R-Integrable.**

**i.e. if $f \in R[a,\ b] \Rightarrow f^{-1} = \frac{1}{f} \in R[a,\ b]$.**

**Proof:** Let $f \in R[a,\ b] \Rightarrow f$ is R − Integrable, hence $f$ is bounded. $\therefore \exists$ positive real number 'k' such that $|f(x)| \le$ k$\forall\ x \in [a,\ b]$.$------\rightarrow$ (1)

Similarly, $\exists$ another positive & non-zero real number 't' such that

$|f(x)| \ge t ----------------------\rightarrow (2) \forall\ x \in [a,\ b]$.

$\Rightarrow \frac{1}{|f(x)|} \le \frac{1}{t}$ i.e. $\frac{1}{|f|}(x) \le \frac{1}{t} \Rightarrow \frac{1}{f}$ is also bounded.

Also as $f$ is R-Integrable for $\varepsilon_1 > 0, \exists$ partition P such that

$$U(P,\ f) - L(P,\ f) < \boldsymbol{\varepsilon}_1 --------\rightarrow (3)$$

let $m_r$ , $m_r{}'$ *and* $M_r, M_r{}'$ be infimum and supremum of of $f$, $\frac{1}{f}$ respectively on $I_r$.

$\therefore$ for $\alpha, \beta \in I_r$, Consider,

$$\left|\frac{1}{f}(\beta) - \frac{1}{f}(\alpha)\right| = \left|\frac{1}{f(\beta)} - \frac{1}{f(\alpha)}\right|$$

$$|M_r{}' - m_r{}'| = \left|\frac{f(\alpha) - f(\beta)}{f(\beta)f(\alpha)}\right|$$

$$M_r{}' - m_r{}' = \frac{|f(\alpha) - f(\beta)|}{|f(\beta)||f(\alpha)|}$$

$$M_r{}' - m_r{}' \le \frac{|f(\alpha)-f(\beta)|}{t.t} \qquad \Rightarrow \frac{1}{|f(x)|} \le \frac{1}{t}\ \ \forall x \in I_r \subseteq I$$

$$M_r{}' - m_r{}' \le \frac{|m_r - M_r|}{t^2}$$

$$M_r{}' - m_r{}' \le \frac{(M_r - m_r)}{t^2}$$

$$\Rightarrow \sum_{r=1}^{n} (M_r{}' - m_r{}')\delta_r \le \frac{1}{t^2} \sum_{r=1}^{n} (M_r - m_r)\delta_r$$

$$U\left(P,\ \frac{1}{f}\right) - L\left(P,\ \frac{1}{f}\right) \le \frac{1}{t^2}[U(P,\ f) - L(P,\ f)]$$

$U\left(P,\ \frac{1}{f}\right) - L\left(P,\ \frac{1}{f}\right) \le \frac{1}{t^2}\boldsymbol{\varepsilon}_1 = \frac{1}{t^2}\varepsilon t^2$ From$\rightarrow$ (3) where $\boldsymbol{\varepsilon}_1 = \varepsilon t^2$ for $\varepsilon > 0$.

$$\Rightarrow U\left(P,\ \frac{1}{f}\right) - L\left(P,\ \frac{1}{f}\right) \le \varepsilon$$

$\Rightarrow \frac{1}{f}$ is R − Integrable. i.e. $\frac{1}{f} \in R[a,\ b]$

**Corollary: If $f$ and $g$ are R-Integrable and $g \neq 0$ then $\frac{f}{g}$ is also R-Integrable.**

**i.e. $f, g \in R[a,\ b]$ *and* $g \neq 0 \Rightarrow \frac{f}{g} \in R[a,\ b]$.**

**Proof:** Let $f, g \in R[a, \ b]$ $and$ $g \neq 0$,

$\therefore$from previous result $\frac{\mathbf{1}}{\boldsymbol{g}} \in R[a, \ b]$and

As $f, \ \frac{\mathbf{1}}{\boldsymbol{g}} \in R[a, \ b] \Rightarrow f.\frac{\mathbf{1}}{\boldsymbol{g}} \in R[a, \ b]$($\because$ as $f, g \in R[a, \ b] \Rightarrow f.g \in R[a, \ b]$ )

$\Rightarrow \frac{\boldsymbol{f}}{\boldsymbol{g}} \in R[a, \ b]$ $\therefore$ If $f$ and $g$ are R-Integrable and $g \neq 0$ then $\frac{f}{g}$ is also R-Integrable.

i.e. $f, g \in R[a, \ b]$ $and$ $g \neq 0 \Rightarrow \frac{f}{g} \in R[a, \ b]$.

**V. Theorem: If $\boldsymbol{f}$ is R-Integrable then $|\boldsymbol{f}|$ is also R-Integrable.**

**i.e. $\boldsymbol{f} \in \boldsymbol{R}[\boldsymbol{a}, \ \boldsymbol{b}] \ \Rightarrow |\boldsymbol{f}| \in \boldsymbol{R}[\boldsymbol{a}, \ \boldsymbol{b}]$. And $\left|\int_a^b \boldsymbol{f}(\boldsymbol{x}).\boldsymbol{dx}\right| \leq \int_a^b |\boldsymbol{f}(\boldsymbol{x})|.\boldsymbol{dx}$.**

**Proof:** Now $f \in R[a, \ b]$

$\Rightarrow f$ is bounded. Then $\exists$ positive real number 'k' such that $|f(x)| \leq$ k$\forall \ x \in [a, \ b]$

i.e. $|f|$ is bounded. And as $f$ is R-Integrable for $\varepsilon > 0\exists$ partition

P= $\left\{x_0 = a, \ x_1, \ x_2, \dots \dots x_{r-1}, \ x_{r\,,} \dots \dots . x_{n-1}, \ x_n = b\right\}$ of $[a, \ b]$

Such that $U(P, \ f) - L(P, \ f) < \boldsymbol{\varepsilon} \rightarrow (\mathbf{1})$**.**

Let $m_r\,, m_r{}'$ $and$ $M_r, \ M_r{}'$be infimum and supremum of of $f$ &$|f|$ respectively on$I_r$.

$\therefore$there exists $\alpha, \beta \in I_r$ such that $M_r{}' = |f|(\beta) and \ m_r{}' = |g|(\beta)$

and, $M_r = \ -m_r$

Consider, $\big||f|(\beta) - |f|(\alpha)\big| = \big||f(\beta)| - |f(\alpha)|\big| \leq |f(\beta) - f(\alpha)|$

$\{\boldsymbol{B'cz}\big||\boldsymbol{x}| - |\boldsymbol{y}|\big| \leq |\boldsymbol{x} - \boldsymbol{y}|\}$

$i.e \ M_r{}' - \ m_r{}' \leq \ M_r - \ m_r$

$$\Rightarrow \sum_{r=1}^{n} (M_r{}' - \ m_r{}')\delta_r \leq \sum_{r=1}^{n} (M_r - \ m_r)\delta_r$$

$U(P, \ |f|) - L(P, \ |f|) \leq U(P, \ f) - L(P, \ f) < \boldsymbol{\varepsilon}$

$\Rightarrow|f|$ is R-Integrable i.e. $|f| \in R[a, \ b]$.

**Next, we have to prove $\left|\int_a^b \boldsymbol{f}(\boldsymbol{x})\boldsymbol{dx}\right| \leq \int_a^b |\boldsymbol{f}(\boldsymbol{x})| \, \boldsymbol{dx}$.**

We know that $|f| = max\{f, -f\}$

$$\Rightarrow \left.\begin{array}{l} f(x) \leq |f(x)| = |f|(x) \\ -f(x) \leq |f|(x) \end{array}\right\} \forall\, x \in [a, \ b] \text{-------------------}(2)$$

$$\Rightarrow \int_a^b f(x)dx \leq \int_a^b |f|(x)\, dx - - - - - - - - - - - -(3)$$

$$and \int_a^b -f(x)dx \leq \int_a^b |f|(x)\, dx \ \Rightarrow -\int_a^b f(x)dx \leq \int_a^b |f|(x)\, dx$$

$$i.e \int_a^b f(x)dx \ \geq -\int_a^b |f|(x)\, dx - - - - - - - - - -(4)$$

∴ from $eqn. \rightarrow (3)$& $eqn. \rightarrow (4)$ we have

$$-\int_a^b |f|(x)\, dx \leq \int_a^b f(x)dx \leq \int_a^b |f|(x)\, dx$$

$$\Rightarrow \left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|\, dx.$$

**Note: Converse of above theorem need not be true. i.e. If $|f|$ is R-Integrable then it is not necessary that $f$ is R-$Integrable. i.e. If$ $|f| \in R[a, \ b] \not\Rightarrow f \in R[a, \ b]$.**

**For Example** : If $f{:}[0, \ 1] \rightarrow R$ defined by $f(x) = \begin{cases} 1 \ \forall \ rational \ x \in [0, \ 1] \\ -1 \forall \ irrational \ x \in [0, \ 1] \end{cases}$

Clearly, $f$ is not R-Integrable.

But $|f(x)| = |f|(x) = 11 \ \forall \ x \in [0, \ 1]$

Which is R- integrable as every constant function is R-Integrable.

**VI. Theorem: If $f$ is R-Integrable i.e. $f \in R[a, \ b]$ and $f(x) \geq 0 \ \forall x \in \ [a, \ b]$ then $i)$ $\int_a^b f(x)dx \geq 0$**

**$ii)$ if $f(x) \geq g(x) \ \forall x \in \ [a, \ b]$then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$ where $g \in R[a, \ b]$.**

**Proof:*i)*** Let $f \in R[a, \ b]$ $f(x) \geq 0 \ \forall x \in \ [a, \ b]$

$\Rightarrow f$ is bounded and hence attains infimum 'm' and supremum 'M' on $[a, \ b]$.
Since $f(x) \geq 0 \ \forall x \in \ [a, \ b] \Rightarrow$ 'm' and 'M' are positive &$(b-a)$isalso positive
∴ $m(b-a) \geq 0$ and also we have , for any partition P ∈ $P[a, \ b]$.

$L(P, f) \geq m(b-a)\ \forall\ \text{P} \in P[a, b]$

$\Rightarrow Sup\{L(P, f)\} \geq 0 \Rightarrow \int_{\underline{a}}^{b} f(x)dx \geq 0$

$\Rightarrow \int_{a}^{b} f(x)dx \geq 0 \because f \in R[a, b]$

$\therefore f(x) \geq 0 \Rightarrow \int_{a}^{b} f(x)dx \geq 0$

***ii)*** Given $f(x) \geq g(x)\ \forall\ x \in [a, b]$

$\Rightarrow f(x) - g(x)\ \forall\ x \in [a, b]$ i.e. $(f-g)(x) \geq 0$

$\Rightarrow \int_{a}^{b} f(x).dx - \int_{a}^{b} g(x)dx \geq 0 \Rightarrow \int_{a}^{b} f(x)dx \geq \int_{a}^{b} g(x)dx$

Hence, if $f(x) \geq g(x)$ and $f, g \in R[a, b]$

$\Rightarrow \int_{a}^{b} f(x)dx \geq \int_{a}^{b} g(x)dx$ .

# UNIT II

# Riemann Integration II

## UNIT − 2. Riemann Integration − II

In this unit we are studying some classes of Riemann Integrable functions, Riemann Sum and another definition of Riemann integration, Mean Value theorems and Fundamental theorem of integral calculus.

### Some Classes of Riemann Integrable Functions:

**Theorem 01:If $f$:$[a,\ b] \to R$ is continuous then $f$ is R-Integrable. OR**

**"Every continuous function on $[a,\ b]$ is R − Integrable."**

**Proof:**To prove the result we use the following theorems on the continuous functions.

$i$) Every continuous function on $[a,\ b]$ is bounded & attains it's supremum and infimum.

$ii$) Every continuous function on $[a,\ b]$ is uniformly continuous.

Let $f$:$[a,\ b] \to R$be an arbitrary continuous function defined on$[a,\ b]$

$\Rightarrow f$ is bounded.

And also$f$ is uniformly continuous on$[a,\ b]$.

$\Rightarrow$ for each $\varepsilon_1 > 0,\ \exists\delta > 0$ such that $|f(x') - f(x'')| < \varepsilon_1 \dashrightarrow (1)$

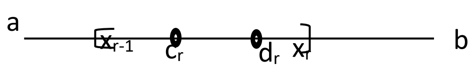
for $all\ x'\&x'' \in [a, b]$ such that $|x' - x''| < \delta - - - - - - \dashrightarrow (2)$

P= $\{x_0 = a,\ x_1,\ x_2, \dots \dots x_{r-1},\ x_{r\,,} \dots \dots . x_{n-1},\ x_n = b\}$ be any partition of $[a,\ \ b]$

Such tha$\|P\| < \delta$.

Since $f$ is continuous on $[a, b] => it\ is\ continuouson\ \ I_r = [x_{r-1},\ x_r]$also.

If $m_r$and$M_r$ be infimum and supremum of $f$ on $I_r \Rightarrow\ \exists c_r$and $d_r$ in $I_r$ such that




$m_r = f(c_r)$ and $M_r = f(d_r)$

$\therefore |c_r -\ d_r| \leq |x_r -\ x_{r-1}| \leq |x' -\ \mathrm{x}''|$

$$i.e\ |c_r -\ d_r| \leq \delta_r < \delta, \|P\| < \delta$$

i.e. $|c_r -\ d_r| \leq \delta$------------------------------------------------------(3)

$\Rightarrow |f(c_r) - f(d_r)| < \varepsilon_1$ *from (1) and (3)*

$i.e\ |m_r - M_r| < \varepsilon_1\ \Rightarrow M_r - m_r < \varepsilon_1$

⇨ $\sum_{r=1}^{n}(M_r - m_r)\delta_r < \varepsilon_1 \sum_{r=1}^{n}\delta_r$

$U(P,\ f) - L(P,\ f) < \boldsymbol{\varepsilon}_1(b-a)$

Now choose $\boldsymbol{\varepsilon}_1 = \frac{\varepsilon}{(b-a)}$ where $\varepsilon > 0 \Rightarrow U(P,\ f) - L(P,\ f) < \boldsymbol{\varepsilon}$

$\Rightarrow f$ is R- Integrable.

$\therefore$ as $f$ is arbitrary then, "Every continuouys function is R- Integrable."

**Theorem 01: If $f:[a,\ b] \to R$ be monotonic on $[a,\ b]$ then $f$ is R-Integrable.**

**Proof:** Let $f$ be an arbitrary monotonically increasing function.

$\therefore \forall\, x \in [a,\ b]\ i.e\ for\ a\ \le x \le b\ ,\ f(a)\ \le f(x) \le f(b) \Rightarrow f$ is bounded.

Let for $\varepsilon > 0,$ be arbitrary &

let P= $\{x_0 = a,\ x_1,\ x_2, \dots \dots x_{r-1},\ x_r, \dots \dots . x_{n-1},\ x_n = b\}$ be any partition of $[a,\ b]$

such that $\delta_r < \frac{\varepsilon}{f(b)-f(a)+1}$ ------------------------→ (1)

Let $m_r$ and $M_r$ be infimum and supremum of $f$ on $I_r = [x_{r-1},\ x_r]$

$\Rightarrow\ \exists c_r$ and $d_r$ in $I_r$ such that $m_r = f(x_{r-1})$ and $M_r = f(x_r)$

Now consider LHS = $U(P,\ f) - L(P,\ f)$

$$\text{LHS} = \sum_{r=1}^{n}(M_r - m_r)\delta_r = \sum_{r=1}^{n}[\,f(x_r) - f(x_{r-1})]\ \frac{\varepsilon}{f(b) - f(a) + 1} \text{ from (1)}$$

$$\text{LHS} = \sum_{r=1}^{n}(M_r - m_r)\delta_r < \frac{\varepsilon}{f(b) - f(a) + 1}\sum_{r=1}^{n}[\,f(x_r) - f(x_{r-1})]$$

$$\text{LHS} = \sum_{r=1}^{n}(M_r - m_r)\delta_r$$

$$< \frac{\varepsilon}{f(b) - f(a) + 1}\begin{bmatrix} f(x_1) - f(x_0) + f(x_2) - f(x_1) + \\ f(x_3) - f(x_2) + \cdots + f(x_n) - f(x_{n-1}) \end{bmatrix}$$

$$\text{LHS} = \sum_{r=1}^{n}(M_r - m_r)\delta_r < \frac{\varepsilon}{f(b) - f(a) + 1}[f(x_n) - f(x_0)]$$

$$\text{LHS} = \sum_{r=1}^{n}(M_r - m_r).\delta_r < \frac{\varepsilon}{f(b) - f(a) + 1}[f(b) - f(a)] < \varepsilon.1$$

LHS = $U(P,\ f) - L(P,\ f) < \varepsilon = \varepsilon$ where $\varepsilon = \varepsilon > 0$.

$\Rightarrow f$ is R-Integrable as $f$ is monotonically increasing function.

Similarly, if $f$ is monotonically decreasing function then $f$ is R-Integrable.

As $f$ is arbitrary then every monotonic function is R-Integrable.

**Theorem 03: If the set of points of discontinuity of a bounded function $f$:[$a$, $b$] → $R$ is finite then $f$ is R-Integrable.**

**Proof:** Even though $f$ is discontinuous in $[a,\ b]$ then also f is R-Integrable provided number of discontinuous points are finite.

**Theorem 04: If the set of points of discontinuity of a bounded function $f$:[$a$, $b$] → $R$ has a finite number of limit points then $f$ is R-Integrable on $[a,\ b]$.**

**Proof:** Even though set of points of discontinuity of $f$ on $[a,\ b]$ is infinite then also f is integrable provided set of discontinuous points has finite number of limit points.

**Riemann Sum:**

Let $f:[a,\ b] \rightarrow R$ be bounded function and

P= {a=$x_0$, $x_1$,$x_2$, --------------------------$x_{r-1}$, $x_r$,------------------$x_n$=b} be a partition of [a,b]

and $x_{r-1} \leq \xi_r \leq x_r$, then the sum $\sum_{r=1}^{n} f\xi_r).\delta_r$ is called Riemann sum and $\lim_{n\to\infty} \sum_{r=1}^{n} f\xi_r).\delta_r$ is called Riemann integral of f and denotd by $\int_a^b f(x)dx$ .

**Theorem: ( Second definition of Riemann Integration):**

**A function $f:[a,\ b] \rightarrow R$ is R-Integrable on $[a,\ b]$ if for $\varepsilon > 0$, $\exists\ \delta > 0$ such that for every partition P= $\{x_0 = a,\ x_1,\ x_2, \ldots x_{r-1},\ x_r, \ldots ., \ x_n = b\}$ of $[a,\ b]$ with $\|P\| < \delta$**

**$\exists\ \xi_r \in [x_{r-1},\ x_r]$ such that $\left|\sum_{r=1}^{n} f(\xi_r).\delta_r - \int_a^b f(x).dx.\right| < \varepsilon$.**

**Proof:** Let a bounded function $f:[a,\ b] \rightarrow R$ be R-Integrable, then by definition we have

$$\int_{\underline{a}}^{b} f(x)dx = \int_{a}^{\bar{b}} f(x)dx = \int_{a}^{b} f(x)dx ------------\rightarrow (1)$$

let $\varepsilon > 0$ be smallest positive real number, by Daurbox theorem, $\exists\ \delta > 0$ such that for every partition P with $\|P\| < \delta$.

$$U(P,\ f) < \int_{a}^{\bar{b}} f(x)dx + \varepsilon \text{ i.e. } U(P,\ f) < \int_{a}^{b} f(x)dx + \varepsilon \rightarrow (2) \text{from } (1)$$

$$\text{Similarly, } L(P,\ f) > \int_{\underline{a}}^{b} f(x).dx - \varepsilon$$

$$i.e. L(P,\ f) > \int_{a}^{b} f(x)dx - \varepsilon --------------\rightarrow (3) \text{ from } (1)$$

Let $m_r$ and $M_r$ be infimum and supremum of $f$ on $I_r = [x_{r-1},\ x_r]$. and

$x_{r-1} \leq \xi_r \leq x_r$, then $m_r \leq f(\xi_r) \leq M_r$

$$\Rightarrow \sum_{r=1}^{n} m_r\delta_r \leq \sum_{r=1}^{n} f(\xi_r)\delta_r \leq \sum_{r=1}^{n} M_r\delta_r \quad \because \delta_r > 0$$

$$\Rightarrow L(P,\ f) \leq \sum_{r=1}^{n} f(\xi_r).\delta_r \leq U(P,\ f)$$

Now from (1)& (2)

$$\Rightarrow \int_{a}^{b} f(x)dx - \varepsilon < L(P,\ f) \leq \sum_{r=1}^{n} f(\xi_r)\delta_r \leq U(P,\ f) < \int_{a}^{b} f(x)dx + \varepsilon$$

$$\Rightarrow \int_{a}^{b} f(x)dx - \varepsilon < \sum_{r=1}^{n} f(\xi_r)\delta_r < \int_{a}^{b} f(x)dx + \varepsilon$$

$\therefore \left|\sum_{r=1}^{n} f(\xi_r)\delta_r - \int_{a}^{b} f(x)dx\right| < \varepsilon$ bcz lx-al< $\varepsilon$ means a- $\varepsilon$<x<a+ $\varepsilon$

As $\varepsilon \rightarrow 0$, we get

$$\Rightarrow \lim_{n\rightarrow\infty} \sum_{r=1}^{n} f(\xi_r)\delta_r = \int_{a}^{b} f(x)dx$$

## I. Examples on some classes of R-Integrable function.

**1. Prove that $f(x) = 2x+3$ is R − Integrable and hence deduce $\int_1^2 f(x)dx = 6$.**

**Proof:** Let given function is $f(x) = 2x+3$ in $[1,\ 2]$.

Clearly, $5 \leq f(x) \leq\ 7\ \forall\ x \in [1,\ 2] \Rightarrow f$ is bounded.

We know that every polynomial function is continuous and hence $f$ is continuous.

$\Rightarrow f$ is R-Integrable on $[1,\ 2]$ as every continuous function on $[a,\ b]$ is R-Integrable.

Let P= $\left\{x_0 = a = 1,\ 1+\frac{1}{n},\ \ 1+\frac{2}{n}, \ldots 1+\frac{(r-1)}{n},\ 1+\frac{r}{n}, \ldots.,\ 1+\frac{n}{n} = 2 = x_n = b\right\}$

be any partition of $[1,\ 2]$.

$x_{r-1} \leq \xi_r \leq\ x_r$, then $f(1+\frac{(r-1)}{n}) \leq f(\xi_r) \leq\ f(1+\frac{r}{n})$

Let $\xi_r = 1+\frac{r}{n} = x_r$ then $f(\xi_r) = 2\left(1+\frac{r}{n}\right)+3$

$$\therefore \text{ Riemann sum } \sum_{r=1}^{n} f(\xi_r)\delta_r = \sum_{r=1}^{n}\left[2+\frac{2r}{n}+3\right]\frac{1}{n} = \frac{2n}{n}+\frac{2n(n+1)}{2n^2}+\frac{3n}{n}$$

$$\text{Riemann sum } \sum_{r=1}^{n} f(\xi_r)\delta_r = 5+\frac{(n+1)}{n}.$$

$$\text{Now Riemann Integral} = \lim_{n\to\infty}\sum_{r=1}^{n} f(\xi_r).\delta_r = \lim_{n\to\infty} 5+1+\frac{1}{n} = 6.$$

**2. Prove by definition of Riemann integration that $\int_0^1 (2x^2-3x+5).dx = \frac{25}{6}$.**

**Proof:** Let $f(x) = (2x^2-3x+5)$. Then $4 \leq f(x) \leq 5\ \forall\ x \in [0,\ 1] \Rightarrow$
$f$ is bounded.

and also function is continuous as it is polynomial. And every continuous function on $[0,\ 1]$ is R-Integrable.

$\therefore\ f(x)$is R-Integrable on $[0,\ 1]$.

Let P= $\left\{x_0 = a = 0, \frac{1}{n},\ \frac{2}{n}, \dots \frac{(r-1)}{n},\ \frac{r}{n}, \dots .,\ \frac{n}{n} = 1 = x_n = b\right\}$be any partition of $[0,\ 1]$

Such that$\|P\| \to 0$ as $n \to \infty$.

By the definition of Riemann integral we have,

$$\lim_{n\to\infty}\sum_{r=1}^{n} f(\xi_r)\delta_r = \int_0^1 f(x)dx\ .\ where\ \frac{(r-1)}{n} \le \xi_r \le \frac{r}{n}$$

Let $\xi_r = \frac{r}{n}$ then $\int_0^1 f(x).dx = \lim_{n\to\infty}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)\frac{1}{n} = \frac{1}{n}\lim_{n\to\infty}\sum_{r=1}^{n}\left[2\frac{r^2}{n^2} - 3\frac{r}{n} + 5\right]$.

$$\int_0^1 f(x)dx = \frac{1}{n}\lim_{n\to\infty}\left[\frac{2}{n^2}\frac{n(n+1)(2n+1)}{6} - \frac{3n(n+1)}{2n} + 5n\right].$$

$$\int_0^1 f(x)dx = \lim_{n\to\infty}\left[\frac{2}{n^2}\frac{n^2\left(1+\frac{1}{n}\right)(2+\frac{1}{n})}{6} - \frac{3n^2(1+\frac{1}{n})}{2n^2} + 5\frac{1}{n}n\right].$$

$$\int_0^1 f(x)dx = \frac{2}{3} - \frac{3}{2} + 5 = \frac{4-9+30}{6} = \frac{25}{6}$$

$$\int_0^1 f(x)dx = \frac{25}{6}.$$

### 3. Evaluate $\int_{-1}^{1} f(x)dx$ where $f(x) = |x|$ by Riemann integration.

**Proof**: Let $f(x) = |x| = \begin{cases} -x & if\ -1 \le x < 0 \\ x & if\ x \ge 0 \end{cases}$

We know that $|x|$ is continuous $\forall\ x \in [-1,\ 1]$ and hence it is R-Integrable as every continuous function is R-Integrable on $[-1,\ 1]$.

Let P= $\left\{x_0 = a = -1,\ -1+\frac{1}{n},\ , \dots -1+\frac{n}{n},\ \frac{1}{n},\ \frac{2}{n} \dots .\frac{n}{n} = 1 = x_n = b\right\}$

be any partition of $[-1,\ 1]$. Such that$\|P\| \to 0$ as $n \to \infty$.

By the definitionof Riemann integral we have,

$$\lim_{n\to\infty}\sum_{r=1}^{n} f(\xi_r)\delta_r = \int_{-1}^{1} f(x)dx\ .\ where x_{r-1} \le \xi_r \le x_r\ and\ \delta_r = \frac{1}{n}$$

$$\therefore \int_{-1}^{1} f(x)dx = \lim_{n\to\infty}\sum_{r=1}^{n} f(\xi_r)\delta_r = \lim_{n\to\infty}\sum_{r=1}^{n} f(x_r)\frac{1}{n}$$

$$\int_{-1}^{1} f(x)dx = \lim_{n\to\infty}\left[\sum_{r=1}^{n} f(x_r)\frac{1}{n} + \sum_{r=n+1}^{2n} f(x_r)\frac{1}{n}\right]$$

$$\int_{-1}^{1} f(x)dx = \lim_{n\to\infty}\left[\sum_{r=1}^{n} f\left(-1+\frac{r}{n}\right)\frac{1}{n} + \sum_{r=n+1}^{2n} f\left(-1+\frac{r}{n}\right)\frac{1}{n}\right]$$

$$\int_{-1}^{1} f(x)dx = \lim_{n\to\infty}\left[\sum_{r=1}^{n} -\left(1-\frac{r}{n}\right)\frac{1}{n} + \sum_{r=n+1}^{2n} \left(-1+\frac{r}{n}\right)\frac{1}{n}\right.$$

$b'cz\ f(x) = -x\ if -1 \le x < 0\ and\ f(x) = x\ if 0 \le x \le 1$

$$\int_{-1}^{1} f(x)dx = \lim_{n\to\infty}\frac{1}{n}\left\{\sum_{r=1}^{n} 1 - \sum_{r=1}^{n}\frac{r}{n} + (-1)\left[\sum_{r=n+1}^{2n} 1 - \sum_{r=n+1}^{2n}\frac{r}{n}\right]\right\}$$

$$= \lim_{n\to\infty}\left\{\frac{1}{n}\sum_{r=1}^{n} 1 - \frac{1}{n}\sum_{r=1}^{n}\frac{r}{n} - \frac{1}{n}\left[\sum_{r=n+1}^{2n} 1 - \sum_{r=n+1}^{n}\frac{r}{n}\right]\right\}$$

$$= \lim_{n\to\infty}\left\{\left(\frac{1}{n}\ n - \frac{1}{n^2}\sum_{r=1}^{n} r\right) - \left(\frac{1}{n}(1+1+ ---(2n-n)times - \frac{1}{n^2}\sum_{r=n+1}^{n} r\right)\right\}$$

$$= \lim_{n\to\infty}\left\{\left(1 - \frac{n(n+1)}{2n^2}\right) - \left(\frac{1}{n}n - \frac{1}{n^2}\ [(n+1)+(n+2)+(n+3) -- \ -+(2n)]\right\}$$

$$= \lim_{n\to\infty}\left\{\left(1 - \frac{(n+1)}{2n}\right) - \left(1 - \frac{1}{n^2}\ \left[\frac{n}{2}\{(n+1)+2n\right]\right\}$$

b'cz in an AP $S_n = \frac{n}{2}\{(a+l)\} where\ l\ is\ nth$ term

$$= \lim_{n\to\infty}\left\{\left(1 - \frac{(n+1)}{2n} - 1 + \frac{1}{n^2}\ \left[\frac{n}{2}\{(3n+1)\right)\right\}$$

$$= \lim_{n\to\infty}\left\{\left(-\frac{(n+1)}{2n} + \frac{1}{n}\ \left[\frac{1}{2}\{(3n+1)\right)\right\} = \lim_{n\to\infty}\left\{\left(-\frac{1}{2}\left(1+\frac{1}{n}\right) + \frac{1}{2}\left(3+\frac{1}{n}\right)\right)\right\}$$

$$= -\frac{1}{2}+\frac{3}{2} = 1$$

$\therefore \int_{-1}^{1} f(x)dx = 1$ .

**4. Show that greatest integer function $f(x)$**

**$= [x]$ is integrable on $[0, \ 4]$ and show that $\int_0^4 f(x)dx = 6$.**

**Soln:** Let $f:[0, \ 4] \to R$ defined by $f(x) = [x]$ (greatest integer function )

***Definition of Greatestintegerfunction***: For real number $x$, greatest integer function is an integer near to $x$ or $\leq x$ and is denoted by $[x]$. $\therefore [x]$ is an integer $\leq x$ and near to $x$ .Examples: $\left[\frac{1}{2}\right] = [0.5] = 0$ , $[99.9] = 99, [-0.5] = \left[-\frac{1}{2}\right] = -1, \left[\frac{100}{3}\right] = [33.33] = 33$

Cleraly $f(x) = [x]$ is not continuous at finite number of points.i,e at 1, 2, 3 and 4

$\Rightarrow f$ is R − Integrable as if $f$ has finite number of discontinuities.

$$\therefore \int_0^4 f(x)dx \text{ exist.}$$

$$\therefore \int_0^4 f(x).dx = \int_0^4 [x].dx = \int_0^1 [x].dx + \int_1^2 [x].dx + \int_2^3 [x].dx + \int_3^4 [x].dx$$

$$\int_0^4 [x].dx = \int_0^1 0.dx + \int_1^2 1.dx + \int_2^3 2.dx + \int_3^4 3.dx$$

$\because \forall x \in [0, \ 1]$then $[x] = 0$ `similarly for remaining intervals.

$$\therefore \int_0^4 [x].dx = 0 + (2-1) + 2(3-2) + 3(4-3) = 1 + 2 + 3 = 6$$

Thus $[x]$ is R − Integrable in $[0, \ 4]$ and $\int_0^4 [x].dx = 6$.

**5. Showthatthefunction$f$definedby$f(x) = \frac{1}{2^n}$where$\frac{1}{2^{n+1}} \leq x \leq \frac{1}{2^n}$is R − Integrablein$[0, \ 1]$althoughithasinfinitenumberofdiscountinuities. andevaluate $\int_0^1 f(x).dx$**

**Soln**: Let $f(x) = \frac{1}{2^n}$ where $\frac{1}{2^{n+1}} \leq x \leq \frac{1}{2^n}$ n $= 0, 1, \ 2, \ldots$

$f(0) = 0$ when x=0

$f(x) = 1$ when $\frac{1}{2} \leq x \leq 1 \ n = 0$

$= \frac{1}{2}$ when $\frac{1}{2^2} \leq x \leq \frac{1}{2} n = 1$

$= \frac{1}{2^2}$ when $\frac{1}{2^3} \leq x \leq \frac{1}{2^2} n = 2$

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$f(x) = \frac{1}{2^{n-1}}$ when $\frac{1}{2^n} \leq x \leq \frac{1}{2^{n-1}} n = n - 1$

$= \frac{1}{2^n}$ when $\frac{1}{2^{n+1}} \leq x \leq \frac{1}{2^n} n = n$

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$f(0) = 0,$

Clearly the function $f(x)$ is discontinuous at the points 1, $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4} \ldots.$

Which are infinite in number but the set $\left\{1, \ \frac{1}{2}, \ \frac{1}{2^2}, \frac{1}{2^3} \ldots\right\}$ has a limit point '0'.

$\therefore$ by one of the property of $f(x)$ is R-Integrable, we consider the integral,

$$\int_{\frac{1}{2^n}}^{1} f(x).dx = \int_{\frac{1}{2^n}}^{\frac{1}{2^{n-1}}} f(x).dx + \int_{\frac{1}{2^{n-1}}}^{\frac{1}{2^{n-2}}} f(x).dx + \cdots + \int_{\frac{1}{2^2}}^{\frac{1}{2}} f(x).dx + \int_{\frac{1}{2}}^{1} f(x).dx$$

$$= \int_{\frac{1}{2}}^{1} f(x).dx + \int_{\frac{1}{2^2}}^{\frac{1}{2}} f(x).dx + \int_{\frac{1}{2^3}}^{\frac{1}{2^2}} f(x).dx + \cdots + \int_{\frac{1}{2^n}}^{\frac{1}{2^{n-1}}} f(x).dx$$

$$= \int_{\frac{1}{2}}^{1} 1.dx + \int_{\frac{1}{2^2}}^{\frac{1}{2}} \frac{1}{2}.dx + \int_{\frac{1}{2^3}}^{\frac{1}{2^2}} \frac{1}{2^2}.dx + \cdots + \int_{\frac{1}{2^n}}^{\frac{1}{2^{n-1}}} \frac{1}{2^{n-1}}.dx$$

$$= 1.\left[1 - \frac{1}{2}\right] + \frac{1}{2}\left[\frac{1}{2} - \frac{1}{2^2}\right] + \frac{1}{2^2}\left[\frac{1}{2^2} - \frac{1}{2^3}\right] + \cdots + \frac{1}{2^{n-1}}\left[\frac{1}{2^{n-1}} - \frac{1}{2^n}\right]$$

$$i.e \int_{\frac{1}{2^n}}^{1} f(x).dx = \left\{\frac{1}{2} + \frac{1}{2}.\frac{1}{2^2} + \frac{1}{2^2}.\frac{1}{2^3} + \cdots + \frac{1}{2^{n-1}}.\frac{1}{2^n}\right\}$$

$$= \frac{1}{2}\left\{1 + \frac{1}{2^2} + \frac{1}{2^4} + \cdots + \frac{1}{(2^{n-1})^2}\right\}$$

$$\int_{\frac{1}{2^n}}^{1} f(x).dx = \frac{1}{2}\left\{\frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{2^2}}\right\} = \frac{1}{2}\left\{\frac{1-\frac{1}{4^n}}{\frac{3}{2^2}}\right\} = \frac{2}{3}\left\{1-\frac{1}{4^n}\right\}$$

$$\int_{\frac{1}{2^n}}^{1} f(x).dx = \frac{2}{3}\left\{1-\frac{1}{4^n}\right\}$$

$$\therefore \int_0^1 f(x).dx = \lim_{n\to\infty}\int_{\frac{1}{2^n}}^{1} f(x).dx = \lim_{n\to\infty}\frac{2}{3}\left\{1-\frac{1}{4^n}\right\} = \frac{2}{3}.$$

**6. Showthatthefunction $f$ definedon $[0,\ 1]$ by**

$$f(x) = \begin{cases} \frac{1}{n} & if\ \frac{1}{n+1} \le x \le \frac{1}{n} \\ 0 & if\ x = 0 \end{cases}$$ **isR − Integrableand hence prove that**

$$\int_0^1 f(x).dx = \frac{\pi^2}{6} - 1.$$

**Soln:** Let given function $f(x) = \frac{1}{n}$ when $\frac{1}{n+1} \le x \le \frac{1}{n}$ n $= 0, 1,\ \ 2, \ldots$

$f(0) = 0$ when $x = 0$

i.e.

$$f(x) = 1 \text{ when } \frac{1}{2} \le x \le 1\ n = 0$$

$$= \frac{1}{2} \text{when } \frac{1}{3} \le x \le \frac{1}{2} n = 1$$

$$= \frac{1}{3} \text{when } \frac{1}{4} \le x \le \frac{1}{3} n = 2$$

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$$f(x) = \frac{1}{n} \text{when } \frac{1}{n+1} \le x \le \frac{1}{n} n = n$$

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$f(0) = 0$, at $x = 0$

Cleraly the function is discountinuous at the points $1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \ldots\frac{1}{n}\ldots.$

Which are infinite in number. But the set $\left\{1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \ldots\frac{1}{n}\ldots.\right\}$ has a limit point '0'(zero). $\therefore$ by one of the property $f(x)$ is R-Integrable.

Now consider the integral,

$$\int_{\frac{1}{n}}^{1} f(x)dx = \int_{\frac{1}{n}}^{\frac{1}{n-1}} f(x)dx + \int_{\frac{1}{n-1}}^{\frac{1}{n-2}} f(x)dx + \cdots + \int_{\frac{1}{3}}^{\frac{1}{2}} f(x)dx + \int_{\frac{1}{2}}^{1} f(x)dx$$

$$\int_{\frac{1}{n}}^{1} f(x)dx = \int_{\frac{1}{2}}^{1} f(x)dx + \int_{\frac{1}{3}}^{\frac{1}{2}} f(x)dx + \cdots + \int_{\frac{1}{n-1}}^{\frac{1}{n-2}} f(x)dx + \int_{\frac{1}{n}}^{\frac{1}{n-1}} f(x)dx$$

$$\int_{\frac{1}{n}}^{1} f(x).dx = \int_{\frac{1}{2}}^{1} 1\,.dx + \int_{\frac{1}{3}}^{\frac{1}{2}} \frac{1}{2}.dx + \cdots + \int_{\frac{1}{n-1}}^{\frac{1}{n-2}} \frac{1}{n-2}.dx + \int_{\frac{1}{n}}^{\frac{1}{n-1}} \frac{1}{n-1}.dx.$$

$$\int_{\frac{1}{n}}^{1} f(x).dx = 1.\left[1-\frac{1}{2}\right] + \frac{1}{2}\left[\frac{1}{2}-\frac{1}{3}\right] + \cdots + \frac{1}{n-2}\left[\frac{1}{n-2}-\frac{1}{n-1}\right]$$
$$+\frac{1}{n-1}\left[\frac{1}{n-1}-\frac{1}{n}\right].$$

$$\int_{\frac{1}{n}}^{1} f(x).dx = \left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{(n-1)^2}\right] - \left[1+\frac{1}{2.3}+\frac{1}{3.4}+\cdots+\frac{1}{n(n-1)}\right]$$

$$\int_{\frac{1}{n}}^{1} f(x).dx = \left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{(n-1)^2}\right]$$
$$-\left[1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{(n-1)}-\frac{1}{n}\right].$$

$$\int_{\frac{1}{n}}^{1} f(x).dx = \left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{(n-1)^2}\right] - \left[1-\frac{1}{n}\right].$$

$$\therefore \int_{\frac{1}{n}}^{1} f(x).dx = \lim_{n\to\infty}\left\{\left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{(n-1)^2}\right] - \left[1-\frac{1}{n}\right]\right\}$$

$$\therefore \lim_{n\to\infty}\int_{\frac{1}{n}}^{1} f(x).dx = \int_{0}^{1} f(x).dx = \lim_{n\to\infty}\left[1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{(n-1)^2}\right]$$
$$-\lim_{n\to\infty}\left[1-\frac{1}{n}\right].$$

$$\int_{0}^{1} f(x).dx = \frac{\pi^2}{6} - 1 \quad \because \sum_{r=1}^{n}\frac{1}{n^2} = \frac{\pi^2}{6}.$$

**7. If f(x) = 2 if 0<x<1**

**8.**

**= 3 if 1<x<2**

**Then prove that f(x) is R-intgrable in [1,2]**

**Soln.:** **Now If f(x) = 2 if 0<x<1**

**= 3 if 1<x<2**

**Clearly LHL = 2 and RHL= 3**

⇨ **LHL≠ RHL at x=1 => f(x) is not continuous at x=1**

**i.e only one point of discontinuity => f(x) is R-intgrable**

**And** $\int_0^2 f(x).dx = \int_0^1 f(x).dx + \int_1^2 f(x).dx$

$= \int_0^1 2\,dx + \int_1^2 3dx$

$= [2x]_0^1 + [3x]_1^2 = 2 + 3 = 5$

$\therefore \int_0^2 f(x).dx = 5$

## First Mean Value Theorem of Integral Calculus.

**Statement: If $f, g \in R[a, b]$ and $g$ keeps the same sign on $[a, b]$ then ∃ a number $\mu$ between $m$ and $M$ of $f$ such that $\int_a^b f(x)g(x).dx = \mu\int_a^b g(x)dx$**

**Proof:** *Now* $f, g \in R[a, b] \Rightarrow f.g \in R[a, b]$

**Case(*i*):** Let $g(x)$ be non-negative $\forall\, x \in [a, b]$ i.e. $g(x) > 0\ \forall x \in [a, b]$

And let $m$ and $M$ be infimum and supremum of $f(x)$ on $[a, b]$.

$\Rightarrow m \le f(x) \le M\ \forall x \in [a, b]$.

$\Rightarrow mg(x) \le f(x)g(x) \le Mg(x) \forall x \in [a, b] \quad \because\ g(x) > 0$

$$\Rightarrow \int_a^b mg(x)dx \le \int_a^b f(x)g(x)dx \le \int_a^b Mg(x)dx$$

$$\Rightarrow m\int_a^b g(x)dx \le \int_a^b f(x)g(x)dx \le M\int_a^b g(x)dx$$

$$\Rightarrow m \leq \frac{\int_a^b f(x)g(x)dx}{\int_a^b g(x)dx} \leq M \rightarrow (\mathbf{1}) \because \int_a^b g(x)dx \neq 0$$

Let $\frac{\int_a^b f(x)g(x).dx}{\int_a^b g(x).dx} = \mu \rightarrow (\mathbf{2}) \Rightarrow \int_a^b f(x)g(x)dx = \mu \int_a^b g(x)dx$

**Then from (1) and (2) we have** $m \leq \mu \leq M$

i.e $\mu$ is between $m$ and $M$ of $f$ such that $\int_a^b f(x)g(x).dx = \mu \int_a^b g(x).dx.$

**Case$(\boldsymbol{ii})$**

Let $g(x)$ be negative $\forall\, x \in [a, \; b]$i.e. $g(x) < 0 \; \forall x \in [a, \; b]$

And let $m$ and $M$ be infimum and supremum of $f(x)$ on$[a, \; b]$.

$\Rightarrow m \leq f(x) \leq M \; \forall x \in [a, \; b]$.

$\Rightarrow mg(x) \geq f(x)g(x) \geq Mg(x) \forall x \in [a, \; b] \quad \because \; g(x) < 0$

$$\Rightarrow \int_a^b Mg(x)dx \leq \int_a^b f(x)g(x)dx \leq \int_a^b mg(x)dx$$

$$\Rightarrow M\int_a^b g(x)dx \leq \int_a^b f(x)g(x)dx \leq m\int_a^b g(x)dx$$

$$\Rightarrow M \geq \frac{\int_a^b f(x)g(x)dx}{\int_a^b g(x)dx} \geq m \rightarrow (\mathbf{3}) \because \int_a^b g(x)dx \neq 0 \text{ and is } < 0$$

Let $\frac{\int_a^b f(x)g(x)dx}{\int_a^b g(x)dx} = \mu \rightarrow (\mathbf{4}) \Rightarrow \int_a^b f(x)g(x)dx = \mu \int_a^b g(x)dx$

**Then from (3) and (4) we have** $M \geq \mu \geq m$ i.e $m \leq \mu \leq M$

i.e $\mu$ is between $m$ and $M$ of $f$ such that $\int_a^b f(x)g(x)dx = \mu \int_a^b g(x)dx$

**Another proof for case (ii)**

Let $g(x)$ be negative $\forall\, x \in [a, \; b]$i.e. $g(x) < 0 \; \forall x \in [a, \; b]$

Now we take $\therefore g(x) = -h(x)$ where $h(x) \geq 0$

then by case$(i)$

$$\Rightarrow \int_a^b mh(x).dx \leq \int_a^b f(x)h(x).dx \leq \int_a^b Mh(x).dx$$

$$\Rightarrow m\int_a^b h(x).dx \leq \int_a^b f(x)h(x).dx \leq M\int_a^b h(x).dx$$

$$\Rightarrow m \leq \frac{\int_a^b f(x)h(x)dx}{\int_a^b h(x)dx} \leq M \qquad if \int_a^b h(x)dx \neq 0$$

$$\Rightarrow -m \geq -\frac{\int_a^b f(x)g(x)dx}{\int_a^b g(x)dx} \geq -M \because g(x) = -h(x) \Rightarrow h(x) = -g(x)$$

$$\Rightarrow m \leq \frac{\int_a^b f(x)g(x)dx}{\int_a^b g(x)dx} \leq M$$

Let $\frac{\int_a^b f(x)g(x)dx}{\int_a^b g(x)dx} = \mu \rightarrow (\mathbf{6})$

Then $\mu$ is between $m$ and $M$ of $f$ such that $\int_a^b f(x)g(x)dx = \mu\int_a^b g(x)dx$.

Thus, "If $f, g \in R[a,\ b]$ and $g$ keeps the same sign on $[a,\ b]$ then $\exists$ a number $\mu$between$m$and$M$ of$f$such that $\int_a^b f(x)g(x)dx = \mu\int_a^b g(x)dx$"

**Corollary: If $f$iscontinuouson$[a,\ b]$ and $g \in R[a,\ b]$ and $g$ keeps the same sign on $[a,\ b]$ then $\int_a^b f(x)g(x)dx = f(c)\int_a^b g(x)dx$.**

**Proof:** Let $f$ is continuous. $\Rightarrow f$ is R-Integrable as every continuous function is R-Integrable. Also by intermediate value theorem, $f$ attains all the values between infimum and supremum. Then $\exists\ c \in [a,\ b]$ such that $f(c) = \mu$.

Then by previous theorem we get

$$\int_a^b f(x)g(x).dx = \mu\int_a^b g(x)dx = f(c)\int_a^b g(x)dx$$

**Examples:**

**1) Prove that $\frac{1}{\pi} \leq \int_0^1 \frac{\sin(\pi x)}{1+x^2}dx \leq \frac{2}{\pi}$**

**Solution**: Let $f(x) = \frac{1}{1+x^2}$ and $g(x) = \sin(\pi x)$.

Clearly $f$ and $g$ are continuous and hence bothe are R-Integrable. And

$g(x) = \sin(\pi x) \geq 0 \ \forall \ x \in [0, \ 1]$. And $f(0) = 1, f(1) = \frac{1}{2} \therefore \ m = \frac{1}{2} \ \& \ M = 1.$

By the first mean value theorem $\exists \ \mu$ between $m = \frac{1}{2} \ \& \ M = 1$ such that

$$\int_0^1 f(x)g(x)dx = \mu \int_0^1 g(x)dx$$

i.e. $\int_0^1 \frac{\sin(\pi x)}{1+x^2} dx = \mu \int_0^1 \sin(\pi x)\, dx = \mu \left[\frac{-\cos(\pi x)}{\pi}\right]_0^1 = \frac{\mu}{\pi}[-(-1-1)] = \frac{2\mu}{\pi}$

$\therefore \mu = \frac{\pi}{2} \int_0^1 \frac{\sin(\pi x)}{1+x^2} dx$ But we have $m = \frac{1}{2} \leq \mu \leq M = 1.$

$\frac{1}{2} \leq \frac{\pi}{2} \int_0^1 \frac{\sin(\pi x)}{1+x^2} \mathrm{dx} \leq 1. \Rightarrow \frac{1}{\pi} \leq \int_0^1 \frac{\sin(\pi x)}{1+x^2} \mathrm{dx} \leq \frac{2}{\pi}$

**2) Byfirstmeanvaluetheoremprovethat** $\boldsymbol{\frac{\pi^3}{24} \leq \int_0^\pi \frac{x^2}{5+3\cos x} dx \leq \frac{\pi^3}{6}}$.

**Solution**: Let $f(x) = \frac{1}{5+3\cos x}$ and $g(x) = x^2$.

Clearly $f$ and $g$ are continuous in $[0, \ \pi]$ and hence both are R-Integrable. And $g(x) = x^2 \geq 0 \ \forall \ x \in [0, \ \pi]$. And $f(0) = \frac{1}{8}, \ f(\pi) = \frac{1}{5-3} = \frac{1}{2} \therefore \ m = \frac{1}{8} \ \& \ M = \frac{1}{2}.$

By the first mean value theorem $\exists \ \mu$ between $m = \frac{1}{8} \ \& \ M = \frac{1}{2}$ $i.e. \frac{1}{8} \leq \mu \leq \frac{1}{2}$

such that $\int_0^\pi f(x)g(x)dx = \mu \int_0^\pi g(x)dx$

i.e. $\int_0^\pi \frac{x^2}{5+3\cos x}.dx = \mu \int_0^\pi x^2.dx = \mu \left[\frac{x^3}{3}\right]_0^\pi = \frac{\mu}{3}[\pi^3 - 0] = \frac{\mu\pi^3}{3}.$

$\therefore \mu = \frac{3}{\pi^3} \int_0^\pi \frac{x^2}{5+3\cos x}.dx.$ But we have $m = \frac{1}{8} \leq \mu \leq M = \frac{1}{2}.$

$\frac{1}{8} \leq \frac{3}{\pi^3} \int_0^\pi \frac{x^2}{5+3\cos x} dx. \leq \frac{1}{2}.$

Multiplying throughout by $\frac{\pi^3}{3}$ we get,

$$\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3cosx}.dx \leq \frac{\pi^3}{6}.$$

**3) Prove that** $\frac{\pi^2}{9} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx \leq \frac{2\pi^2}{9}.$

**Solution**: Let $f(x) = \frac{1}{sinx}$ and $g(x) = x$.

Clearly $f$ and $g$ are continuous in $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ and hence bothe are R − Integrable. And

$g(x) = x > 0 \ \forall \ x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$. And $f\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{6}\right)} = 2, \ f\left(\frac{\pi}{2}\right) = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = 1$

$\therefore \ m = 1 \ \& \ M = 2.$

By the first mean value theorem $\exists \ \mu$ between $m = 1 \ \& \ M = 2 \ i.e. 1 \leq \mu \leq 2$

such that $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(x)g(x).dx = \mu \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} g(x).dx$

i.e. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx = \mu \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x.dx = \mu \left[\frac{x^2}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{\mu}{2}\left[\frac{\pi^2}{4} - \frac{\pi^2}{36}\right] = \frac{\mu}{2}.\frac{8\pi^2}{36} = \frac{\mu\pi^2}{9}.$

i.e $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx = \mu\frac{\pi^2}{9}$

$\therefore \mu = \frac{9}{\pi^2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx$. But we have $m = 1 \leq \mu \leq M = 2$

$$1 \leq \frac{9}{\pi^2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx. \leq 2.$$

$$\frac{\pi^2}{9} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx \leq \frac{2\pi^2}{9}.$$

**OR**

$1 \leq f(x) \leq 2$

$1.g(x) \leq f(x).g(x) \leq 2.g(x)$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} g(x).dx \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f(x)g(x).dx \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2.g(x).dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x.dx \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2.x.dx$$

$$\left[\frac{x^2}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx \leq \left[\frac{2x^2}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$\frac{1}{2}\left[\frac{\pi^2}{4} - \frac{\pi^2}{36}\right] \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx \leq \left[\frac{\pi^2}{4} - \frac{\pi^2}{36}\right]$$

$$\frac{1}{2}\left[\frac{8\pi^2}{36}\right] \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx \leq \left[\frac{8\pi^2}{36}\right]$$

$$\frac{\pi^2}{9} \leq \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{sinx}.dx \leq \frac{2\pi^2}{9}.$$

**4) Provethat $\frac{\pi^2}{2b} \leq \int_0^{\pi} \frac{x}{acos^2x+bsin^2x}.dx \leq \frac{\pi^2}{2a}$., where a<b**

**Solution**: Let $f(x) = \dfrac{1}{acos^2x + bsin^2x}$ and $g(x) = x$.

Clearly $f$ and $g$ are continuous in $[0,\ \pi]$and hence bothe are R-Integrable. And

$g(x) = x \geq 0\ \forall\ x \in [0,\ \pi]$. And $f(0) = \dfrac{1}{a},\ \ f(\pi) = \dfrac{1}{a}$

But $0 \leq \mu \leq \dfrac{\pi}{2}, f\left(\dfrac{\pi}{2}\right) = \dfrac{1}{b}$ since $0 < a < b \Rightarrow \dfrac{1}{b} < \dfrac{1}{a}$

$\therefore\ m = \dfrac{1}{b}$ & $M = \dfrac{1}{a}$.

By the first mean value theorem $\exists\ \mu$ between $m = \dfrac{1}{b}$ & $M = \dfrac{1}{a}$ $i.e. \dfrac{1}{b} \leq \mu \leq \dfrac{1}{a}$

such that $\displaystyle\int_0^{\pi} f(x)g(x)dx = \mu \int_0^{\pi} g(x)dx$

i.e. $\displaystyle\int_0^{\pi} \frac{x}{acos^2x + bsin^2x}.dx = \mu \int_0^{\pi} xdx = \mu \left[\frac{x^2}{2}\right]_0^{\pi} = \frac{\mu}{2}[\pi^2 - 0] = \frac{\mu\pi^2}{2}$.

i.e $\int_0^{\pi} \frac{x}{acos^2x+bsin^2x}.dx = \mu \frac{\pi^2}{2}$

$\therefore \mu = \dfrac{2}{\pi^2}\displaystyle\int_0^{\pi} \frac{x}{acos^2x + bsin^2x}.dx$. But we have $m = \dfrac{1}{b} \leq \mu \leq M = \dfrac{1}{a}$.

$$\frac{1}{b} \leq \frac{2}{\pi^2}\int_0^{\pi} \frac{x}{acos^2x + bsin^2x}.dx. \leq \frac{1}{a}.$$

$$\frac{\pi^2}{2b} \leq \int_0^{\pi} \frac{x}{acos^2x + bsin^2x}dx. \leq \frac{\pi^2}{2a}.$$

**5) Provethat $\frac{1}{\sqrt{2}} \leq \int_0^1 \frac{3x^2}{\sqrt{(1+x)}}.dx \leq 2\sqrt{2}.$**

**Solution**: Let $f(x) = \frac{1}{\sqrt{(1+x)}}$ and $g(x) = 3x^2$.

Clearly $f$ and $g$ are continuous in $[0,\ 1]$and hence bothe are R-Integrable. And

$g(x) = 3x^2 \geq 0\ \forall\ x \in [0,\ 1]$. And $f(0) = 1,\ f(1) = \frac{1}{\sqrt{2}}\ \therefore\ m = \frac{1}{\sqrt{2}}\ \&\ M = 1.$

By the first mean value theorem $\exists\ \mu$ between $m = \frac{1}{\sqrt{2}}\ \&\ M = 1$ $i.e. \frac{1}{\sqrt{2}} \leq \mu \leq 1$

such that $\int_0^1 f(x)g(x).dx = \mu\int_0^1 g(x).dx$

i.e. $\int_0^1 \frac{3x^2}{\sqrt{(1+x)}}.dx = \mu\int_0^1 3x^2.dx = \mu 3\left[\frac{x^3}{3}\right]_0^1 = \mu[1-0] = \mu.$

$\therefore\ \mu = \int_0^1 \frac{3x^2}{\sqrt{(1+x)}}.dx.$ But we have $m = \frac{1}{\sqrt{2}} \leq \mu \leq M = 1.$

$$\frac{1}{\sqrt{2}} \leq \int_0^1 \frac{3x^2}{\sqrt{(1+x)}}.dx \leq 1.$$

$$\frac{1}{\sqrt{2}} \leq \int_0^1 \frac{3x^2}{\sqrt{(1+x)}}.dx \leq 1 < 2\sqrt{2}. \text{ as } 1 < 2\sqrt{2}$$

$$\therefore\ \frac{1}{\sqrt{2}} \leq \int_0^1 \frac{3x^2}{\sqrt{(1+x)}}.dx < 2\sqrt{2}.$$

**6) Provethat $\frac{\pi}{4} \leq \int_0^{\frac{\pi}{4}} secx.dx \leq \frac{\pi}{2\sqrt{2}}.$**

**Solution**: Let $f(x) = secx = \frac{1}{cosx}$ and $g(x) = 1$.

Clearly $f$ and $g$ are continuous in $\left[0,\ \frac{\pi}{4}\right]$and hence bothe are R-Integrable. And

$g(x) = 1 > 0 \;\forall\; x \in [0, \;\; 1]$. And $f(0) = 1, \;\; f\left(\frac{\pi}{4}\right) = \sqrt{2} \therefore\; m = 1 \,\&\, M = \sqrt{2}$.

By the first mean value theorem $\exists\, \mu$ between $m = 1 \,\&\, M = \sqrt{2}$ $i.e. 1 \;\leq \mu \leq \sqrt{2}$

such that $\int_0^{\frac{\pi}{4}} f(x)g(x).dx = \mu \int_0^{\frac{\pi}{4}} g(x).dx$

i.e. $\int_0^{\frac{\pi}{4}} secx.dx = \mu \int_0^{\frac{\pi}{4}} 1.dx = \mu[x]_0^{\frac{\pi}{4}} = \mu\left[\frac{\pi}{4} - 0\right] = \frac{\mu\pi}{4}$.

$\therefore \mu = \frac{4}{\pi}\int_0^{\frac{\pi}{4}} secx.dx = \frac{4}{\pi}\int_0^{\frac{\pi}{4}} \frac{1}{cosx}.dx$ But we have $m = 1 \;\leq\; \mu \;\leq\; M = \sqrt{2}$

$$\frac{\pi}{4} \leq \int_0^{\frac{\pi}{4}} \frac{1}{cosx}.dx \leq \frac{\sqrt{2}.\pi}{4}.$$

$$\frac{\pi}{4} \leq \int_0^{\frac{\pi}{4}} \frac{1}{cosx}.dx \leq \frac{\pi}{2\sqrt{2}} \qquad \Rightarrow \frac{\pi}{4} \leq \int_0^{\frac{\pi}{4}} secx.dx \leq \frac{\pi}{2\sqrt{2}}$$

**OR**

Let $f(x) = secx = \frac{1}{cosx}$ and $g(x) = 1$.

Clearly $f$ and $g$ are continuous in $\left[0, \;\; \frac{\pi}{4}\right]$ and hence bothe are R-Integrable. And

$g(x) = 1 > 0 \;\forall\; x \in [0, \;\; 1]$. And $f(0) = 1, \;\; f\left(\frac{\pi}{4}\right) = \sqrt{2}$

$\therefore 1 \;\leq\; f(x) \leq \sqrt{2}$

$1.g(x) \leq\; f(x).g(x) \leq \sqrt{2}.g(x)$

$1.1 \leq\; f(x).g(x) \leq \sqrt{2}.1 \qquad \because\; g(x) = 1 > 0 \;\forall\; x \in [0, \;\; 1]$

$$\int_0^{\frac{\pi}{4}} 1.dx \leq \int_0^{\frac{\pi}{4}} f(x)g(x).dx \leq \int_0^{\frac{\pi}{4}} \sqrt{2}.1.dx$$

$$[x]_0^{\frac{\pi}{4}} \leq \int_0^{\frac{\pi}{4}} \frac{1}{cosx}.dx \leq \left[\sqrt{2}.x\right]_0^{\frac{\pi}{4}}$$

$$\left[\frac{\pi}{4} - 0\right] \leq \int_0^{\frac{\pi}{4}} \frac{1}{cosx}.dx \leq \sqrt{2}\left[\frac{\pi}{4} - 0\right]$$

$$\Rightarrow \frac{\pi}{4} \leq \int_0^{\frac{\pi}{4}} secx.dx \leq \frac{\pi}{2\sqrt{2}}.$$

**7) Provethat** $\frac{1}{3\sqrt{2}} \le \int_0^1 \frac{x^2}{\sqrt{(1+x^2)}} dx \le \frac{1}{3}$.

**Solution**: Let $f(x) = \frac{1}{\sqrt{(1+x^2)}}$ and $g(x) = x^2$.

Clearly $f$ and $g$ are continuous in $[0,\ 1]$and hence bothe are R-Integrable. And

$g(x) = x^2 \ge 0\ \forall\ x \in [0,\ 1]$. And $f(0) = 1,\ \ f(1) = \frac{1}{\sqrt{2}}$ $\therefore\ m = \frac{1}{\sqrt{2}}$ & $M = 1$.

$$\therefore \frac{1}{\sqrt{2}} \le f(x) \le 1$$

$$\frac{1}{\sqrt{2}}.g(x) \le f(x).g(x) \le 1.g(x)$$

$$\frac{1}{\sqrt{2}}.x^2 \le \frac{1}{\sqrt{(1+x^2)}}.x^2 \le 1.x^2 \qquad \because\ g(x) = x^2 \ge 0\ \forall\ x \in [0,\ 1]$$

$$\int_0^1 \frac{x^2}{\sqrt{2}}.dx \le \int_0^1 \frac{x^2}{\sqrt{(1+x^2)}}.dx \le \int_0^1 x^2.dx.$$

$$\frac{1}{\sqrt{2}}\left[\frac{x^3}{3}\right]_0^1 \le \int_0^1 \frac{x^2}{\sqrt{(1+x^2)}}.dx \le \left[\frac{x^3}{3}\right]_0^1$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left[\frac{1}{3} - 0\right] \le \int_0^1 \frac{x^2}{\sqrt{(1+x^2)}}.dx \le \left[\frac{1}{3} - 0\right].$$

$$\therefore \frac{1}{3\sqrt{2}} \le \int_0^1 \frac{x^2}{\sqrt{(1+x^2)}}.dx \le \frac{1}{3}.$$

**8)Using first mean value theorem prove that** $\int_0^{\pi} \frac{6x^2}{1+cosx} dx > \pi^3$.

**Solution**: Let $f(x) = \frac{1}{1+cosx}$ and $g(x) = 6x^2$.

Clearly $f$ and $g$ are continuous in $[0,\ \pi]$and hence both are R-Integrable. And

$g(x) = 6x^2 \ge 0\ \forall\ x \in [0,\ \pi]$. And $f(0) = \frac{1}{2},\ \ f(\pi) = \infty$

$\therefore\ f$ is not bounded above.

By the first mean value theorem $\exists\ \mu$ between $\frac{1}{2} \le \mu$

$$\Rightarrow f(x) > \frac{1}{2} \Rightarrow \frac{1}{1+cosx} > \frac{1}{2}$$

$$\frac{6x^2}{1+cosx} > \frac{6x^2}{2} \Rightarrow \int_0^{\pi} \frac{6x^2}{1+cosx}.dx > \int_0^{\pi} \frac{6x^2}{2}.dx$$

$$\int_0^{\pi} \frac{6x^2}{1+cosx}.dx > \left[\frac{6x^3}{3.2}\right]_0^{\pi} \Rightarrow \int_0^{\pi} \frac{6x^2}{1+cosx}.dx > [\pi^3 - 0].$$

$$\therefore \int_0^{\pi} \frac{6x^2}{1+cosx}.dx > \pi^3.$$

**9) Using first mean value theorem provethat** $\frac{1}{2} < \int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^{2n}}}. < \frac{\pi}{6}.$

**Solution**: Let $\forall\ x \in \left[0,\ \frac{1}{2}\right] 0 \le x \le \frac{1}{2} < 1 \ \Rightarrow$ and $0 \le x < 1$

$\Rightarrow\ 0 \le x^2 < 1$ and $x^2 \ge (x^2)^n$ and also $x^{2n} < 1\ \forall\ x \in \left[0,\ \frac{1}{2}\right]$

$\Rightarrow\ -x^2 \le -x^{2n}$

$\Rightarrow 1 - x^2 \le 1 - x^{2n} \le 1$

$\Rightarrow \sqrt{1-x^2} \le \sqrt{1-x^{2n}} \le \sqrt{1}$

If any number 0<x<1 then its any positive power also greater than 0 and less than one. i.e 0<x<1 then (i) $0<x^n<1$ (ii) $x>x^2>x^3$------------------ (iii) 0<1-x<1. For example: If x =1/2 then $x^2$ = ¼ , $x^3$ =1/8 , -------so that ½>1/4>1/8-- and 1-1/2 = ½<1, 1-1/4 = ¾<1--------

$\Rightarrow \frac{1}{\sqrt{1-x^2}} \ge \frac{1}{\sqrt{1-x^{2n}}} \ge 1 \because$ by the proprties of real numbers

i.e. $1 \le \frac{1}{\sqrt{1-x^{2n}}} \le \frac{1}{\sqrt{1-x^2}}$

Now integrating throughout with respect to $x$ between the limits 0 to $\frac{1}{2}$

$$\int_0^{\frac{1}{2}} 1.dx \ \le \ \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^{2n}}}.dx \le \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}.dx.$$

$$\frac{1}{2}[x]_0^{\frac{1}{2}} \ \le \ \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^{2n}}}.dx \le [sin^{-1}(x)]_0^{\frac{1}{2}}.$$

$$\frac{1}{2}[1-0] \ \le \ \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^{2n}}}.dx \le \left[sin^{-1}\left(\frac{1}{2}\right) - sin^{-1}(0)\right]$$

$$\frac{1}{2} \ \le \ \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^{2n}}}.dx \le \left[sin^{-1}sin\left(\frac{\pi}{6}\right) - sin^{-1}\sin(0)\right]$$

$$\therefore \frac{1}{2} \leq \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^{2n}}}.dx \leq \frac{\pi}{6}.$$

**10) Provethat** $\boldsymbol{\frac{1}{2} < \int_0^1 \frac{dx}{\sqrt{4-x^2+x^3}}. < \frac{\pi}{6}}$.

**Solution**: Let$\forall\ x \in [0,\ \ 1] 0 \leq x^2 \leq 1$ and$0 \leq 1-x \leq 1$

$$\Rightarrow\ 0 \leq x^2(1-x) \leq x^2 \quad \Rightarrow\ 0 \geq (x^3-x^2) \geq -x^2$$

$$\Rightarrow\ 4 \geq 4+(x^3-x^2) \geq 4-x^2$$

$$\Rightarrow\ 2 \geq \sqrt{4+(x^3-x^2)} \geq \sqrt{4-x^2}$$

$$\Rightarrow\ \frac{1}{2} \leq \frac{1}{\sqrt{4+(x^3-x^2)}} \leq \frac{1}{\sqrt{4-x^2}}$$

Now integrating throughout with respect to $x$ between the limits 0 to 1

$$\int_0^1 \frac{1}{2}.dx \ \leq \ \int_0^1 \frac{1}{\sqrt{4+(x^3-x^2)}}.dx \leq \int_0^1 \frac{1}{\sqrt{4-x^2}}.dx.$$

$$\frac{1}{2}[x]_0^{\ \ 1} \ \leq \ \int_0^1 \frac{1}{\sqrt{4+(x^3-x^2)}}.dx \leq \left[sin^{-1}\left(\frac{x}{2}\right)\right]_0^1.$$

$$\frac{1}{2}[1-0]_0^{\ \ 1} \ \leq \ \int_0^1 \frac{1}{\sqrt{4+(x^3-x^2)}}.dx \leq \left[sin^{-1}\left(\frac{1}{2}\right) - sin^{-1}(0)\right]$$

$$\frac{1}{2} \ \leq \ \int_0^1 \frac{1}{\sqrt{4+(x^3-x^2)}}.dx \leq \left[sin^{-1}sin\left(\frac{\pi}{6}\right) - sin^{-1}\sin(0)\right]$$

$$\therefore \frac{1}{2} \ \leq \ \int_0^1 \frac{1}{\sqrt{4+(x^3-x^2)}}.dx \leq \frac{\pi}{6}.$$

**11) Using first mean value theorem, if $\boldsymbol{f}$is continuous on $[\boldsymbol{a},\ \ \boldsymbol{b}]$ then provethat** $\boldsymbol{\int_a^b f(x)dx. = f(c)(b-a).}$ **wherec** $\boldsymbol{\in (a,\ \ b)}$

**Proof:** Let $g(x) = 1$, and$f(x)$ is continuous.$\Rightarrow f$ and $g$ both are R-integrable. And $g(x) > 0 \forall x \in [a,\ \ b]$.$\therefore$ By corollary on first mean value theorem $\exists c \in (a,\ \ b)$ that $\int_a^b f(x)g(x)dx. = f(c)\int_a^b g(x)dx.$

i.e. $\int_a^b f(x)1dx. = f(c)\int_a^b 1.dx. = f(c).(b-a)$

$\int_a^b f(x)1dx. = f(c).(b-a)$ where $c \in (a,\ b)$.

**12) Prove that $\frac{\pi}{1536} < \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} sin^8 x.dx < \frac{27\pi}{512}$ Using first mean value theorem.**

**Proof:** We know that $\frac{\pi}{6} \le x \le \frac{\pi}{3} \Rightarrow sin\frac{\pi}{6} \le sin\,x \le sin\frac{\pi}{3}$.

$$\frac{1}{2} \le sin\,x \le \frac{\sqrt{3}}{2} \Rightarrow \frac{1}{2^8} \le sin^8 x \le \frac{\left(3^{\frac{1}{2}}\right)^8 \sqrt{3}}{2^8}.$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{256}dx \le \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} sin^8 x.dx \le \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{81}{256}.dx$$

$$\frac{1}{256}\left[\frac{\pi}{3}-\frac{\pi}{6}\right] \le \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} sin^8 x.dx \le \frac{81}{256}\left[\frac{\pi}{3}-\frac{\pi}{6}\right]$$

$$\frac{\pi}{1536} < \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} sin^8 x.dx < \frac{27\pi}{512}.$$

**13) Verify the first mean value theorem for the function $f(x) = x$ and $g(x) = e^x$ in $[-1,\ 1]$.**

**Proof:** Let $f(x) = x$ and $g(x) = e^x$ clearly $f$ and $g$ are continuous in $[-1,\ 1]$ and hence $f, g \in R[-1,\ 1]$ also $g(x)$ is positive $\forall\, x \in [-1,\ 1]$.

∴ By the corollary on first mean value theorem $\exists\, c \in (-1,\ 1)$ such that

$$\int_{-1}^{1} f(x)g(x)dx. = f(c)\int_{-1}^{1} g(x)dx. \Rightarrow \int_{-1}^{1} x.e^x dx. = c\int_{-1}^{1} g(x)dx.$$

$$[x.e^x - 1.e^x]_{-1}^{\ 1} = c[e^x]_{-1}^{\ 1} = c[e^1 - e^{-1}]$$

$$(e^1 - e^1) - (-e^{-1} - e^{-1}) = c\frac{[e^2-1]}{e} \Rightarrow c = \frac{2}{[e^2-1]}$$

Clearly $c = \frac{2}{[e^2-1]} \in (-1,\ 1)$.

**Second Mean Value Theorem:(Bonnet's form):.**

**Statement: If $f$ and $g$ are bounded andintegrable on $[a, \ b]$ and $g$isdecreasingand positive in $[a, b]$functionthen$\exists\xi \in [a, \ b]$suchthat**

$$\int_a^b f(x)g(x)dx. = g(a)\int_a^{\xi} f(x)dx.$$

**Proof:**Let $f$and $g$be bounded and R-Integrable.

Without loss of generality let $f(x)$ *be non negative* i.e $f(x) \geq 0 \ \forall \ x \in [a, \ b]$.

***i*) Let *g* be decreasing function of *x* in $[a, \ b]$.**

$$\forall \ x, \qquad a \leq x \ \Rightarrow g(a) \geq g(x)$$

i.e $g(x) \leq g(a)$ , $\forall \ x \geq a$

$$\Rightarrow f(x)g(x) \leq f(x)g(a) \because \ f(x) \geq 0$$

$$\therefore \int_a^b f(x)g(x)dx \leq g(a)\int_a^b f(x)dx. ---------------\rightarrow (1)$$

$\exists\xi \in [a, \ b]$ such that from(1) $\int_a^b f(x)g(x)dx \leq g(a)\int_a^{\xi} f(x)dx.$

***ii*) Let *g* be increasing function of *x* in$[a, \ b]$**

$$\forall \ x, \qquad a \leq x \leq b, \Rightarrow g(a) \leq g(x) \leq g(b)$$

Now consider $g(x) \leq g(b)$for $x \leq b \ \ \Rightarrow f(x)g(x) \leq f(x)g(b)$

$$\int_a^b f(x)g(x)dx \leq g(b)\int_a^b f(x)dx ---------------\rightarrow (2)$$

$\exists t \in [a, \ b]$ such that from(2) $\int_a^b f(x)g(x)dx \leq g(b)\int_t^b f(x)dx.$

**Wiestrass form of second mean value theorem:( Generalised second mean value theorem or simply second mean value theorem).**

**Statement:Let f and g be bounded and integrable on $[a, \ b]$ and let g be and monotonic on $[a, \ b]$ then there exists$\xi \in [a, \ b]$**

$$\int_a^b f(x)g(x)dx1 = g(a)\int_a^{\xi} f(x)dx + g(b)\int_{\xi}^b f(x)dx$$

**Proof:**Let f and g be bounded and integrable on $[a, \ b]$

**Let g be monnotonically decreasing function on $[a, \ b]$.**

$\therefore$ for $x \le b,\ \ g(x) \ge g(b) \Rightarrow g(x) - g(b) \ge 0$

As g is decreasing $g(x) - g(b)$ is also decreasing.

If any h(x) is increasing or decreasing then h(x) $\pm$ constant is also increasing or decreasing.

Let $G(x) = g(x) - g(b)$. Thus $G(x)$ is decreasing and positive.

$\therefore$ By Bonnet's form of second mean value theorem $\exists \xi \in [a,\ \ b]$ such that

$$\int_a^b G(x)f(x)dx = G(a)\int_a^{\xi} f(x)dx$$

$$\Rightarrow \int_a^b [g(x) - g(b)]f(x)dx = [g(a) - g(b)]\int_a^{\xi} f(x)dx$$

$$i.e. \int_a^b f(x)g(x)dx - \int_a^b f(x)g(b)dx = g(a)\int_a^{\xi} f(x)dx. - g(b)\int_a^{\xi} f(x)dx$$

$$\int_a^b f(x)g(x)dx = g(b)\left[\int_a^b f(x)dx - \int_a^{\xi} f(x)dx\right] + g(a)\int_a^{\xi} f(x)dx$$

i.e $\int_a^b f(x)g(x)dx = g(b)\left[\int_a^b f(x)dx + \int_{\xi}^a f(x)dx\right] + g(a)\int_a^{\xi} f(x)dx$

$$= g(b)\left[\int_{\xi}^a f(x)dx + \int_a^b f(x)dx\right] + g(a)\int_a^{\xi} f(x)dx$$

$$\therefore \int_a^b f(x)g(x)dx = g(b)\left[\int_{\xi}^b f(x)dx\right] + g(a)\int_a^{\xi} f(x)dx$$

$$\therefore \int_a^b f(x)g(x)dx = g(a)\int_a^{\xi} f(x)dx + g(b)\int_{\xi}^b f(x)dx \text{ where } \xi \in [a,\ \ b].$$

Similarly if g is increasing function we get (Hint: if g is increasing then -g is decreasing and replcace g by -g in above result we get)

$$\int_a^b f(x)(-g(x))dx = f(a)\int_a^{\xi} -g(x)dx + f(b)\int_{\xi}^b -g(x)dx$$

i.e $\int_a^b f(x)g(x)dx = g(a)\int_a^{\xi} f(x)dx + g(b)\int_{\xi}^b f(x)dx$

**Examples on Bonnet's form of second mean value theorem:**

**Provethat** $\left|\int_a^b \frac{sinx}{x}dx\right| \le \frac{2}{a}$ **ifb > a > 0byusingBonnet'sfromofsecond meanvaluetheorem.**

**Proof:** Let $f(x) = sinx \& g(x) = \frac{1}{x}$ clearly $f$ is continuous and $g(x) = \frac{1}{x}$ is monotonic decreasing $\Rightarrow f \& g$ are integrtable. By Bonnet's form of second mean value theorem

$\exists t \in [a,\ b]$ such that $\int_a^b f(x)g(x)dx = g(a)\int_a^t f(x)dx.$

$$\text{i. e.} \int_a^b \frac{sinx}{x}dx = \frac{1}{a}\int_a^t sinx.dx = \frac{1}{a}[-cosx]_a{}^t = \frac{1}{a}[-cost + cosa]$$

$$\left|\int_a^b \frac{sinx}{x}dx\right| = \left|\frac{1}{a}[-cost + cosa]\right| \le \frac{1}{a}\{|-cost| + |cosa|\}$$

$$\left|\int_a^b \frac{sinx}{x}dx\right| \le \frac{1}{a}\{1 + 1\}$$

$$\therefore \left|\int_a^b \frac{sinx}{x}dx\right| \le \frac{2}{\mathrm{a}}$$

**Provethat** $\left|\int_p^q \boldsymbol{sin^2x\,dx}\right| \le \frac{\mathbf{1}}{\mathbf{p}}$ **if** $\mathbf{q > p > 0}$

**Proof**: Let $sin^2x = \frac{2x}{2x}sin^2x$

Now $f(x) = 2xsin^2x$is continuous and $g(x) = \frac{1}{2x}$ is monotonically decreasing.

$\Rightarrow f \& g$ are integrable. By Bonnet's form of second mean value theorem as $g(x)$ is monotonically decreasing function then $\exists t \in [p,\ q]$ such that,

$$\int_p^q f(x)g(x)dx = g(p)\int_p^t f(x)dx.$$

$$\int_p^q \frac{2xsin^2x}{2x}dx = \frac{1}{2p}\int_p^t 2xsin^2xdx = \frac{1}{2p}[-cos^2x]_p{}^t = \frac{1}{2p}[-cos^2t + cos^2p]$$

$$\left|\int_p^q \frac{2xsin^2x}{2x}dx\right| = \left|\frac{1}{2p}[-cos^2t + cos^2p]\right| \le \frac{1}{2p}[|-cos^2t| + |cos^2p|]$$

$$\left|\int_p^q \frac{2xsin^2x}{2x}dx\right| \le \frac{1}{2p}[1+1]$$

$$\left|\int_p^q \frac{2xsin^2x}{2x}dx\right| \le \frac{1}{p}.$$

**Primitive of a function:**

**Definition: If $f(x)$ and $F(x)$ are two functions defined on an interval $[a,\ b]$ such that $F'(x) = f(x)$ or $\int f(x)dx = F(x) + c\ \forall x \in [a,\ b]$ then $F(x)$ is called primitive of $f(x)$.**

**Example**: 1) $x^3$ is primitive of $3x^2$ $\because \int 3x^2dx = x^3 + c$

2) $sinx$ is primitive of $cosx$ $\because \int cosxdx = sinx + c$

**Fundamental Theorem of Integral Calculus:**

**Statement: If $f$ is R-Integrable on $[a,\ b]$ and $F$ is primitive of $f$ on $[a,\ b]$ then**

$$\int_a^b f(x)dx. = F(b) - F(a).$$

**Proof**: Given that $f$ is integrable

$$\Rightarrow \int_a^b f(x)dx.\ \text{is exists. and } F(x) \text{ is primitive of } f(x).$$

$\therefore F'(x) = f(x) \to (1) \forall x \in [a,\ b]$. Now consider partition $P = \{x_0 = a, x_1, \ldots x_{r-1}, x_r, \ldots . x_{n-1}, x_n = b\}$ since $F(x)$ is diffedrentiable in $[a,\ b] \Rightarrow F(x)$ is differntiable at each sub-interval $I_r = [x_{r-1},\ x_r]$ of $[a,\ b]$.

$$\left\{ \begin{array}{c} \textbf{Lagrange'sMeanValueTheorem}: \\ \textbf{If } f(x) \textbf{ iscontinuosin}[a,\ b] \textbf{anddifferentiablein}(a,\ b) \textbf{then} \exists \textbf{atleast} \\ \textbf{onepointc}]\ suchthat f'(c) = \frac{f(b) - f(a)}{b - a}\ . \end{array} \right\}$$

Now by using Lagrange's mean value theorem for one variable ∃ atleast oner point $\xi_r \in (x_{r-1},\ x_r)$such that $F'(\xi_r) = \dfrac{F(x_r) - F(x_{r-1})}{x_r - x_{r-1}}$

i. e. $(x_r - x_{r-1})\ F'(\xi_r) = F(x_r) - F(x_{r-1})$

i.e $\delta_r F'(\xi_r) = F(x_r) - F(x_{r-1})$

But from(1)$F'(x) = f(x) => F'(\xi_r) = f(\xi_r)$as$\xi_r \in I_r \subseteq [a,\ b]$

$$\therefore \delta_r f(\xi_r) = F(x_r) - F(x_{r-1})$$

$$\Rightarrow \sum_{r=1}^{n} f(\xi_r)\,\delta_r = \sum_{r=1}^{n} [F(x_r) - F(x_{r-1})]$$

$$\Rightarrow \lim_{n\to\infty} \sum_{r=1}^{n} f(\xi_r)\,\delta_r = \lim_{n\to\infty} \sum_{r=1}^{n} [F(x_r) - F(x_{r-1})]$$

$$\int_a^b f(x)dx = \lim_{n\to\infty} [F(x_1) - F(x_0) + F(x_2) - F(x_1) + \cdots F(x_n) + F(x_{n-1})]$$

$$\int_a^b f(x)dx. = \lim_{n\to\infty} [F(x_n) - F(x_0)]$$

$$= \lim_{n\to\infty} [F(b) - F(a)] \text{as } x_n = \text{b } \& x_0 = \text{b}$$

$$Thus \int_a^b f(x)dx = F(b) - F(a).$$

Examples: 1. Evaluate$\int_0^2 \boldsymbol{x}\ \boldsymbol{dx}$. **By FThm of IC**

**2.** Evaluate$\int_1^4 (\boldsymbol{x} - \frac{1}{x})\ \boldsymbol{dx}$. **By FThm of IC**

**3.** Evaluate$\int_0^{\frac{\pi}{2}} \boldsymbol{sinx}\ \boldsymbol{dx}$. **By FThm of IC**

**Example: If $\boldsymbol{f(x)}$ is continuous in $[\boldsymbol{a}, \boldsymbol{b}]$ then prove that $\emptyset(\boldsymbol{t}) = \int_{\boldsymbol{a}}^{\boldsymbol{t}} \boldsymbol{f(x)dx}.\ \forall\ \boldsymbol{t} \in [\boldsymbol{a}, \boldsymbol{b}]$ is**

**$\boldsymbol{i}$) Continuous in $[\boldsymbol{a}, \boldsymbol{b}]$ $\boldsymbol{ii}$) differentiable in $(\boldsymbol{a}, \boldsymbol{b})$ & $\emptyset'(\boldsymbol{t}) = \boldsymbol{f(t)}\ \forall \boldsymbol{t} \in [\boldsymbol{a}, \boldsymbol{b}]$.**

**Proof**: **Given that** $\emptyset(\boldsymbol{t}) = \int_{\boldsymbol{a}}^{\boldsymbol{t}} \boldsymbol{f(x)dx}. \rightarrow (\mathbf{1})$.

$\emptyset(t+h) = \int_a^{t+h} f(x)dx. \rightarrow (2)$.

$\emptyset(t+h) - \emptyset(t) = \int_a^{t+h} f(x)dx. - \int_a^{t} f(x)dx. \qquad from (1) \& (2)$

$\emptyset(t+h) - \emptyset(t) = \int_a^{t+h} f(x)dx. + \int_t^{a} f(x)dx. = \int_t^{t+h} f(x)dx.$

$|\emptyset(t+h) - \emptyset(t)| = \left|\int_t^{t+h} f(x)dx.\right| \leq \int_t^{t+h} |f(x)|dx.$

$|\emptyset(t+h) - \emptyset(t)| \leq \int_t^{t+h} k.dx.$ as f is continuous, it is bounded and hence $|f(x)| \leq k$

$|\emptyset(t+h) - \emptyset(t)| \leq k[x]_t^{t+h}$

$|\emptyset(t+h) - \emptyset(t)| \leq kh \rightarrow 0$ as $h \rightarrow 0$

$\therefore \lim_{h\rightarrow 0}|\emptyset(t+h) - \emptyset(t)| \leq \lim_{h\rightarrow 0} kh = 0$

$\lim_{h\rightarrow 0}|\emptyset(t+h)| = \emptyset(t)$

$\Rightarrow \emptyset$ is continnuous.

$ii$) Now $\emptyset(t+h) - \emptyset(t) = \int_t^{t+h} f(x)dx.$

$\emptyset(t+h) - \emptyset(t) = f(t+\theta h)h$ where $t+\theta h \in [t, t+h]$

$\emptyset(t+h) - \emptyset(t) = f(t+\theta h)h$

$i.e\ f(t+\theta h) = \dfrac{\emptyset(t+h) - \emptyset(t)}{h}$

$\Rightarrow \lim_{h\rightarrow 0} f(t+\theta h) = \lim_{h\rightarrow 0} \dfrac{\emptyset(t+h) - \emptyset(t)}{h}$

$\Rightarrow f(t) = \emptyset'(t) \therefore \emptyset(t)$ is integrable.

**P**

# Topic 8 Notes

Jeremy Orloff

# 8 Residue Theorem

## 8.1 Poles and zeros

We remind you of the following terminology: Suppose $f(z)$ is analytic at $z_0$ and

$$f(z) = a_n(z - z_0)^n + a_{n+1}(z - z_0)^{n+1} + \ldots,$$

with $a_n \neq 0$. Then we say $f$ has a zero of order $n$ at $z_0$. If $n = 1$ we say $z_0$ is a simple zero.

Suppose $f$ has an *isolated* singularity at $z_0$ and Laurent series

$$f(z) = \frac{b_n}{(z - z_0)^n} + \frac{b_{n-1}}{(z - z_0)^{n-1}} + \ldots + \frac{b_1}{z - z_0} + a_0 + a_1(z - z_0) + \ldots$$

which converges on $0 < |z - z_0| < R$ and with $b_n \neq 0$. Then we say $f$ has a pole of order $n$ at $z_0$. If $n = 1$ we say $z_0$ is a simple pole.

There are several examples in the Topic 7 notes. Here is one more

**Example 8.1.**

$$f(z) = \frac{z + 1}{z^3(z^2 + 1)}$$

has isolated singularities at $z = 0, \pm i$ and a zero at $z = -1$. We will show that $z = 0$ is a pole of order 3, $z = \pm i$ are poles of order 1 and $z = -1$ is a zero of order 1. The style of argument is the same in each case.

At $z = 0$:

$$f(z) = \frac{1}{z^3} \cdot \frac{z + 1}{z^2 + 1}.$$

Call the second factor $g(z)$. Since $g(z)$ is analytic at $z = 0$ and $g(0) = 1$, it has a Taylor series

$$g(z) = \frac{z + 1}{z^2 + 1} = 1 + a_1 z + a_2 z^2 + \ldots$$

Therefore

$$f(z) = \frac{1}{z^3} + \frac{a_1}{z^2} + \frac{a_2}{z} + \ldots.$$

This shows $z = 0$ is a pole of order 3.

At $z = i$: $f(z) = \frac{1}{z-i} \cdot \frac{z+1}{z^3(z+i)}$. Call the second factor $g(z)$. Since $g(z)$ is analytic at $z = i$, it has a Taylor series

$$g(z) = \frac{z + 1}{z^3(z + i)} = a_0 + a_1(z - i) + a_2(z - i)^2 + \ldots$$

where $a_0 = g(i) \neq 0$. Therefore

$$f(z) = \frac{a_0}{z - i} + a_1 + a_2(z - i) + \ldots.$$

This shows $z = i$ is a pole of order 1.

The arguments for $z = -i$ and $z = -1$ are similar.

## 8.2 Words: Holomorphic and meromorphic

**Definition.** A function that is analytic on a region $A$ is called holomorphic on $A$.

A function that is analytic on $A$ except for a set of poles of finite order is called meromorphic on $A$.

**Example 8.2.** Let

$$f(z) = \frac{z + z^2 + z^3}{(z-2)(z-3)(z-4)(z-5)}.$$

This is meromorphic on $\mathbf{C}$ with (simple) poles at $z = 2, 3, 4, 5$.

## 8.3 Behavior of functions near zeros and poles

The basic idea is that near a zero of order $n$, a function behaves like $(z - z_0)^n$ and near a pole of order $n$, a function behaves like $1/(z - z_0)^n$. The following make this a little more precise.

**Behavior near a zero.** If $f$ has a zero of order $n$ at $z_0$ then near $z_0$,

$$f(z) \approx a_n (z - z_0)^n,$$

for some constant $a_n$.

*Proof.* By definition $f$ has a Taylor series around $z_0$ of the form

$$\begin{aligned} f(z) &= a_n(z - z_0)^n + a_{n+1}(z - z_0)^{n+1} + \ldots \\ &= a_n(z - z_0)^n \left(1 + \frac{a_{n+1}}{a_n}(z - z_0) + \frac{a_{n+2}}{a_n}(z - z_0)^2 + \ldots\right) \end{aligned}$$

Since the second factor equals 1 at $z_0$, the claim follows.

**Behavior near a finite pole.** If $f$ has a pole of order $n$ at $z_0$ then near $z_0$,

$$f(z) \approx \frac{b_n}{(z - z_0)^n},$$

for some constant $b_n$.

*Proof.* This is nearly identical to the previous argument. By definition $f$ has a Laurent series around $z_0$ of the form

$$\begin{aligned} f(z) &= \frac{b_n}{(z - z_0)^n} + \frac{b_{n-1}}{(z - z_0)^{n-1}} + \ldots + \frac{b_1}{z - z_0} + a_0 + \ldots \\ &= \frac{b_n}{(z - z_0)^n} \left(1 + \frac{b_{n-1}}{b_n}(z - z_0) + \frac{b_{n-2}}{b_n}(z - z_0)^2 + \ldots\right) \end{aligned}$$

Since the second factor equals 1 at $z_0$, the claim follows.

### 8.3.1 Picard's theorem and essential singularities

Near an essential singularity we have Picard's theorem. We won't prove or make use of this theorem in 18.04. Still, we feel it is pretty enough to warrant showing to you.

**Picard's theorem.** If $f(z)$ has an essential singularity at $z_0$ then in every neighborhood of $z_0$, $f(z)$ takes on all possible values infinitely many times, with the possible exception of one value.

**Example 8.3.** It is easy to see that in any neighborhood of $z = 0$ the function $w = \mathrm{e}^{1/z}$ takes every value except $w = 0$.

### 8.3.2 Quotients of functions

We have the following statement about quotients of functions. We could make similar statements if one or both functions has a pole instead of a zero.

**Theorem.** Suppose $f$ has a zero of order $m$ at $z_0$ and $g$ has a zero of order $n$ at $z_0$. Let

$$h(z) = \frac{f(z)}{g(z)}.$$

Then

- If $n > m$ then $h(z)$ has a pole of order $n - m$ at $z_0$.
- If $n < m$ then $h(z)$ has a zero of order $m - n$ at $z_0$.
- If $n = m$ then $h(z)$ is analytic and nonzero at $z_0$.

We can paraphrase this as $h(z)$ has 'pole' of order $n - m$ at $z_0$. If $n - m$ is negative then the 'pole' is actually a zero.

*Proof.* You should be able to supply the proof. It is nearly identical to the proofs above: express $f$ and $g$ as Taylor series and take the quotient.

**Example 8.4.** Let

$$h(z) = \frac{\sin(z)}{z^2}.$$

We know $\sin(z)$ has a zero of order 1 at $z = 0$ and $z^2$ has a zero of order 2. So, $h(z)$ has a pole of order 1 at $z = 0$. Of course, we can see this easily using Taylor series

$$h(z) = \frac{1}{z^2}\left(z - \frac{z^3}{3!} + \ldots\right)$$

## 8.4 Residues

In this section we'll explore calculating residues. We've seen enough already to know that this will be useful. We will see that even more clearly when we look at the residue theorem in the next section.

We introduced residues in the previous topic. We repeat the definition here for completeness.

**Definition.** Consider the function $f(z)$ with an isolated singularity at $z_0$, i.e. defined on the region $0 < |z - z_0| < r$ and with Laurent series (on that region)

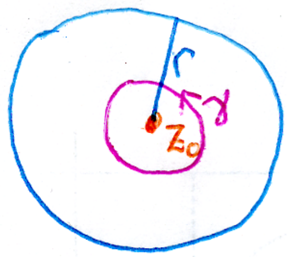
$$f(z) = \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n} + \sum_{n=0}^{\infty} a_n (z - z_0)^n.$$

The residue of $f$ at $z_0$ is $b_1$. This is denoted

$$\operatorname{Res}(f, z_0) = b_1 \quad \text{or} \quad \operatorname*{Res}_{z=z_0} f = b_1.$$

What is the importance of the residue? If $\gamma$ is a small, simple closed curve that goes counterclockwise around $b_1$ then

$$\int_\gamma f(z) = 2\pi i b_1.$$




$\gamma$ small enough to be inside $|z - z_0| < r$, surround $z_0$ and contain no other singularity of $f$.

This is easy to see by integrating the Laurent series term by term. The only nonzero integral comes from the term $b_1/z$.

**Example 8.5.**

$$f(z) = e^{1/2z} = 1 + \frac{1}{2z} + \frac{1}{2(2z)^2} + \ldots$$

has an isolated singularity at 0. From the Laurent series we see that $\operatorname{Res}(f, 0) = 1/2$.

**Example 8.6.**

(i) Let

$$f(z) = \frac{1}{z^3} + \frac{2}{z^2} + \frac{4}{z} + 5 + 6z.$$

$f$ has a pole of order 3 at $z = 0$ and $\operatorname{Res}(f, 0) = 4$.

(ii) Suppose

$$f(z) = \frac{2}{z} + g(z),$$

where $g$ is analytic at $z = 0$. Then, $f$ has a simple pole at 0 and $\operatorname{Res}(f, 0) = 2$.

(iii) Let

$$f(z) = \cos(z) = 1 - z^2/2! + \ldots.$$

Then $f$ is analytic at $z = 0$ and $\operatorname{Res}(f, 0) = 0$.

(iv) Let

$$f(z) = \frac{\sin(z)}{z} = \frac{1}{z}\left(z - \frac{z^3}{3!} + \ldots\right) = 1 - \frac{z^2}{3!} + \ldots$$

So, $f$ has a removable singularity at $z = 0$ and $\operatorname{Res}(f, 0) = 0$.

**Example 8.7.** Using partial fractions. Let

$$f(z) = \frac{z}{z^2+1}.$$

Find the poles and residues of $f$.

*Solution:* Using partial fractions we write

$$f(z) = \frac{z}{(z-i)(z+i)} = \frac{1}{2}\cdot\frac{1}{z-i} + \frac{1}{2}\cdot\frac{1}{z+i}.$$

The poles are at $z = \pm i$. We compute the residues at each pole:

At $z = i$:

$$f(z) = \frac{1}{2}\cdot\frac{1}{z-i} + \text{ something analytic at } i.$$

Therefore the pole is simple and $\text{Res}(f, i) = 1/2$.

At $z = -i$:

$$f(z) = \frac{1}{2}\cdot\frac{1}{z+i} + \text{ something analytic at } -i.$$

Therefore the pole is simple and $\text{Res}(f, -i) = 1/2$.

**Example 8.8.** Mild warning! Let

$$f(z) = -\frac{1}{z(1-z)}$$

then we have the following Laurent expansions for $f$ around $z = 0$.

On $0 < |z| < 1$:

$$f(z) = -\frac{1}{z}\cdot\frac{1}{1-z} = -\frac{1}{z}(1 + z + z^2 + \ldots).$$

Therefore the pole at $z = 0$ is simple and $\text{Res}(f, 0) = -1$.

On $1 < |z| < \infty$:

$$f(z) = \frac{1}{z^2}\cdot\frac{1}{1-1/z} = \frac{1}{z^2}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \ldots\right).$$

Even though this is a valid Laurent expansion you must not use it to compute the residue at 0. This is because the definition of residue requires that we use the Laurent series on the region $0 < |z - z_0| < r$.

**Example 8.9.** Let

$$f(z) = \log(1 + z).$$

This has a singularity at $z = -1$, but it is not isolated, so not a pole and therefore there is no residue at $z = -1$.

### 8.4.1 Residues at simple poles

Simple poles occur frequently enough that we'll study computing their residues in some detail. Here are a number of ways to spot a simple pole and compute its residue. The justification for all of them goes back to Laurent series.

Suppose $f(z)$ has an isolated singularity at $z = z_0$. Then we have the following properties.

**Property 1.** If the Laurent series for $f(z)$ has the form

$$\frac{b_1}{z - z_0} + a_0 + a_1(z - z_0) + \dots$$

then $f$ has a simple pole at $z_0$ and $\text{Res}(f, z_0) = b_1$.

**Property 2** If

$$g(z) = (z - z_0)f(z)$$

is analytic at $z_0$ then $z_0$ is either a simple pole or a removable singularity. In either case $\text{Res}(f, z_0) = g(z_0)$. (In the removable singularity case the residue is 0.)

*Proof.* Directly from the Laurent series for $f$ around $z_0$.

**Property 3.** If $f$ has a simple pole at $z_0$ then

$$\lim_{z \to z_0} (z - z_0)f(z) = \text{Res}(f, z_0)$$

This says that the limit exists and equals the residue. Conversely, if the limit exists then either the pole is simple, or $f$ is analytic at $z_0$. In both cases the limit equals the residue.

*Proof.* Directly from the Laurent series for $f$ around $z_0$.

**Property 4.** If $f$ has a simple pole at $z_0$ and $g(z)$ is analytic at $z_0$ then

$$\text{Res}(fg, z_0) = g(z_0)\,\text{Res}(f, z_0).$$

If $g(z_0) \neq 0$ then

$$\text{Res}(f/g, z_0) = \frac{1}{g(z_0)}\,\text{Res}(f, z_0).$$

*Proof.* Since $z_0$ is a simple pole,

$$f(z) = \frac{b_1}{z - z_0} + a_0 + a_1(z - z_0)$$

Since $g$ is analytic,

$$g(z) = c_0 + c_1(z - z_0) + \dots,$$

where $c_0 = g(z_0)$. Multiplying these series together it is clear that

$$\text{Res}(fg, z_0) = c_0 b_1 = g(z_0)\,\text{Res}(f, z_0). \qquad \text{QED}$$

The statement about quotients $f/g$ follows from the proof for products because $1/g$ is analytic at $z_0$.

**Property 5.** If $g(z)$ has a simple zero at $z_0$ then $1/g(z)$ has a simple pole at $z_0$ and

$$\text{Res}(1/g, z_0) = \frac{1}{g'(z_0)}.$$

*Proof.* The algebra for this is similar to what we've done several times above. The Taylor expansion for $g$ is

$$g(z) = a_1(z - z_0) + a_2(z - z_0)^2 + \dots,$$

where $a_1 = g'(z_0)$. So

$$\frac{1}{g(z)} = \frac{1}{a_1(z-z_0)} \left( \frac{1}{1 + \frac{a_2}{a_1}(z-z_0) + \ldots} \right)$$

The second factor on the right is analytic at $z_0$ and equals 1 at $z_0$. Therefore we know the Laurent expansion of $1/g$ is

$$\frac{1}{g(z)} = \frac{1}{a_1(z-z_0)} (1 + c_1(z-z_0) + \ldots)$$

Clearly the residue is $1/a_1 = 1/g'(z_0)$. QED.

**Example 8.10.** Let

$$f(z) = \frac{2+z+z^2}{(z-2)(z-3)(z-4)(z-5)}.$$

Show all the poles are simple and compute their residues.

*Solution:* The poles are at $z = 2, 3, 4, 5$. They are all isolated. We'll look at $z = 2$ the others are similar. Multiplying by $z - 2$ we get

$$g(z) = (z-2)f(z) = \frac{2+z+z^2}{(z-3)(z-4)(z-5)}.$$

This is analytic at $z = 2$ and

$$g(2) = \frac{8}{-6} = -\frac{4}{3}.$$

So the pole is simple and $\mathrm{Res}(f, 2) = -4/3$.

**Example 8.11.** Let

$$f(z) = \frac{1}{\sin(z)}.$$

Find all the poles and their residues.

*Solution:* The poles of $f(z)$ are the zeros of $\sin(z)$, i.e. $n\pi$ for $n$ an integer. Since the derivative

$$\sin'(n\pi) = \cos(n\pi) \neq 0,$$

the zeros are simple and by Property 5 above

$$\mathrm{Res}(f, n\pi) = \frac{1}{\cos(n\pi)} = (-1)^n.$$

**Example 8.12.** Let

$$f(z) = \frac{1}{z(z^2+1)(z-2)^2}.$$

Identify all the poles and say which ones are simple.

*Solution:* Clearly the poles are at $z = 0, \pm i, 2$.

At $z = 0$:

$$g(z) = zf(z)$$

is analytic at 0 and $g(0) = 1/4$. So the pole is simple and the residue is $g(0) = 1/4$.

At $z = i$:

$$g(z) = (z - i)f(z) = \frac{1}{z(z+i)(z-2)^2}$$

is analytic at $i$, the pole is simple and the residue is $g(i)$.

At $z = -i$: This is similar to the case $z = i$. The pole is simple.

At $z = 2$:

$$g(z) = (z - 2)f(z) = \frac{1}{z(z^2+1)(z-2)}$$

is not analytic at 2, so the pole is not simple. (It should be obvious that it's a pole of order 2.)

**Example 8.13.** Let $p(z)$, $q(z)$ be analytic at $z = z_0$. Assume $p(z_0) \neq 0$, $q(z_0) = 0$, $q'(z_0) \neq 0$. Find

$$\operatorname*{Res}_{z=z_0} \frac{p(z)}{q(z)}.$$

*Solution:* Since $q'(z_0) \neq 0$, $q$ has a simple zero at $z_0$. So $1/q$ has a simple pole at $z_0$ and

$$\text{Res}(1/q, z_0) = \frac{1}{q'(z_0)}$$

Since $p(z_0) \neq 0$ we know

$$\text{Res}(p/q, z_0) = p(z_0)\,\text{Res}(1/q, z_0) = \frac{p(z_0)}{q'(z_0)}.$$

### 8.4.2 Residues at finite poles

For higher-order poles we can make statements similar to those for simple poles, but the formulas and computations are more involved. The general principle is the following

**Higher order poles.** If $f(z)$ has a pole of order $k$ at $z_0$ then

$$g(z) = (z - z_0)^k f(z)$$

is analytic at $z_0$ and if

$$g(z) = a_0 + a_1(z - z_0) + \ldots$$

then

$$\text{Res}(f, z_0) = a_{k-1} = \frac{g^{(k-1)}(z_0)}{(k-1)!}.$$

*Proof.* This is clear using Taylor and Laurent series for $g$ and $f$.

**Example 8.14.** Let

$$f(z) = \frac{\sinh(z)}{z^5}$$

and find the residue at $z = 0$.

*Solution:* We know the Taylor series for

$$\sinh(z) = z + z^3/3! + z^5/5! + \ldots$$

(You can find this using $\sinh(z) = (\mathrm{e}^z - \mathrm{e}^{-z})/2$ and the Taylor series for $\mathrm{e}^z$.) Therefore,

$$f(z) = \frac{1}{z^4} + \frac{1}{3!z^2} + \frac{1}{5!} + \ldots$$

We see $\mathrm{Res}(f, 0) = 0$.

Note, we could have seen this by realizing that $f(z)$ is an even function.

**Example 8.15.** Let

$$f(z) = \frac{\sinh(z)\mathrm{e}^z}{z^5}.$$

Find the residue at $z = 0$.

*Solution:* It is clear that $\mathrm{Res}(f, 0)$ equals the coefficient of $z^4$ in the Taylor expansion of $\sinh(z)\mathrm{e}^z$. We compute this directly as

$$\sinh(z)\mathrm{e}^z = \left(z + \frac{z^3}{3!} + \ldots\right)\left(1 + z + \frac{z^2}{2} + \frac{z^3}{3!} + \ldots\right) = \ldots + \left(\frac{1}{4!} + \frac{1}{3!}\right)z^4 + \ldots$$

So

$$\mathrm{Res}(f, 0) = \frac{1}{3!} + \frac{1}{4!} = \frac{5}{24}.$$

**Example 8.16.** Find the residue of

$$f(z) = \frac{1}{z(z^2+1)(z-2)^2}$$

at $z = 2$.

*Solution:* $g(z) = (z-2)^2 f(z) = \frac{1}{z(z^2+1)}$ is analytic at $z = 2$. So, the residue we want is the $a_1$ term in its Taylor series, i.e. $g'(2)$. This is easy, if dull, to compute

$$\mathrm{Res}(f, 2) = g'(2) = -\frac{13}{100}$$

### 8.4.3 $\cot(z)$

The function $\cot(z)$ turns out to be very useful in applications. This stems largely from the fact that it has simple poles at all multiples of $\pi$ and the residue is 1 at each pole. We show that first.

**Fact.** $f(z) = \cot(z)$ has simple poles at $n\pi$ for $n$ an integer and $\mathrm{Res}(f, n\pi) = 1$.

*Proof.*

$$f(z) = \frac{\cos(z)}{\sin(z)}.$$

This has poles at the zeros of sin, i.e. at $z = n\pi$. At poles $f$ is of the form $p/q$ where $q$ has a simple zero at $z_0$ and $p(z_0) \neq 0$. Thus we can use the formula

$$\mathrm{Res}(f, z_0) = \frac{p(z_0)}{q'(z_0)}.$$

In our case, we have

$$\mathrm{Res}(f, n\pi) = \frac{\cos(n\pi)}{\cos(n\pi)} = 1,$$

as claimed.

Sometimes we need more terms in the Laurent expansion of $\cot(z)$. There is no known easy formula for the terms, but we can easily compute as many as we need using the following technique.

**Example 8.17.** Compute the first several terms of the Laurent expansion of $\cot(z)$ around $z = 0$.

*Solution:* Since $\cot(z)$ has a simple pole at 0 we know

$$\cot(z) = \frac{b_1}{z} + a_0 + a_1 z + a_2 z^2 + \dots$$

We also know

$$\cot(z) = \frac{\cos(z)}{\sin(z)} = \frac{1 - z^2/2 + z^4/4! - \dots}{z - z^3/3! + z^5/5! - \dots}$$

Cross multiplying the two expressions we get

$$\left(\frac{b_1}{z} + a_0 + a_1 z + a_2 z^2 + \dots\right)\left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots\right) = 1 - \frac{z^2}{2} + \frac{z^4}{4!} - \dots$$

We can do the multiplication and equate the coefficients of like powers of $z$.

$$b_1 + a_0 z + \left(-\frac{b_1}{3!} + a_1\right) z^2 + \left(-\frac{a_0}{3!} + a_2\right) z^3 + \left(\frac{b_1}{5!} - \frac{a_1}{3!} + a_3\right) z^4 = 1 - \frac{z^2}{2!} + \frac{z^4}{4!}$$

So, starting from $b_1 = 1$ and $a_0 = 0$, we get

$$\begin{aligned} -b_1/3! + a_1 &= -1/2! & \Rightarrow \quad & a_1 = -1/3 \\ -a_0/3! + a_2 &= 0 & \Rightarrow \quad & a_2 = 0 \\ b_1/5! - a_1/3! + a_3 &= 1/4! & \Rightarrow \quad & a_3 = -1/45. \end{aligned}$$

As noted above, all the even terms are 0 as they should be. We have

$$\cot(z) = \frac{1}{z} - \frac{z}{3} - \frac{z^3}{45} + \dots$$

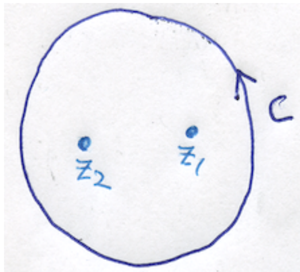
## 8.5 Cauchy Residue Theorem

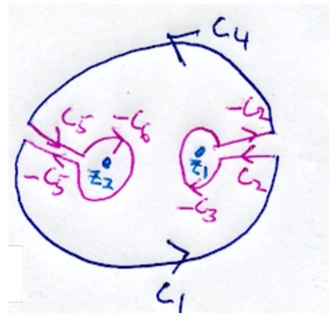
This is one of the major theorems in 18.04. It will allow us to make systematic our previous somewhat ad hoc approach to computing integrals on contours that surround singularities.

**Theorem.** (**Cauchy's residue theorem**) Suppose $f(z)$ is analytic in the region $A$ except for a set of isolated singularities. Also suppose $C$ is a simple closed curve in $A$ that doesn't go through any of the singularities of $f$ and is oriented counterclockwise. Then

$$\int_C f(z)\,dz = 2\pi i \sum \text{ residues of } f \text{ inside } C$$

*Proof.* The proof is based of the following figures. They only show a curve with two singularities inside it, but the generalization to any number of signularities is straightforward. In what follows we are going to abuse language and say pole when we mean isolated singularity, i.e. a finite order pole or an essential singularity ('infinite order pole').




The left figure shows the curve $C$ surrounding two poles $z_1$ and $z_2$ of $f$. The right figure shows the same curve with some cuts and small circles added. It is chosen so that there are no poles of $f$ inside it and so that the little circles around each of the poles are so small that there are no other poles inside them. The right hand curve is

$$\tilde{C} = C_1 + C_2 - C_3 - C_2 + C_4 + C_5 - C_6 - C_5$$

The left hand curve is $C = C_1 + C_4$. Since there are no poles inside $\tilde{C}$ we have, by Cauchy's theorem,

$$\int_{\tilde{C}} f(z)\, dz = \int_{C_1+C_2-C_3-C_2+C_4+C_5-C_6-C_5} f(z)\, dz = 0$$

Dropping $C_2$ and $C_5$, which are both added and subtracted, this becomes

$$\int_{C_1+C_4} f(z)\, dz = \int_{C_3+C_6} f(z)\, dz \tag{1}$$

If

$$f(z) = \ldots + \frac{b_2}{(z-z_1)^2} + \frac{b_1}{z-z_1} + a_0 + a_1(z-z_1) + \ldots$$

is the Laurent expansion of $f$ around $z_1$ then

$$\begin{aligned}\int_{C_3} f(z)\, dz &= \int_{C_3} \ldots + \frac{b_2}{(z-z_1)^2} + \frac{b_1}{z-z_1} + a_0 + a_1(z-z_1) + \ldots\, dz \\ &= 2\pi i b_1 \\ &= 2\pi i \operatorname{Res}(f, z_1)\end{aligned}$$

Likewise

$$\int_{C_6} f(z)\, dz = 2\pi i \operatorname{Res}(f, z_2).$$

Using these residues and the fact that $C = C_1 + C_4$, Equation 1 becomes

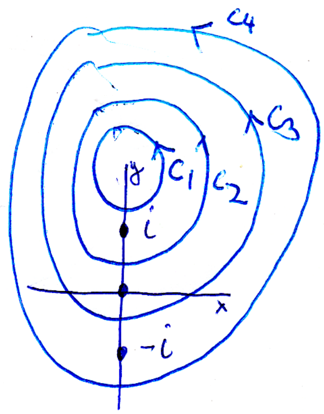
$$\int_C f(z)\, dz = 2\pi i \left[\operatorname{Res}(f, z_1) + \operatorname{Res}(f, z_2)\right].$$

That proves the residue theorem for the case of two poles. As we said, generalizing to any number of poles is straightforward.

**Example 8.18.** Let

$$f(z) = \frac{1}{z(z^2+1)}.$$

Compute $\int f(z)\,dz$ over each of the contours $C_1$, $C_2$, $C_3$, $C_4$ shown.




*Solution:* The poles of $f(z)$ are at $z = 0, \pm i$. Using the residue theorem we just need to compute the residues of each of these poles.

At $z = 0$:

$$g(z) = zf(z) = \frac{1}{z^2+1}$$

is analytic at 0 so the pole is simple and

$$\text{Res}(f, 0) = g(0) = 1.$$

At $z = i$:

$$g(z) = (z-i)f(z) = \frac{1}{z(z+i)}$$

is analytic at $i$ so the pole is simple and

$$\text{Res}(f, i) = g(i) = -1/2.$$

At $z = -i$:

$$g(z) = (z+i)f(z) = \frac{1}{z(z-i)}$$

is analytic at $-i$ so the pole is simple and

$$\text{Res}(f, -i) = g(-i) = -1/2.$$

Using the residue theorem we have

$$\int_{C_1} f(z)\,dz = 0 \;\; (\text{since } f \text{ is analytic inside } C_1)$$

$$\int_{C_2} f(z)\,dz = 2\pi i\,\text{Res}(f, i) = -\pi i$$

$$\int_{C_3} f(z)\,dz = 2\pi i\,[\text{Res}(f, i) + \text{Res}(f, 0)] = \pi i$$

$$\int_{C_4} f(z)\,dz = 2\pi i\,[\text{Res}(f, i) + \text{Res}(f, 0) + \text{Res}(f, -i)] = 0.$$

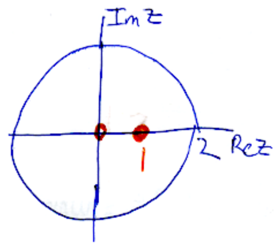
**Example 8.19.** Compute

$$\int_{|z|=2} \frac{5z-2}{z(z-1)}\,dz.$$

*Solution:* Let

$$f(z) = \frac{5z-2}{z(z-1)}.$$

The poles of $f$ are at $z = 0, 1$ and the contour encloses them both.




At $z = 0$:

$$g(z) = zf(z) = \frac{5z-2}{(z-1)}$$

is analytic at 0 so the pole is simple and

$$\text{Res}(f, 0) = g(0) = 2.$$

At $z = 1$:

$$g(z) = (z-1)f(z) = \frac{5z-2}{z}$$

is analytic at 1 so the pole is simple and

$$\text{Res}(f, 1) = g(1) = 3.$$

Finally

$$\int_C \frac{5z-2}{z(z-1)}\,dz = 2\pi i\,[\text{Res}(f,0) + \text{Res}(f,1)] = 10\pi i.$$

**Example 8.20.** Compute

$$\int_{|z|=1} z^2 \sin(1/z)\,dz.$$

*Solution:* Let

$$f(z) = z^2 \sin(1/z).$$

$f$ has an isolated singularity at $z = 0$. Using the Taylor series for $\sin(w)$ we get

$$z^2 \sin(1/z) = z^2 \left(\frac{1}{z} - \frac{1}{3!z^3} + \frac{1}{5!z^5} - \ldots\right) = z - \frac{1/6}{z} + \ldots$$

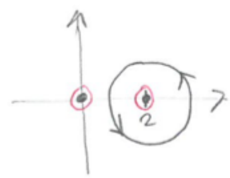
So, $\text{Res}(f, 0) = b_1 = -1/6$. Thus the residue theorem gives

$$\int_{|z|=1} z^2 \sin(1/z)\,dz = 2\pi i\,\text{Res}(f, 0) = -\frac{i\pi}{3}.$$

**Example 8.21.** Compute

$$\int_C \frac{dz}{z(z-2)^4}\,dz,$$

where, $C : |z-2| = 1$.




*Solution:* Let

$$f(z) = \frac{1}{z(z-2)^4}.$$

The singularity at $z = 0$ is outside the contour of integration so it doesn't contribute to the integral.

To use the residue theorem we need to find the residue of $f$ at $z = 2$. There are a number of ways to do this. Here's one:

$$\begin{aligned}\frac{1}{z} &= \frac{1}{2+(z-2)} \\ &= \frac{1}{2}\cdot\frac{1}{1+(z-2)/2} \\ &= \frac{1}{2}\left(1 - \frac{z-2}{2} + \frac{(z-2)^2}{4} - \frac{(z-2)^3}{8} + \ldots\right)\end{aligned}$$

This is valid on $0 < |z-2| < 2$. So,

$$f(z) = \frac{1}{(z-2)^4}\cdot\frac{1}{z} = \frac{1}{2(z-2)^4} - \frac{1}{4(z-2)^3} + \frac{1}{8(z-2)^2} - \frac{1}{16(z-2)} + \ldots$$

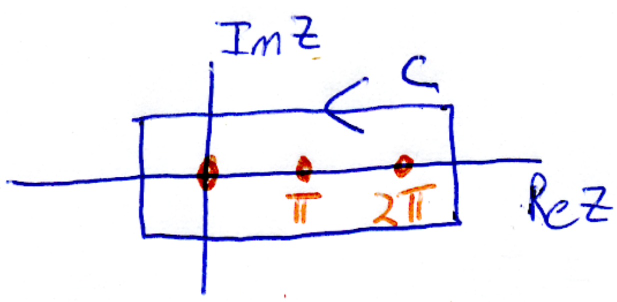
Thus, $\text{Res}(f, 2) = -1/16$ and

$$\int_C f(z)\,dz = 2\pi i\,\text{Res}(f,2) = -\frac{\pi i}{8}.$$

**Example 8.22.** Compute

$$\int_C \frac{1}{\sin(z)}\,dz$$

over the contour $C$ shown.



*Solution:* Let

$$f(z) = 1/\sin(z).$$

There are 3 poles of $f$ inside $C$ at 0, $\pi$ and $2\pi$. We can find the residues by taking the limit of $(z - z_0)f(z)$. Each of the limits is computed using L'Hospital's rule. (This is valid, since the rule is just a statement about power series. We could also have used Property 5 from the section on residues of simple poles above.)

At $z = 0$:

$$\lim_{z\to 0} \frac{z}{\sin(z)} = \lim_{z\to 0} \frac{1}{\cos(z)} = 1.$$

Since the limit exists, $z = 0$ is a simple pole and

$$\text{Res}(f, 0) = 1.$$

At $z = \pi$:

$$\lim_{z\to \pi} \frac{z-\pi}{\sin(z)} = \lim_{z\to \pi} \frac{1}{\cos(z)} = -1.$$

Since the limit exists, $z = \pi$ is a simple pole and

$$\text{Res}(f, \pi) = -1.$$

At $z = 2\pi$: The same argument shows

$$\text{Res}(f, 2\pi) = 1.$$

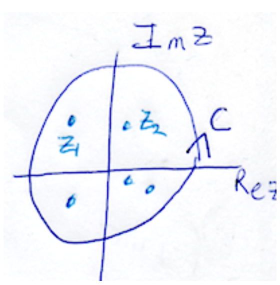
Now, by the residue theorem

$$\int_C f(z)\, dz = 2\pi i\, [\text{Res}(f, 0) + \text{Res}(f, \pi) + \text{Res}(f, 2\pi)] = 2\pi i.$$

## 8.6 Residue at $\infty$

The residue at $\infty$ is a clever device that can sometimes allow us to replace the computation of many residues with the computation of a single residue.

Suppose that $f$ is analytic in **C** except for a finite number of singularities. Let $C$ be a positively oriented curve that is large enough to contain all the singularities.




All the poles of $f$ are inside $C$

**Definition.** We define the residue of $f$ at infinity by

$$\text{Res}(f, \infty) = -\frac{1}{2\pi i} \int_C f(z)\, dz.$$

We should first explain the idea here. The interior of a simple closed curve is everything to left as you traverse the curve. The curve $C$ is oriented counterclockwise, so its interior contains all the poles of $f$. The residue theorem says the integral over $C$ is determined by the residues of these poles.

On the other hand, the interior of the curve $-C$ is everything outside of $C$. There are no poles of $f$ in that region. If we want the residue theorem to hold (which we do –it's that important) then the only option is to have a residue at $\infty$ and define it as we did.

The definition of the residue at infinity assumes all the poles of $f$ are inside $C$. Therefore the residue theorem implies

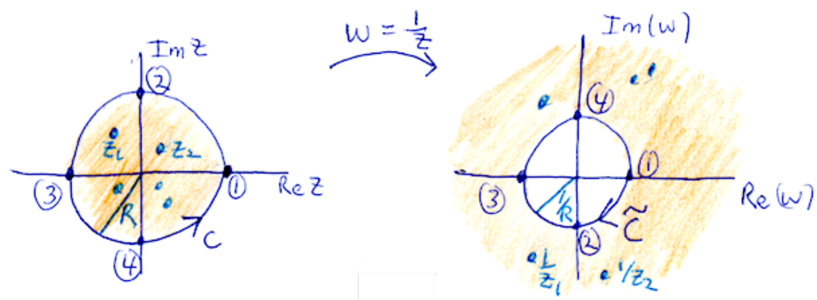
$$\text{Res}(f, \infty) = -\sum \text{ the residues of } f.$$

To make this useful we need a way to compute the residue directly. This is given by the following theorem.

**Theorem.** If $f$ is analytic in $\mathbf{C}$ except for a finite number of singularities then

$$\text{Res}(f, \infty) = -\,\text{Res}\left(\frac{1}{w^2} f(1/w), 0\right).$$

*Proof.* The proof is just a change of variables: $w = 1/z$.




Change of variable: $w = 1/z$

First note that $z = 1/w$ and

$$dz = -(1/w^2)\, dw.$$

Next, note that the map $w = 1/z$ carries the positively oriented $z$-circle of radius $R$ to the negatively oriented $w$-circle of radius $1/R$. (To see the orientiation, follow the circled points 1, 2, 3, 4 on $C$ in the $z$-plane as they are mapped to points on $\tilde{C}$ in the $w$-plane.) Thus,

$$\text{Res}(f, \infty) = -\frac{1}{2\pi i}\int_C f(z)\, dz = \frac{1}{2\pi i}\int_{\tilde{C}} f(1/w)\frac{1}{w^2}\, dw$$

Finally, note that $z = 1/w$ maps all the poles inside the circle $C$ to points outside the circle $\tilde{C}$. So the only possible pole of $(1/w^2)f(1/w)$ that is inside $\tilde{C}$ is at $w = 0$. Now, since $\tilde{C}$ is oriented clockwise, the residue theorem says

$$\frac{1}{2\pi i}\int_{\tilde{C}} f(1/w)\frac{1}{w^2}\, dw = -\,\text{Res}\left(\frac{1}{w^2} f(1/w), 0\right)$$

Comparing this with the equation just above finishes the proof.

**Example 8.23.** Let

$$f(z) = \frac{5z-2}{z(z-1)}.$$

Earlier we computed

$$\int_{|z|=2} f(z)\, dz = 10\pi i$$

by computing residues at $z = 0$ and $z = 1$. Recompute this integral by computing a single residue at infinity.

*Solution:*

$$\frac{1}{w^2} f(1/w) = \frac{1}{w^2} \frac{5/w - 2}{(1/w)(1/w - 1)} = \frac{5 - 2w}{w(1 - w)}.$$

We easily compute that

$$\mathrm{Res}(f, \infty) = -\,\mathrm{Res}\left(\frac{1}{w^2} f(1/w), 0\right) = -5.$$

Since $|z| = 2$ contains all the singularities of $f$ we have

$$\int_{|z|=2} f(z)\, dz = -2\pi i\, \mathrm{Res}(f, \infty) = 10\pi i.$$

This is the same answer we got before!

# Structure of Accounting theory – III

## Meaning of structure:

Structure refers to a frame work where in, the elements of accounting are put together for building an accounting theory.

## Definitions:

"The Type of frame work used to house a theory of accounting may be formed, as in the axiomatic approach to theory, or it may be implicit, as in the pragmatic & ethical approaches."

## Elements of the Structure:

The structure of accounting theory is based on a set of elements and their relationships that, govern the development of accounting techniques, approaches and methodologies.

## Types of Elements:

1. Objectives
2. Postulates
3. Concepts
4. Principles
5. Techniques & systems
6. Accounting standards.

1. **Objectives:**

   objectives are the goals or purposes for which the accounts are prepared & presented to the users.

2. **Postulates:**

   Postulates are the basic assumptions made by the accountant while preparing accounts and

Presenting it to the users. They are accepted by all, so ~~these assets under~~ They need not to be proved or verified before adopting them.

3. Concepts:

Concepts are also called as Theories. The accounting concepts are linked with the types of business organisation & the ownership of property.

Concepts tell us the nature of accounting entities and the ownership of property.

4. Principles:

The accounting Techniques & methods are developed on the basis of general rules & principles. These principles are derived from both the objectives and the theoretical concepts of accounting.

5. Techniques and Systems:

Accounting techniques ~~and~~ are the "specific rules derived from the accounting principles to prepare & present the accounts for specific events faced by accounting entity.

6. Accounting Standards:

Accounting standards are written policy documents issued by professional accounting bodies ~~are~~ or by government or other regulatory bodies covering aspects of recognisation, measurement, treatment, presentation and disclosure of accounting transaction in the financial statements.

## Postulates of Accounting Theory: [2016 - 5mks]

**Meaning**: [2mks: 2018]

Postulates are the basic assumptions on which the accounts are kept and presented. They are accepted by all, so they need not be proved or verified.

**Definition**: [2mks: 2012, 2014]

* Hendriksen, E.S Defines "The accounting postulates are basic assumptions or fundamental propositions concerning the economic, political & sociological environment in which accounting must operate".

* The Term postulate as defined by Longman Dictionary means "something belived to be true, but not proven, on which an argument or scientific discussion is based".

**Features of Postulates**: [2mks: 2019]

1. Basic Assumptions: Postulates are basic and fundamental assumptions.
2. Environmental influences: They are arising out of economic, political and sociological environment.
3. No necessity of verification: Postulates are the assumptions accepted by all so they are not to be verified to know whether it is right or wrong.
4. Basis for other Elements: They are basis for formulating the theoretical concept; principles, standards, Techniques and methods of accounting.
5. Universally accepted: These are universally accepted and self proved statements that is they need not be proved with evidence & documents.

The following are the different Postulates of Accounting: [15mks; 2015]

1. Entity Postulates
2. Going concern Postulates
3. The Monetary unit Postulates
4. The Accounting Period Postulates

Explanation:

1. Entity Postulates: [5 mks: 2018]

Entity means existance. The businessman starts business, so there are two entities: first one is business and second one is businessman. So the first postulate is business & it is having a separate entity from businessman. With this assumption we should start keeping accounts on behalf of business & not business-man. The accountant should separate the business Transactions from personal transactions & he has to record only business transactions in the books of accounts.

Business is treated as a separate & distinct entity from its owners. If there are two or more business units or branches, each of such unit is also treated as a separate entity.

There can be two approaches to the definition of an accounti-ng entity:

a. The Firm-oriented approach
b. The users-oriented approach.

a. The Firm-oriented approach:

According to FASB, an accounting entity is the economic unit responsible for the economic activities & administrative control of the unit. So such an accounting entity may be a sole trader, partnership, a joint stock company, a corporation or a co-operative society etc. Thus, separate entity is applicable to all forms of organization.

B] The Users-Oriented Approach:

The interests of the users rather than the economic activities and administrative control of the business unit are taken into consideration.

Advantages of Entity Postulates:

1. It helps for recording only the business transactions. The personal Transaction are not recorded in the books of accounts.
2. separation of business Transactions from personal transaction helps for ascertaining the real profitability of the business separating business & personal funds.
3. It ascertains the real financial position of the business accurately by separation of business funds from personal funds.
4. The true & fair view of the view of state of affairs of the business is displayed by the balance sheet ~~from~~ of the business.

Disadvantages:

1. It is not possible to distinguish between business entity & out side entity.
2. In case of small scale organisation it is very difficult task to distinguish between business entity and the owners.
3. If owners are the managers of the firm then a separate responsibility accounting cannot be implemen -ted effectively.

2. Going concern Postulate: [2mks : 2016, 2013] [5mks : 2017]

Accounting in this method is based on the assumption that firm will continue to operate for indefinite period of time.

"The financial statements should be based on the assumption that an entity will continue its operations for the foreseeable future period of time."

The going concern or continuity postulate holds that the business entity will continue its operations long enough to realize its projects, commitments, and ongoing activities.

According to International Accounting Standard "The enterprise is normally viewed as a going concern that is as continuing in operation for the foreseeable future". It is view that the enterprise has an intention to be carried on for long period.

Advantages:

1. Fair basis of accounting: Going concern postulate is the fair basis for preparation of annual financial statements.
2. Revenue & capital: It helps for making distinction between revenue expenditure & capital expenditure.
3. Fixed & current: This postulate helps for classification of assets as fixed assets & current assets. Similarly liabilities are classified as fixed liabilities & current liabilities.
4. Valuation of fixed assets: It helps for valuation of fixed assets at historical cost price after depreciation for the purpose of balance sheet.
5. Accrual basis of income & Expenses: This concept takes into consideration the adjustment of outstanding incomes & expenses, prepaid expenses & pre-received incomes while preparing the final account.

Limitations:

1. Not an assumption but a condition: According to many authorities, going concern is not an assumption, but it is an condition

2. Time value of money: At the times of changing prices, valuation of fixed assets at their historical cost, is not relevant & practical.

3. The assumption that a concern will continue its operations for an indefinite period does not hold good in case there is forced liquidity.

3. Monetary Unit Postulate or Money Measurment Postulate:

[2 mks 2014]

Exchange of money or money's worth is called a transaction in accounts. Every business transaction resulting from events is recorded ~~as~~ in terms of money value in accounting. Money is regarded as the Language of accounting. The events, which cannot be measured in terms of money, cannot be recorded in books of accounts.

Events which cannot be expressed in terms of money, are not recorded in books of accounts.

According to welsch & Anothony, "accounting focuses on the measurement and reporting in monetary terms, of the flows of resources into and out of an organization or the resources controlled by the organisation, and of the ~~measure~~ claims against those resources".

Advantages:

1. It helps to express the transactions in terms of money.
2. The profitability of the business is also ascertained properly.
3. The prosperity of business is also made known to users.
4. The solvency of the business can be determined.
5. It helps to knows the "state of affairs of the business".

Limitations:

1. Non-monetary Transactions of relevance & significant to business are not considered.
2. competence of leadership is ignored.
3. Financial statements do not reflect whether the Profitability & prosperity is based on effective or ineffective management.
4. Annual reports do not reflect the financial implications

4. Accounting Period Postulate: [2mks: 2017] [5mks: 2014, 2018]

"Business units are required to report the changes in their wealth periodically."

Firm is assumed to undertake business for an indefinite period of time. The items of revenue and expenses & there by income & position of assets & liabilities are determined at specific intervals during life time of enterprise. Most of the business enterprises choose 12 moths period for which operating results are obtained & financial statements are prepared.

According to E. L. Kohler, Accounting period may be defined as "the period of time for which an operation statement is customarily prepared. Accounting should provide the information about the economic activities of the enterprises for a specified period of time that is for a short period.

According to going concern postulate the business entity will continue for an indefinite period but the owner of the business cannot wait for such a long period to know the profitability of the business at the end of the life of business.

Advantages:

1. Helps in decision making
2. It fulfills legal requirements.
3. To know Annual results.
4. Solvency of business.
5. Facilitates comparison.

6. Facilitates the preparation of financial statements on accrual basis.

Limitations:

1. Valuation Problem
2. Treatment of pre paid expenses, pre received income & outstanding expenses & income of business poses a problem.
3. Writing off of the bad Debts & creating of reserve for future bad and Doubtful Debts poses certain problems.

# Theoretical Concepts of Accounting:

## Concepts:

Every Theory has its own concepts. Various issues or aspects in a theory are coined or phrased in the form of a concept. The concept gives meaning of the term in precise manner.

Concepts of accounting tell us the nature of accounting entities operating in a free economy characterised by private or public ownership of property.

The following are the theoretical concepts of accounting:

1. The Proprietary Theory ⟵ ] -> 15mks 2017
2. The Entity Theory ⟵ ]
3. The Residual Equity theory
4. The Enterprise theory
5. The Fund Theory.

### 1. The Proprietary Theory [Owners Equity] [2mks 2016]

This concept of theory studies the subject of accounting from owners point of view.

The Proprietary Theory of accounting treats the owners of an entity as an extension of the firm itself & focuses on determining and evaluating the owner's net worth. It is based on following Equation.

Owners Equity = Assets - Liabilities

In this formula, owned materials are assets, outstanding Debts are Liabilities and net income equals revenue minus Expenses. Proprietary Theory is most applicable to Partnership and sole Proprietorships. It does not Differentiate between owner and firm. It advocates unlimited Liability That is owner is personally liable for Liabilities or Debts of firm.

According to Kenneth Most "This theory seeks to explain the content and measurement principles underlying in financial statements by placing the owner of the enterprise in the centre of the accounting universe. All observations are made from this view point the accountant sees only what the proprietor wishes to see and values objects according to his Interest".

Features:

1. Ownership: The proprietary theory places the owners of the business enterprise at the centre of accounting universe.
2. Net worth: The main object of this theory is to determine proprietor's net worth that is proprietors capital + Reserves & surplus.
3. Net income: All the profit or Loss of the business belongs to the proprietors. Therefore They are adjusted to capital accounts.
4. Owners Equity: The net income affects the owners Equity.
5. Suitability: This theory is basically suitable to Sole Trade concern and partnership firms.

**Illustration : 1** Problem on proprietory Theory:

1) Find out the owners equity from the given balance sheet under proprietary theory.

**Balance Sheet**

| Liabilities | Amount | Assets | | Amount |
|---|---|---|---|---|
| Capital | 8,00,000 | Fixed Assets | | |
| Retained earnings (Reserve) | 4,00,000 | Plant & Machinery | 7,00,000 | |
| | | Land & Building | 5,00,000 | 12,00,000 |
| Profit and Loss A/c | 3,00,000 | Current Asset | | 10,00,000 |
| Creditors | 5,00,000 | | | |
| Bill Payable | 2,00,000 | | | |
| | 22,00,000 | | | 22,00,000 |

**Solution**

Owners equity = Capital + Reserve + Profit & Loss A/c
= 8,00,000 + 4,00,000 + 3,00,000
= Rs. 15,00,000

OR

| | | |
|---|---|---|
| Fixed Assets | 12,00,000 | |
| Add Current Assets | 10,00,000 | |
| | | 22,00,000 |
| Less Current Liabiliteis | | |
| Creditors | 5,00,000 | |
| Bills payable | 2,00,000 | |
| | | 7,00,000 |
| Net worth of shareholders owners equity. | | 15,00,000 |

OR

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According to this theory, corporate income i.e. EAIT (Earnings After Interest and Tax) means Net income to shareholders. This theory, helps for EPS (Earnings per Share) and Dividend per share (DPS).

## How to calculate EAIT

**Statement showing EAIT (Earnings after interest and tax)**

| | | |
|---|---|---|
| | Sales | xxxxxxx |
| Less | Variable cost | x |
| | **Contribution** | xxxxxx |
| Less | Fixed cost | x |
| | **Earnings before interest & tax (EBIT)** | xxxxx |
| Less | Interest on debt | x |
| | **Earnings before tax (EBT)** | xxxx |
| Less | Tax on EBT | x |
| | **(EAIT)** Earnings after interest & tax | xxx |
| | (Amount available to equity shareholders) | |

$$\text{Earnings per share (EPS)} = \frac{\text{Amount available}}{\text{Number of equity shares}}$$

**Illustration : 3**

The capital structure of Hubli, Ltd consists of equity share capital of Rs. 1000000 (shares of Rs. 10 each) and Rs. 1000000 of 20% debentures.

The other particulars are as below

| | |
|---|---|
| Sales | 200000 units and 250000 units |
| Selling price | Rs. 10 per unit |
| Variable cost | Rs. 6 per unit |
| Fixed cost | Rs. 250000 |

Rate of tax 50%

Your are required to calculate EPS

## Calculation of EAIT and EPS

| Particulars | For 200000 Units | For 250000 Units |
|---|---|---|
| Sales price Rs. 10 per unit | 2000000 | 2500000 |
| Less Variable cost Rs. 6 per unit | 1200000 | 1500000 |
| **Contribution** | 800000 | 1000000 |
| Less Fixed cost | 250000 | 250000 |
| **Operaing profit (EBIT)** (Earnings before int. & tax) | 550000 | 750000 |
| Less Interest on debentures | 200000 | 200000 |
| Profit before tax (PBT) | 350000 | 550000 |
| Less Tax at 50% in PBT | 175000 | 275000 |
| Amount available for - equity shareholders (EAIT) (Earning After Interest and Tax) | 175000 | 275000 |

$$EPS = \frac{\text{Amount available}}{\text{Number of equity shares}} = \frac{175000}{100000} \quad \frac{175000}{100000}$$

=1.75 times 2.75 times

2. The Entity Theory or Entity concept: [5mks - 2013,15] [2mks - 2019]

The concept considers owners as separate entity or organization. Firm has independent existence distinct from its owners. It is based on the concept of Limited Liability. that is firm being liable for its borrowing or Debts.

The business concern is a legal entity. All the assets & Liabilities are owned by the business. The Transactions, contracts are entered by business by in its name. Hence it is the business which is Liable to pay the Liabilities out of the assets possessed by it. According to entity theory, owners are different from its business.

The accounting equation in this case is:

Assets = Liabilities + Shareholder's Equity + Retained earnings.

Definition:

According to Keneth Most "the entity is something separate and distinct from those who provide capital to the entity. The providers of capital beome a kind of creditor interest Assets, Liabilities, Expenses & Revenues are determined from the interest of the company rather than that of shareholders".

The Theory assumes that the economic activities of a business are distinct from those of its owners the entity theory maintains that the activities of a business can be accounted for separately from the activities of its owners. Therefore the owners are not personally responsible for Loans or other liabilities taken on by the company.

U

Features:

1. Separate Legal entity:
Business has separate entity from its owners who provide capital to the business unit.

2. Liabilities of company: The business unit is responsible for paying the third party liabilities & not the share holders of a company.

3. Profit belongs to company: This theory is income centered that ~~p~~ is profitability of business is determined from the interest of the company rather than that of the shareholders.

4. Divisible profit: out of total profits of business unit, only the divisible profits belong to equity and preference shareholders

5. Suitability: This theory mainly applies to the corporate form of business enterprises.

3. Residual Equity Theory. [2mks - 2012]

It is an extension of proprietary theory. According to this theory equity shareholders are considered as residual owners of the firm. Here residual equity refers to 'residue or remaining" part of the assets available to equity shareholders after meeting specific equities. that is outside liabilities and preference shareholders. Residual owners, namely equity shareholders have to bear the residual risk as well.

It means the position of the ordinary shareholders is "the bearers of residual risk & the owners of the residual award'. It is a concept some where between the proprietary theory & equity theory.

The main object of this theory is to provide better information to the equity shareholders for making investment decisions. on the basis of information equity shareholders expect the future dividends.

Formula of calculating Residual equity:

Assets - Specific Equities = Residual Equities.

Features of Residual Equity Theory:

1. It is an extension of proprietary theory. but it this theory divides the proprietor that is shareholders into equity shareholders & preference shareholders.
2. This theory defines liability side of balance sheet as specific equities & residual equities.
3. Residual equity refers to equity share capital and Reserves & Surplus.
4. This theory is based on the risk bearing. The risk bearers are the equity shareholders and not the preference shareholders & creditors.

4. Enterprise Theory:

The enterprise is a broader concept and is working in an environment of socially, politically & economically interested groups. These groups influence the functioning of the enterprise to satisfy their needs and ambitions. Therefore, an enterprise is responsible not only to shareholders but also to the employees, government, customers, creditors, and the general public at large.

The role, responsibilities & obligations of business have now become broader. This accounting theory is also called as a social theory of Accounting. This theory protects the interest of social groups such as employees, Government, creditors, customers, and society at large apart from that of the shareholders.

According to Hendriksen, E.S. "The large corporations can no longer operate solely in the interests of shareholders and it can not be assumed that the force of competition will necessarily protect the interest of other groups".

Features:

1. The business unit is working in an environment of socially, politically and economically interested groups.
2. In order to contribute value to society, as business uses certain amount of inputs.
3. The interested groups are the stakeholders namely shareholders as well as social groups in the value addition of the business.
4. The enterprise theory has given birth to new accounting practices called Human Resource Accounting, Environment Accounting, social accounting, corporate & Governance Accounting etc.

5. Fund Theory:

This theory focuses on administrative and appropriate use of assets. Hence, it can be said that it is an asset centered theories discusses earlier. Under this theory balance sheet is not the primary statement.

Instead the funds statement that is a statement of sources and users of funds in the main statement.

The fund theory is applicable to non-trading concerns and government departmental undertakings whose aim is to serve society and not to earn profits.

# Accounting Principles unit-4

**Meaning:**

* The Term 'Principle' means 'fundamental belief or 'general truth.

**Accounting Principle:** [2mks: 2018]

Accounting Principle are rules of action or conduct which are adopted by accountants universally, while recording accounting transactions.

**Definitions:**

Accounting Principle are generally decision rules. They are adopted or professed as a guide to action or a basis of conduct or practice. They are evolved from assumptions and concepts.

**Need for Accounting Principles:** [2mks: 2012, 2013, 2016] [5mks: 2014]

1. To meet the requirements of both internal as well as external users of financial information.
2. To maintain uniformity in accounting principles and Techniques among business units.
3. For proper analysis and interpretation of financial Techniques.
4. For better comparison of performance of inter firms and intrafirms based on standard principles.
5. For presentation of true and fair position of the enterprise through the preparation of financial statements.

**Definition of Accounting Principles:**

American Institute of Certified Public Accountants USA:

"Accounting Principles are "general Laws or rules adopted or professed as a guide of action, a settled ground or basis of conduct or practice".

## Features of Accounting Principles: [2mks:2015]

1. Principles are general rules of action universally accepted as a guide of action.
2. The accounting principles are man made.
3. They are not original They are arising out of objectives, Postulates and theoretical concepts.
4. They are associated with the accounting theory and accounting procedures.
5. They serves as an explanation.
6. verification of principles.

## Types of Accounting Principles:

Basic principles are the general decision rules derived from the objectives and theoretical concepts of accounting which are used to develop the accounting techniques. These are the principles which are followed, traditionally to maintain the books of accounts and to prepare financial statements and reports.

There are mainly two types of accounting principles: [15mks - 2014]

I. Basic Principles/Traditional.

1. The cost principles
2. The dual aspect principle
3. The matching principle
4. The revenue principle
5. The objective principle
6. The full disclosure principle.

II. Modified Principles

1. cost benefit principles
2. Consistancy principles
3. Conservatism principles
4. Materiality Principles
5. uniformity Principles
6. Industry Principles

(15mks 2018 — items 2–4)

I. Traditional or Basic Principles: [5 mks : 2016, 2017]

These are old principles those that are developed out of practice, custom, tradition. These are the principles which are followed traditionally to maintain the books of accounts and to prepare financial statements and reports, and they have become principle by ritual & practice.

1. Cost Principle:

These principle highlights on recording Transaction at the cost that is incurred on any item. It may be towards an item of expense or on any asset.

[Rule: Transactions should be initially valued at historical costs, rather than at market or appraised values]

Cost means amount spent an asset or expense measured in terms of money.

According to AICPA "cost is the amount, measured in money, of cash expended or other property transfered capital stock issued, services performed or a liability incurred in consideration of goods and services received or to be received".

The cost principle is closely related to or rather derived from the "going concern postulate" because the asset is recorded at acquisition cost and not at market value with the basic assumption that the business entity will continue along with the existing assets and the assets will not be sold in near future to take them at market price.

The cost principle is also derived from the 'money measurement postulate' because all assets are recorded in terms of money spent on their acquisition;

Advantages:

1. Easy to record:

Cost is measurable in terms of money, so it is very easy to record the transaction.

2. Verification of assets:

The value of assets shown in accounting books can be verified easily with the help of vouchers.

3. No Personal basis:

costs prices are free from personal ~~ba~~ bias of the values because the cost is ascertained at the time of transaction and it cannot be altered later on.

4. It helps for auditing:

costs prices hope to determine the periodic net profit & to draw the current financial statement.

5. No Forecast:

As the cost principle is based on an actual transaction, there is no scope for guessing the anticipated cost or probable cost.

6. Easy to apply:

cost price is very easy to apply. It is less time-consuming and easy to value the assets under the cost principal price method.

Disadvantages:

1. Difficult to ascertain the present worth of business.
2. It is not suitable for planning: Actual costs are irrelevant for ~~to~~ the purpose of planning for the future.
3. Historical cost method fails to determine the present worth of business as the present market value of assets are not taken int consideration.
4. Networth of assets is not recognized:

Inflation affects the net worth of the enterprise but the principle does not recognize it. therefore net worth of business cannot be determined accurately under inflationary conduct.

5. Fixed method of Depreciation is not applicable:

under the historical cost system, depreciation on assets is calculated on the basis of the original cost of the assets. All the methods of Depreciation are based on some assumptions.

2. Dual Aspect Principle: [2mks:2017, 5mks:2016]

This principle is followed in Double entry system of accounting. According to this principle, all business transactions ~~in~~ involve two elements or two fold aspects, namely Debit & Credit. Both these aspects are to be recorded in the books of accounts to call it as dual aspect principle.

The capital, reserves and liabilities are equal to "assets and properties" as it is evident from the balance sheet. The sources of funds are always equal to application of funds. While recording the business transactions in accounting books, we always record dual aspect of the Transaction that is receiving of the benefit and giving benefit as credit then debit is always equal to credit. Every business transaction also effects necessarily the two accounts, of which one account is Debited and the other one is to be credited.

According to this principle the amount invested by the owner is the owners capital which is shown on the liabilities side of the balance sheet. Assets acquired by the business out of such capital are shown on the assets side in terms of money.

This Doub entry system is also called as the accounting equivalence principle.

3. Matching Principle: [2mks: 2013]

The principle considers revenue and expenses of an accounting period for the purpose of recording the transaction of that year.

This principle is mainly based on accounting period postulate and is also closely related to the accrual & realization principle. According to this principle "the net trading profit of a business is calculated by matching the total amount of revenues earned during a given period with the total amount of expenses incurred in the same period.

In prepare in final accounts of a concern the outstanding and prepaid items of incomes and expenses should be adjusted before preparing final accounts.

This accounting principle requires companies to use the accrual basis of accounting. The matching principle requires that expenses be matched with revenues.

**Advantages:**

1. It helps in distinguishing the revenue expenses with the capital expenses.
2. Only the revenue expenses and revenues of the current financial year are taken to know the real profitability of the concern.
3. It also takes into consideration the allocated costs on the basis of benefits derived
4. Expenses paid in advance are excluded from total costs, but outstanding expenses are added to the total cost to find out the real profitability of that period.

**Disadvantages:**

1. Some of the expenses are not readily indentifiable against the revenue of a particular period.
2. Difficulty in ascertaining the amount of the capital expenditure.
3. It does not consider further developments in the field of business like inflation, Human Resource, social accounting.
4. Recording is based on cost and revenue incurred or earned.

~~4) Revenue Principle~~ : [illegible]

According to Robert N. Anthony: "the matching principle holds that all of the expenses incurred in generating revenue should be identified or matched with the revenue generated period by period".

4. Revenue Principle: [2mks: 2018] or Realization Principle:

Revenue arises when a sale is made or the amounts received for sale of output are called Revenue.

Revenue are recognized when they are realized or realisable. Further revenue are recognized only when they are earned. The revenue principle holds that revenue should be measured in the period in which it is earned or realized or recognized. The whole principle of realization is revolting around the term revenue.

According to Robert Anthony: "Revenue is considered as being earned on the Date at which it is realised that is, date when goods or services are furnished to the customers in exchange of cash or for other valuable consideration".

Advantages:

1. It provides a sound basis for the ascertainment of the correct Profit or Loss of the business for a year on the basis of sales or service provided.
2. Convenient and sample method o' to determine revenue.
3. Authentical method based on documentary evidence. that is sale invoice or service voucher.

Disadvantages:

1. ~~user~~ unrealized gains are not considered in accounting because property in goods has not yet been transferred.
2. It is criticized by economists on the ground that if an asset has increased in value then it is ~~irrele~~ irrelevant because it has not been sold.

5. Objectivity Principles:

Objectivity refers to reliability, trust worthiness and verifiability, which means that there is some evidence in ascertaining the correctness of records and statements reported The accounting should contain the factual and verified data, which is free from bias.

The objectivity rather than subjectivity is considered in Objectivity principle.

If the fair value of the fixed assets is to be estimated, There is a possibility of different valuers valuing differently, on the basis of the information available, skill of forecasting and their knowledge. The valuation is influenced by various factors which are subject to the personal bias.

**Advantages:**

1. Free from personal bias.
2. Transactions are recorded only when there are supporting evidential documents. so manipulation of accounts is avoided.
3. The evidences are verifiable. so it facilitates better auditing practices.
4. qualified accounting practitioners have consensus amongst them regarding this principle.
5. It is measurable through statistical tools.

**Disadvantages:**

1. objectivity needs evidences.
2. Every transaction needs verifiable evidence.
3. Estimation leads to personal bias. therefore it is avoided under this principle.

6. **Full Disclosure Principle:** [2mks: 2013, 2016, 2019]

Accounting records of business are more meant for the firm only. with increasing role of public investment in companies and Government it is mandatory for the firms to Public Publish. The accounting records in the public interest the account books have to disclose maximum information in which investor, Government and public is Interested

According to Belkaoui: "Full disclosure requires that financial statement be designed and prepared to portray accurately the economic events that affected the firm for the period & to contain information sufficient to make them useful and not misleading to the average investor".

According to this principle "all the important information should be fully disclosed in the financial statements of a concern, since such statements are meant for the use of various parties". It is on the basis of information conveyed to them by the statements they will be able to know whatever is necessary for them about the business. such significant information is the basis of valuation of fixed assets, valuation of investments & closing stock etc.

II. Modified Principles: [15mks 2013]

Modify means to alter or changing the principles as per the present requirement. sometime the basic principles are altered, changed and improved while preparing the financial reports to suit needs of users. such altered basic principle are known as modified principles.

1. cost benefit Principle:

This modified principle states that the cost of applying an accounting postulate, concept, policy or principle etc, should not exceed benefit derived from it. The benefit derived from providing additional information should be greater than the cost of providing it. Then only it would be economi-cally Justified. The benefit must be measured from the users point of view.

This principles does not state to save the cost by giving very little information to users. In fact the reliable information given to internal & external users should be sufficient even at higher cost in order to reduce uncertain-ty in decision-making.

2. Consistency Principle:

In accounting all concepts, principles, policies, practices and methods should remain same, that is unchanged from one accounting year to another to ensure that the accounting data reported in financial statements are reasonably comparable over a period of time. If frequent changes are made in accounting process or policies, then comparison of financial statements of one period to another would be misleading. The consistent use of accounting methods & procedures over time will check the distortion of profit & loss account & balance sheet. This will control possible manipulation of the accounts or financial statements.

According to IAS-1 consistency is a fundamental assumption & it is assumed that accounting policies are consistent from period to another. Where this assumption is not followed, the fact should be disclosed together with reasons.

Types of Consistency:

A. According to Kolher three types of consistencies:

1. Vertical Consistency:

It is maintained within the interrelated financial statement to the same date

2. Horizontal Consistency:

It helps the comparison of performance of an organisa-tion in one year with its performance in the next year.

3. Third Dimensional Consistency:

It helps the comparison of the performance of one organisation with the performance of other organisation in the same industry.

Advantages of consistency principle:

1. To make inter period compression possible.
2. To make inter-company comparison.
3. To eliminate personal bias.
4. Rare possibility for manipulation.
5. Comparison in auditing.
6. Trend analysis.

Disadvantages:

1. This principle goes against the principle of full disclosure.
2. It encourages accountant to create secret reserves by making excessive.
3. It does not give the true & fair picture of the statement of affairs of the concern.

3. Conservatism Principle:

The business exists in a world of uncertainty. Therefore, to safeguard its interest it should make provision for all possible future losses without taking into consideration the future anticipated profits. This principle is called the principle of conservatism. This view is the result of 'minimax' managerial philosophy that is 'minimise' the chance of maximum losses.

This principle follows the policy of 'anticipate no profit & provide for all possible losses'.

It is a general tendency in every ~~an~~ field and particularly in business to maintain a fairly stable of affairs without sudden or considerable changes. Such a tradition is known as convention of conservatism.

Advantages:

1. To safeguard against possible future loses.
2. To show better financial position of the concern.
3. To counter the over optimism of managers & owners.

4. To avoid identification of income until it is realized.
5. To avoid the increase or appreciation in the value of assets unless they are sold.

Limitations:

1. It creates secret reserves
2. Misleads the external users regarding sate of affairs of the business due to secret reserves.
3. It break the principle of full disclosure.
4. This principle brings inconsistency to the accounting policies & methods.

4. Materiality Principle:

Materiality refers to 'very important' item when compared to other items. Materiality means the important facts or an item or a statement capable of influencing the judgement of the rational users in the decisions.

Definitions:

American Accounting Association [AAA]: "an item should be regarded as material, if there is a reason to believe that knowledge of it would influence the decision of informed investor".

Under this principle, the financial statements are expected to cover only the material, or significant or important facts, ~~or an item or~~ either ignoring all other unimportant and insignificant items. or by merging them with the material or significant items.

The Accountant, while preparing the financial statement is expected to mention important details & ignore less important or insignificant facts. If such insignificant information cannot be avoided, then either it should be recorded as footnotes or merged with the more important fact.

## Defination of Materiality Principle:

American Accounting Association [AAA] defines "materiality as following." An item should be regarded as material if there is reason to belive that knowledge of it would influence of the decision of information informed investor."

### Benefits:

1. The System offers only those accounting records that are material. This will save the cost of keeping record & sharing it.
2. Decision making is made simple. Only relevant financial information of the organisation is accounted.
3. With growing awarness of environment it can save the resources by creating unnecessary expenses on paper, stationery & avoid wastage of labor.
4. Only relevant information is stored that will b help the reader to understand such information.

### Limitations:

1. Objectivity principle is overlooked in determining whether the accounting information is material or not.
2. The determination of immateriality is subjective. It means the meaning chages from firm to firm.
3. This principle is opposed to the full disclosure principle.

## 5. Uniformity Principle:

There should be uniformity in financial information provided by different business organisations. These institutions have investors from different parts of world. ~~Thesf~~ These firms may be conducting their operations in different countries. Accounting information provided by them should be uniform or same as it is used world wide.

Every commercial enterprise is required to prepare & present the financial statements at the end of every accounting period. The information available in financial statements is used by the internal & external users in making their relevant decisions. It depends on the credibility & realiability of the enterprise as shown by their financial statements. It is but natural, therefore to expect that these statements provide true, fair & unbiased position of the business & project a perspective view of the enterprise.

However, such a thing is possible only when the information gathered & presented is based on some uniform postulates, concepts, principles & policies.

**Advantages:**

1. It makes the organisation's results comparable at the national & international level.
2. The verification of accounting records becomes easy & standard auditing practices can be adopted easily.
3. To help investors in decision making.
4. uniformity principles gives less scope for using their own judgement in selecting the accounting policies.
5. Determining managerial accountability by measuring the effectiveness of managements.

**Disadvantages:**

1. The heterogeneous nature of organizations may make it difficult to adopt uniform accounting principle.
2. The presence of different environmental factors, laws, governments makes it difficult to adopt uniformity principle.
3. choice between alternative methods is not available to accountants.
4. Difficulties in bringing uniformity due to the existence of various conflicting accounting theories.

6. Industry Principle:

Every industry or business organization has its own style of recording, maintaining & reporting accounting information, Nature, size, scale of business differ from one firm to other firm. Each firm depending on its own convenience may have its own practice/style of keeping accounts. Basically they may follow some principles but application to extent differs.

This is more specific in relation to valuation models. The valuation of stock investment, asset, etc may be based on different methods in the same accounting report.

Advantages:

1. It is flexible.
2. Helps in decision making.

Disadvantages:

1. It neglects the application of standard practices among all the industries.
2. Such flexibility in accounting records may not be acceptable by league authorities.

Difference between accounting Principle & Accounting Policies: [2mks : 2014 ~~20~~ 5mks: 2015, 2019]

| Points | Accounting Principles | Accounting Policies |
|---|---|---|
| 1. Presumption | * Presumptions are assumed to have been used & accepted in the financial statements. | * No presumptions can be made in respect of accounting policies. |
| 2. Application | * Principles can be used simultaneously. | * At a time only one type of policy can be used. |
| 3. Disclosure | They need not be disclosed but can be detected from the accounting records. | All significant policies must be disclosed. |
| 4. Choice | These principles are selected by accountants and accounts practitioners. | Policies are selected by management & decided in consultation with staff |
| 5. Modifications | Principles can be easily modified as per the requirement by the accountant. | Policies must be consistent change has to be done as per Accounting Standards. |
| 6. Development | These can be developed as per the environmental needs or the user. | These are developed & adopted by management to suit the situation. |
| 7. Type of Theory | They satisfies the positive theory of accounting. | They satisfies the normative theory of accounting. |
| 8. Nature | These are general Principles. | They are ~~selected by~~ the specific principles. |

# Accounting Standards - 5

**Meaning of Accounting standards:** [2mks: 2012, 2016, 2018]

Accounting standards are written policy documents, issued by Professional accounting bodies or by Government or other regulatory bodies covering the aspects of recognition, measurement, treatment, presentation and disclosure of accounting transaction in the financial statements.

**Definitions of Accounting standards:** [2mks: 2015]

* **According to Kohler's dictionary:**

"Accounting standard is a mode of conduct imposed on accountants by custom, law or professional body and accounting principle".

* **According to Anthony:**

"Standards, are referred to as those accounting Policies that are design to govern the nature and content of financial statements".

**Need for Accounting standards:** [5mks: 2016, 2019 2mks: 2017]

1. To make the financial statements to depict true & fair Position & progress of an enterprise.
2. To bring uniformity in preparation & presentation of the financial statements.
3. To ensure consistency & comparability of financial Data contained in financial statements.
4. To raise the confidence of the users by means of reliable & credible financial data reported through statements based on set standards.

5. To provide an important mechanism in solving potenti-ial conflicts of interest between various interested parties through reliable published financial reports.

Objectives of Accounting Standards:

1. To standardize the diverse accounting policies & practices.
2. To facilitate comparability of financial statements.
3. To assume the reliability to the financial statements to its users.
4. To provide information to the users as to the basis on which the accounts have been prepared & the financial statements have been presented.
5. To make financial statements more meaningful and comparable.
6. To guide the judgement of professional accountants in dealing with those items which are to be followed consistently from year to year.

Importance of Accounting standards: or Advantages [2mks: 2015]

1. Uniform Accounting Policies.
2. Practices of comparable business Records.
3. Accurate & scientific.
4. Help in globalization of business.
5. Credibility & Reliability.
6. Commitment of professionals.
7. Raises the standards of Auditing.
8. Statutory Acceptance.

# International Accounting Standards Committee [IASC]

In order to maintain uniformity in accounting principles throughout the world, international accounting standards committee [IASC] came into being on 29/6/1973. The IASC comprises the professional accountancy bodies of over 75 countries.

Objectives of IASC are to develop accounting standards which are to be observed in the presentation of audited financial statements and to promote their word-wide acceptance. So far IASC has issued 40 IASs. However, the IASs, are not accepted worldwide. In 1978 another professio-nal body. The international Federation of Accountants [IFAC] was established.

The objections of IASC which are set out in its revised agreement & constitution [November 1982] are:

1. To formulate & publish in the public interest accounting standards to be observed in presentation of financial Statements & to promote their world wide acceptances & observations.
2. To work for the improvement & harmonisation of regulating accounting standards & procedures relating to the presentation of financial statements.

## Uses of IASC Standards:

1. As a basis for national accounting requirements in many countries.
2. As an international bench mark by certain countries which develop their own requirments.
3. By stock exchange & regulatory authorities that allow or require foreign domestic companies to present financial statements in accordance with international accounting standards.

## International Financial Reporting Standards [IFRS]:

**Meaning:** Statement of IAS [International Accounting Standards] issued by the board of the IASC [1973-2001] are designated as IAS. However, the IASB announced in April, 2001 that its Accounting standards would be designated as 'International Financial Reporting standards [IFRS]. IASB publishes its standards in a series of pronouncements called international Financial Reporting Standards [IFRS].

It also adopted the body of standards issued by the board of the International Accounting standards committee. Those pronouncements continue to be designated as "International Accounting standards" [IAS].

**Benefits of IFRS:**

1. It benefits the economy by increasing growth of its international business.
2. It facilitates maintenance of orderly & efficient capital markets & also helps to increase the capital formation.
3. It encourages international investing & there by leads to more foreign capital flows to the country.
4. The accounting professionals are benefitted in a way that they are able to sell their services as experts in different parts of the world.

## Generally Accepted Accounting Principles [GAAP]:

**Meaning:**

GAAP are the rules or action or conduct which are derived from experience and practice & when they prove useful, they become accepted as principles of accounting.

## The List of accounting standards in India :

The following is the list of accounting standards issued by the Institute c charted accountants of India (ICAI) :

**Accounting standards**

| AS No. | Title | Date from which it commences / becomes mandatory |
|---|---|---|
| AS-1 | Disclosure of accounting policies | 01-04-1991<br>01-04-1991 |
| AS-2 | Valuation of inventories (revised) | 01-04-1999 |
| AS-3 | Cash flow statements (revised) | 01-04-2001 |
| As-4 | Contingencies and events occurring after the balance sheet date | 01-04-1995 |
| AS-5 | Net profit or loss for the period, Prior period items and changes in accounting policies. | 01-04-1996 |
| AS-6 | Depreciation accounting | 01-04-1995 |
| AS-7 | Accounting for construction contracts | 01-04-1991 |
| AS-8 | Accounting for research and development | 01-04-1994 |
| AS-9 | Revenue recognition | 01-04-1993 |
| AS-10 | Accounting for fixed assets | 01-04-1993 |
| As-11 | Accounting for effects of changes in Foreign exchange rates | 01-04-1995 |
| AS-12 | Accounting for Government Grants | 1-04-1994 |
| AS-13 | Accounting for investments | 1-04-1995 |
| AS-14 | Accounting for amalgamations | 1-04-1995 |
| AS-15 | Accounting for retirement benefits in the financial statements of employers | 1-04-1995 |
| AS-16 | Borrowing costs | 1-04-2000 |
| AS-17 | Segment reporting | 1-04-2001 |
| AS-18 | Related party disclosures | 1-04-2001 |
| AS-19 | "Leases" | 1-04-2001 |
| AS-20 | Earnings per share | 1-04-2001 |
| AS-21 | Consolidated financial statements | 1-04-2001 |
| AS-22 | Accounting for taxes on income | 1-04-2001 |
| AS-23 | Accounting for investments in associates in consolidated financial statements | |
| AS-24 | Discontinuing operations | 1-04-2002 |
| AS-25 | Interim financial reporting | 1-04-2002 |
| AS-26 | Intangible assets | 1-04-2002 |
| AS-27 | Financial reporting of interest in joint ventures | 1-04-2003 |
| AS-28 | Impairment of assets | 1-4-2002<br>2004-05<br>2005-06 |

| | | |
|---|---|---|
| AS-29 | Provisions, contingent liabilities and contingent assets | 1-4-2004 |
| AS-30 | Financial instruments : Recognition and measurement | |
| AS-31 | Financial instruments : Presentation | 1-4-2009 |
| AS-32 | Financial instruments : Disclosures | |

## Accounting Standards Provisions:

1. AS-2 Inventory valuation
2. AS-6 Depreciation
3. AS-10 Fixed Asset
4. AS-29 Provisions for contingent Liabilities & Assets.

### 1. AS-2 Inventory valuation:

Inventory means stocks. All the business firms Keep the stock to meet its future requirements of production and sales.

**Definition of Inventory:**

According to accounting standard, inventories are the tangible property or assets -

a. Held for sale in the ordinary course of business.

b. In the process of production for such ~~scale~~ sale.

c. In the form of materials or supplies to be consumed in the production process or in the rendering of services.

**Importance / objectives of Inventory valuation:**

1. Ascertainment of income.
2. True financial position
~~3. Sufficient~~
3. Sufficient inventory for production sales process.

**Methods for valuation of stock:**

The following are the various methods for valuation of stock under historical cost:

1. Specific Identification methods
2. First in first out Method [FIFO] } [Imp]
3. Last in first out Method [LIFO] } [Imp]
4. Highest in first out Method [HIFO]
5. Base stock Method
6. Next in first out Method [NIFO]

7. Simple average method
8. Standard cost Method
9. Adjusted selling price Method
10. Latest purchase price Method.

* First in Firstout: [FIFO]

Under this system, materials are issued in the order in which they are received in the store. The material received first will be issued first. "First come first served."

In other words old stocks are issued first and new stoks will be issued afterwards. As a result of this system, when we value the closing stock of materials, that will be at the latest price.

Merits:

1. It is simple.
2. It is based on a realistic assumption that materials are issued in the order of their receipt.
3. It proves or repreents good inventory management.
4. There is no danger of over or under recovery of cost.
5. This method is good where the prices are falling.

Demerits:

1. Cost flunctuation in different Jobs.
2. It involves complicated calculation.
3. In periods of rising prices, the FIFO method gives more profit and results in higher tax Liability.

* Last in First out: [LIFO]

This method is opposite to FIFO. Under this method the last purchased are issued would be the cost price of the last Lot of materials purchased.

Merits of LIFO:

1. No danger of over or under recovery of cost
2. Uptodate profit picture.
3. It is boom to stock pillers.
4. It helps in reducing the tax burden.
5. It is simple.

Demerits:

1. It leads to more clerical mistake.
2. Job cost will differ.
3. Bad inventory management.
4. Balance sheet fails to give true picture.
5. It calls for tedious calculation.

2. AS-6 Depreciation Accounting:

Depreciation is reduction in the value of an asset due to tits used or obsolescence [out of Date]

Definition:

According to ICAI, defines depreciation as a measure of vering out, consumption or other loss of value of depreciable assets arising from use time/abulance throught technology & market change.

Objectives/Needs for depreciation:

1. To show the true profit of business
2. To show the financial position
3. To accumilate funds for replacement for asset.
4. To follow the companies ~~act~~ act.
5. To ascertain the true cost of production.
6. To keep nominal capital invested in different fixed assets.

3. AS-10 - Accounting for fixed Assets:

Assets are that purchased for carring operations like manufacturing, trading and to provide for some kind of service. business are classified under fixed assets.

Example: Land, building, base hold Land, plant, machinery, furniture etc.

It is concerned with retarement & discloser of assets in this case the following guidline are issued by the council of institute of charted accountant of india.

1. An item of fixed asset is eleminated from the financial statement on discloser.
2. Historical cost financial statement gains or Losess arising from dispsalls are genarally recognised in the profit & Loss statement.
3. Items of fixed asset that have been retained from active use & are held for disposal are stated at the Lower of there book & net realisable value are shown saperately in the financial statement.

4] AS-29 Provisions, contingent Liabilities & contingent Assets:

A provision is an estimated amount of expenditure that a firm may have incur due to it's a operational other financial deals.

Provisions for doubtful debts is the most commonly accepted provision a firm is allowed to create such providing by treating it as onitem of expense, that is deducted to P&L A/c.

Amount towards such provisions are created out of past experience of finding similar kind of business/act.

A contingent Liability is an expected amount of liability that a firm may to incur due to its present or past acts. It is obligation/duty/responsibility to pay towards such liability.

[redacted] **(23225/B 250)**

## II Semester B.Com.2 Examination, May 2012
## ACCOUNTING THEORY

Time : 3 Hours Max. Marks : 80

***Instructions :*** *1) Answer the questions with strict observance of internal choices offered.*

*2)* ***Neatness*** *and* ***clarity*** *carry due weightage.*

### SECTION – A

1. Answer **any ten** of the following : (10×2=20)

a) What is accounting theory ?

b) What is deductive approach ?

c) What is socio-economic environment ?

d) Define accounting postulates.

e) What are accounting standards ?

f) What is decision model approach ?

g) What is FIFO ?

h) What is reducing balance method of depreciation ?

i) Explain in brief cost-benefit principle.

j) What is normative theory ?

k) What is the principle of materiality ?

l) State two needs for accounting principles.

### SECTION – B

Answer **any three** of the following : (3×5=15)

2. Distinguish between accounting theory and accounting practice.

3. Explain, in brief, the nature of accounting theory.

4. State the advantages and disadvantages of structural theory.

P.T.O.

5. Find out owners equity from the given balance sheet under proprietary theory.

**Balance Sheet**

**As 31-3-2012**

| Liabilities | Amt. | Assets | Amt. |
|---|---|---|---|
| Capital | 8,00,000 | Fixed Assets | |
| General Reserve | 4,00,000 | Plant and machinery | 7,00,000 |
| P and L Account (Surplus) | 3,00,000 | Land and buildings | 5,00,000 |
| Creditors | 5,00,000 | Current Assets | 10,00,000 |
| Bills payable | 2,00,000 | | |
| | **22,00,000** | | **22,00,000** |

6. Mr. Ram commences business with an investment of his own of Rs. 1,00,000; borrowed amount of Rs. 50,000 from his friend Krishna. He deposits Rs. 1,00,000 in to a bank account opened in the name of business. Purchased furniture of Rs. 40,000 for the business.

Prepare a Balance Sheet of Mr. Ram by applying Dual Aspect Principle.

SECTION – C

Answer any two of the following : (2×15=30)

7. Discuss information theory with its advantages and disadvantages.

8. Discuss briefly the effects of various environments on accounting theory.

9. From the following information, calculate the profit by matching principles

a) Purchases of goods : 1000 units at Rs. 250 per unit

b) Sales of goods : 800 units at Rs. 425 per unit

c) Rent of shop paid : Rs. 4,000 p.m, for 11 months

d) Salary to staff : Rs. 5,000 p.m, for 11 months

e) Commission on sales payable at 1%

f) Miscellaneous income received : Rs. 525

g) Received cash on account of sales from sundry customers Rs. 2,00,000

h) Paid cash to sundry suppliers for purchase Rs. 1,50,000

i) Period of transaction 1st April 2008 to 31st March 2009

j) Capital introduced in business Rs. 2,00,000.

10. From the following transactions prepare separately stores ledger account using the LIFO method.

| | |
|---|---|
| 1 Jan. | Purchased 100 units @ Rs. 5 per unit |
| 5 Jan. | Purchased 500 units @ Rs. 6 per unit |
| 20 Jan. | Issued 300 units |
| 5 Feb. | Issued 200 units |
| 6 Feb. | Bought 300 units @ Rs. 7 per unit |
| 10 March | Issued 300 units |
| 15 March | Purchased 400 units @ Rs. 6.50 per unit |
| 31 March | Issued 250 units. |

## SECTION – D

Case study question is **compulsory** : (1×15=15)

11. Anita and Sunita are partners in a partnership firm. They converted their firm in to a private Ltd. Co. and registered under Indian Companies Act 1956. At the end of the financial year they presented the final accounts in the manner in which it was presented earlier (as a partnership firm). The Registrar of companies objected to it and directed the company to present it as per schedule mentioned in the Indian Companies Act 1956. He also directed them to transfer the surplus profit i.e. profit after interest, tax and dividend to reserves and surplus account.

**Questions** :

a) Under what approach the Registrar was asking to present final accounts as per schedule mentioned in the Indian Companies Act 1956 ?

b) What type of environment is forcing the company to change the accounting system ? legal

c) Whether the accounts as per schedule mentioned in the Indian Companies Act 1956 are as per positive or normative accounting theory ?

d) Which principle has forced the Government of India to compel the companies to prepare the final accounts as per schedule mentioned in the Indian Companies Act, 1956 ? uniformity

e) Is there any difference in theories (Proprietary, Entity, Residential Equity etc) while maintaining accounts for partnership firm and for a private company ?

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# II Semester B.Com. 2 Examination, April/May 2014
## (Regular and Repeater)
## ACCOUNTING THEORY

Time : 3 Hours Max. Marks : 80

***Instructions :*** *1) Answer the questions with strict observance of the* ***Internal Choice*** *offered.*

*2)* ***Neatness*** *and clarity carry due* ***weightage.***

SECTION – A

1. Answer **any ten** of the following : (10×2=20)

a) Name the parties interested in accounting information. Direct interested users information

b) Write any four features of accounting theory.

c) What is structural theory ?

d) What is deductive approach ? Give an example.

e) Mention the three values stressed in Ethical Approach.

f) Define postulates.

g) Write any two advantages of unit of measurement postulate.

h) Write two needs of accounting principles.

i) Expand the following :

1) IASB – International Accounting Standards Board

2) IFRS. – International Financial Reporting Standards.

P.T.O.

j) What are depreciable assets as per AS-6 ?

k) What things are included in inventories ?

l) Write any two differences between accounting principles and accounting policies.

## SECTION – B

(3×5=15)

Answer **any three** of the following :

2. Distinguish between Positive Theory and Normative Theory.

3. Briefly explain the decision theory as a root of accounting theory.

4. Write a note on Accounting period postulate.

5. Briefly discuss the need for Accounting Principles.

6. Find out the Owner's Equity under the 'Residual Equity Theory' from the particulars given below.

OE = Equity share Capital + Reserve + Surplus + Dividend on equity share
= 10,00,000 + 14,00,000 + 1,00,000 + 4,00,000
= 16,00,000

| | Rs. |
|---|---|
| Equity share capital | 10,00,000 |
| Preference share capital | 4,00,000 |
| General reserves | 1,00,000 |
| P & L (Credit) Balance | 1,00,000 |
| Sundry Creditors | 2,00,000 |
| Fixed Assets | 14,00,000 |
| Current Assets | 4,00,000 |

## SECTION – C

(2×15=30)

Answer **any two** of the following :

7. Define accounting theory. Mention the various objectives of accounting theory and discuss the role of accounting theory.

8. Briefly discuss the traditional approaches to accounting theory.

9. Define accounting principles. Briefly discuss the various principles of accounting.

10. The particulars of purchases and issues of material 'P' in a factory in January, 2014 are as given below :

| 2014 | | | |
|---|---|---|---|
| Jan. | 1 | Opening balance | 1500 kgs @ Rs. 24 per kg |
| " | 2 | Issued to production centre | 650 kgs |
| " | 5 | Purchased | 400 kgs @ Rs. 25 per kg |
| " | 9 | Issued | 300 kgs |
| " | 10 | Purchased from suppliers | 200 kgs @ Rs. 27 per kg |
| " | 12 | Sent to production centre | 300 kgs |
| " | 14 | Issued | 410 kgs |
| " | 17 | Bought from Suppliers | 500 kgs @ Rs. 26 per kg |
| " | 20 | Received from Suppliers | 300 kgs @ Rs. 25 per kg |
| " | 25 | Issued to production centre | 200 kgs |
| " | 31 | Purchased | 200 kgs @ Rs. 28 per kg |

Pricing of issues is to be done on 'FIFO' basis. Write-up stores ledger in suitable format.

## SECTION – D

11. Case Study – Compulsory : (1×15=15)

Shri Shraddhanand has presented his balance sheet as on 31-3-2013 :

**Balance Sheet as on 31-3-2013**

| Liabilities | | Amount | Assets | | Amount |
|---|---|---|---|---|---|
| Capital | 8,00,000 | | Buildings | 8,00,000 | |
| Add. profit | 1,00,000 | 9,00,000 | Less : Deprn. | 80,000 | 7,20,000 |
| Reserve fund | | 3,00,000 | Vehicles | 4,00,000 | |
| Sundry creditors | | 4,00,000 | Less : Deprn. | 80,000 | 3,20,000 |
| Bills payable | | 20,000 | Investments | | |
| Outstanding Exps. | | 10,000 | (Market value 1,20,000) | | 1,00,000 |
| (Contingent liabilities | | | Stock | | 2,40,000 |
| 1,00,000) | | | Debtors | 2,00,000 | |
| | | | Less : R.D.D. | 20,000 | 1,80,000 |
| | | | Cash at Bank | | 70,000 |
| | | **16,30,000** | | | **16,30,000** |

You are required to identify the postulates, concepts and principles applied in preparing the given balance sheet. Give brief explanation for each of them.

ಕನ್ನಡ ಆವೃತ್ತಿ

ವಿಭಾಗ – ಅ

1. ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ಹತ್ತು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ : (10×2=20)

   a) ಲೆಕ್ಕಗಳ ಮಾಹಿತಿಯಲ್ಲಿ ಆಸಕ್ತಿ ಹೊಂದಿದ ಕಕ್ಷಿದಾರರನ್ನು ಹೆಸರಿಸಿರಿ.

   b) ಲೆಕ್ಕ ಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಯಾವುದಾದರೂ ನಾಲ್ಕು ಲಕ್ಷಣಗಳನ್ನು ಬರೆಯಿರಿ.

   c) ರಚನಾತ್ಮಕ ಸಿದ್ಧಾಂತ ಎಂದರೇನು ?

   d) ಋಣಾತ್ಮಕ ಪಥವೆಂದರೇನು ? ಉದಾಹರಣೆ ನೀಡಿರಿ.

   e) ನೀತಿ ಪಥದಲ್ಲಿ ಒತ್ತು ನೀಡಿದ ಮೂರು ಮೌಲ್ಯಗಳನ್ನು ಹೆಸರಿಸಿರಿ.

   f) ಮೂಲಭೂತ ನಿಯಮಗಳ ವ್ಯಾಖ್ಯೆ ನೀಡಿರಿ.

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Reg. No.

## II Semester B.Com. 2 Degree Examination, April/May 2013
## (Freshers/Repeaters)
## ACCOUNTING THEORY

Time : 3 Hours | Max. Marks : 80

***Instructions :*** *1) Answer the questions with* ***strict*** *observance of internal choices* ***offered.***

*2)* ***Neatness*** *and* ***clarity*** *carry due* ***weightage.***

### SECTION – A

Answer **any ten** of the following. (10×2=20)

1. a) Define accounting theory.

   b) Write two needs of accounting theory ?

   c) What is positive accounting theory ?

   d) What is information theory ?

   e) What is going concern postulates ?

   f) What is proprietory theory ?

   g) What is matching principle ?

   h) What is meaning of full disclosure principle ?

   i) What is GAAP ?

   j) What is LIFO ?

   k) What is the need of Accounting Standards ?

   l) Write two features of Depreciation.

P.T.O.

## SECTION – B

Answer **any three** of the following. (3×5=15)

2. Explain the nature of accounting theory.

3. Briefly explain the measurement theory.

4. Explain entity theory of accounting.

5. Find out the owner's equity from the following information under proprietary theory :

| | | ₹ |
|---|---|---|
| 1-4-2011 | Capital | 10,00,000 |
| 1-4-2011 | Reserves | 5,00,000 |
| 31-3-2012 | Fixed Assets | 20,00,000 |
| 31-3-2012 | Current Assets | 5,00,000 |
| 31-3-2012 | Liabilities (outsiders) | 8,00,000 |

Profit earned during the year 31-3-2012 ₹ 2,00,000.

6. M/s Dayanand Mills Ltd., purchased plant ₹ 5,00,000 on 1-4-2007. Depreciation is charged at the rate of 10% p.a. as per reducing balance method. Accounts are closed on 31st March each year.

Prepare Plant A/c for the five years.

## SECTION – C

Answer **any two** of the following. (2×15=30)

7. What is economic approach ? Write the advantages and disadvantages of economic approach.

8. Describe briefly the modifying principles of accounting.

9. From the following data, prepare the stores ledger according to FIFO method.

1-3-2012— 400 units @ ₹ 5 each [opening stock].

**Purchases :**

| | | |
|---|---|---|
| 5th March 2012 | 600 | @ ₹ 6 |
| 15 March 2012 | 500 | @ ₹ 7 |
| 25 March 2012 | 400 | @ ₹ 8 |
| 30 March 2012 | 300 | @ ₹ 9 |

**Issues :**

| | |
|---|---|
| 3rd March 2012 | 300 units |
| 10th March 2012 | 500 units |
| 17 March 2012 | 500 units |
| 26 March 2012 | 500 units |
| 31 March 2012 | 200 units |

10. Shamala Ltd., Belgavi has provided the following information on 31st March 2012.

| | ₹ |
|---|---|
| **Share Capital :** | |
| 40,000 Equity shares of Rs. 20 each | 8,00,000 |
| 4000-10% Preference shares of Rs. 100 each | 4,00,000 |
| General Reserves | 50,000 |
| Profit as on 1-4-2011 | 2,00,000 |
| Net Profit of the year (2011-12) | 4,00,000 |

**Adjustments :**

i) Directors recomeded a dividend of 15% on equity share.

ii) Transfer ₹ 30,000 to General Reserve.

Prepare final accounts of the company.

## SECTION – D

11. **Case study** question is **compulsory**. **(1×15=15)**

Syndicate Bank, Belgavi have advanced a Loan of ₹ 3,00,000 to be repaid in 5 years to Shri Gangadhra. He paid the first two years installments. Thereafter he failed to pay the installments. Inspite of continous reminders for four years from the date of Default Mr. Gangadhra did not pay a single rupee.

The Bank Manager of Syndicate Bank was worried about recovery of loan. So he asked the accountant to create a provisions for bad debts and not to show interest on loan as "Income earned but not received" in P & L A/c.

The accountant agreed to create the reserve but did not agree for not to show interest in P & L A/c.

The Manager told him to follow Banking Regulation Act, 1949 and Amendment Act which states that if loan is overdue you have to create not only reserve for NPA but also not to show such interest as receivables.

**Questions :**

1) *State what environment is applicable.*

2) What principle the accountant was violating ?

3) Under what authority government can force the bank to accept the principles.

## ಕನ್ನಡ ಆವೃತ್ತಿ

*ಸೂಚನೆಗಳು: 1) ಆಂತರಿಕ ಆಯ್ಕೆಗೆ ಒಳಪಟ್ಟು ಎಲ್ಲಾ ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ.*
*2) ಶುದ್ಧತೆಗೆ ಹೆಚ್ಚಿನ ಮಹತ್ವವಿದೆ.*

## ವಿಭಾಗ - ಅ

1. ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ಹತ್ತು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ. (10×2=20)

a) ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ವ್ಯಾಖ್ಯೆ ಕೊಡಿ.

b) ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಎರಡು ಅವಶ್ಯಕತೆಯನ್ನು ಬರೆಯಿರಿ.

Reg. No. C 1 4 2 9 0 6 7

# II Semester B.Com.2 Degree Examination, April/May 2015

## (RCU – Regular/ Repeater)

## ACCOUNTING THEORY

Time : 3 Hours] [Max. Marks : 80

*Instructions/ಸೂಚನೆಗಳು :*

1) *Answer the questions with strict observance of the internal choices given.*
2) *Neatness and clarity carry due weightage.*

SECTION – A/ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following : (10 × 2 = 20)

ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ಹತ್ತು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :

(a) What is a theory?
ಸಿದ್ಧಾಂತ ಎಂದರೇನು?

(b) Mention any four features of accounting theory.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಯಾವುದಾದರೂ ನಾಲ್ಕು ಲಕ್ಷಣಗಳನ್ನು ಹೇಳಿರಿ.

(c) What is structural or syntactical theory?
ರಚನಾತ್ಮಕ ಸಿದ್ಧಾಂತ ಎಂದರೇನು?

(d) What are the environments surrounding the accounting?
ಲೆಕ್ಕಶಾಸ್ತ್ರವನ್ನು ಆವರಿಸಿದ ಪರಿಸರಗಳು ಯಾವುವು?

(e) Name any four modern accounting theory approaches.
ಯಾವುದಾದರೂ ನಾಲ್ಕು ಆಧುನಿಕ ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಪಥಗಳನ್ನು ಹೆಸರಿಸಿರಿ.

(f) Write any four general objectives of accounting.
ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಸಾಮಾನ್ಯ ಉದ್ದೇಶಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ನಾಲ್ಕು ಬರೆಯಿರಿ.

P.T.O.

(g) Name any four features of accounting principles.

ಲೆಕ್ಕಶಾಸ್ತ್ರದ ತತ್ವಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ನಾಲ್ಕು ಲಕ್ಷಣಗಳನ್ನು ಸೂಚಿಸಿರಿ.

(h) Define accounting standard.

ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಪ್ರಮಾಣಕದ ವ್ಯಾಖ್ಯೆ ನೀಡಿರಿ.

(i) Name the parties benefited by accounting standards.

ಲೆಕ್ಕಶಾಸ್ತ್ರ ಪ್ರಮಾಣಕಗಳ ಪ್ರಯೋಜನ ಪಡೆಯುವವರನ್ನು ಹೆಸರಿಸಿರಿ.

(j) Mention any two methods identified by AS-2 for inventory valuation.

ದಾಸ್ತಾನು ಮೌಲ್ಯೀಕರಣದಲ್ಲಿ AS-2ರಲ್ಲಿ ಗುರುತಿಸಿದ ಎರಡು ಪದ್ಧತಿಗಳನ್ನು ಸೂಚಿಸಿರಿ.

(k) Write two features of depreciation.

ಸವಕಳಿಯ ಎರಡು ಲಕ್ಷಣಗಳನ್ನು ಬರೆಯಿರಿ.

(l) Expand – (i) ASB (ii) IASC.

ವಿಸ್ತರಿಸಿ ಬರೆಯಿರಿ – (i) ASB (ii) IASC.

## SECTION – B/ವಿಭಾಗ - ಬ

Answer **any three** of the following : (3 × 5 = 15)

ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ಮೂರಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :

2. What is Normative theory? Mention its features.

ಪ್ರಮಾಣಿತ ಸಿದ್ಧಾಂತ ಎಂದರೇನು? ಅದರ ಲಕ್ಷಣಗಳನ್ನು ಬರೆಯಿರಿ.

3. Write a note on role of an Accountant.

ಲೆಕ್ಕಿಗನ ಪಾತ್ರ ಕುರಿತು ಟಿಪ್ಪಣಿ ಬರೆಯಿರಿ.

4. Distinguish between accounting principles and accounting policies.

ಲೆಕ್ಕಶಾಸ್ತ್ರದ ತತ್ವಗಳು ಮತ್ತು ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಧೋರಣೆಗಳ ನಡುವಿನ ವ್ಯತ್ಯಾಸಗಳನ್ನು ಬರೆಯಿರಿ.

5. Write a note on Entity concept.

ಅಸ್ತಿತ್ವದ ಪರಿಕಲ್ಪನೆ ಕುರಿತು ಟಿಪ್ಪಣಿ ಬರೆಯಿರಿ.

6. Find out the 'Owner's Equity' from the following under Proprietary Theory.

Balance Sheet as on 31.3.2014

| Liabilities | Rs. | Assets | Rs. |
|---|---|---|---|
| Capital | 10,00,000 | Fixed Assts | 18,00,000 |
| General Reserves | 5,00,000 | Current Assets | 7,00,000 |
| Profit & Loss A/c | 3,00,000 | | |
| Sundry Creditors | 4,00,000 | | |
| Outstanding expenses | 1,00,000 | | |
| Bills Payables | 2,00,000 | | |
| Total | 25,00,000 | Total | 25,00,000 |

ಕೆಳಗಿನ ಆಢಾವೆ ಪತ್ರಿಕೆಯಿಂದ ಸ್ವಾಮಿತ್ವ ಸಿದ್ಧಾಂತದ ಪ್ರಕಾರ ಒಡೆತನದ ಪಾಲನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :

ಅಢಾವೆ ಪತ್ರಿಕೆ ದಿ. 31.3.2014 ರಂದು ಇದ್ದಂತೆ

| ಹೊಣೆಗಾರಿಕೆಗಳು | ರೂ. | ಆಸ್ತಿಗಳು | ರೂ. |
|---|---|---|---|
| ಬಂಡವಾಳ | 10,00,000 | ಸ್ಥಿರಾಸ್ತಿಗಳು | 18,00,000 |
| ಸಾಮಾನ್ಯ ನಿಧಿ | 5,00,000 | ಚರಾಸ್ತಿಗಳು | 7,00,000 |
| ಲಾಭ-ನಷ್ಟ ಖಾತೆ | 3,00,000 | | |
| ಸಾಲಿಗರು | 4,00,000 | | |
| ಪಾವತಿಸಬೇಕಾದ ವೆಚ್ಚಗಳು | 1,00,000 | | |
| ಕೊಡತಕ್ಕ ಹುಂಡಿಗಳು | 2,00,000 | | |
| ಒಟ್ಟು | 25,00,000 | ಒಟ್ಟು | 25,00,000 |



P.T.O.

## SECTION – C/ವಿಭಾಗ – ಕ

Answer **any two** of the following : (2 × 15 = 30)

ಈ ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ಎರಡಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :

7. Explain the need for accounting theory.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಅವಶ್ಯಕತೆಯನ್ನು ವಿವರಿಸಿರಿ.

8. Discuss the various traditional approaches to accounting theory.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಸಾಂಪ್ರದಾಯಿಕ ವಿವಿಧ ಪಥಗಳನ್ನು ಚರ್ಚಿಸಿರಿ.

9. Define accounting postulate. Briefly discuss the various accounting postulates.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಮೂಲಭೂತ ನಿಯಮಗಳ ವ್ಯಾಖ್ಯೆ ನೀಡಿರಿ. ಮತ್ತು ಅವುಗಳನ್ನು ಸಂಕ್ಷಿಪ್ತವಾಗಿ ಚರ್ಚಿಸಿರಿ.

10. Giving the suitable format, write-up Stores Ledger under FIFO method :

Opening Stock of material on 1.2.2014 – 500 units @ Rs. 10 each

| Purchases | Issues |
|---|---|
| 2.2.14 1000 units @ Rs. 15 each | 7.2.14 – 800 units |
| 5.2.14 1500 units @ Rs. 17 each | 10.2.14 – 200 units |
| 12.2.14 1200 units @ Rs. 16 each | 18.2.14 – 1500 units |
| 19.2.14 900 units @ Rs. 20 each | 22.2.14 – 300 units |
| 24.2.14 2000 units @ Rs. 13 each | 27.2.14 – 600 units |

Show the total value of stock on 28.2.2014.

'FIFO' ಪದ್ಧತಿಯನ್ನು ಅನ್ವಯಿಸಿ, ಸಮರ್ಪಕ ನಮೂನೆಯಲ್ಲಿ 'ಸ್ಟೋರ್ಸ್ ಖಾತೆ'ಯನ್ನು ಸಿದ್ಧಗೊಳಿಸಿರಿ.

ದಿನಾಂಕ 1.2.2014 ರಂದು ಪ್ರಾರಂಭ ಶಿಲ್ಕು – 500 ಯೂನಿಟ್ ರೂ. 10 ಪ್ರತಿ ಯೂನಿಟ್

| ಖರೀದಿಸಿದ್ದು | ನೀಡಿದ್ದು |
|---|---|
| 2.2.14 1000 ಯೂನಿಟ್ ರೂ. 15 ರಂತೆ | 7.2.14 – 800 ಯೂನಿಟ್ |
| 5.2.14 1500 ಯೂನಿಟ್ ರೂ. 17 ರಂತೆ | 10.2.14 – 200 ಯೂನಿಟ್ |
| 12.2.14 1200 ಯೂನಿಟ್ ರೂ. 16 ರಂತೆ | 18.2.14 – 1500 ಯೂನಿಟ್ |
| 19.2.14 900 ಯೂನಿಟ್ ರೂ. 20 ರಂತೆ | 22.2.14 – 300 ಯೂನಿಟ್ |
| 24.2.14 2000 ಯೂನಿಟ್ ರೂ. 13 ರಂತೆ | 27.2.14 – 600 ಯೂನಿಟ್ |

ದಿನಾಂಕ 28.2.2014 ರಂದು ಇದ್ದಂತಹ ದಾಸ್ತಾನು ಮೌಲ್ಯವನ್ನು ನೀಡಿರಿ.

11. Case study (Compulsory) : **(6 + 9 = 15)**

ಪ್ರಕರಣ ಅಧ್ಯಯನ (ಕಡ್ಡಾಯವಾದದ್ದು) :

(a) Mention the postulate, concept and principle of Accounting on which the following accounting statements or comments are based :

(i) A company provided depreciation on its fixed assets.

(ii) Amount of Rs. 5,00,000 invested in business by the proprietor is treated as liability of business.

(iii) Provision is made for contingent losses.

(iv) The land owned by a company is shown at cost price even though the market price is ten times higher than cost.

(v) The Trading and Profit and Loss Account is prepared for a period covering 12 months.

(vi) Contingent liabilities are shown on liability side of a Balance Sheet.

ಈ ಕೆಳಗಿನ ಹೇಳಿಕೆಗಳ ಅಥವಾ ಅಭಿಪ್ರಾಯಗಳು ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಮೂಲಭೂತ ನಿಯಮಗಳನ್ನು, ಪರಿಕಲ್ಪನೆ ಅಥವಾ ತತ್ವಗಳ ಆಧಾರಗಳನ್ನು ಸೂಚಿಸಿರಿ.

(i) ಕಂಪನಿಯು ತನ್ನ ಸ್ಥಿರಾಸ್ತಿಗಳ ಮೇಲೆ ಸವಕಳಿಯನ್ನು ಲೆಕ್ಕಿಸಿದೆ

(ii) ವ್ಯವಹಾರದ ಮಾಲೀಕನು ವ್ಯವಹಾರದಲ್ಲಿ ಹೂಡಿದ ರೂ. 5,00,000 ಗಳನ್ನು ವ್ಯವಹಾರದ ಹೊಣೆಗಾರಿಕೆಯೆಂದು ಪರಿಗಣಿಸಲಾಗಿದೆ.

(iii) ಸಂಭವಿಸಬಹುದಾದ ನಷ್ಟಕ್ಕಾಗಿ ನಿಧಿಯನ್ನು ಸೃಷ್ಟಿಸಲಾಗಿದೆ.

(iv) ಕಂಪನಿಯು ತಾನು ಹೊಂದಿದ 'ಭೂಮಿ'ಯ ಮೌಲ್ಯವನ್ನು ಮೂಲ ವೆಚ್ಚದ ಪ್ರಕಾರ ತೋರಿಸಿದೆ, ಆದರೆ ಅದರ ಮಾರುಕಟ್ಟೆ ಬೆಲೆಯು ಹತ್ತು ಪಟ್ಟು ಹೆಚ್ಚಾಗಿದೆ.

(v) ಹನ್ನೆರಡು ತಿಂಗಳ ಅವಧಿಯನ್ನು ಹೊಂದಿದ ವ್ಯಾಪಾರ ಮತ್ತು ಲಾಭ-ನಷ್ಟ ಖಾತೆಯನ್ನು ಸಿದ್ಧಗೊಳಿಸಲಾಗಿದೆ.

(vi) ಸಂಭವಿಸಬಹುದಾದ ಹೊಣೆಗಾರಿಕೆಗಳನ್ನು ಆಢಾವೆ ಪತ್ರಿಕೆಯಲ್ಲಿ ಹೊಣೆಗಾರಿಕೆ ಬದಿಗೆ ತೋರಿಸಲಾಗಿದೆ.

(b) M/s. Janaki and Bros. bought 100 shares of TCS Co. Ltd. of face value of Re. 1 each of a market value of Rs. 1,100 each by paying Rs. 1,000 each.

The firm also purchased 10,000 shares of USW Ltd. of the face value of Rs. 10 each issued at par by the company. But, the market value of shares is Rs. 20 each.

Answer the following questions :

(i) At what value the shares of TCS Co. Ltd. are shown in Balance Sheet? What type of principles are applied?

(ii) What value is recorded in Balance Sheet for shares of USW Ltd.? Identify the applicable accounting principle.

Reg. No. C 1 5 1 5 6 0 1

# II Semester B.Com.2/B.Com.3 Degree Examination, May 2016

## (RCU – Regular/ Repeater)

## ACCOUNTING THEORY

Time : 3 Hours] [Max. Marks : 80

***Instructions/ಸೂಚನೆಗಳು :***

*1) Answer all questions subject to internal choice.*
*ಆಂತರಿಕ ಆಯ್ಕೆಗಳಿಗೆ ಒಳಪಟ್ಟು ಎಲ್ಲಾ ಪ್ರಶ್ನೆಗಳನ್ನು ಉತ್ತರಿಸಿರಿ.*

*2) Question No. – **11** (Case Study) is **compulsory**.*
*ಪ್ರಶ್ನೆ – **11** ಕಡ್ಡಾಯ (ಪ್ರಕರಣ ಅಧ್ಯಯನ).*

SECTION – A/ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following questions : (10 × 2 = 20)
ಕೆಳಗಿನ ಬೇಕಾದ ಹತ್ತಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :

(a) Who are the users of accounting information?
ಲೆಕ್ಕಬರಹ ಮಾಹಿತಿಯನ್ನು ಉಪಯೋಗಿಸುವವರು ಯಾರು?

(b) What is Proprietary theory?
ಒಡೆತನದ ಕಲ್ಪನೆ ಎಂದರೇನು?

(c) Name the theories that represent roots of accounting theory.
ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಮೂಲಗಳನ್ನು ಪ್ರತಿನಿಧಿಸುವ ಸಿದ್ಧಾಂತಗಳನ್ನು ಹೆಸರಿಸಿರಿ.

(d) State two needs of accounting principles.
ಲೆಕ್ಕಬರಹ ತತ್ವಗಳ ಎರಡು ಅವಶ್ಯಕತೆಗಳನ್ನು ತಿಳಿಸಿರಿ.

(e) What do you mean by accounting standards?
ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಪ್ರಮಾಣಗಳು ಎಂದರೇನು?

P.T.O.

(f) What is accounting environment?
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಪರಿಸರ ಎಂದರೇನು?

(g) What do you mean by socio-economic approach?
ಸಾಮಾಜಿಕ ಆರ್ಥಿಕ ಪಥ ಎಂದರೇನು?

(h) What is normative accounting theory?
ಪ್ರಮಾಣಿತ ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತವೆಂದರೇನು?

(i) Give the meaning of full disclosure principle.
ಪೂರ್ಣ ಬಹಿರಂಗಗೊಳಿಸುವಿಕೆ ತತ್ವದ ಅರ್ಥವನ್ನು ನೀಡಿರಿ.

(j) Name any two names of contributors to accounting theory.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತಗಳ ಬಗ್ಗೆ ಕೊಡುಗೆ ನೀಡಿದ ಯಾವುದಾದರೂ ಇಬ್ಬರನ್ನು ಹೆಸರಿಸಿರಿ.

(k) What is Inventory?
ದಾಸ್ತಾನು (Inventory) ಎಂದರೇನು?

(l) What is going concern postulate?
ವ್ಯವಹಾರದ ನಿರಂತರ ಮುಂದುವರಿಕೆ ಮೂಲಭೂತ ನಿಯಮು ಎಂದರೇನು?

SECTION – B/ವಿಭಾಗ – ಬ

Answer **any three** of the following questions : (3 × 5 = 15)
ಬೇಕಾದ ಮೂರು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :

2. Explain the role of accounting theory.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಪಾತ್ರ ವಿವರಿಸಿರಿ.

3. Explain the Dual aspect principle.
ದ್ವಿಸಮೂದು ತತ್ವವನ್ನು ವಿವರಿಸಿರಿ.

4. What is behavioural approach? State the its advantages and disadvantages.
ವರ್ತನೆಯ ಪಥ ಎಂದರೇನು? ಅದರ ಅನುಕೂಲತೆಗಳು ಹಾಗೂ ಅನಾನುಕೂಲತೆಗಳನ್ನು ತಿಳಿಸಿರಿ.

5. Briefly explain the different postulates of accounting.
ಲೆಕ್ಕಶಾಸ್ತ್ರದ ವಿವಿಧ ಮೂಲಭೂತ ನಿಯಮಗಳನ್ನು ಸಂಕ್ಷಿಪ್ತವಾಗಿ ವಿವರಿಸಿರಿ.

6. Explain the need for accounting standards.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಪ್ರಮಾಣಗಳ ಅವಶ್ಯಕತೆಗಳನ್ನು ವಿವರಿಸಿರಿ.

SECTION – C/ವಿಭಾಗ – ಠ

Answer **any two** of the following questions : (2 × 15 = 30)
**ಬೇಕಾದ ಎರಡು** ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :

7. Discuss the effects of various environments on accounting theory.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಮೇಲೆ ವಿವಿಧ ಪರಿಸರಗಳು ಬೀರುವ ಪರಿಣಾಮಗಳನ್ನು ಚರ್ಚಿಸಿರಿ.

8. Explain in brief the basic accounting principles.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಾಂಪ್ರದಾಯಿಕ ತತ್ವಗಳನ್ನು ಸಂಕ್ಷಿಪ್ತದಲ್ಲಿ ವಿವರಿಸಿರಿ.

9. Explain in brief the different traditional approaches to the formulation of accounting theory (Non-theoretical).
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತವನ್ನು ಮಂಡಿಸುವ ಅನೇಕ ಸಾಂಪ್ರದಾಯಿಕ ಪಥಗಳನ್ನು ಸಂಕ್ಷಿಪ್ತದಲ್ಲಿ ವಿವರಿಸಿರಿ.

10. From the following transactions, prepare stores ledger account using the FIFO method for January 2016 :

| On | |
|---|---|
| 1.1.2016 | Opening balance 850 units at Rs. 5 per unit |
| 4.1.2016 | Purchased 300 units at Rs. 6 per unit |
| 8.1.2016 | Purchased 600 units at Rs. 8 per unit |
| 10.1.2016 | Issued 500 units |
| 14.1.2016 | Issued 300 units |
| 15.1.2016 | Purchased 700 units at Rs. 8 per unit |
| 17.1.2016 | Purchased 500 units at Rs. 8.50 per unit |
| 18.1.2016 | Issued 800 units |
| 25.1.2016 | Issued 300 units |
| 27.1.2016 | Purchased 300 units at Rs. 10 per unit |

'FIFO' (ಫಿಫೋ) ವಿಧಾನವನ್ನು ಅನುಸರಿಸಿ ಜನವರಿ 2016ರ ಸ್ಟೋರ್ಸ್ ಖಾತೆಯನ್ನು ತಯಾರಿಸಿರಿ :

| | |
|---|---|
| 1.1.2016 | ಪ್ರಾರಂಭಿಕ ಶಿಲ್ಕು ಕಚ್ಚಾ ಸರಕುಗಳು 850 ಯೂನಿಟ್‌ಗೆ ರೂ. 5 ರಂತೆ |
| 4.1.2016 | ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ ರೂ. 6 ರಂತೆ 300 ಯೂನಿಟ್‌ಗಳನ್ನು ಖರೀದಿಸಿದ |
| 8.1.2016 | ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ ರೂ. 8 ರಂತೆ 600 ಯೂನಿಟ್‌ಗಳನ್ನು ಖರೀದಿಸಿದ |
| 10.1.2016 | 500 ಯೂನಿಟ್ಸ್‌ಗಳನ್ನು ನೀಡಲಾಗಿದೆ |
| 14.1.2016 | 300 ಯೂನಿಟ್ಸ್‌ಗಳನ್ನು ನೀಡಲಾಗಿದೆ |
| 15.1.2016 | ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ ರೂ. 8 ರಂತೆ 700 ಯೂನಿಟ್‌ಗಳನ್ನು ಖರೀದಿಸಿದ |
| 17.1.2016 | ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ ರೂ. 8.50 ರಂತೆ 500 ಯೂನಿಟ್‌ಗಳನ್ನು ಖರೀದಿಸಿದ |
| 18.1.2016 | 800 ಯೂನಿಟ್ಸ್‌ಗಳನ್ನು ನೀಡಲಾಗಿದೆ |
| 25.1.2016 | 300 ಯೂನಿಟ್ಸ್‌ಗಳನ್ನು ನೀಡಲಾಗಿದೆ |
| 27.1.2016 | ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ ರೂ. 10 ರಂತೆ 300 ಯೂನಿಟ್‌ಗಳನ್ನು ಖರೀದಿಸಿದ |

SECTION – D/ವಿಭಾಗ – ಡ

11. Case study (**Compulsory** question) : **(1 × 15 = 15)**
ಪ್ರಕರಣ ಅಧ್ಯಯನ (ಕಡ್ಡಾಯವಾದದ್ದು) :

State with explanation the principles, concepts, postulates that have been used to prepare the following Balance Sheet in the books of Bharat :

Balance Sheet on 31.3.2016

| Liabilities | | Rs. | Assets | | Rs. |
|---|---|---|---|---|---|
| Capital | 4,00,000 | | Fixed Assets | 6,00,000 | |
| Add : Net Profit | 1,00,000 | 5,00,000 | Less : Depr. | 60,000 | 5,40,000 |
| Creditors | 6,00,000 | | Investments | | 1,60,000 |
| Less : R.D. on Crs. | 40,000 | 5,60,000 | Stock at Cost | | 1,40,000 |
| Bills Payable | | 40,000 | (Market price Rs. 2,00,000) | | |
| Contingent Liabilities | | | Debtors | 2,00,000 | |
| (Rs. 40,000) | | | Less : R.D.D. — | 20,000 | 1,80,000 |
| | | | Cash | | 80,000 |
| | | 11,00,000 | | | 11,00,000 |

ಭರತ್ ಅವರ ಪುಸ್ತಕದಲ್ಲಿ ಈ ಕೆಳಗಿನ ಅಢಾವೆ ಪತ್ರ ಸಿದ್ಧಪಡಿಸುವಲ್ಲಿ ಬಳಸಲಾದ ತತ್ವಗಳು, ಪರಿಕಲ್ಪನೆಗಳು ಹಾಗೂ ಮೂಲಭೂತ ಕಲ್ಪನೆಗಳನ್ನು ವಿವರವಾಗಿ ನಿರೂಪಿಸಿರಿ :

ಅಢಾವೆ ಪತ್ರಿಕೆ 31.3.2016

| ಹೊಣೆಗಾರಿಕೆಗಳು | | ರೂ. | ಆಸ್ತಿಗಳು | | ರೂ. |
|---|---|---|---|---|---|
| ಬಂಡವಾಳ | 4,00,000 | | ಸ್ಥಿರಾಸ್ತಿಗಳು | 6,00,000 | |
| Add : ನಿವ್ವಳ ಲಾಭ | 1,00,000 | 5,00,000 | Less : ಸವಕಳಿ | 60,000 | 5.40,000 |
| ಸಾಲಿಗರು | 6,00,000 | | ಹೂಡಿಕೆಗಳು | | 1.60,000 |
| Less : ಕಾಯ್ದಿಟ್ಟ ಸೋಡಿ | 40,000 | 5,60,000 | ದಾಸ್ತಾನು ವೆಚ್ಚ ಬೆಲೆಯಲ್ಲಿ | | |
| ಪಾವತಿಸುವ ಬಿಲ್‌ಗಳು | | 40,000 | (ಮಾರುಕಟ್ಟೆ ಬೆಲೆ - 2,00,000) | | 1.40.000 |
| ಸಂಶಯಾಸ್ಪದ ಹೊಣೆಗಾರಿಕೆಗಳು | | | ಸಾಲಗಾರರು | 2,00,000 | |
| ರೂ. 40,000 | | | Less : ಸಂಶಯಾಸ್ಪದ ನಿಧಿ | 20,000 | 1,80,000 |
| | | | ನಗದು | | 80,000 |
| | | 11,00,000 | | | 11,00,000 |

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**23225/36225/B 250**

Reg. No. C11630213

**II Semester B.Com. 2/B.Com. 3 Examination, May/June 2017**
**(Regular and Repeater)**
**ACCOUNTING THEORY**
**(2011-2012 Onwards)**

Time : 3 Hours | Max. Marks : 80

***Instructions*** : *1) Answer **all** questions subject to **internal** choice.*
*2) Question No. **11** (case study) **compulsory**.*

SECTION – A

1. Answer **any ten** of the following questions :
   a) Define accounting theory.
   b) What is eclectic approach ?
   c) What is dual aspect principle ?
   d) What do you understand by accounting period postulate ?
   e) What is legal environment ?
   f) Expand GAAP and FASB. → Financial Accounting Standards Board
   g) What is ethical approach ?
   h) How do you findout owners equity under proprietary theory ?
   i) Mention two needs of accounting standards.
   j) What is accounting practice ?
   k) What do you mean by specific equities in-residual equity theory ?
   l) Name the roots of accounting theory. **(10×2=20)**

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SECTION – B

Answer **any three** of the following questions :

2. Explain in brief the need for accounting theory. 3
3. Explain the decision model approach.
4. Explain briefly the traditional principles of accounting theory. 5
5. What is pragmatic approach ? State its advantages. 2
6. Write a short note on going concern postulate. **(3×5=15)**

**P.T.O.**

## SECTION – C

Answer **any two** of the following questions :

7. What is accounting environment ? Explain in brief the impact of socio-economic and legal environments on accounting system.

8. Describe the modern approaches of accounting theory.

9. What do you mean by concept of accounting ? Explain the proprietary theory and the entity theory of accounting.

10. The following transactions occur in the purchase and issue of materials in February 2017 :

| | |
|---|---|
| 1st Feb. | Stock of raw materials 500 tons at Rs. 200 |
| 5th Feb. | Issued 250 tons |
| 10th Feb. | Returns from departments 15 tons |
| 14th Feb. | Issued 180 tons |
| 16th Feb. | Received 200 tons at Rs. 190 |
| 20th Feb. | Received 320 tons at Rs. 200 |
| 26th Feb. | Issued 150 tons |
| 27th Feb. | Returned from department 30 tons |
| 28th Feb. | Received 250 tons @ Rs. 220 |
| 28th Feb. | Issued 300 tons |

From the above information prepare the stores ledger adopting FIFO method.

(2×15=30)

## SECTION – D

**Compulsory** question (**Case Study**) :

11. Name and explain the principles and postulates involved in the following cases and give reasons :

a) Assets are always equal to liabilities plus capital.

b) Mr. Ramesh made non business investment using his surplus fund in hand of Rs. 1 lakh.

c) Mr. Shankar paid for taxes Rs. 1 lakh, donations Rs. 50,000 and charity Rs. 25,000.

d) Provision is made for anticipated losses.

e) Machinery worth Rs. 2,00,000 was purchased and recorded at Rs. 1,80,000.

(1×15=15)

Reg. No.

**II Semester B.Com.2/B.Com.3 Degree Examination, May/June 2018**
**ACCOUNTING THEORY**

Time : 3 Hours Max. Marks : 80

***Instructions :*** *1) Answer the questions with **strict** observance of the internal choices given.*
*2) Question No. **11** (case study) is **compulsory.***

SECTION – A

1. Answer **any ten** of the following : **(10×2=20)**
   a) Who are the users of accounting information ?
   b) Name the different branches of accounting.
   c) What is sociological approach ?
   d) What is ethical approach ?
   e) What is accounting postulates ?
   f) What is accounting principles ?
   g) Give the equation of owners equity.
   h) What is monetary unit postulate ?
   i) What is realisation principle ?
   j) Expand : IFRS, ICAI.
   k) What do you mean by accounting standard ?
   l) Mention any two methods identified by AS-2 for inventory valuation.

SECTION – B

Answer **any three** of the following : **(3×5=15)**

2. What is normative theory ? Mention its features.
3. State the distinction between accounting theory and accounting practice.
4. Write note on ethical approach.
5. Write note on cost principle.
6. What is accounting period postulate ?

**P.T.O.**

## SECTION – C

Answer **any two** of the following : **(2×15=30)**

7. What do you understand by accounting theory ? Explain its role.

8. What is entity postulate ? Explain its advantages.

9. Explain the materiality and consistency principles of accounting.

10. Giving the suitable format, write-up stores ledger under FIFO method :

Opening stock of material on 1-2-2017-1000 units @ Rs. 10 each

| **Purchases** | | **Issues** | |
|---|---|---|---|
| 2-2-2017 | 2000 units @ Rs. 15 each | 7-2-17 | 1600 units |
| 5-2-2017 | 3000 units @ Rs. 17 each | 10-2-17 | 400 units |
| 12-2-2017 | 2400 units @ Rs. 16 each | 18-2-17 | 3000 units |
| 19-2-2017 | 1800 units @ Rs. 20 each | 22-2-17 | 600 units |
| 24-2-2017 | 4000 units @ Rs. 13 each | 27-2-17 | 1200 units |

Show the total value of stock on 28-2-2017.

## SECTION – D

11. Case Study (**compulsory**) : **(1×15=15)**

State with explanation the principles, concepts, postulates that have been used to prepare the following Balance Sheet in the books of Anand.

**Balance Sheet as on 31-3-2017**

| **Liabilities** | | **Rs.** | **Assets** | | **Rs.** |
|---|---|---|---|---|---|
| Capital | | 2,00,000 | Fixed assets | 3,00,000 | |
| Add : Net profit | | 50,000 | Less : Depreciation | 30,000 | 2,70,000 |
| | | 2,50,000 | | | |
| Creditors | 3,00,000 | | Investments | | 80,000 |
| Less : Reserve for discount | 20,000 | 2,80,000 | Stock at cost (Market price Rs. 1,00,000) | | 90,000 |
| Bills payable | | 20,000 | Debtors | 1,00,000 | |
| Contingent liabilities | | | Less : RDD | 10,000 | 90,000 |
| | | | Cash | | 20,000 |
| | | **5,50,000** | | | **5,50,000** |

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No. of Printed Pages : 5

**36225/23225/B250**

Reg. No. [ | | | | | | | | ]

## II Semester B.Com.2/B.Com.3 Degree Examination, May - 2019

## ACCOUNTING THEORY (Repeaters and Regular)

Time : 3 Hours

Max. Marks : 80

**Instructions : (1)** *Answer the questions according to the internal choices.*

**ಸೂಚನೆಗಳು :** *ಆಂತರಿಕ ಆಯ್ಕೆಗಳಿಗೆ ಅನುಗುಣವಾಗಿ ಉತ್ತರಿಸಿರಿ.*

(2) *Section-D Case Study question is compulsory.*

*ವಿಭಾಗ-'ಡ' ಪ್ರಕರಣ ಅಧ್ಯಯನ ಪ್ರಶ್ನೆ ಕಡ್ಡಾಯವಾಗಿದೆ.*

(3) *COPYING OF ANY KIND IS STRICTLY PROHIBITED.*

*ಯಾವುದೇ ತರಹದ ನಕಲು ಮಾಡುವುದನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.*

### SECTION - A / ವಿಭಾಗ – ಅ

**1.** Answer any ten of the following : **10x2=20**

ಈ ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಬೇಕಾದ ಹತ್ತಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :

(a) Define a Theory.

ಸಿದ್ಧಾಂತದ ವ್ಯಾಖ್ಯೆ ನೀಡಿರಿ.

(b) Write any four features of accounting theory.

ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಯಾವುದಾದರೂ ನಾಲ್ಕು ಲಕ್ಷಣಗಳನ್ನು ಬರೆಯಿರಿ.

(c) Name the roots of accounting theory.

ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಮೂಲಗಳನ್ನು ಹೆಸರಿಸಿರಿ.

(d) What do you mean by accounting environment ?

ಲೆಕ್ಕಶಾಸ್ತ್ರ ಪರಿಸರ ಎಂದರೇನು ?

(e) What is pragmatic approach ?

ಲೌಕಿಕ ಮಾರ್ಗ ಎಂದರೇನು ?

(f) What do you mean by accounting policies ?
ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಧೋರಣೆಗಳೆಂದರೇನು ?

(g) Write any two features of postulates.
ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಮೂಲಭೂತ ನಿಯಮಗಳ ಯಾವುದಾದರೂ ಎರಡು ಲಕ್ಷಣಗಳನ್ನು ಬರೆಯಿರಿ.

(h) What is entity concept ?
ಅಸ್ತಿತ್ವದ ಪರಿಕಲ್ಪನೆ ಎಂದರೇನು ?

(i) What is the principle of full disclosure ?
ಪೂರ್ಣ ಬಹಿರಂಗಗೊಳಿಸುವ ತತ್ವವೆಂದರೇನು ?

(j) Name the parties benefited by accounting standards.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ಪ್ರಮಾಣಕಗಳಿಂದ ಪ್ರಯೋಜನ ಪಡೆಯುವ ಕಕ್ಷಿದಾರರನ್ನು ಹೆಸರಿಸಿರಿ.

(k) Write any two features of depreciation.
ಸವಕಳಿಯ ಯಾವುದಾದರೂ ಎರಡು ಲಕ್ಷಣಗಳನ್ನು ಬರೆಯಿರಿ.

(l) Expand (i) GAAP and (ii) IASB.
ವಿಸ್ತರಿಸಿರಿ (i) GAAP ಮತ್ತು (ii) IASB

## SECTION - B / ವಿಭಾಗ – ಬ

Answer any three of the following : 3x5=15
ಈ ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ಮೂರಕ್ಕೆ ಉತ್ತರಿಸಿರಿ.

2. Distinguish between Positive theory and Normative theory.
ಧನಾತ್ಮಕ ಮತ್ತು ಪ್ರಮಾಣಿತ ಲೆಕ್ಕಶಾಸ್ತ್ರಗಳ ವ್ಯತ್ಯಾಸ ಬರೆಯಿರಿ.

3. Explain the decision-model approach.
ನಿರ್ಣಯಿಸುವ ಮಾದರಿಯ ಮಾರ್ಗವನ್ನು ವಿವರಿಸಿರಿ.

4. Distinguish between Accounting principles and Accounting policies.
ಲೆಕ್ಕಶಾಸ್ತ್ರ ತತ್ವಗಳು ಮತ್ತು ಧೋರಣೆಗಳಲ್ಲಿಯ ವ್ಯತ್ಯಾಸ ಬರೆಯಿರಿ.

5. Write a note on deductive approach.
ನಿರ್ಗಮನ ಪಥದ ಕುರಿತು ಟಿಪ್ಪಣಿ ಬರೆಯಿರಿ.

6. Find out the owner's equity under the 'Residual Equity Theory' from the below given information :

| | |
|---|---|
| Equity Share capital | ₹ 10,00,000 |
| Preference Shares | ₹ 4,00,000 |
| Reserve Funds | ₹ 4,00,000 |
| Sundry Creditors | ₹ 2,00,000 |
| Fixed Assets | ₹ 14,00,000 |
| Current Assets | ₹ 6,00,000 |

ಈ ಕೆಳಗಿನ ಮಾಹಿತಿಯಿಂದ 'ಶೇಷ ಈಕ್ವಿಟಿ ಸಿದ್ಧಾಂತದ' ಆಧಾರದಂತೆ ಬಡತನದ ಪಾಲನ್ನು ಕಂಡುಹಿಡಿಯಿರಿ :

| | |
|---|---|
| ಸಾಮಾನ್ಯ ಶೇರು ಬಂಡವಾಳ | ₹ 10,00,000 |
| ಪಾಶಸ್ತ್ಯ ಶೇರುಗಳು | ₹ 4,00,000 |
| ಕಾಯ್ದಿಟ್ಟ ನಿಧಿಗಳು | ₹ 4,00,000 |
| ಸಾಲಿಗರು | ₹ 2,00,000 |
| ಸ್ಥಿರಾಸ್ತಿಗಳು | ₹ 14,00,000 |
| ಚರಾಸ್ತಿಗಳು | ₹ 6,00,000 |

## SECTION - C / ವಿಭಾಗ – ಕ

Answer **any two** of the following : **2x15=30**

ಈ ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ಎರಡಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :

7. Define Accounting Theory. Explain the features and objectives of accounting theory.

ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತವನ್ನು ವ್ಯಾಖ್ಯಾಯಿಸಿರಿ. ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಲಕ್ಷಣಗಳನ್ನು ಮತ್ತು ಉದ್ದೇಶಗಳನ್ನು ವಿವರಿಸಿರಿ.

8. Explain the various traditional approaches to accounting theory.

ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ಸಾಂಪ್ರದಾಯಿಕ ಪಥಗಳನ್ನು ವಿವರಿಸಿರಿ.

9. Define Accounting Principles. Explain the need for accounting principles.

ಲೆಕ್ಕಶಾಸ್ತ್ರದ ತತ್ವಗಳನ್ನು ವ್ಯಾಖ್ಯಾಯಿಸಿರಿ. ಲೆಕ್ಕಶಾಸ್ತ್ರ ತತ್ವಗಳ ಅವಶ್ಯಕತೆಗಳನ್ನು ವಿವರಿಸಿರಿ.

10. With the help of prescribed format, prepare the Stores Ledger under FIFO method :

| RECEIPTS | ISSUES |
|---|---|
| 2.4.19 - 2000 units @ ₹ 30 each | 7.4.19 - 1600 units |
| 5.4.19 - 3000 units @ ₹ 34 each | 10.4.19 - 400 units |
| 12.4.19 - 2400 units @ ₹ 32 each | 18.4.19 - 3000 units |
| 19.4.19 - 1800 units @ ₹ 40 each | 22.4.19 - 600 units |
| 24.4.19 - 4000 units @ ₹ 26 each | 27.4.19 - 1200 units |

Also show the total value of stock on 30.4.2019.

ನಿಗದಿತ ನಮೂನೆಯನ್ನು ಅಳವಡಿಸಿ, FIFO ಪದ್ಧತಿಯಂತೆ ಸ್ಟೋರ್ಸ್ ಖಾತೆಯನ್ನು ತಯಾರಿಸಿರಿ :

| ಸ್ವೀಕರಿಸಿದ್ದು | ನೀಡಿದ್ದು |
|---|---|
| 2.4.19 - ₹ 30 ರಂತೆ 2000 ಯುನಿಟ್‌ಗಳು | 7.4.19 - 1600 ಯುನಿಟ್‌ಗಳು |
| 5.4.19 - ₹ 34 ರಂತೆ 3000 ಯುನಿಟ್‌ಗಳು | 10.4.19 - 400 ಯುನಿಟ್‌ಗಳು |
| 12.4.19 - ₹ 32 ರಂತೆ 2400 ಯುನಿಟ್‌ಗಳು | 18.4.19 - 3000 ಯುನಿಟ್‌ಗಳು |
| 19.4.19 - ₹ 40 ರಂತೆ 1800 ಯುನಿಟ್‌ಗಳು | 22.4.19 - 600 ಯುನಿಟ್‌ಗಳು |
| 24.4.19 - ₹ 26 ರಂತೆ 4000 ಯುನಿಟ್‌ಗಳು | 27.4.19 - 1200 ಯುನಿಟ್‌ಗಳು |

ಅಲ್ಲದೆ, ದಿನಾಂಕ 30.4.2019 ರಂದು ಇರುವ ಒಟ್ಟು ಸಂಗ್ರಹದ ಮೌಲ್ಯವನ್ನು ಲೆಕ್ಕಿಸಿರಿ.

## SECTION - D / ವಿಭಾಗ – ಡ

11. Case-Study Compulsory question. 7+8=15

ಪ್ರಕರಣ–ಅಧ್ಯಯನ ಕಡ್ಡಾಯ ಪ್ರಶ್ನೆ.

(a) State the appropriate approach for the following statements.

ಈ ಕೆಳಗಿನ ಹೇಳಿಕೆಗಳಲ್ಲಿ ಅಳವಡಿಸಿದ ಸೂಕ್ತ ಪಥಗಳನ್ನು ಸೂಚಿಸಿರಿ.

(i) Accounting techniques are imposed by regulating bodies.

ಲೆಕ್ಕಶಾಸ್ತ್ರದ ತಂತ್ರಗಳು ನಿಯಂತ್ರಣ ಸಂಸ್ಥೆಗಳಿಂದ ಒತ್ತಾಯಿಸಲ್ಪಡುತ್ತವೆ.

(ii) Real world practices are the basis for theory

ನೈಜ ಜಗತ್ತಿನ ಅಚರಣೆಗಳು ಸಿದ್ಧಾಂತಕ್ಕೆ ಆಧಾರಗಳು.

(iii) Emphasis on what ought to be.

ಏನಿರಬೇಕೆಂಬುದರ ಮೇಲೆ ಒತ್ತು ನೀಡುವರು.

(iv) Fairness, Justice and Faith are the basis.

ಶುದ್ಧತೆ, ನ್ಯಾಯ ಮತ್ತು ನಂಬಿಕೆಗಳೇ ಆಧಾರ.

(v) Mixture of good thoughts of different approaches.
ವಿವಿಧ ಪಥಗಳಲ್ಲಿಯ ಉತ್ತಮ ವಿಚಾರಗಳ ಮಿಶ್ರಣ.

(vi) Accounting information should have a predictable factors.
ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಮಾಹಿತಿಯು ಊಹಿಸಬಹುದಾದ ಅಂಶಗಳನ್ನು ಹೊಂದಿರಬೇಕು.

(vii) National Economic Goodness should be considered.
ರಾಷ್ಟ್ರೀಯ ಆರ್ಥಿಕ ಉತ್ತಮಿಕೆಯನ್ನು ಪರಿಗಣಿಸುವುದು.

(b) State the postulate or concept or principles of accounting theory for the following statements :
ಈ ಕೆಳಗಿನ ಹೇಳಿಕೆಗಳಲ್ಲಿ ಅಳವಡಿಸಿದ ಮೂಲಭೂತ ನಿಯಮ ಅಥವಾ ಪರಿಕಲ್ಪನೆ ಅಥವಾ ಲೆಕ್ಕಶಾಸ್ತ್ರ ಸಿದ್ಧಾಂತದ ತತ್ವಗಳನ್ನು ಸೂಚಿಸಿರಿ :

(i) Paises are rounded off to the nearest rupee.
ಪೈಸೆಗಳನ್ನು ಸಮೀಪದ ರೂಪಾಯಿಗೆ ಹೊಂದಿಸುವುದು.

(ii) Provision is made for anticipated losses.
ನಿರೀಕ್ಷಿಸಿದ ನಷ್ಟಗಳಿಗೆ ನಿಧಿ ಕಾಯ್ದಿರಿಸಿದೆ.

(iii) Provision for depreciation is made for fixed assets.
ಸ್ಥಿರಾಸ್ತಿಗಳಿಗೆ ಸವಕಳಿಯ ನಿಧಿ ಕಾಯ್ದಿರಿಸಿದೆ.

(iv) Assets total amount is always equal to total of capital plus-liabilities.
ಆಸ್ತಿಗಳ ಒಟ್ಟು ಮೊತ್ತವು ಯಾವಾಗಲೂ ಬಂಡವಾಳ ಮತ್ತು ಹೊಣೆಗಾರಿಕೆಗಳ ಮೊತ್ತಕ್ಕೆ ಸಮನಾಗಿರುತ್ತದೆ.

(v) Accounting year begins on 1st April and ends on 31st March.
ಲೆಕ್ಕಪತ್ರದ ವರ್ಷವು 1ನೇ ಎಪ್ರಿಲ ರಂದು ಪ್ರಾರಂಭವಾಗಿ, 31ನೇ ಮಾರ್ಚಕ್ಕೆ ಅಂತ್ಯಗೊಳ್ಳುತ್ತದೆ.

(vi) A machinery worth ₹ 5 lakh is purchased and recorded in books with ₹ 4,50,000 as the subsidy of ₹ 50,000/- was received from government.
₹ 5 ಲಕ್ಷದ ಯಂತ್ರೋಪಕರಣ ಖರೀದಿಸಲಾಗಿದ್ದು, ಸರ್ಕಾರದಿಂದ ₹ 50,000 ಸಹಾಯ ಧನ ಪಡೆದಿದ್ದರಿಂದ ₹ 4,50,000 ವನ್ನು ಲೆಕ್ಕಪತ್ರ ಪುಸ್ತಕದಲ್ಲಿ ನಮೂದಿಸಿದೆ.

(vii) Amount of ₹ 50 lakh invested in business by the proprietor is treated as liability of business.
ಮಾಲೀಕರು ವ್ಯವಹಾರದಲ್ಲಿ ಹೂಡಿದ ₹ 50 ಲಕ್ಷಗಳನ್ನು ವ್ಯವಹಾರದ ಹೊಣೆಗಾರಿಕೆಗಳೆಂದು ಲೆಕ್ಕಿಸಲಾಗಿದೆ.

(viii) Contingent liabilities are shown on liability side of a Balance sheet.
ಸಂಭವನೀಯ ಹೊಣೆಗಾರಿಕೆಗಳನ್ನು ಆಢಾವೆ ಪತ್ರಿಕೆಯ ಹೊಣೆಗಾರಿಕೆಯ ಬದಿಗೆ ತೋರಿಸಲಾಗಿದೆ.

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# V Semester B.Com.2 Degree Examination, November 2015

## (RCU Syllabus – Regular/Repeaters)

## ECONOMIC DEVELOPMENT OF INDIA

Time : 3 Hours] [Max. Marks : 80

***Instructions/ಸೂಚನೆಗಳು :***

1) *All sub-questions should be answered at one place continuously. Quote facts and figures to support your answer.*
*ಎಲ್ಲಾ ಉಪ ಪ್ರಶ್ನೆಗಳಿಗೆ ಒಂದೇ ಕಡೆಗೆ ಉತ್ತರ ಬರೆಯಿರಿ. ಆವಶ್ಯಕ ಅಂಕಿ ಅಂಶಗಳೊಂದಿಗೆ ನಿಮ್ಮ ಉತ್ತರಗಳನ್ನು ಸಮರ್ಥಿಸಿರಿ.*

2) *Answer to all the sections should be written in the same answer book.*
*ಎಲ್ಲಾ ವಿಭಾಗಗಳ ಉತ್ತರಗಳನ್ನು ಒಂದೇ ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಬರೆಯಿರಿ.*

SECTION – A/ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following questions in 2-3 sentences each : **(10 × 2 = 20)**
ಕೆಳಗಿನ **ಬೇಕಾದ ಹತ್ತು** ಪ್ರಶ್ನೆಗಳಿಗೆ 2-3 ವಾಕ್ಯಗಳಲ್ಲಿ ಉತ್ತರಿಸಿರಿ :

(a) Give the meaning of a developed economy.
ಅಭಿವೃದ್ಧಿ ಅರ್ಥ ವ್ಯವಸ್ಥೆಯ ಅರ್ಥ ನೀಡಿರಿ.

(b) Mention any two features of underdeveloped economy.
ಅನಭಿವೃದ್ಧಿ ಅರ್ಥ ವ್ಯವಸ್ಥೆಯ ಯಾವುದಾದರೂ ಎರಡು ಲಕ್ಷಣಗಳನ್ನು ಹೇಳಿರಿ.

(c) Define tertiary sector.
ತೃತೀಯ ವಲಯವನ್ನು ವ್ಯಾಖ್ಯಾನಿಸಿರಿ.

(d) What is National Income?
ರಾಷ್ಟ್ರೀಯ ಆದಾಯ ಎಂದರೇನು?

(e) Expand HDI.
HDIಯನ್ನು ವಿಸ್ತರಿಸಿರಿ.

(f) What is Family Planning?
ಕುಟುಂಬ ಯೋಜನೆ ಎಂದರೇನು?

(g) Define Economic Planning.
ಆರ್ಥಿಕ ಯೋಜನೆ ವ್ಯಾಖ್ಯಾನಿಸಿರಿ.

(h) State the meaning of VAT.
VATನ ಅರ್ಥ ಹೇಳಿರಿ.

(i) Give the meaning of Budget.
ಮುಂಗಡ ಪತ್ರಿಕೆಯ ಅರ್ಥ ನೀಡಿರಿ.

(j) What is Money Market?
ಹಣದ ಮಾರುಕಟ್ಟೆ ಎಂದರೇನು?

(k) Give the meaning of deficit budget.
ಕೊರತೆ ಮುಂಗಡ ಪತ್ರದ ಅರ್ಥ ನೀಡಿರಿ.

(l) Who is the Deputy Chairman of NITI Commission?
ನೀತಿ ಆಯೋಗದ ಉಪಾಧ್ಯಕ್ಷರು ಯಾರು?

SECTION – B/**ವಿಭಾಗ – ಬ**

Answer **any three** of the following : **(3 × 5 = 15)**
**ಕೆಳಗಿನ ಬೇಕಾದ ಮೂರು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :**

2. Explain the reasons for India's underdevelopment.
ಭಾರತದ ಅನಭಿವೃದ್ಧಿಯ ಕಾರಣಗಳನ್ನು ವಿವರಿಸಿರಿ.

3. Describe the structure of Indian Economy.
ಭಾರತದ ಆರ್ಥಿಕ ವ್ಯವಸ್ಥೆಯ ರಚನೆಯನ್ನು ವರ್ಣಿಸಿರಿ.

4. Explain the determinants of Density of Population.
ಜನ ಸಾಂದ್ರತೆ ನಿರ್ಧರಿಸುವ ಅಂಶಗಳನ್ನು ವಿವರಿಸಿರಿ.

5. Explain the objectives of XIth Five Year Plan.
XIನೇ ಪಂಚ ವಾರ್ಷಿಕ ಯೋಜನೆಯ ಉದ್ದೇಶಗಳನ್ನು ವಿವರಿಸಿರಿ.

6. Examine the objectives of Monetary Policy.
ಹಣಕಾಸು ನೀತಿಯ ಉದ್ದೇಶಗಳನ್ನು ಪರೀಕ್ಷಿಸಿರಿ.

SECTION – C/ವಿಭಾಗ – ಕ

Answer **any two** of the following : **(2 × 15 = 30)**

ಕೆಳಗಿ ಬೇಕಾದ ಎರಡು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :

7. Explain the characteristics of underdeveloped Economy.
ಅನಭಿವೃದ್ಧಿ ಆರ್ಥಿಕ ವ್ಯವಸ್ಥೆಯ ಲಕ್ಷಣಗಳನ್ನು ವಿವರಿಸಿರಿ.

8. Discuss the methods and difficulties in estimation of National Income in India.
ಭಾರತದಲ್ಲಿ ರಾಷ್ಟ್ರೀಯ ಆದಾಯದ ಅಂದಾಜು ಮಾಡುವ ವಿಧಾನಗಳು ಮತ್ತು ತೊಂದರೆಗಳನ್ನು ಚರ್ಚಿಸಿರಿ.

9. Explain the various functions of R.B.I.
ಭಾರತೀಯ ರಿಜರ್ವ ಬ್ಯಾಂಕಿನ ವಿವಿಧ ಕಾರ್ಯಗಳನ್ನು ವಿವರಿಸಿರಿ.

10. Critically examine monetary policy of India.
ಭಾರತದ ಹಣಕಾಸಿನ ನೀತಿಯನ್ನು ವಿಮರ್ಶಿಸಿರಿ.

SECTION – D/ವಿಭಾಗ – ಡ

Case Let : **(1 × 15 = 15)**

ಸ್ಥಿತಿ ವಿದ್ಯಮಾನ :

11. The most important problem that India is facing today is the huge size of the population and alarming rate at which it has been increasing. Rapidly growing population has a negative impact on the economic development of our country. The growth of population has slowed down the rise in percapita income. Under the circumstances an appropriate population policy is absolutely necessary to tackle India's population problem.

Questions :

(a) Explain the size and growth of India's population.

(b) How population growth has a negative impact on the economic development?

(c) Suggest an appropriate population policy to check the growth of population.

ಬೃಹತ್ ಹಾಗೂ ಆತಂಕಕಾರಿ ದರದಲ್ಲಿ ಏರುತ್ತಿರುವ ಜನಸಂಖ್ಯೆಯು ಭಾರತ ಎದುರಿಸುತ್ತಿರುವ ಇಂದಿನ ಪ್ರಮುಖ ಸಮಸ್ಯೆಯಾಗಿದೆ. ತ್ವರಿತ ಗತಿಯಲ್ಲಿ ಏರುತ್ತಿರುವ ಜನಸಂಖ್ಯೆ ಭಾರತದ ಆರ್ಥಿಕ ಬೆಳವಣಿಗೆಯ ಮೇಲೆ ನಕಾರಾತ್ಮಕ ಪರಿಣಾಮ ಬೀರಿದೆ. ಜನಸಂಖ್ಯೆಯ ಬೆಳವಣಿಗೆಯಿಂದ ತಲಾ ವರಮಾನ ಬೆಳವಣಿಗೆಯು ನಿಧಾನಗೊಂಡಿದೆ. ಇಂತಹ ಸಂದರ್ಭದಲ್ಲಿ ಈ ಸಮಸ್ಯೆಯನ್ನು ಬಗೆಹರಿಸಲು ಒಂದು ಸೂಕ್ತ ಜನಸಂಖ್ಯಾ ನೀತಿ ಭಾರತಕ್ಕೆ ಅತ್ಯಾವಶ್ಯಕವಾಗಿದೆ.

ಪ್ರಶ್ನೆಗಳು :

(a) ಭಾರತದ ಜನಸಂಖ್ಯೆಯ ಗಾತ್ರ ಹಾಗೂ ಬೆಳವಣಿಗೆಯನ್ನು ವಿವರಿಸಿರಿ.

(b) ಹೇಗೆ ಜನಸಂಖ್ಯಾ ಬೆಳವಣಿಗೆ ಆರ್ಥಿಕ ಬೆಳವಣಿಗೆಯ ಮೇಲೆ ನಕಾರಾತ್ಮಕ ಪರಿಣಾಮ ಬೀರಿದೆ?

(c) ಜನಸಂಖ್ಯಾ ಬೆಳವಣಿಗೆಯ ನಿಯಂತ್ರಣಕ್ಕೆ ಸೂಕ್ತ ಜನಸಂಖ್ಯಾ ನೀತಿಯ ಸಲಹೆ ನೀಡಿರಿ.

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# V Semester B.Com.2 Degree Examination, November 2015

## (KUD – Regular and Repeaters)

## INDIAN FINANCIAL MARKETS

## Paper – I

Time : 3 Hours] [Max. Marks : 80

***Instructions/ಸೂಚನೆಗಳು :***

*1) Answer all questions subject to choice.*
*ಆಂತರಿಕ ಆಯ್ಕೆಗಳಿಗೆ ಒಳಪಟ್ಟು ಎಲ್ಲಾ ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ.*

*2) Q. No.* ***13*** *is* ***compulsory*** *(Case Study).*
*ಪ್ರಶ್ನೆ ಸಂಖ್ಯೆ* ***13*** ***ಕಡ್ಡಾಯವಾಗಿದೆ*** *(ಪ್ರಕರಣ ವಿಶ್ಲೇಷಣೆ).*

SECTION – A/ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following : **(10 × 2 = 20)**
ಕೆಳಗಿನವುಗಳಲ್ಲಿ **ಬೇಕಾದ ಹತ್ತು** ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :

(a) State two objectives of financial system.
ಹಣಕಾಸಿನ ವ್ಯವಸ್ಥೆಯ ಎರಡು ಉದ್ದೇಶಗಳನ್ನು ಹೇಳಿರಿ.

(b) What is meant by financial markets?
ಹಣಕಾಸಿನ ಮಾರುಕಟ್ಟೆಗಳೆಂದರೇನು?

(c) What is capital market?
ಬಂಡವಾಳ ಪೇಟೆಯೆಂದರೇನು?

(d) What is meant by commercial papers?
ವಾಣಿಜ್ಯದ ಪತ್ರಗಳೆಂದರೇನು?

(e) Define Stock Exchange.
ಶೇರು ಪೇಟೆಯ ವ್ಯಾಖ್ಯೆ ನೀಡಿರಿ.

(f) Define the bulls and bears of Stock Exchange.
ಶೇರು ಪೇಟೆಯಲ್ಲಿ ಗೂಳಿಗಳು ಹಾಗೂ ಕರಡಿಗಳು ಎಂದರೆ ಯಾರು?

(g) What is Merchant Banking?
ವರ್ತಕ ಬ್ಯಾಂಕಿಂಗ್ ಎಂದರೇನು?

(h) What is Mutual Funds?
ಪರಸ್ಪರ ನಿಧಿಗಳು ಎಂದರೇನು?

(i) Expand NABARD and CRISIL.
NABARD ಹಾಗೂ CRISILಗಳನ್ನು ವಿಸ್ತರಿಸಿರಿ.

(j) Expand OTCEI and IDBI.
OTCEI ಹಾಗೂ IDBIಗಳನ್ನು ವಿಸ್ತರಿಸಿರಿ.

(k) What is meant by open ended funds?
ಪರಸ್ಪರ ನಿಧಿಗಳ ತೆರೆದ ಆಯ್ಕೆಗೊಂಡ ಯೋಜನೆ ಎಂದರೇನು?

(l) What is NAV? How it is determined?
NAV ಎಂದರೇನು? NAVಯನ್ನು ಹೇಗೆ ಅಳತೆ ಮಾಡುವಿರಿ?

## SECTION – B/ವಿಭಾಗ – ಬ

Answer **any three** of the following : **(3 × 5 = 15)**
**ಬೇಕಾದ ಮೂರು** ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :

2. Define and explain the structure of financial system.
ಹಣಕಾಸಿನ ವ್ಯವಸ್ಥೆಯ ಅರ್ಥ ಹಾಗೂ ರಚನೆಯನ್ನು ವಿವರಿಸಿರಿ.

3. Distinguish between money market and capital market.
ಹಣದ ಪೇಟೆ ಹಾಗೂ ಬಂಡವಾಳ ಪೇಟೆಗಳ ನಡುವಿನ ವ್ಯತ್ಯಾಸಗಳನ್ನು ತಿಳಿಸಿರಿ.

4. Explain the objectives of Stock Exchange.
ಶೇರು ವಿನಿಮಯ ಕೇಂದ್ರದ ಉದ್ದೇಶಗಳನ್ನು ವಿವರಿಸಿರಿ.

5. Explain various mutual funds schemes.
ಪರಸ್ಪರ ನಿಧಿಗಳ ವಿವಿಧ ಯೋಜನೆಗಳನ್ನು ವಿವರಿಸಿರಿ.

6. Briefly state the mechanism of issuing instruments in capital market.
ಬಂಡವಾಳ ಪೇಟೆಯ ಬಂಡವಾಳ ಪತ್ರಗಳನ್ನು ನೀಡುವ ವಿವಿಧ ವಿಧಾನಗಳನ್ನು ವಿವರಿಸಿರಿ.

7. What is preference share? Explain different types of preference shares.
ಪ್ರಾಶಸ್ತ್ಯದ ಶೇರುಗಳೆಂದರೇನು? ಪ್ರಾಶಸ್ತ್ಯಶೇರುಗಳ ಪ್ರಕಾರಗಳನ್ನು ವಿವರಿಸಿರಿ.

## SECTION – C/ವಿಭಾಗ – ಕ

Answer **any two** of the following : **(2 × 15 = 30)**
**ಬೇಕಾದ ಎರಡು** ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :

8. Explain the progress of mutual funds in India.
ಭಾರತದಲ್ಲಿ ಪಾರಸ್ಪರಿಕ ನಿಧಿಗಳ ಬೆಳವಣಿಗೆಯನ್ನು ವಿವರಿಸಿರಿ.

9. Explain the weakness of Stock Exchanges in India.
ಭಾರತದ ಶೇರು ಮಾರುಕಟ್ಟೆಯ ನ್ಯೂನತೆಗಳನ್ನು ವಿವರಿಸಿರಿ ಹಾಗೂ ಅದರ ಸುಧಾರಣೆಗೆ ಸಲಹೆಗಳನ್ನು ತಿಳಿಸಿರಿ.

10. Explain the money market instruments and state the dealers dealing in money market.
ಹಣದ ಪೇಟೆಯಲ್ಲಿ ವ್ಯವಹರಿಸಲ್ಪಡುವ ಪತ್ತಿನ ಪತ್ರಗಳನ್ನು ವಿವರಿಸಿರಿ ಹಾಗೂ ಹಣದ ಪೇಟೆಯಲ್ಲಿ ವ್ಯವಹಾರಿಸುವವರು ಯಾರು?

11. State the objectives and functions of financial market.
ಹಣಕಾಸು ಮಾರುಕಟ್ಟೆಯ ಉದ್ದೇಶಗಳನ್ನು ಹಾಗೂ ಕಾರ್ಯಗಳನ್ನು ವಿವರಿಸಿರಿ.

12. Write short notes :

(a) Depository system

(b) Risks of investing in mutual funds

(c) Issues in Indian financial system

ಟಿಪ್ಪಣಿ ಬರೆಯಿರಿ :

(a) ಡಿಪಾಜಿಟರ ವ್ಯವಸ್ಥೆ

(b) ಪರಸ್ಪರ ನಿಧಿಗಳಲ್ಲಿ ಬಂಡವಾಳ ತೊಡಗಿಸುವಲ್ಲಿ ಇರುವ ಅಪಾಯಗಳನ್ನು ವಿವರಿಸಿರಿ.

(c) ಭಾರತೀಯ ಹಣಕಾಸಿನ ವ್ಯವಸ್ಥೆಯ ಎದುರಿಸುತ್ತಿರುವ ಸಮಸ್ಯೆಗಳನ್ನು ತಿಳಿಸಿರಿ.

## SECTION – D/ವಿಭಾಗ – ಡ

13. Case study **(Compulsory question)** : **(1 × 15 = 15)**
ಪ್ರಕರಣ ವಿಶ್ಲೇಷಣೆ **(ಕಡ್ಡಾಯ ಪ್ರಶ್ನೆ)** :

Mr. Murugesh is successful entrepreneur. He has established number of agro based sugar mills. He and his team of directors have promoted a new public company called "BADAMI SUGARI PUBLIC LIMITED" with an authorised capital of Rs. 500 crore. The company has decided to mobilise the necessary capital.

Questions :

(a) As management expert, suggest the securities to be used for mobilizing fixed and working capital.

(b) What amount of ownership capital and borrowed capital is advisable?

(c) Name the industrial development Banks which will finance the necessary capital.

ಶ್ರೀ ಮುರಗೇಶ ಯಶಸ್ವಿ ಪ್ರವರ್ತಕರಾಗಿ ಅನೇಕ ಸಕ್ಕರೆ ಕಾರ್ಖಾನೆಗಳನ್ನು ಸ್ಥಾಪಿಸಿರುತ್ತಾರೆ. ಅವರು ಮತ್ತು ಅವರ ನಿರ್ದೇಶಕರೆಲ್ಲರೂ ಸೇರಿ 500 ಕೋಟಿ ರೂಪಾಯಿ ಬಂಡವಾಳದ ಹೊಸ ಸಾರ್ವಜನಿಕ ಬದಾಮಿ ಶುಗರ್ಸ್ ಕಂಪನಿಯನ್ನು ಪ್ರಾರಂಭಿಸಿರುತ್ತಾರೆ ನಿರ್ದೇಶಕರು ಬೇಕಾಗುವ ಬಂಡವಾಳವನ್ನು ಸಂಗ್ರಹಿಸುವ ನಿರ್ಧಾರವನ್ನು ಮಾಡಿರುತ್ತಾರೆ.

ಪ್ರಶ್ನೆಗಳು :

(a) ಆಡಳಿತ ಕಾರ್ಯ ನಿರ್ವಹಣೆಯ ಪ್ರವೀಣನಾಗಿ, ಬಂಡವಾಳ ಸಂಗ್ರಹಿಸಲು ಯಾವ ಹಣಕಾಸಿನ ಕಾಗದ ಪತ್ರಗಳನ್ನು ಮಾರಾಟ ಮಾಡಲು ಸಲಹೆ ನೀಡುವಿರಿ?

(b) ಒಡೆತನದ ಹಾಗೂ ಸಾಲದ ಬಂಡವಾಳ ಮೊತ್ತವನ್ನು ಸಂಗ್ರಹಿಸಲು ಸಲಹೆ ನೀಡುವಿರಿ?

(c) ಬಂಡವಾಳ ಒದಗಿಸುವ ಅಭಿವೃದ್ಧಿ ಬ್ಯಾಂಕುಗಳ ಹೆಸರುಗಳನ್ನು ತಿಳಿಸಿರಿ.

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Reg. No. | | | | | | | | |

# V Semester B.Com.2 Degree Examination, November 2015

## (Regular/Repeaters)

## ELEMENTS OF COSTING – I

## Paper – I

Time : 3 Hours] [Max. Marks : 80

***Instructions/ಸೂಚನೆಗಳು :***

*1) Attempt questions according to internal choice in each section.*
*ಪ್ರತಿಯೊಂದು ವಿಭಾಗದ ಆಂತರಿಕ ಆಯ್ಕೆಗಳನುಸಾರ ಎಲ್ಲಾ ಪ್ರಶ್ನೆಗಳಿಗೂ ಉತ್ತರಿಸಿರಿ.*

*2) Question No. 11 (Case Study) is **compulsory**.*
*ಪ್ರಶ್ನೆ ಸಂಖ್ಯೆ 11, (ಪ್ರಕರಣ ಅಧ್ಯಯನ) **ಕಡ್ಡಾಯವಾಗಿದೆ.***

*3) Working notes must be given wherever necessary.*
*ಅವಶ್ಯವಿದ್ದಲ್ಲಿ ಲೆಕ್ಕದ ಟಿಪ್ಪಣಿಗಳನ್ನು ಒದಗಿಸುವದು.*

*2) Non-programmable calculator is allowed.*
*ನಾನ್-ಪ್ರೋಗ್ರಾಮ ಇದ್ದ ಕ್ಯಾಲ್ಕುಲೇಟರನ್ನು ಮಾನ್ಯ ಮಾಡಲಾಗಿದೆ.*

SECTION – A/ ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following : **(10 × 2 = 20)**
ಕೆಳಗಿನವುಗಳಲ್ಲಿ **ಬೇಕಾದ ಹತ್ತಕ್ಕೆ** ಉತ್ತರಿಸಿರಿ :

(a) Define Cost Accounting.
ವೆಚ್ಚಶಾಸ್ತ್ರವನ್ನು ವ್ಯಾಖ್ಯಾನಿಸಿರಿ.

(b) What is cost unit? Give two examples of composite cost unit.
ವೆಚ್ಚ ಘಟಕವೆಂದರೇನು? ಸಂಯುಕ್ತ ವೆಚ್ಚ ಘಟಕಗಳ ಎರಡು ಉದಾಹರಣೆ ಕೊಡಿರಿ.

(c) What is direct material cost? Give two examples.
ಸರಕುಗಳ ನೇರ ವೆಚ್ಚವೆಂದರೇನು? ಎರಡು ಉದಾಹರಣೆ ಕೊಡಿರಿ.

(d) What is perpetual inventory system?
ನಿರಂತರ ತಪಶೀಲು ಪದ್ಧತಿ ಎಂದರೇನು?

(e) What is purchase requisition?
ಖರೀದಿ ಮನವಿ ಎಂದರೇನು?

(f) What is Idle time?
ನಿಷ್ಪ್ರಯೋಜಕ ವೇಳೆ ಎಂದರೇನು?

(g) What is indirect labour cost? Give two examples.
ನೇರವಲ್ಲದ ಕೂಲಿ ವೆಚ್ಚವೆಂದರೇನು? ಎರಡು ಉದಾಹರಣೆಗಳನ್ನು ಕೊಡಿರಿ.

(h) Write the formula used to calculate the earnings under Rowan Plan.
ರೋವನ ಪದ್ಧತಿಯಂತೆ ಕೂಲಿಯ ಗಳಿಕೆಯನ್ನು ಕಂಡು ಹಿಡಿಯುವ ಸೂತ್ರವನ್ನು ಬರೆಯಿರಿ.

(i) What do you mean by functional classification of overhead?
ಮೇಲು ವೆಚ್ಚಗಳ ಕಾರ್ಯಾತ್ಮಕ ವರ್ಗೀಕರಣ ಎಂದರೇನು?

(j) What is secondary distribution summary?
ಸೇವಾ ವಿಭಾಗಗಳ ಮೇಲು ವೆಚ್ಚಗಳನ್ನು ಮರು ಹಂಚಿಕೆ ಮಾಡುವ ದ್ವಿತೀಯ ಸಾರಾಂಶ ಪಟ್ಟಿ ಎಂದರೇನು?

(k) What is machine hour rate?
ಯಂತ್ರ ಗಂಟೆ ಎಂದರೇನು?

(l) Cost price is Rs. 20,000, the profit on sale is 20%. Ascertain the profit when the closing stock is Rs. 4,000.
ಉತ್ಪಾದನಾ ವೆಚ್ಚ ರೂ. 20,000, ಮಾರಾಟ ಬೆಲೆ ಮೇಲೆ ಶೇಕಡಾ 20 ರಂತೆ ಲಾಭ ಮಾಡಲಾಗಿದೆ. ಅಂತಿಮ ಸರಕು ಬೆಲೆ ರೂ. 4,000 ಆದರೆ ಆಗುವ ಲಾಭವನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

## SECTION – B / ವಿಭಾಗ – ಬ

Answer **any three** of the following : **(3 × 5 = 15)**
ಕೆಳಗಿನ **ಬೇಕಾದ ಮೂರಕ್ಕೆ** ಉತ್ತರಿಸಿರಿ :

2. What are the classification of overhead?
ಮೇಲು ವೆಚ್ಚಗಳ ವರ್ಗೀಕರಣ ತಿಳಿಸಿರಿ.

3. Calculate EOQ and No. of orders to be placed from the following information :
Consumption of material – 10,000 kgs p.a.
Cost per kg of raw materials – Rs. 2
Order placing cost per order – Rs. 50
Storage cost – 8% on average inventory.
ಈ ಕೆಳಗಿನ ಮಾಹಿತಿಯಿಂದ ಮಿತವ್ಯಯ ಖರೀದಿ ಮೊತ್ತ ಮತ್ತು ಆದೇಶಗಳ ಸಂಖ್ಯೆಯನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :
ವಾರ್ಷಿಕ ಸರಕುಗಳ ಅಗತ್ಯವು – 10,000 ಕಿಲೋ
ಪ್ರತಿ ಕಿಲೋದ ಸರಕಿನ ಬೆಲೆ – ರೂ. 12
ಪ್ರತಿ ಆದೇಶದ ಖರ್ಚು – ರೂ. 50
ದಾಸ್ತಾನು ಖರ್ಚು – ಸರಾಸರಿ ಶೇಕಡಾ 8.

4. An SSI unit produced 500 units of a commodity in the month of April 2014. Its cost details are as below :

| | ₹ |
|---|---|
| Material consumed | 4,000 |
| Productive wages | 3,500 |
| Direct expenses | 500 |
| Works on cost charged @ 50% of the Direct wages | |
| Administrative expenses | 149 |

The proprietor expects 25% profit on selling price. Prepare a Cost Sheet showing selling price and total profit.

ಒಂದು ಲಘು ಉದ್ಯಮ ಘಟಕವು ಏಪ್ರೀಲ 2014ರಲ್ಲಿ ಒಂದು ವಸ್ತುವಿನ 500 ಯೂನಿಟ್‌ಗಳನ್ನು ಉತ್ಪಾದಿಸಿದ ಅದರ ವೆಚ್ಚ ವಿವರಗಳು ಈ ಕೆಳಗಿನಂತಿವೆ :

| | ರೂ. |
|---|---|
| ಉಪಯೋಗಿಸಿದ ಕಚ್ಚಾ ಸರಕು | 4,000 |
| ಉತ್ಪಾದಕ ಕೂಲಿಗಳು | 3,500 |
| ಅಪರೋಕ್ಷ ಖರ್ಚುಗಳು | 500 |
| ಉತ್ಪಾದನೆಯ ಪರೋಕ್ಷ ವೆಚ್ಚಗಳನ್ನು ನೇರ ಕೂಲಿಯ ಶೇ. 50 ರಂತೆ ಆಕರಿಸಲಾಗಿದೆ | |
| ಆಡಳಿತಾತ್ಮಕ ಖರ್ಚುಗಳು | 149 |

ಮಾಲಿಕರು ಮಾರಾಟದ ಮೇಲೆ ಶೇ. 25 ರಷ್ಟು ಲಾಭ ನಿರೀಕ್ಷಿಸಿದ್ದಾರೆ. ಮಾರಾಟ ಬೆಲೆ ಹಾಗೂ ಒಟ್ಟು ಲಾಭ ತೋರಿಸುವ ವೆಚ್ಚ ಪತ್ರಿಕೆಯನ್ನು ಸಿದ್ಧಪಡಿಸಿರಿ.

5. Furnished below are the particulars relating to Material J 123 :

| | |
|---|---|
| Minimum consumption per day | 75 units |
| Maximum consumption per day | 125 units |
| Delivery period | 4 to 6 days |
| Reordering quantity | 500 units |
| Consumption during May 2013 | 3000 units |

Calculate
(a) Average stock held and
(b) Material Turnover Index

ಕೆಳಗಿನ ವಿವರಗಳ ಸಾಮಾಗ್ರಿ J 123ಗೆ ಸಂಬಂಧಿಸಿವೆ :

| | |
|---|---|
| ಪ್ರತಿ ದಿನಕ್ಕೆ ಕನಿಷ್ಟ ಬಳಕೆ | 75 ಯೂನಿಟ್‌ಗಳು |
| ಪ್ರತಿ ದಿನಕ್ಕೆ ಗರಿಷ್ಟ ಬಳಕೆ | 125 ಯೂನಿಟ್‌ಗಳು |
| ಸರಕು ಸರಬರಾಜು ಅವಧಿ | 4 ರಿಂದ 6 ದಿನಗಳು |
| ಪುನರ್ ಖರೀದಿ ಗಾತ್ರ | 500 ಯೂನಿಟ್‌ಗಳು |
| ಮೇ 2013ರಲ್ಲಿ ಒಟ್ಟು ಬಳಕೆ | 3000 ಯೂನಿಟ್‌ಗಳು |

ಸರಾಸರಿ ದಾಸ್ತಾನು ಮಟ್ಟ ಮತ್ತು ಸಾಮಾಗ್ರಿ ಖರೀದಿ ಮತ್ತು ಮಾರಾಟದ ಸೂಚ್ಯಾಂಕಗಳನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

6. The following particulars relate to Anand an employee of India Tools Ltd. :

| | |
|---|---|
| Standard time allowed | 45 hours |
| Actual time taken | 36 hours |
| Wage rate per day of 8 hours | Rs. 120 |

Find out Anand's total earnings and effective wage rate per hour under Rowan's plan. What will be difference in effective wage rate if he is paid according to Halsey Plan?

ಈ ಕೆಳಗಿನ ವಿವರಗಳು ಇಂಡಿಯಾ ಟೂಲ್ಸ್ ಲಿ.ನ ಕಾರ್ಮಿಕ ಆನಂದನಿಗೆ ಸಂಬಂಧಿಸಿವೆ :

| | |
|---|---|
| ನೀಡಿದ ಸ್ಟ್ಯಾಂಡರ್ಡ್ ಕಾಲ | 45 ಗಂಟೆಗಳು |
| ನೈಜವಾಗಿ ತೆಗೆದುಕೊಂಡ ಕಾಲ | 36 ಗಂಟೆಗಳು |
| 8 ಗಂಟೆಗಳ ಒಂದು ದಿನದ ದಿನಗೂಲಿ | ರೂ. 120 |

ರೋವನ್‌ರ ಯೋಜನೆ ಉಪಯೋಗಿಸಿ ಆನಂದನ ಒಟ್ಟು ಗಳಿಕೆ ಹಾಗೂ ಪ್ರತಿ ಗಂಟೆಯ ಪರಿಣಾಮಕಾರಿ ಕೂಲಿ ದರವನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ. ಒಂದು ವೇಳೆ ಹಾಲ್ಸೆ ಯೋಜನೆ ಪ್ರಕಾರ ಕೂಲಿ ನೀಡಿದರೆ ಆತರ ಪರಿಣಾಮಕಾರಿ ಕೂಲಿ ದರದಲ್ಲಿ ಆಗುವ ಬದಲಾವಣೆ ಏನು?

SECTION – C / ವಿಭಾಗ – ಕ

Answer **any two** of the following : **(2 × 15 = 30)**

ಕೆಳಗಿನವುಗಳಲ್ಲಿ **ಬೇಕಾದ ಎರಡಕ್ಕೆ** ಉತ್ತರಿಸಿರಿ :

7. Explain the steps involved in installation of costing system.
ವೆಚ್ಚಶಾಸ್ತ್ರ ಪದ್ಧತಿಯನ್ನು ಅಳವಡಿಸಲು ಬೇಕಾದ ಹಂತಗಳನ್ನು ವಿವರಿಸಿರಿ.

8. From the following transactions, prepare a stores ledger under LIFO pricing method :

Receipts :

| | |
|---|---|
| 6.3.2012 | 1600 units @ Rs. 20 each |
| 12.3.2012 | 600 units @ Rs. 24 each |
| 18.3.2012 | 200 units @ Rs. 25 each |
| 25.3.2012 | 400 units @ Rs. 20 each |

Issues :

| | |
|---|---|
| 3.3.2012 | 100 units |
| 7.3.2012 | 600 units |
| 15.3.2012 | 640 units |
| 20.3.2012 | 240 units |
| 29.3.2012 | 500 units |

The opening balance was 500 units @ Rs. 21 each. On 8.3.2012 40 units were received to Stores, which were issued on 3.3.2012. Further on 22.3.2012, 80 units were returned to vendor which were received on 18.3.2012. For purchase on 25.3.2012 freight charges of Rs. 200 were paid. There was a surplus on 31.3.2012 of 50 units as per stock audit.

ಈ ಕೆಳಗೆ ಕಾಣಿಸಿದ ಮಾಹಿತಿ ಆಧರಿಸಿರಿ LIFO ಬೆಲೆ ಪದ್ಧತಿ ಅಳವಡಿಸಿ ಸರಕಿನ ಖಾತೆಯನ್ನು ತಯಾರಿಸಿರಿ :

ಖರೀದಿಗಳು :

| | |
|---|---|
| 6.3.2012 | 1600 ಯೂನಿಟ್‌ಗಳು ರೂ. 20 ರಂತೆ |
| 12.3.2012 | 600 ಯೂನಿಟ್‌ಗಳು ರೂ. 24 ರಂತೆ |
| 18.3.2012 | 200 ಯೂನಿಟ್‌ಗಳು ರೂ. 25 ರಂತೆ |
| 25.3.2012 | 400 ಯೂನಿಟ್‌ಗಳು ರೂ. 20 ರಂತೆ |

ವಿತರಣೆಗಳು :

| | |
|---|---|
| 3.3.2012 | 100 ಯೂನಿಟ್‌ಗಳು |
| 7.3.2012 | 600 ಯೂನಿಟ್‌ಗಳು |
| 15.3.2012 | 640 ಯೂನಿಟ್‌ಗಳು |
| 20.3.2012 | 240 ಯೂನಿಟ್‌ಗಳು |
| 29.3.2012 | 500 ಯೂನಿಟ್‌ಗಳು |

ತಿಂಗಳ ಪ್ರಾರಂಭದ ಶಿಲ್ಕು 500 ಯೂನಿಟ್‌ಗಳು ದರ ರೂ. 21 ಪ್ರತಿ ಯೂನಿಟ್ಟಿಗೆ ಇರುತ್ತದೆ. 3.3.2012ರ ಹಂಚಿಕೆಯ ಪೈಕಿ 40 ಯೂನಿಟ್‌ಗಳು 8.3.2012 ರಂದು ಮರಳಿಸಲಾಗಿದೆ. 18.3.2012 ರಂದು ಖರೀದಿಸಿದ ಯೂನಿಟ್‌ಗಳಲ್ಲಿ 80 ಯೂನಿಟ್‌ಗಳನ್ನು 22.3.2012 ರಂದು ವ್ಯಾಪಾರಿಗೆ ಮರಳಿಸಿದೆ 25.3.2012 ರಂದು ಖರೀದಿಸಿದ ಯೂನಿಟ್‌ಗಳಿಗೆ ಸಾಗಾಣಿಕೆ ವೆಚ್ಚ ರೂ. 200 ಕೊಡಲಾಗಿದೆ. 31.3.2012 ರಂದು ನಡೆದ ದಾಸ್ತಾನು ಪರಿಶೀಲನೆಯಿಂದ 50 ಯೂನಿಟ್‌ಗಳು ಹೆಚ್ಚುವರಿ ಕಂಡು ಬಂದಿವೆ.

9. From the following details compute earnings of Mr. Amar under Halsey and Rowan Plan and effective Labour hour rates also :

Job commenced – Monday 23rd February 2013 at 8 a.m.

Job completed – Saturday 28th February 2013 at 12 noon

Quantity produced and approved – 400 units

Wage rate – Rs. 2 per hour

Time allowed – 8 units per hour

Shift timings – 8 am to 4 pm

ಈ ಕೆಳಗಿನ ಮಾಹಿತಿ ಮೇರೆಗೆ ಕಾರ್ಮಿಕ ಅಮರನ ಗಳಿಕೆಯನ್ನು ಹಾಲ್ಸೆ ಮತ್ತು ರೋವನ್ ಯೋಜನೆ ಪ್ರಕಾರ ಕಂಡು ಹಿಡಿದು ಪರಿಣಾಮಕಾರಿ ದರಗಳನ್ನು ಸಹ ಕಂಡು ಹಿಡಿಯಿರಿ :

ಕೆಲಸದ ಪ್ರಾರಂಭ – ಸೋಮವಾರ 23.2.2013ರ ಮುಂಜಾನೆ 8 ಗಂಟೆಗೆ

ಕೆಲಸದ ಮುಕ್ತಾಯ – ಶನಿವಾರ 28.2.2013ರ ಮಧ್ಯಾಹ್ನ 12 ಗಂಟೆ

ಉತ್ಪಾದನೆ – 400 ಯೂನಿಟ್‌ಗಳು

ಕೂಲಿ ದರ – ಪ್ರತಿ ಗಂಟೆಗೆ ರೂ. 2

ನಿಗದಿತ ಅವಧಿ – ಪ್ರತಿ ಗಂಟೆಗೆ 8 ಯೂನಿಟ್‌ಗಳು

ಶಿಫ್ಟ್ ವೇಳೆ – ಮುಂಜಾನೆ 8 ಗಂಟೆಯಿಂದ ಮಧ್ಯಾಹ್ನ 4 ಗಂಟೆಗೆ

10. The following particulars relate to a Company's manufacturing operations. There are three production departments viz. $P_1$, $P_2$ and $P_3$ and two service departments viz. X and Y.

| Departments | Overhead (Rs.) |
|---|---|
| $P_1$ | 1980 |
| $P_2$ | 2920 |
| $P_3$ | 2540 |
| X | 680 |
| Y | 1500 |

The overhead charged to Service Departments are charged out on percentage basis as under :

| Particulars | Production Departments | | | Service Departments | |
|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | X | Y |
| Department X | 40% | 20% | 30% | – | 10% |
| Department Y | 20% | 40% | 20% | 20% | – |

Re-apportion the Service Departments overhead charged to production departments using "Simultaneous Equation Method" also determine overhead recovery rate if direct labour hours worked are 2,000, 3,000 and 4,000 respectively in the three Production Departments.

ಕೆಳಗಿನ ವಿವರಗಳು ಒಂದು ಕಂಪನಿಯ ಉತ್ಪಾದನೆಯ ಚಟುವಟಿಕೆಗಳಿಗೆ ಸಂಬಂಧಿಸಿವೆ $P_1$, $P_2$ ಮತ್ತು $P_3$ ಮೂರು ಉತ್ಪಾದನಾ ವಿಭಾಗಗಳು ಮತ್ತು X ಮತ್ತು Y ಗಳು ಸೇವಾ ವಿಭಾಗಗಳಾಗಿರುತ್ತವೆ

| ವಿಭಾಗಗಳು | ಮೇಲು ವೆಚ್ಚಗಳು (ರೂ.) |
|---|---|
| $P_1$ | 1980 |
| $P_2$ | 2920 |
| $P_3$ | 2540 |
| X | 680 |
| Y | 1500 |

ಸೇವಾ ವಿಭಾಗಗಳ ಮೇಲು ವೆಚ್ಚಗಳನ್ನು ಶೇಕಡಾವಾರು ಈ ಕೆಳಗಿನಂತೆ ಆಕರಿಸಲಾಗುತ್ತಿದೆ :

| ವಿವರಗಳು | ಉತ್ಪಾದನಾ ವಿಭಾಗಗಳು | | | ಸೇವಾ ವಿಭಾಗಗಳು | |
|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | X | Y |
| ವಿಭಾಗ X | 40% | 20% | 30% | – | 10% |
| ವಿಭಾಗ Y | 20% | 40% | 20% | 20% | – |

ಏಕಕಾಲಿಕ ಸಮೀಕರಣ ಪದ್ಧತಿ ಅನುಸರಿಸಿ ಸೇವಾ ವಿಭಾಗಗಳ ಮೇಲು ವೆಚ್ಚಗಳನ್ನು ಮರು ಹಂಚಿಕೆ ಮಾಡಿರಿ. ಮೂರು ಉತ್ಪಾದನಾ ವಿಭಾಗಗಳಲ್ಲಿ ಅಪರೋಕ್ಷ ಕೂಲಿ ಗಂಟೆಗಳು ಅನುಕ್ರಮವಾಗಿ 2,000, 3,000 ಹಾಗೂ 4,000 ಇದ್ದರೆ ಮೇಲು ವೆಚ್ಚ ಆಕರಣೆ ದರವನ್ನು ನಿರ್ಧರಿಸಿರಿ.

SECTION – D/ವಿಭಾಗ – ಡ

Case study **(Compulsory)** :

ಪ್ರಕರಣ ಅಧ್ಯಯನ **(ಕಡ್ಡಾಯ ಪ್ರಶ್ನೆ)** : **(1 × 15 = 15)**

11. (a) A Chemical Industry used chemical 'X' Mark II as a raw material. This chemical costs Rs. 10 per kg and Input-Output ratio is 125%.

On account of sudden shortage of this material the company is compelled to use substitutes and following two grades of chemical 'X' were found suitable, keeping in view quality factors.

| Materials | Rate per kg | I.O ratio |
|---|---|---|
| X Mark 1 | Rs. 12 | 110% |
| X Mark 3 | Rs. 9 | 150% |

Recommend which of the Grades is to be used.

ಒಂದು ರಾಸಾಯನಿಕ ಉದ್ದಿಮೆ ಸಂಸ್ಥೆಯು X ಮಾರ್ಕ್ II ಎಂಬ ಕಚ್ಚಾ ವಸ್ತುವನ್ನು ಉಪಯೋಗಿಸುತ್ತಿದೆ. ಅದರ ಬೆಲೆಯು ಪ್ರತಿ ಕಿಲೋಗೆ ರೂ. 10 ಮತ್ತು ಉಪಯೋಗ–ತಯಾರಿಕಾ ಅನುಪಾತವು ಶೇಕಡಾ 125 ಇರುತ್ತದೆ. ಮಾರುಕಟ್ಟೆಯಲ್ಲಿ ರಾಸಾಯನಿಕ ಕಚ್ಚಾ ವಸ್ತುವು ಕೊರತೆಯಾದ ಕಾರಣ ಇದಕ್ಕೆ ಪರ್ಯಾಯವಾದ ಕಚ್ಚಾ ವಸ್ತುಗಳ ವಿವರ ಈ ಕೆಳಗಿನಂತೆ ಇದೆ.

| ಕಚ್ಚಾ ವಸ್ತು | ಪ್ರತಿ ಕಿಲೋ ದರ | ಉಪಯೋಗ ತಯಾರಿಕೆ ಅನುಪಾತ |
|---|---|---|
| X ಮಾರ್ಕ್ 1 | ರೂ. 12 | 110% |
| X ಮಾರ್ಕ್ 3 | ರೂ. 9 | 150% |

ಮೇಲಿನವುಗಳಲ್ಲಿ ಯಾವ ಗ್ರೇಡಿನ ವಸ್ತುವನ್ನು ಉಪಯೋಗಿಸಬೇಕು ಎಂಬುದರ ಬಗ್ಗೆ ಸೂಕ್ತ ಶಿಫಾರಸು ನೀಡಿರಿ.

(b) Annual consumption of a material in a company is 10,000 units. Purchase price of each unit is Rs. 5. Ordering and receiving cost is Rs. 50 per order. Storage cost is 20% of average inventory, the company is following optimum policy. A supplier has offered 0.4% discount, if annual requirement is brought in five lots in a year.

You are required to advise with your argument.

(i) Should the offer be accepted?

(ii) If not what should be the counter offer?

ಒಂದು ಕಂಪನಿಯಲ್ಲಿ ಒಂದು ಕಚ್ಚಾ ಸರಕಿನ ವಾರ್ಷಿಕ ಉಪಯೋಗ 10,000 ಯುನಿಟ್ಟುಗಳಾಗಿವೆ, ಪ್ರತಿ ಯೂನಿಟ್ಟಿನ ಖರೀದಿ ಬೆಲೆ ರೂ. 5 ಇದೆ. ಆದೇಶ ನೀಡಿ ಸರಕುಗಳನ್ನು ಪಡೆಯುವ ವೆಚ್ಚ ಪ್ರತಿ ಆದೇಶಕ್ಕೆ ರೂ. 50 ಇದೆ. ಸಂಗ್ರಹಣಾ ವೆಚ್ಚ ಸರಾಸರಿ ದಾಸ್ತಾನಿನ ಶೇ. 20 ಇದೆ. ಕಂಪನಿಯು ಕನಿಷ್ಠ ಖರೀದಿ ಧೋರಣೆಯನ್ನು ಅನುಸರಿಸುತ್ತಿದೆ. ವಾರ್ಷಿಕ ಅವಶ್ಯಕತೆಯನ್ನು ಐದು ಕಂತುಗಳಲ್ಲಿ ಕೊಳ್ಳುವದಾದರ ಶೇ. 0.4 ರಷ್ಟು ಸೋಡಿಯನ್ನು ನೀಡುವದಾಗಿ ಒಬ್ಬ ಪೂರೈಕೆದಾರನು ಆಹ್ವಾನ ನೀಡಿದ್ದಾನೆ.

(i) ಈ ಆಹ್ವಾನವನ್ನು ಸ್ವೀಕರಿಸಬೇಕೆ?

(ii) ಇಲ್ಲವಾದರೆ ಪ್ರತಿ ಆಹ್ವಾನ ಯಾವದಿರಬೇಕು?

ಈ ಕುರಿತು ನೀವು ನಿಮ್ಮ ವಾದದ ಜೊತೆ ಸಲಹೆ ನೀಡಬೇಕಾಗಿದೆ.

23522/E 220

Reg. No. | | | | | | | | |

# V Semester B.Com.2 Degree Examination, November 2015

## INCOME TAX – I

## (Compulsory)

Time : 3 Hours] [Max. Marks : 80

***Instructions/ಸೂಚನೆಗಳು :***

*1) Answer all the questions with strict observations of inner choice in each Section.*
*ಆಂತರಿಕ ಆಯ್ಕೆ ಅನ್ವಯ ಎಲ್ಲಾ ವಿಭಾಗಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ.*

*2) Question No. 11 under Section-D is* ***compulsory****.*
*ವಿಭಾಗ-ಡದಲ್ಲಿ ಪ್ರಶ್ನೆ 11* ***ಕಡ್ಡಾಯವಾಗಿ*** *ಉತ್ತರಿಸಿರಿ.*

*3) Use of calculators is allowed.*
*ಕ್ಯಾಲಕುಲೇಟರ ಉಪಯೋಗಕ್ಕೆ ಅನುಮತಿ ಇದೆ.*

SECTION – A/ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following : **(10 × 2 = 20)**
ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಯಾವುದಾದರೂ ಹತ್ತಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :

(a) Who is a person?
ವ್ಯಕ್ತಿ ಎಂದರೆ ಯಾರು?

(b) Define previous year.
ಆದಾಯ ವರ್ಷ ವಿವರಿಸಿರಿ.

(c) How do you treat agricultural income for tax purpose?
ಕೃಷಿ ಆದಾಯವನ್ನು ತೆರಿಗೆ ಉದ್ದೇಶಕ್ಕಾಗಿ ಹೇಗೆ ಪರಿಗಣಿಸುತ್ತೀರಿ?

(d) State the provisions of Sec. 80 u.
ಕಲಂ 80 uದ ಅವಕಾಶಗಳನ್ನು ತಿಳಿಸಿರಿ.

(e) What are casual incomes? Give any two examples.
ಆಕಸ್ಮಿಕ ಆದಾಯಗಳೆಂದರೇನು? ಯಾವುದಾದರೂ ಎರಡು ಉದಾಹರಣೆಗಳನ್ನು ಕೊಡಿರಿ.

(f) What are perquisites? State any two examples.
ಸವಲತ್ತುಗಳೆಂದರೇನು? ಯಾವುದಾದರೂ ಎರಡು ಉದಾಹರಣೆಗಳನ್ನು ಕೊಡಿರಿ.

(g) What is Annual Value?
ಮನೆ ಆಸ್ತಿಯ ವಾರ್ಷಿಕ ಮೌಲ್ಯ ಎಂದರೇನು?

(h) Name any four items of investments or savings which qualify for deduction u/s 80 C.
ಹೂಡಿಕೆ ಅಥವಾ ಉಳಿತಾಯಗಳನ್ನು ಕಲಂ 80 C ಪ್ರಕಾರ ಕಡಿತಗೊಳಿಸಲು ಅರ್ಹತೆ ಹೊಂದಿರುವ ಯಾವುದಾದರೂ ನಾಲ್ಕು ಹೇಳಿರಿ.

(i) What is standard rent of property?
ಆಸ್ತಿಯ ಪ್ರಮಾಣೀಕೃತ ಬಾಡಿಗೆ ಎಂದರೇನು?

(j) What is Gratuity?
ಗೌರವಧನ ಎಂದರೇನು?

(k) What do you mean by Foreign Income?
ವಿದೇಶಿ ಆದಾಯ ಎಂದರೇನು?

(l) What is a Statutory Provident Fund?
ಶಾಸನಬದ್ಧ ಭವಿಷ್ಯ ನಿಧಿ ಎಂದರೇನು?

SECTION – B/ವಿಭಾಗ – ಬ

Answer **any three** of the following : **(3 × 5 = 15)**
ಕೆಳಗಿನವುಗಳಲ್ಲಿ **ಬೇಕಾದ ಮೂರಕ್ಕೆ** ಉತ್ತರಿಸಿರಿ :

2. Mr. Raveendra of Bangalore leaves India for the first time on 1st June 2010 to Germany to live with his eldest son. He came back to India on 1st July 2011 and left to Japan on 1st May 2012 to live with his daughter. He returned to India on 28th February 2015.

Determine his Residential Status for the Assessment Year 2015-16.

ದಿನಾಂಕ 1.6.2010 ರಂದು ಶ್ರೀ ರವೀಂದ್ರ ಮೊದಲ ಬಾರಿಗೆ ಭಾರತವನ್ನು ಬಿಟ್ಟು ಜರ್ಮನಿಗೆ ಮಗನೊಂದಿಗೆ ವಾಸಿಸಲು ಹೋಗಿರುತ್ತಾನೆ ಮತ್ತು ದಿನಾಂಕ 1.7.2011 ಮರಳಿ ಭಾರತಕ್ಕೆ ಬಂದಿರುತ್ತಾನೆ. ಮತ್ತೆ ದಿನಾಂಕ 1.5.2012 ರಂದು ಮಗಳೊಂದಿಗೆ ವಾಸಿಸಲು ಜಪಾನ ದೇಶಕ್ಕೆ ಹೋಗಿರುತ್ತಾನೆ ಮತ್ತು ದಿನಾಂಕ 28.2.2015 ರಂದು ಮರಳಿ ಭಾರತಕ್ಕೆ ಬಂದಿರುತ್ತಾನೆ.

ಅವನ ವಾಸದ ಸ್ಥಿತಿಯನ್ನು ಕರಾಕರಣೆಯ ವರ್ಷ 2015-16ಕ್ಕೆ ನಿರ್ಧರಿಸಿರಿ.

3. Mrs. Revati received Rs. 3,00,000 as gratuity on her retirement on 30th September 2014. She was working in a private firm. At the time of retirement she was getting per month Rs. 10,000 as Basic Salary and Rs. 5,000 as Dearness Allowances (75% of it forms part of salary for service benefits)

She has served for 25 years and 8 months. She is not covered by Payment of Gratuity Act. Calculate exempted gratuity for Assessment Year 2015-16.

ಖಾಸಗಿ ಕಂ. ಲಿ. ಉದ್ಯೋಗಿ ಶ್ರೀಮತಿ ರೇವತಿ 30.9.2014 ರಂದು ರೂ. 3,00,000 ಗ್ರಾಚ್ಯುಟಿಯನ್ನು ಪಡೆದು ನಿವೃತ್ತಿಯಾಗಿದ್ದಾರೆ. ಅವರ ನಿವೃತ್ತಿಯ ಹಿಂದಿನ ತಿಂಗಳ ಮೂಲ ವೇತನ ರೂ. 10.000 ಮಾಸಿಕ ದಿನಭತ್ಯೆ ರೂ. 5,000 (75% ನಿವೃತ್ತಿ ಲಾಭಾಂಶ).

ಶ್ರೀಮತಿ ರೇವತಿರವರು 25 ವರ್ಷ 8 ತಿಂಗಳ ಸೇವೆಯನ್ನು ಸಲ್ಲಿಸಿದ್ದಾರೆ. ಸದರಿಯವರು ಗೌರವಧನ ವ್ಯತ್ತಿಗೆ ಒಳಪಡುವದಿಲ್ಲ. ಅವರ ವಿನಾಯ್ತಿ ಮತ್ತು ತೆರಿಗೆ ಒಳಪಡುವ ಗೌರವಧನವನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

4. The Gross Total Income of Mr. Janardhan for A.Y. 2015-16 is Rs. 2,00,000. He has made the following donations. Calculate deduction u/s 80 G.

(a) National defence fund Rs. 50,000

(b) Karnataka University (Institute of National Eminence) Rs. 20,000

(c) District Literacy Committee Rs. 40,000

(d) National Blood Transfusion Council Rs. 20,000

(e) PM's National Relief Fund Rs. 40,000

(f) Indira Gandhi Memorial Trust Rs. 50,000

(g) Govt for Family Planning Rs. 1,00,000

(h) Approved Charitable Institution Rs. 1,20,000

ಶ್ರೀ ಜನಾರ್ಧನ ರವರ ಆದಾಯ ವರ್ಷ 2015-16 ಒಟ್ಟು ಆದಾಯ ರೂ. 2,00,000 ಆಗಿತ್ತು. ಅವರು ಈ ಕೆಳಗಿನ ನಿಧಿಗಳು ಮತ್ತು ಸಂಸ್ಥೆಗಳಿಗೆ ದೇಣಿಗೆಯನ್ನು ನೀಡಿರುತ್ತಾರೆ. ಕಲಂ 80 Gದ ಅಡಿಯಲ್ಲಿ ಅವರಿಗೆ ಸಿಗುವ ಕಡತ ಮೊತ್ತವನ್ನು ಲೆಕ್ಕಿಸಿರಿ.

(a) ರಾಷ್ಟ್ರೀಯ ರಕ್ಷಣಾ ನಿಧಿ ರೂ. 50,000

(b) ಕರ್ನಾಟಕ ವಿಶ್ವವಿದ್ಯಾಲಯ (ಇನ್ಸ್ಟ್ಯೂಟ್ ಆಫ್ ನ್ಯಾಷನಲ್ ಎಮಿನೆನ್ಸ್) ರೂ. 20,000

(c) ಜಿಲ್ಲಾ ಸಾಕ್ಷರತಾ ಕಮಿಟಿ ರೂ. 40,000

(d) ರಾಷ್ಟ್ರೀಯ ರಕ್ತ ಭಂಡಾರ ಕೌನ್ಸಿಲ ರೂ. 20,000

(e) ಪ್ರಧಾನ ಮಂತ್ರಿಗಳ ರಾಷ್ಟ್ರೀಯ ಪರಿಹಾರ ನಿಧಿ ರೂ. 40,000

(f) ಇಂದಿರಾ ಗಾಂಧಿ ಸ್ಮಾರಕ ಟ್ರಸ್ಟ ರೂ. 50,000

(g) ಸರಕಾರಿ ಕುಟುಂಬ ಯೋಜನೆ ರೂ. 1,00,000

(h) ಅನುಮೋದಿತ ಚಾರಿಟೇಬಲ್ ಟ್ರಸ್ಟ್ ರೂ. 1,20,000

5. From the following information submitted by Mr. Megharaj compute the deduction allowable u/s 80 C for Assessment Year 2015-16 :

(a) Gross Salary Rs. 5,40,000

(b) Repayment of housing loan from SBI for construction of house completed in 2007-08 (Principal amount Rs. 35,000 and Interest Rs. 65,000)

(c) Royalty (gross) Rs. 40,000 and expenses of manuscript Rs. 10,000.

(d) Interest on Fixed deposits of Rs. 25,000 for 5 years in Canara Bank Rs. 2,000.

(e) Premium on life insurance on his own life (policy amount Rs. 25,000) Rs. 7,000.

(f) LIP on life of wife Rs. 3,000.

(g) Equal contribution of his and his employer to RPF Rs. 10,000 each.

(h) Contribution to ULIP Rs. 15,000.

(i) Subscription to units of a Mutual Fund (notified) Rs. 20,000.

(j) Amount paid for Education of his children for Higher Education

(i) Adhya Rs. 20,000

(ii) Sampreet Rs. 25,000 of which Rs. 10,000 as donation.

ಶ್ರೀ ಮೇಘರಾಜ ಕೆಳಗಿನಂತೆ ಕರಾಕರಣೆಯ ವರ್ಷ 2015-16ಕ್ಕೆ ವಿವರಣೆಗಳನ್ನು ನೀಡಿರುತ್ತಾರೆ. ಕಲಂ 80 C ಅಡಿಯಲ್ಲಿ ಅವಕಾಶವಿರುವ ಕಡತಗಳನ್ನು ಲೆಕ್ಕ ಹಾಕಿರಿ :

(a) ಒಟ್ಟು ಸಂಬಳ ರೂ. 5,40,000

(b) ಮನೆ ಸಾಲ SBI ಇವರಿಂದ, ಮನೆ ಪೂರ್ಣಗೊಂಡ ವರ್ಷ 2007-08 (ಪ್ರಧಾನ ಮೊತ್ತ ರೂ. 35,000 ಮತ್ತು ಬಡ್ಡಿ ರೂ. 65,000)

(c) ರಾಯಧನ (ಒಟ್ಟು) ರೂ. 40,000, ಮತ್ತು ದಸ್ತಾಪತ್ರಗಳ ವೆಚ್ಚ ರೂ. 10,000.

(d) ನಿಶ್ಚಿತ ಠೇವಣಿ ರೂ. 25,000, 5 ವರ್ಷಗಳ ಅವಧಿ ಕೆನರಾ ಬ್ಯಾಂಕು ಬಡ್ಡಿ ರೂ. 2,000.

(e) ಜೀವ ವಿಮಾ ಕಂತು ರೂ. 7,000 (ಪಾಲಿಸಿಯ ರೂ. 25,000).

(f) ಶ್ರೀಮತಿ ಮೇಘರಾಜ ಜೀವ ವಿಮಾ ಕಂತು ರೂ. 3,000.

(g) ನೌಕರನ ಮತ್ತು ಮಾಲೀಕರ ಸಮನಾದ RPFಗೆ ವಂತಿಗೆ ರೂ. 10,000 ದಂತೆ.

(h) ULIP ವಂತಿಗೆ ರೂ. 15,000.

(i) ಪರಸ್ಪರ ನಿಧಿ (ನೊಂದಾಯಿತ) ಪಾವತಿ ರೂ. 20,000.

(j) ಶಿಕ್ಷಣ ಭತ್ಯ

(i) ಆದ್ಯಾ ರೂ. 20,000

(ii) ಸಂಪ್ರೀತ ರೂ. 25,000 (ರೂ. 10,000 ದೇಣಿಗೆ) ಒಳಗೊಂಡಿದೆ.

6. Mrs. Shashirekha of Chennai is employed as a Manager of TVS Company. The particulars of her salary for the P.Y. 2014-15 are as under :

(a) Basic Salary Rs. 7,000 p.m.

(b) D.A. (taken for R.B.) Rs. 3,000 p.m.

(c) Conveyance allowance Rs.1,000 p.m.

(d) Commission at 1% on turnover of Rs. 7,00,000 achieved by her

(e) HRA Rs. 3,000 p.m.

(f) CCA Rs. 500 p..m

(g) Medical allowances Rs. 1,000 p.m. The actual paid by her is Rs. 2,500 p.m.

Calculate taxable HRA.

TVS ಕಂಪನಿ ಮದ್ರಾಸ ಉದ್ಯೋಗಿ ಶ್ರೀಮತಿ ಶಶಿರೇಖಾ ಕೆಳಗಿನಂತೆ ವಿವರಗಳನ್ನು ಸಲ್ಲಿಸಿದ್ದಾರೆ. ಆದಾಯ ವರ್ಷ 2014-15 ವಿವರಗಳು :

(a) ಮೂಲ ವೇತನ ರೂ. 7,000 ಪ್ರತಿ ತಿಂಗಳು.

(b) ದಿನಭತ್ಯೆ (ನಿವೃತ್ತಿ ಲಾಭಾಂಶ) ರೂ. 3,000 ಪ್ರತಿ ತಿಂಗಳು.

(c) ಕನವೆಯನ್ಸ ಭತ್ಯೆ ರೂ.1,000 ಪ್ರತಿ ತಿಂಗಳು.

(d) ವ್ಯವಹಾರದ ಮೇಲೆ ಕಮೀಶನ್ 1% ಒಟ್ಟು ವ್ಯವಹಾರ ರೂ. 7,00,000.

(e) ಮನೆ ಬಾಡಿಗೆ ಭತ್ಯೆ ರೂ. 3,000 ಪ್ರತಿ ತಿಂಗಳು.

(f) ನಗರ ಭತ್ಯೆ ರೂ. 500 ಪ್ರತಿ ತಿಂಗಳು.

(g) ವೈದ್ಯಕೀಯ ವೆಚ್ಚ ರೂ. 1,000 ಪ್ರತಿ ತಿಂಗಳು. ಖರ್ಚು ಮಾಡಿದ ವೈದ್ಯಕೀಯ ವೆಚ್ಚ ರೂ. 2,500.

ಮನೆ ಬಾಡಿಗೆಯ ಭತ್ಯೆಯನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

SECTION – C/ವಿಭಾಗ – ಕ

Answer **any two** of the following : **(2 × 15 = 30)**
**ಬೇಕಾದ ಎರಡಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :**

7. Mr. Prasad is an employee in a limited company at Mumbai. He gives you the following information for the Previous Year 2014-15 :

(a) Basic Salary Rs. 8,000 per month

(b) D.A. (enter into RB) Rs. 2,000 p.m.

(c) Bonus (equal to two months basic salary)

(d) Project allowances Rs. 800 p.m.

(e) Travelling allowances Rs. 400 p.m.

(f) House Rent allowances Rs. 2,000 p.m. (Rent paid Rs. 18,000)

(g) Contribution to RPF by company and by him at 15% of Salary.

(h) Interest credited to RPF balance at 8.50% p.a. amount to Rs. 1,400.

He paid a profession tax of Rs. 1,500 during the year. Compute the Income from Salary for the Assessment Year 2015-16.

ನಿಯಮಿತ ಕಂಪನಿಯ ಮುಂಬೈಯ ಉದ್ಯೋಗಿ ಶ್ರೀ ಪ್ರಸಾದರವರು ಕರಾಕರಣೆಯ ವರ್ಷ 2014-15ರ ವಿವರಗಳನ್ನು ಒದಗಿಸಿದ್ದಾರೆ :

(a) ಬೇಸಿಕ್ ವೇತನ ರೂ. 8,000 ಪ್ರತಿ ತಿಂಗಳು.

(b) ದಿನಭತ್ಯೆ (ನಿವೃತ್ತಿ ಲಾಭ) ರೂ. 2,000 ಪ್ರತಿ ತಿಂಗಳು.

(c) ಬೋನಸ್ (ಎರಡು ತಿಂಗಳ ಬೇಸಿಕ್ ವೇತನಕ್ಕೆ ಸಮ).

(d) ಪ್ರೊಜೆಕ್ಟ್ ಭತ್ಯ ರೂ. 800 ಪ್ರತಿ ತಿಂಗಳು.

(e) ಸಂಚಾರಿ ಭತ್ಯ ರೂ. 400 ಪ್ರತಿ ತಿಂಗಳು.

(f) ಮನೆ ಬಾಡಿಗೆ ಭತ್ಯ ರೂ. 2,000 ಪ್ರತಿ ತಿಂಗಳು (ವಾಸ್ತವಿಕ ಬಾಡಿಗೆ ಕೊಟ್ಟದ್ದು ರೂ. 18,000)

(g) ನೌಕರ ಮತ್ತು ಮಾಲೀಕರ RPF ವಂತಿಗೆ 15% ವೇತನ.

(h) RPF ಮೇಲಿನ ಬಡ್ಡಿ 8.50% ಪ್ರತಿ ವರ್ಷ ರೂ. 1,400.

ಪ್ರಸಾದನು ರೂ. 1,500 ವೃತ್ತಿ ತೆರಿಗೆ ಕೊಟ್ಟಿದ್ದಾನೆ. ಮೇಲಿನ ವಿವರಗಳಿಂದ ವೇತನ ಮೂಲದ ಆದಾಯ ಕಂಡು ಹಿಡಿಯಿರಿ.

8. The Gross Total Income of Shilpa for the A.Y. 2015-16 is Rs. 4,00,000. She has made the following donations :

(a) National Defence Fund Rs. 10,000

(b) National fund for communal harmony Rs. 6,000

(c) Rajeev Gandhi Memorial Fund Rs. 8,000

(d) Prime Minister's National Relief Fund Rs. 16,000

(e) Beds to Orphanage worth Rs. 4,000

(f) Government for Family Planning Rs. 28,000

(g) District Literacy Committee Rs. 5,000

She has also incurred Rs. 14,000 as expenditure on medical treatment of dependent who is a person with disability. Compute her total income.

ಶ್ರೀಮತಿ ಶಿಲ್ಪಾರವರ ಕರಾಕರಣೆಯ ವರ್ಷ 2015-16ರ ಒಟ್ಟು ಆದಾಯ ರೂ. 4,00,000, ಮತ್ತು ಅವರು ಈ ಕೆಳಗಿನ ನಿಧಿಗಳು ಮತ್ತು ಸಂಸ್ಥೆಗಳಿಗೆ ದೇಣಿಗೆಯನ್ನು ನೀಡಿರುತ್ತಾರೆ :

(a) ರಾಷ್ಟ್ರೀಯ ರಕ್ಷಣೆ ನಿಧಿ ರೂ. 10,000

(b) ರಾಷ್ಟ್ರೀಯ ಕೋಮು ಸೌಹಾರ್ದ ಪ್ರತಿಷ್ಠಾನಕ್ಕೆ ರೂ. 6,000

(c) ಶ್ರೀ ರಾಜೀವ ಗಾಂಧಿ ಸ್ಮಾರಕ ನಿಧಿ ರೂ. 8,000

(d) ಪ್ರಧಾನ ಮಂತ್ರಿಗಳ ರಾಷ್ಟ್ರೀಯ ಪರಿಹಾರ ನಿಧಿ ರೂ. 16,000

(e) ಬೆಡ್ಸ್ ಟು ಆರ್ಫನೇಜ ರೂ. 4,000

(f) ಸರಕಾರಿ ಕುಟುಂಬ ಯೋಜನಾ ರೂ. 28,000

(g) ಜಿಲ್ಲಾ ಸಾಕ್ಷರತಾ ಸಮಿತಿ ರೂ. 5,000

ಶ್ರೀ ಶಿಲ್ಪಾರವರು ಅಂಗವಿಕಲ ಅವಲಂಬಿತನಿಗೆ ವೈದ್ಯಕೀಯ ಉದ್ದೇಶಕ್ಕೆ ರೂ. 14,000 ವೆಚ್ಚ ಮಾಡಿದ್ದಾರೆ. ಒಟ್ಟು ಆದಾಯವನ್ನು ಲೆಕ್ಕ ಮಾಡಿರಿ.

9. From the particulars given below, compute salary income of Mr. John for P.Y. 2014-15 who is working in Bangalore in a firm and receives the following during P.Y. :

Basic Pay Rs. 20,000 p.m.

D.A. Rs. 2,500 p.m. (Enter into RB)

Educational Allowances Rs. 200 per month per child for his two children

Tribal Area Allowances Rs. 300 p.m.

CCA Rs. 125 p.m.

Conveyance Rs. 600 p.m.

Helper Allowance Rs. 500 p.m.

Entertainment Allowance received Rs. 36,000 (spent Rs. 30,000)

Travelling Allowances received for official duty Rs. 20,000 (spent Rs. 20,000)

The Employer paid the wages to the following servants provided him :

(a) A cook Rs. 1,000 p.m.

(b) A sweeper Rs. 300 p.m.

(c) A watchman Rs. 500 p.m.

(d) Gardener salary Rs. 1,000 p.m.

The Employer has provided him a car of 14 H.P. He is allowed to use it for both official as well as personal purposes. However he himself met all running and maintenance expenses including driver salary.

ಜಾನ್‌ರವರು ಖಾಸಗಿ ಕಂಪನಿ ಬೆಂಗಳೂರು ಉದ್ಯೋಗಿ ಆದಾಯ ವರ್ಷ 2014-15ರ ವಿವರಗಳನ್ನು ಕೆಳಗಿನಂತೆ ಸಲ್ಲಿಸಿದ್ದಾರೆ :

ಮೂಲ ವೇತನ ರೂ. 20,000 ಪ್ರತಿ ತಿಂಗಳು

ದಿನಭತ್ಯೆ ರೂ. 2,500 ಪ್ರತಿ ತಿಂಗಳು (ನಿವೃತ್ತಿ ಸೌಲಭ್ಯ)

ಶಿಕ್ಷಣ ಭತ್ಯ ರೂ. 200 ಪ್ರತಿ ತಿಂಗಳು, ಪ್ರತಿ ಮಗುವಿಗೆ, ಒಟ್ಟು ಇಬ್ಬರು ಮಕ್ಕಳಿಗೆ

ಗುಡ್ಡಗಾಡು ಪ್ರದೇಶದ ಭತ್ಯ ರೂ. 300 ಪ್ರತಿ ತಿಂಗಳು

ನಗರ ಭತ್ಯ ರೂ. 125 ಪ್ರತಿ ತಿಂಗಳು

ಕನವೆಯನ್ಸ್ ಭತ್ಯ ರೂ. 600 ಪ್ರತಿ ತಿಂಗಳು

ಸಹಾಯಕನ ಭತ್ಯ ರೂ. 500 ಪ್ರತಿ ತಿಂಗಳು

ಮನರಂಜನಾ ಭತ್ಯ ರೂ. 36,000 (ಖರ್ಚು ಮಾಡಿದ್ದು ರೂ. 30,000)

ಸಂಚಾರಿ ಭತ್ಯ ಕಛೇರಿ ಕೆಲಸ ರೂ. 20,000 (ಖರ್ಚು ರೂ. 20,000)

ಇತರ ಸೌಲಭ್ಯಗಳು :

(a) ಅಡಿಗೆ ತಯಾರಕ ರೂ. 1,000 ಪ್ರತಿ ತಿಂಗಳು

(b) ಸ್ವೀಪರ ರೂ. 300 ಪ್ರತಿ ತಿಂಗಳು

(c) ವಾಚಮನ್ ರೂ. 500 ಪ್ರತಿ ತಿಂಗಳು

(d) ತೋಟದ ಮಾಲಿ ರೂ. 1,000 ಪ್ರತಿ ತಿಂಗಳು

ಮೋಟಾರ ಕಾರ 14 H.P ಕಛೇರಿ ಮತ್ತು ವೈಯಕ್ತಿಕ ಕೆಲಸಕ್ಕಾಗಿ ರನ್ನಿಂಗ, ಮೆಂಟೆನೆನ್ಸ್ ಚಾಲಕ ವೆಚ್ಚಗಳನ್ನು ಜಾನ್‌ರವರು ಮಾಡುತ್ತಾರೆ. ಕರಾಕರಣೆಯ ವರ್ಷ 2015-16ಕ್ಕೆ ಮೂಲ ಆದಾಯ ಲೆಕ್ಕ ಹಾಕಿರಿ.

10. Mr. Vijaya gives following particulars for the A.Y. 2015-16 :

He is the owner of a house consisting two equal units. He retained the first chit for his use and 2nd unit is let out at a monthly rent of Rs. 24,000.

The Municipal value of the house is Rs. 5,50,000 and Municipal tax paid is 10% of Municipal value.

During the year house loan repaid amounted Rs. 30,000 which included interest on loan of Rs. 10,000. The let out unit remained vacant for the month of March, 2014.

Other Information :

Donation to Prime Minister National Relief Fund Rs. 2,000

Life Insurance Premium paid Rs. 4,000

Compute the total income for A.Y. 2015-16.

ಶ್ರೀ ವಿಜಯ ಅವರ ಕೆಳಗೆ ವಿರಗಳಿಂದ ಕರಾಕರಣೆಯ ವರ್ಷ 2015-16ಕ್ಕೆ ಮನೆಯ ಆದಾಯನ್ನು ಲೆಕ್ಕ ಮಾಡಿರಿ :

ಶ್ರೀ ವಿಜಯರವರು ಎರಡು ಸಮಾಂತರ ಮನೆಗಳನ್ನು ಹೊಂದಿದ್ದಾರೆ. ಒಂದು ಮನೆಯನ್ನು ಸ್ವಂತಕ್ಕೆ ಮತ್ತು ಇನ್ನು ಒಂದು ಮನೆಯನ್ನು ಪ್ರತಿ ತಿಂಗಳು ರೂ. 24,000 ರಂತೆ ಬಾಡಿಗೆ ನೀಡಿರುತ್ತಾರೆ.

ನಗರ ಸಭೆಯ ಮೌಲ್ಯ ರೂ. 5,50,000, ನಗರ ಸಭೆ ತೆರಿಗೆ ಮೌಲ್ಯದ 10%.

ಈ ವರ್ಷದಲ್ಲಿ ಸಾಲ ಮರುಪಾವತಿ ಮಾಡಿದ್ದು ರೂ. 30,000 (ರೂ. 10,000 ಬಡ್ಡಿ ಒಳಗೊಂಡಿದೆ). ಬಾಡಿಗೆ ಮನೆಯು ಮಾರ್ಚ್ 2014 ತಿಂಗಳಲ್ಲಿ ಬಾಡಿಗೆ ಖಾಲಿ ಇತ್ತು.

ಇತರ ವಿವರ :

ಪ್ರಧಾನ ಮಂತ್ರಿ ರಾಷ್ಟ್ರೀಯ ವಿಪತ್ತು ಪರಿಹಾರ ನಿಧಿಗೆ ರೂ. 2,000

ಜೀವ ವಿಮಾ ನಿಗಮದ ಪ್ರೀಮಿಯಂ ರೂ. 4,000

ಮೇಲಿನ ವಿವರಗಳಿಂದ ಮನೆಯ ಆದಾಯ ಲೆಕ್ಕ ಮಾಡಿರಿ.

## SECTION – D/ವಿಭಾಗ – ಡ

**Compulsory** question : **(1 × 15 = 15)**
ಕಡ್ಡಾಯ ಪ್ರಶ್ನೆ :

11. Mr. Murugan a cricketer of Sri Lanka provides you the following information for the P.Y. 2014-15 :

(1) Agriculture income from Sri Lanka Rs. 60,000 of which half amount is received in India.

(2) Income from Business in Sri Lanka Rs. 80,000 this business is controlled from Bangalore.

(3) Dividend declared in Sri Lanka but received in India Rs. 5,000.

(4) Remuneration for cricket coaching in India Rs. 1,00,000.

(5) Incomes from units of UTI Rs. 15,000.

(6) Rent from property in Sri Lanka received there Rs. 60,000 of which Rs. 40,000 remitted to India.

Questions :

(a) Determine the Taxable income of Mr. Murugan, a Sri Lanka cricketer, if he has been coming to Bangalore for 100 days every year since Jan 2002.

(b) Will your answer be different if he has been coming to India for 110 days every year?

(c) What will be the total income if Mr. Murugan comes to India only for 50 days every year?

(d) Do you think that the incidence of tax depends on the residential status? If yes prove it by taking the above information and if not give the reasons.

ಮುರಗನ್‌ರವರು ಶ್ರೀಲಂಕಾ ಕ್ರಿಕೇಟ ಆಟಗಾರ, ಇವರಿಗೆ ಸಂಬಂಧಿಸಿದ ಆದಾಯ ವರ್ಷದ ವಿವರಗಳನ್ನು ಸಲ್ಲಿಸಿದ್ದಾರೆ. 2014-15 :

(1) ಶ್ರೀಲಂಕಾ ಕೃಷಿ ಆದಾಯ ರೂ. 60,000, ಅರ್ಧ ಹಣವನ್ನು ಭಾರತದಲ್ಲಿ ಪಡೆಯಲಾಗಿದೆ.

(2) ಶ್ರೀಲಂಕಾ ವ್ಯವಹಾರದ ಆದಾಯ ರೂ. 80,000, ವ್ಯವಹಾರ ಭಾರತದಿಂದ ನಿಯಂತ್ರಿಸಲ್ಪಟ್ಟಿದೆ.

(3) ಶ್ರೀಲಂಕಾದಲ್ಲಿ ಲಾಭಾಂಶ ಘೋಷಣೆ ರೂ. 5,000. ಮೊತ್ತವನ್ನು ಭಾರತದಲ್ಲಿ ಪಡೆಯಲಾಗಿದೆ.

(4) ಭಾರತದಲ್ಲಿ ಕೋಚಿಂಗ ಮೂಲಕ ಆದಾಯ ರೂ. 1,00,000.

(5) UTI ಆದಾಯ ರೂ. 15,000.

(6) ಶ್ರೀಲಂಕಾದ ಮನೆ ಬಾಡಿಗೆ ಆದಾಯ ರೂ. 60,000, ಶ್ರೀಲಂಕಾದಲ್ಲಿ ಹಣ ಪಡೆಯಲಾಗಿದೆ. ಅದರಲ್ಲಿ ರೂ. 40,000 ಹಣ ಭಾರತಕ್ಕೆ ತರಲಾಗಿದೆ.

ಪ್ರಶ್ನೆಗಳು :

(a) ಒಟ್ಟು ಆದಾಯವನ್ನು ಲೆಕ್ಕ ಮಾಡಿರಿ, ಒಂದು ವೇಳೆ ಮುರುಗನ್‌ರವರು ವರ್ಷಕ್ಕೆ 100 ದಿನ ಬೆಂಗಳೂರಿನಲ್ಲಿ ನೆಲೆಸಿದರೆ ಜನೇವರಿ 2002 ರಿಂದ.

(b) ಒಂದು ವೇಳೆ ವರ್ಷಕ್ಕೆ 100 ದಿನ ಬದಲಾಗಿ 110 ದಿನ ಬೆಂಗಳೂರಿನಲ್ಲಿ ನೆಲೆಸಿದರೆ ಅವರ ಆದಾಯವೆಷ್ಟು?

(c) ಒಂದು ವೇಳೆ ವರ್ಷಕ್ಕೆ 100 ದಿನ ಬದಲಾಗಿ 50 ದಿನ ಬೆಂಗಳೂರಿನಲ್ಲಿ ನೆಲೆಸಿದರೆ ಅವರ ಆದಾಯವೆಷ್ಟು?

(d) ತೆರಿಗೆ ಆಕರಣೆಯ ವಾಸಸ್ಥಾನ ಅವಲಂಬಿಸಿದದೆ. ಹೌದು ಎಂದಾದರೆ ಖಚಿತ ಪಡಿಸಿರಿ. ಒಂದು ವೇಳೆ ಇಲ್ಲ ಎಂದಾದರೆ ಸಮಂಜಸ ಕಾರಣ ನೀಡಿರಿ.

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Reg. No. | | | | | | | | |

# V Semester B.Com.2 Degree Examination, November 2015

## (Regular)

## INDIRECT TAXES

## Paper – II

Time : 3 Hours] [Max. Marks : 80

***Instructions/ಸೂಚನೆಗಳು :***

*1) Attempt all Sections according to internal choice.*
*ಆಂತರಿಕ ಆಯ್ಕೆಗೆ ಅನುಗುಣವಾಗಿ ಎಲ್ಲಾ ವಿಭಾಗಗಳನ್ನು ಉತ್ತರಿಸಿರಿ.*

*2) Use of simple calculator is allowed.*
*ಸಾದಾ ಕ್ಯಾಲ್ಕುಲೇಟರ್ ಬಳಕೆ ಮಾಡಬಹುದು.*

SECTION – A/ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following : **(10 × 2 = 20)**
ಕೆಳಗಿನ **ಬೇಕಾದ ಹತ್ತಕ್ಕೆ** ಉತ್ತರಿಸಿರಿ :

(a) What is indirect taxes?
ಅಪರೋಕ್ಷ ತೆರಿಗೆ ಎಂದರೇನು?

(b) What is the purpose of Customs Act?
ಸೀಮಾ ತೆರಿಗೆಯ ಎರಡು ಉದ್ದೇಶಗಳನ್ನು ಬರೆಯಿರಿ.

(c) Write two functions of Customs department.
ಸೀಮಾಶುಲ್ಕ ವಿಭಾಗದ ಎರಡು ಕಾರ್ಯಗಳನ್ನು ಬರೆಯಿರಿ.

(d) What is Special Economic Zones in excise?
ಅಬಕಾರಿಯಲ್ಲಿರುವ ವಿಶೇಷ ಆರ್ಥಿಕ ಕೇಂದ್ರ ಎಂದರೇನು?

(e) What is Basic Excise Duty?
ಅಬಕಾರಿಯ ಮೂಲ ಸುಂಕ ಎಂದರೇನು?

(f) Write two objectives of CST.
ಸಿ.ಎಸ್.ಟಿ.ಯ ಯಾವುದಾದರೂ ಎರಡು ಉದ್ದೇಶಗಳನ್ನು ಬರೆಯಿರಿ.

(g) What are the different forms for registration under CST?
ಸಿ.ಎಸ್.ಟಿ.ಯಲ್ಲಿ ನೊಂದಾಯಿಸುವದಕ್ಕಾಗಿ ಯಾವ ನಮೂನೆಯನ್ನು ಉಪಯೋಗಿಸುವದು?

(h) Write two merits of VAT.
ವ್ಯಾಟ್‌ದ ಎರಡು ಉದ್ದೇಶಗಳನ್ನು ಬರೆಯಿರಿ.

(i) What is input and output tax?
ಸರಕು ತೆರಿಗೆ ಮತ್ತು ವಸ್ತುಗಳ ತೆರಿಗೆ ಎಂದರೇನು?

(j) What is service tax?
ವೃತ್ತಿ ಸುಂಕ ಎಂದರೇನು?

(k) What is penalty on non payment of service tax u/s 76?
ಕಲಂ–76ದಲ್ಲಿರುವಂತೆ ವೃತ್ತಿ–ಸುಂಕ ಪಾವತಿಸದೇ ಇದ್ದಾಗ ಯಾವ ದಂಡ ವಿಧಿಸಬಹುದು?

(l) Give four examples of services.
ವೃತ್ತಿಯ ಯಾವುದಾದರೂ ನಾಲ್ಕು ಉದಾಹರಣೆಗಳನ್ನು ಹೇಳಿರಿ.

## SECTION – B/ವಿಭಾಗ – ಬ

Answer **any three** of the following : **(3 × 5 = 15)**
**ಬೇಕಾದ ಮೂರಕ್ಕೆ** ಉತ್ತರಿಸಿರಿ :

2. State the powers of customs officer.
ಸೀಮಾ ಶುಲ್ಕ ಅಧಿಕಾರಿಯ ವಿವಿಧ ಅಧಿಕಾರಗಳು (Powers)ನ್ನು ನಮೂದಿಸಿರಿ.

3. What are the features of central excise?
ಕೇಂದ್ರ ಅಬಕಾರಿಯ ಲಕ್ಷಣಗಳನ್ನು ಕುರಿತು ಬರೆಯಿರಿ.

4. State the methods for computation of tax under VAT.
ವ್ಯಾಟ್‌ನ್ನು ಕಂಡು ಹಿಡಿಯುವ ವಿವಿಧ ಪ್ರಕಾರಗಳನ್ನು ಬರೆಯಿರಿ.

5. Miss Priti reported following data for the financial year 2012-13 :
   (a) Total interstate sales (inclusive of CST) ₹ 1,02,55,000
   (b) Above sales include excise duty ₹ 13,60,000
   (c) Incentive on sales ₹ 2,00,000
   (d) Deposits for returnable containers ₹ 8,00,000

   Determine the turnover and CST assuming the rate is 2%.

   ಮಿಸ್ ಪ್ರೀತಿಯು 2012-13 ಹಣಕಾಸಿನ ವರ್ಷದ ಮಾಹಿತಿಯನ್ನು ಈ ಕೆಳಗಿನಂತೆ ಕೊಟ್ಟಿರುವರು :
   (a) ಒಟ್ಟು ಅಂತರಾಜ್ಯ ಮಾರಾಟ (ಸಿ.ಎಸ್.ಟಿ. ಸಹಿತ) ₹ 1,02,55,000
   (b) ಅಬಕಾರಿಯ ಹೆಚ್ಚಿನ ಮಾರಾಟದ ಶುಲ್ಕು ಸಹಿತ ₹ 13,60,000
   (c) ಮಾರಾಟದ ಮೇಲೆ ಕೊಟ್ಟಿರುವ ಹೆಚ್ಚುವರಿ ಭತ್ಯೆ ₹ 2,00,000
   (d) ತಿರಿಗಿ ಕೊಡುವ ಡಬ್ಬಿಗಳ ಠೇವು ₹ 8,00,000

   ಮೇಲಿನ ಮಾಹಿತಿಯಿಂದ ಆಯವ್ಯಯ ಮತ್ತು ಶೇಕಡಾ 2 ರಂತೆ ಸಿ.ಎಸ್.ಟಿ.ಯನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ

6. Mr. Sharique is a consulting engineer raised a bill ₹ 4,00,000 (including service tax) on his client for the consulting services rendered by him in the month of June 2013. A partial payment of ₹ 2,00,000 was received by him in the month of March 2014.

Compute the service tax amount payable at 12.36% and the due date by which service tax can be deposited.

ಶ್ರೀ ಶಾರಿಕ್ ಎಂಬ ಸಂಬಂಧಿಸಿದ ಅಭಿಯಂತರರು ಕೆಲಸ ಮಾಡುವದಕ್ಕಾಗಿ ದಿನಾಂಕ ಜೂನ್ 2012ಕ್ಕೆ ತಮ್ಮ ಕಕ್ಷಿದಾರರಿಂದ ರೂ. 4,00,000ದ ಹುಂಡಿಯನ್ನು ತೆಗೆದುಕೊಂಡಿರುವರು (ವೃತ್ತಿ ತೆರಿಗೆ ಸಹಿತ). ಇದರಲ್ಲಿ ಮಾರ್ಚ್ 2013 ರಂದು ಅರ್ಧ ಶುಲ್ಕದಂತೆ ರೂ. 2,00,000 ತೆಗೆದುಕೊಂಡಿರುವರು. ಈ ಮೇಲಿನ ಮಾಹಿತಿಯಿಂದ ಶೇಕಡಾ 12.36 ರಂತೆ ವೃತ್ತಿ ಸುಂಕ ಕಂಡು ಹಿಡಿಯಿರಿ ಮತ್ತು ಯಾವ ತಾರೀಖಿಗೆ ವೃತ್ತಿ ಸುಂಕವನ್ನು ಠೇವಣಿ ಮಾಡಬೇಕಿದ್ದು ಹೇಳಿರಿ.

SECTION – C/ವಿಭಾಗ – ಕ

Answer **any two** of the following : **(2 × 15 = 30)**

**ಯಾವುದಾದರೂ ಎರಡಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :**

7. Mr. Arvind returned from US after 3 day stay. He brought with him

(a) Personal effect ₹ 40,000

(b) 2 laptop of ₹ 20,000 each

(c) 1 LED ₹ 36,000 and sundry gift item ₹ 20,000

He returns with his wife and one child of 6 year age.

Find out the import duty payable.

ಶ್ರೀ ಅರವಿದಂದವರು ಅಮೇರಿಕಾದಿಂದ 4 ದಿವಸದ ಮೇಲಿದ್ದು ಬರುವಾಗಿ ಈ ಕೆಳಗೆ ಕೊಟ್ಟ ಸಾಮಾನುಗಳು ತಂದಿರುವರು

(a) ವೈಯಕ್ತಿಕ ವಸ್ತುಗಳು ರೂ. 40,000

(b) ಎರಡು ಲ್ಯಾಪಟ್ಯಾಪ್ ರೂ. 20,000 ಒಂದಕ್ಕೆದಂತೆ

(c) ಒಂದು LED ರೂ. 36,000 ಮತ್ತು ಬಹುಮಾನ ಸಾಮಾನು ರೂ. 20,000

ಇವರು ತಮ್ಮ ಹೆಂಡತಿ ಮತ್ತು ಚಿರಂಜಿ (6 ವರ್ಷದ) ಕೂಡಿ ತಿರುಗಿ ಬಂದಿರುವರು.

ಮೇಲ ಮಾಹಿತಿಯಂತೆ ಆಮದು ಸುಂಕನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.

8. M/s Harish Mumbai sells iron rods to M/s Hyderabad Ltd. for value ₹ 10,00,000 inclusive of CST 2% and local sales is 3%. What will be the CST if both are registered dealers? Find out the CST if Hyderabad Ltd., Hyderabad is unregistered dealer. Assume that iron rods are not declared goods.

M/s ಹರೀಶ್ ಮುಂಬಯಿ ಇವರು M/s ಹೈದ್ರಾಬಾದ್ ಲಿ. ಹೈದ್ರಾಬಾದ್‌ವರಿಗೆ ಕಬ್ಬಿಣ ತಂತಿಯನ್ನು ರೂ. 10,00,000 (ಸಿ.ಎಸ್.ಟಿ. ಶೇ. 2 ರಂತೆ ಮತ್ತು ಸ್ಥಳೀಯ ಮಾರಾಟ ಕರ ಶೇ. 3 ರಂತೆ) ಮಾರಾಟ ಮಾಡಿರುವರು. ಹೈದ್ರಾಬಾದ್ ಲಿ. ಮತ್ತು ಮುಂಬಯಿ ಕಂಪನಿಯು ನೊಂದಾಯಿತ ಮಾರಾಟಗಾರ ಇದ್ದಂತೆ ಶುಲ್ಕನ್ನು (C.S.T.) ಕಂಡು ಹಿಡಿಯಿರಿ ಮತ್ತು ಹೈದ್ರಾಬಾದ್‌ವರು ನೊಂದಾಯಿತ ಇಲ್ಲದಿದ್ದರ ಸಿ.ಎಸ್.ಟಿ. ಏನಾಗಬಹುದು ಎಂದು ಕಂಡು ಹಿಡಿಯಿರಿ. ಕಬ್ಬಿಣದ ತಂತಿಯನ್ನು ಭಾವಿಸಲಾರದ ವಸ್ತುಗಳನ್ನು ಎಂದು ತಿಳಿಯಿರಿ.

9. Find Assessable value and duty payable from the following :

MRP – ₹ 2,200

Sales Tax, Surcharge, Octroi and other taxes – 10%

Cash discount – 2%

Trade discount – 8%

Excise duty – 8%

Abatement – 40%

What will be your answer if the product is not covered under MRP provisions?

ಕೆಳಗೆ ಕೊಟ್ಟ ಮಾಹಿತಿಯಿಂದ ಅಸೆಸಬಲ್ ಬೆಲೆ ಮತ್ತು ತೆರಿಗೆಯನ್ನು ಏನಾಗಬಹುದೆಂದು ಕಂಡು ಹಿಡಿಯಿರಿ :

ನಿರ್ಧಿಷ್ಟ ಪಡಿಸಿದ ಮಾರಾಟ ಬೆಲೆ (MRP) ರೂ. 2,200

ಮಾರಾಟ ತೆರಿಗೆ ಮತ್ತು ಇತರ ತೆರಿಗೆ ಶೇ. 10 ರಂತೆ

ಹಣ ಕೊಟ್ಟದ ಮೇಲೆ ರಿಯಾಯಿತಿ ಶೇ. 2 ರಂತೆ

ಮಾರಾಟ ರಿಯಾಯಿತಿ ಶೇ. 8 ರಂತೆ

ಅಬಕಾರಿ ಸುಂಕ ಶೇ. 8 ರಂತೆ

ಅಬೇಟಮೆಂಟ (Abatement) ಶೇ. 40 ರಂತೆ

ವಸ್ತುವಿನ ನಿರ್ಧಿಷ್ಟ ಪಡಿಸಿದ ಮಾರಾಟ ಬೆಲೆಯು ಕಾನೂನಬದ್ಧವಾಗಿ ಇಲ್ಲದಿದ್ದಲ್ಲಿ ನಿಮ್ಮ ಉತ್ತರ ಏನು ಇರುವದು ತಿಳಿಸಿರಿ.

10. Golden company is engaged in the services of site preparation and clearance, excavation earth moving and demolition services. The gross amount received for the quarter ended 30.6.2013 for the services are given below :

(a) Core extraction services for construction ₹ 1,90,000

(b) Land reclamation work ₹ 90,000

(c) Services in relation to agriculture ₹ 2,00,000

(d) Renovating and restoring water sources ₹ 3,00,000

(e) Horizontal drilling of passage of cable ₹ 1,00,000

(f) Soil stabilization ₹ 90,000

(g) Construction of transport terminals ₹ 55,000

Calculate the value of taxable services under Finance Act 1994. Service tax payable 12.36%. Give working note.

ಗೋಲ್ಡನ್ ಕಂಪನಿಯು ಭೂಮಿಯನ್ನು ಸ್ವಚ್ಛಗೊಳಿಸಿ ಅನುಕೂಲವಾಗದಂತೆ ಬೇಕಾದ ಸೈಟನ್ನು ಪರಿವರ್ತಿಸುವಲ್ಲಿ ತೊಡಗಿರುವರು. ಇವರು ಇದಕ್ಕೆ ತೆಗೆದುಕೊಂಡ ಒಟ್ಟು ಹಣವು ಈ ಕೆಳಗಿನಂತಿರುವದು (ಮೂರು ತಿಂಗಳ ಮುಗಿದ 30.6.2013 ರಂತೆ ಇರುವದು).

(a) ಭೂಮಿಯನ್ನು ಸಮತಟ್ಟ ಮಾಡುವ ಖರ್ಚು ರೂ. 1,90,000

(b) ರೀಕ್ಲೆಮೇಶನ್ (reclamation work) ಕೆಲಸಕ್ಕೆ ರೂ. 90,000

(c) ಕೃಷಿ ಮತ್ತು ವೃತಿಗೆ ಸಂಬಂಧಿಸಿದ ರೂ. 2,00,000

(d) ಜಲ ಸಂಪನ್ಮೂಲ ಬದಲಾವಣೆ ಮತ್ತು ಶೇಖರಣೆ ರೂ. 3,00,000

(e) ಕೇಬಲ್‌ಗಳನ್ನು ಹಾಕಲಿಕ್ಕೆ ಕಾಲುವೆ ತೆಗೆಯುವದು ರೂ. 1,00,000

(f) ಮಣ್ಣು ಸಮತಟ್ಟು ಮಾವದ ಬಗ್ಗೆ ರೂ. 90,000

(g) ವಾಹನ ನಿಲ್ದಾಣ ಕಟ್ಟಡದ ಬಗ್ಗೆ ರೂ. 55,000

ಈ ಮೇಲೆ ಕೊಟ್ಟಿರುವ ಮಾಹಿತಿಯಿಂದ ವೃತ್ತಿಯ ಬೆಲೆಯನ್ನು ಹಣಕಾಸು 1994 ಕಲಂ ತಕ್ಕಂತೆ ಕಂಡು ಹಿಡಿಯಿರಿ ಮತ್ತು ವೃತ್ತಿ ತೆರಿಗೆ ಶೇ. 12.36 ದಂತೆ ಎಷ್ಟಾಗುವುದು ಹೇಳಿರಿ. ಇದಕ್ಕೆ ನಿಮ್ಮ ಟಿಪ್ಪಣಿಯನ್ನು ಬರೆಯಿರಿ.

SECTION – D/ವಿಭಾಗ – ಡ

Case study **(Compulsory)** : **(15)**

ಪ್ರಕರಣ ಅಧ್ಯಯನ **(ಕಡ್ಡಾಯ ಪ್ರಶ್ನೆ)** :

11. Monaka Ltd. a registered dealer has furnished you the following details to compute VAT Liability (4%)

Raw material ₹ 2,25,000 (inclusive of VAT @ 12.5%)

Wages ₹ 60,000 and

Overheads ₹ 1,00,000

Calculate VAT as per :

(a) Additional method (assuming profit ₹ 40,000)

(b) Subtraction method (assuming that S Price including VAT ₹ 4,16,000)

(c) Invoice method (assuming profit ₹ 40,000)

Which method is beneficial?

ಮೊನ್ಯಾಕೊ ಲಿ. ನೊಂದಾಯಿತ ಮಾರಾಟಗಾರರ ಶೇ. 4 ರಂತೆ ವ್ಯಾಟನ್ನು ಈ ಕೆಳಗೆ ಕೊಟ್ಟ ಮಾಹಿತಿಯಿಂದ ಕಂಡು ಹಿಡಿಯಿರಿ.

ಸರಕು ಸಾಗಾಣಿಕೆ ರೂ. 2,25,000 (ವ್ಯಾಟ ಶೇ.12.5 ಸಹಿತ)

ಸಂಬಳ ರೂ. 60,000

ಇತರ ಖರ್ಚು ರೂ. 1,00,000

ಮೇಲೆ ಕಾಣಿಸಿದ ಮಾಹಿತಿಯಿಂದ ಈ ಕೆಳಗೆ ಕೊಟ್ಟದ ಪ್ರಕಾರದಿಂದ ವ್ಯಾಟನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :

(a) ಭಿನ್ನ ಪದ್ಧತಿಯಿಂದ (ಕಾಲ್ಪನಿಕ ಲಾಭ ರೂ. 40,000)

(b) ನಕಾರಾತ್ಮಕ ಪದ್ಧತಿಯಿಂದ (ಕಾಲ್ಪನಿಕ ಮಾರಾಟ ಬೆಲೆ ವ್ಯಾಟ ರೂ. 4,16,000 ಸಹಿತ)

(c) ರಸೀದಿ ಪದ್ಧತಿ (Invoice method) (ಕಾಲ್ಪನಿಕ ಲಾಭ ರೂ. 40,000 ಇದ್ದಂತೆ)

ಯಾವ ಪದ್ಧತಿಯು ಲಾಭದಾಯಕವಾಗುವುದು ವಿವರಿಸಿರಿ.

23521/E 210

Reg. No. | | | | | | | | |

# V Semester B.Com.2 Degree Examination, November 2015

## (Regular and Repeaters)

## FUNDAMENTALS OF MANAGEMENT ACCOUNTING

Time : 3 Hours] [Max. Marks : 80

***Instructions/ಸೂಚನೆಗಳು :***

*1) Attempt the questions according to internal choice.*
*ಆಂತರಿಕ ಆಯ್ಕೆಯುತ ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ.*

*2) Non-programmable calculator may be used.*
*ನಾನ್‌-ಪ್ರೋಗ್ರಾಮೇಬಲ್ ಕ್ಯಾಲ್ಕುಲೇಟರ ಉಪಯೋಗಿಸಬಹುದು.*

SECTION – A / ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following : **(10 × 2 = 20)**
**ಬೇಕಾದ ಹತ್ತಕ್ಕೆ** ಉತ್ತರಿಸಿರಿ :

(a) State any two objectives of Management Accounting.
ನಿರ್ವಹಣಾ ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಎರಡು ಉದ್ದೇಶಗಳನ್ನು ಸೂಚಿಸಿರಿ.

(b) Mention any two advantages of Management Accounting.
ನಿರ್ವಹಣ ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಎರಡು ಅನುಕೂಲತೆಗಳನ್ನು ತಿಳಿಸಿ.

(c) Give the types of Financial Analysis.
ಹಣಕಾಸಿನ ವಿಶ್ಲೇಷಣೆಯ ವಿಧಗಳನ್ನು ಕೊಡಿರಿ.

(d) What are Comparative Financial Statements?
ತುಲನಾತ್ಮಕ ಹಣಕಾಸಿನ ಪಟ್ಟಿಗಳು ಎಂದರೇನು?

(e) Give the meaning of Inventory Turnover Ratio.
ದಾಸ್ತಾನು ಆವರ್ತನ ಗತಿ ಎಂದರೇನು?

(f) From the following calculate current assets :
Current ratio = 2 : 0,
Net working capital = Rs. 1,60,000 and
Current Liabilities = Rs. 80,000
ಈ ಕೆಳಗಿನ ಮಾಹಿತಿಯಿಂದ ಚಾಲ್ತಿ ಆಸ್ತಿಗಳ ಮೊತ್ತವನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :
ಚಾಲ್ತಿ ಪ್ರಮಾಣ = 2 : 0,
ನಿವ್ವಳ ದುಡಿಯುವ ಬಂಡವಾಳ = ರೂ. 1,60,000 ಮತ್ತು
ಚಾಲ್ತಿ ಹೊಣೆಗಾರಿಕೆ = ರೂ. 80,000



P.T.O.

(g) Calculate total debtors from the following :
Debtors velocity – 3 months and
Credit Sales – Rs. 4,50,000
ಈ ಕೆಳಗಿನ ಮಾಹಿತಿಯಿಂದ ಸಾಲಗಾರರ ಮೊತ್ತವನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :
ಬರತಕ್ಕ ಬಾಕಿಗಳ ಚಲನೆ – 3 ತಿಂಗಳು,
ಉದ್ದರಿ ಮಾರಾಟ – ರೂ. 4,50,000

(h) State any four sources of funds.
ನಿಧಿ ಮೂಲಗಳ ಯಾವುದೇ ನಾಲ್ಕು ಅಂಶಗಳನ್ನು ವಿವರಿಸಿ.

(i) What is Cash Flow Statement?
ನಗದು ಚಲನ ಪಟ್ಟಿ ಎಂದರೇನು?

(j) What are funds from operations?
ವ್ಯವಹಾರ ಚಟುವಟಿಕೆಯ ನಿಧಿಗಳು ಎಂದರೇನು?

(k) Give any four advantages of ratio analysis.
ಅನುಪಾತ ವಿಶ್ಲೇಷಣೆಯ ನಾಲ್ಕು ಅನುಕೂಲತೆಗಳನ್ನು ತಿಳಿಸಿರಿ.

(l) What do you mean by Cash Flows from Investing Activities?
ಹಣವನ್ನು ತೊಡಗಿಸುವ ಚಟುವಟಿಕೆಗಳಿಂದಾಗುವ ನಗದು ಹರಿವಿನ ಅರ್ಥ ಕೊಡಿರಿ.

## SECTION – B/ವಿಭಾಗ – ಬ

Answer **any three** of the following : **(3 × 5 = 15)**
**ಬೇಕಾದ ಮೂರು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :**

2. Write the differences between Management Accounting and Financial Accounting.
ನಿರ್ವಹಣಾ ಲೆಕ್ಕಶಾಸ್ತ್ರ ಹಾಗೂ ಹಣಕಾಸಿನ ಲೆಕ್ಕಶಾಸ್ತ್ರಗಳ ನಡುವಿನ ವ್ಯತ್ಯಾಸಗಳನ್ನು ಬರೆಯಿರಿ.

3. Following is the Assets side of the Balance Sheet of Mahesh Co. Ltd. as on 31st December, 2013 and 2014. Prepare asset side common size Balance Sheet.

| Assets | 2013 (Rs.) | 2014 (Rs.) |
|---|---|---|
| Cash | 20,000 | 40,000 |
| Debts | 55,000 | 60,000 |
| Stock | 70,000 | 80,000 |
| Bills Receivable | 20,000 | 20,000 |
| Prepaid expenses | 5,000 | 6,000 |
| Land and Building | 1,00,000 | 1,20,000 |
| Plant and Machinery | 1,50,000 | 1,80,000 |
| Furniture | 10,000 | 15,000 |
| | 4,30,000 | 5,21,000 |

ಮಹೇಶ ಕಂಪನಿ ನಿಯಮಿತ ಇವರ 31ನೇ ಡಿಸೆಂಬರ 2013 ಮತ್ತು 2014ರ ಆಢಾವೆ ಪತ್ರಿಕೆಯ ಆಸ್ತಿಗಳ ಮಾಹಿತಿಯಿಂದ 'ಸಾಮಾನ್ಯ ಗಾತ್ರದ ಆಸ್ತಿಗಳ ಆಢಾವೆ ಪತ್ರಿಕೆಯನ್ನು ತಯಾರಿಸಿರಿ :

| ಆಸ್ತಿಗಳು | 2013 (ರೂ.) | 2014 (ರೂ.) |
|---|---|---|
| ನಗದು | 20,000 | 40,000 |
| ಸಾಲಗಾರರು | 55,000 | 60,000 |
| ಸರಕು ಶಿಲ್ಕು | 70,000 | 80,000 |
| ಬರತಕ್ಕ ಹುಂಡಿಗಳು | 20,000 | 20,000 |
| ಮುಂಗಡವಾಗಿ ಪಾವತಿಸಿದ ವೆಚ್ಚಗಳು | 5,000 | 6,000 |
| ಭೂಮಿ ಹಾಗೂ ಕಟ್ಟಡ | 1,00,000 | 1,20,000 |
| ಸ್ಥಾವರ ಹಾಗೂ ಯಂತ್ರೋಪಕರಣ | 1,50,000 | 1,80,000 |
| ಪೀಠೋಪಕರಣ | 10,000 | 15,000 |
| | 4,30,000 | 5,21,000 |

4. Calculate Current ratio and Liquidity ratio from the information given below :

| | Rs. |
|---|---|
| Debtors | 80,000 |
| Stock | 80,000 |
| Advances | 50,000 |
| Cash | 40,000 |
| Prepaid expenses | 20,000 |
| Outstanding income | 10,000 |
| Creditors | 80,000 |
| Provision for taxation | 60,000 |

ಕೆಳಗಿನ ವಿವರಗಳಿಂದ ಚಾಲ್ತಿ ಅನುಪಾತ ಹಾಗೂ ದ್ರವ ಅನುಪಾತ ಕಂಡು ಹಿಡಿಯಿರಿ :

| | ರೂ. |
|---|---|
| ಸಾಲಗಾರರು | 80,000 |
| ಸರಕು ಶಿಲ್ಕು | 80,000 |
| ಮುಂಗಡ ಕೊಟ್ಟದ್ದು | 50,000 |
| ನಗದು | 40,000 |
| ಮುಂಗಡ ಸಂದಾಯವಾದ ವೆಚ್ಚಗಳು | 20,000 |
| ಬರತಕ್ಕ ಆದಾಯ | 10,000 |
| ಸಾಹುಕಾರರು | 80,000 |
| ತೆಗೆದಿಟ್ಟ ತೆರಿಗೆ | 60,000 |



P.T.O.

5. From the following data of Liability side of the Balance Sheet of Shanti Ltd., calculate trend percentages taking 2011 as base year :

| Liabilities | 2011 (Rs.) | 2012 (Rs.) | 2013 (Rs.) | 2014 (Rs.) |
|---|---|---|---|---|
| Creditors | 45,000 | 65,000 | 55,000 | 75,000 |
| Bills payable | 25,000 | 35,000 | 38,000 | 44,000 |
| Tax payable | 36,000 | 28,000 | 34,000 | 46,000 |
| Debentures | 65,000 | 75,000 | 70,000 | 80,000 |
| Share Capital | 2,00,000 | 2,25,000 | 2,10,000 | 2,50,000 |
| Reserve | 30,000 | 30,000 | 23,000 | 25,000 |
| Profit & Loss a/c | 20,000 | 20,000 | 18,000 | 19,000 |

ಶಾಂತಿ ಕಂಪನಿ ನಿಯಮಿತ ಆಢಾವೆ ಪತ್ರಿಕೆಯ ಹೊಣೆಗಾರಿಕೆಗಳ ಮಾಹಿತಿ ಕೆಳಗಿನಂತಿರುತ್ತದೆ. 2011ನ್ನು ಮೂಲವಾಗಿ ಇಟ್ಟುಕೊಂಡು ಗತಿಯ ಶೇಕಡಾವಾರಗಳನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :

| ಹೊಣೆಗಾರಿಕೆಗಳು | 2011 (ರೂ.) | 2012 (ರೂ.) | 2013 (ರೂ.) | 2014 (ರೂ.) |
|---|---|---|---|---|
| ಸಾಹುಕಾರರು | 45,000 | 65,000 | 55,000 | 75,000 |
| ಕೊಡತಕ್ಕ ಹುಂಡಿಗಳು | 25,000 | 35,000 | 38,000 | 44,000 |
| ಕೊಡತಕ್ಕ ತೆರಿಗೆ | 36,000 | 28,000 | 34,000 | 46,000 |
| ಸಾಲಪತ್ರಗಳು | 65,000 | 75,000 | 70,000 | 80,000 |
| ಶೇರು ಬಂಡವಾಳ | 2,00,000 | 2,25,000 | 2,10,000 | 2,50,000 |
| ನಿಧಿಗಳು | 30,000 | 30,000 | 23,000 | 25,000 |
| ಲಾಭ ಮತ್ತು ನಷ್ಟ ಖಾತೆ | 20,000 | 20,000 | 18,000 | 19,000 |

6. Following is the Balance Sheet of 'Shiva Shakti Ltd.' as on 31st December 2013 and 2014. Prepare schedule of changes in working capital.

| Liabilities | 2013 Rs. | 2014 Rs. | Assets | 2013 Rs. | 2014 Rs. |
|---|---|---|---|---|---|
| Share Capital | 4,00,000 | 6,00,000 | Land & Buildings | 3,00,000 | 5,00,000 |
| Debentures | 1,00,000 | 1,20,000 | Machinery | 1,00,000 | 1,50,000 |
| P & L a/c | 60,000 | 80,000 | Cash | 30,000 | 50,000 |
| Sundry Creditors | 2,00,000 | 1,00,000 | Debtors | 3,00,000 | 3,20,000 |
| Bills Payable | 1,10,000 | 1,40,000 | Stock | 1,60,000 | 30,000 |
| Provision for Taxation | 20,000 | 10,000 | | | |
| | 8,90,000 | 10,50,000 | | 8,90,000 | 10,50,000 |

2015 (6)



ದಿನಾಂಕ 31ನೇ ಡಿಸೆಂಬರ, 2013 ಹಾಗೂ 2014ರಂದು ಇರುವ 'ಶಿವಶಕ್ತಿ ನಿಯಮಿತ' ಇದರ ಆಢಾವೆ ಪತ್ರಗಳಿಂದ ದುಡಿಯುವ ಬಂಡವಾಳದ ಬದಲಾವಣೆ ಪಟ್ಟಿಯನ್ನು ತಯಾರಿಸಿ.

| ಹೊಣೆಗಾರಿಕೆಗಳು | 2013 ರೂ. | 2014 ರೂ. | ಆಸ್ತಿಗಳು | 2013 ರೂ. | 2014 ರೂ. |
|---|---|---|---|---|---|
| ಶೇರು ಬಂಡವಾಳ | 4,00,000 | 6,00,000 | ಭೂಮಿ ಮತ್ತು ಕಟ್ಟಡ | 3,00,000 | 5,00,000 |
| ಸಾಲಪತ್ರಗಳು | 1,00,000 | 1,20,000 | ಯಂತ್ರೋಪಕರಣ | 1,00,000 | 1,50,000 |
| ಲಾಭ ಹಾನಿ ಖಾತೆ | 60,000 | 80,000 | ನಗದು | 30,000 | 50,000 |
| ಇತರೆ ಸಾಹುಕಾರರು | 2,00,000 | 1,00,000 | ಸಾಲಗಾರರು | 3,00,000 | 3,20,000 |
| ಕೊಡತಕ್ಕ ಹುಂಡಿ | 1,10,000 | 1,40,000 | ಸರಕು ದಾಸ್ತಾನು | 1,60,000 | 30,000 |
| ತೆರಿಗೆಗಾಗಿ ಮೀಸಲಿಟ್ಟ ಹಣ | 20,000 | 10,000 | | | |
| | 8,90,000 | 10,50,000 | | 8,90,000 | 10,50,000 |

SECTION – C/ವಿಭಾಗ – ಕ

Answer **any two** of the following : **(2 × 15 = 30)**

**ಈ ಕೆಳಗಿನವುಗಳಲ್ಲಿ ಬೇಕಾದ ಎರಡಕ್ಕೆ ಉತ್ತರಿಸಿರಿ :**

7. Following is the Balance Sheet of XYZ Co. Ltd.
Balance Sheet as on 31.12.2014. (figures are in multiples of 100)

| Liabilities | Rs. | Assets | Rs. |
|---|---|---|---|
| Equity share capital | 5,00,000 | Goodwill | 40,000 |
| 8% Preference share capital | 3,00,000 | Land and Building | 5,00,000 |
| General Reserve | 40,000 | Machinery | 3,50,000 |
| Profit & Loss a/c | 30,000 | Furniture | 20,000 |
| 6% Debentures | 2,00,000 | Stock | 1,80,000 |
| Sundry Creditors | 56,000 | Sundry Debtors | 42,000 |
| Bills Payable | 24,000 | Cash at Bank | 10,000 |
| | | Preliminary expenses | 8,000 |
| | 11,50,000 | | 11,50,000 |



P.T.O.

Other information is as follows :

Total sales for the year is Rs. 8,00,000. Gross Profit and Net Profit (after tax) for the year ended amounted to Rs. 1,60,000 and Rs. 40,000 respectively.

Calculate :

(a) Current ratio

(b) Liquidity ratio

(c) Proprietary ratio

(d) Debt Equity ratio

(e) Gross Profit ratio

(f) Net Profit ratio and

(g) Stock Turnover ratio

'ಎಕ್ಸ್.ವಾಯ್.ಜಡ್. ಕಂಪನಿ ನಿಯಮಿತ' ಇವರ ಆಢಾವೆ ಪತ್ರಿಕೆ ಕೆಳಗಿನಂತಿರುತ್ತದೆ :

ಆಢಾವೆ ಪತ್ರಿಕೆ 31.12.2014. (ಸಂಖ್ಯೆಗಳು 100 ರಿಂದ ಗುಣಿಸಲ್ಪಡುತ್ತವೆ)

| ಹೊಣೆಗಾರಿಕೆಗಳು | ರೂ. | ಆಸ್ತಿಗಳು | ರೂ. |
|---|---|---|---|
| ಸಾಮಾನ್ಯ ಶೇರು ಬಂಡವಾಳ | 5,00,000 | ಸುನಾಮ | 40,000 |
| 8% ಪ್ರಥಮ ಪ್ರಾಶಸ್ತ್ಯದ ಶೇರು ಬಂಡವಾಳ | 3,00,000 | ಭೂಮಿ ಮತ್ತು ಕಟ್ಟಡ | 5,00,000 |
| ಮೀಸಲು ನಿಧಿ | 40,000 | ಯಂತ್ರೋಪಕರಣ | 3,50,000 |
| ಲಾಭ ಮತ್ತು ಹಾನಿ ಖಾತೆ | 30,000 | ಪೀಠೋಪಕರಣ | 20,000 |
| 6% ಸಾಲ ಪತ್ರಗಳು | 2,00,000 | ಸರಕು ದಾಸ್ತಾನು | 1,80,000 |
| ಇತರೆ ಸಾಹುಕಾರರು | 56,000 | ಸಾಲಗಾರರು | 42,000 |
| ಕೊಡತಕ್ಕ ಹುಂಡಿಗಳು | 24,000 | ಬ್ಯಾಂಕಿನಲ್ಲಿರುವ ನಗದು | 10,000 |
| | | ಪೂರ್ವಭಾವಿ ವೆಚ್ಚ | 8,000 |
| | 11,50,000 | | 11,50,000 |

ಹೆಚ್ಚಿನ ಮಾಹಿತಿ ಕೆಳಗಿನಂತಿರುತ್ತದೆ :

ಪ್ರಸಕ್ತ ವರ್ಷದಲ್ಲಿ ಒಟ್ಟು ಮಾರಾಟ ರೂ. 8,00,000.ವರ್ಷದ ಕೊನೆಯಲ್ಲಿ ಆಗಿರುವ ಒಟ್ಟು ಲಾಭ ಮತ್ತು ನಿವ್ವಳ ಲಾಭ (ತೆರಿಗೆ ನಂತರ) ಕ್ರಮವಾಗಿ ರೂ. 1,60,000 ಹಾಗೂ ರೂ. 40,000 ಇರುತ್ತವೆ.

ಕಂಡು ಹಿಡಿಯಿರಿ :

(a) ಚಾಲ್ತಿ ಅನುಪಾತ

(b) ದ್ರವ ಅನುಪಾತ

(c) ಮಾಲಕತ್ವದ ಅನುಪಾತ

(d) ಸಾಲ–ಸಾಮಾನ್ಯ ಶೇರು ಅನುಪಾತ

(e) ವ್ಯಾಪಾರ ಲಾಭ ಅನುಪಾತ

(f) ನಿವ್ವಳ ಲಾಭ ಅನುಪಾತ ಹಾಗೂ

(g) ಶಿಲ್ಕು ಆವರ್ತನ ಅನುಪಾತ

8. From the following Balance Sheets of 'Prakriti Co. Ltd.' prepare Comparative Balance Sheet and comment on the Liquidity and Profitability of the company.

| Liabilities | 2013 Rs. | 2014 Rs. | Assets | 2013 Rs. | 2014 Rs. |
|---|---|---|---|---|---|
| Equity share capital | 4,00,000 | 4,00,000 | Cash | 1,00,000 | 1,40,000 |
| 8% Pref. share capital | 3,00,000 | 3,00,000 | Sundry Debtors | 2,00,000 | 3,00,000 |
| Reserves | 2,00,000 | 2,45,000 | Stock | 2,00,000 | 3,00,000 |
| 8% Debentures | 1,00,000 | 1,50,000 | Land | 1,00,000 | 1,00,000 |
| Bills Payable | 50,000 | 75,000 | Buildings | 3,00,000 | 2,70,000 |
| Sundry Creditors | 1,50,000 | 2,00,000 | Plant & Machinery | 3,00,000 | 2,70,000 |
| Tax payable | 1,00,000 | 1,50,000 | Furniture | 1,00,000 | 1,40,000 |
| | 13,00,000 | 15,20,000 | | 13,00,000 | 15,20,000 |

'ಪ್ರಕೃತಿ ನಿಯಮಿತ' ಇವರ ಈ ಕೆಳಗಿನ ಆಢಾವೆ ಪತ್ರಿಕೆಗಳಿಂದ ತುಲನಾತ್ಮಕ ಆಢಾವೆ ಪತ್ರಿಕೆಗಳನ್ನು ತಯಾರಿಸಿರಿ ಹಾಗೂ ಕಂಪನಿಯ ದ್ರವತ್ವದ ಮತ್ತು ಲಾಭ ಗಳಿಸುವ ಸ್ಥಿತಿಗತಿಯ ಬಗ್ಗೆ ವಿಮರ್ಶಿಸಿ ಬರೆಯಿರಿ :

| ಹೊಣೆಗಾರಿಕೆಗಳು | 2013 ರೂ. | 2014 ರೂ. | ಆಸ್ತಿಗಳು | 2013 ರೂ. | 2014 ರೂ. |
|---|---|---|---|---|---|
| ಸಾಮಾನ್ಯ ಶೇರು ಬಂಡವಾಳ | 4,00,000 | 4,00,000 | ನಗದು | 1,00,000 | 1,40,000 |
| 8% ಪ್ರಥಮ ಪ್ರಾಶಸ್ತ್ಯದ | | | ಇತರೆ ಸಾಲಗಾರರು | 2,00,000 | 3,00,000 |
| ಶೇರು ಬಂಡವಾಳ | 3,00,000 | 3,00,000 | ಸರಕು ದಾಸ್ತಾನು | 2,00,000 | 3,00,000 |
| ಮೀಸಲು ನಿಧಿ | 2,00,000 | 2,45,000 | ಭೂಮಿ | 1,00,000 | 1,00,000 |
| 8% ಸಾಲ ಪತ್ರಗಳು | 1,00,000 | 1,50,000 | ಕಟ್ಟಡ | 3,00,000 | 2,70,000 |
| ಕೊಡತಕ್ಕ ಹುಂಡಿಗಳು | 50,000 | 75,000 | ಸ್ಥಾವರ ಹಾಗೂ | | |
| ಇತರ ಸಾಹುಕಾರರು | 1,50,000 | 2,00,000 | ಯಂತ್ರೋಪಕರಣ | 3,00,000 | 2,70,000 |
| ಕೊಡತಕ್ಕ ತೆರಿಗೆ | 1,00,000 | 1,50,000 | ಪೀಠೋಪಕರಣ | 1,00,000 | 1,40,000 |
| | 13,00,000 | 15,20,000 | | 13,00,000 | 15,20,000 |

9. Write short notes on the following :

   (a) Operating Income and Operating Expenses

   (b) Cash Flow Statement as per AS-3

   (c) Debtors Velocity and Creditors Velocity

ಲಘು ಟಿಪ್ಪಣಿ ಬರೆಯಿರಿ :

   (a) ಕ್ರಿಯಾತ್ಮಕ ಲಾಭ ಹಾಗೂ ಕ್ರಿಯಾತ್ಮಕ ಖರ್ಚುಗಳು

   (b) ನಗದು ಹರಿವು ಪಟ್ಟಿ AS-3 ಪ್ರಕಾರ

   (c) ಬರತಕ್ಕ ಬಾಕಿಗಳ ಚಾಲನೆ ಹಾಗೂ ಕೊಡತಕ್ಕ ಬಾಕಿಗಳ ಚಾಲನೆ

10. The Balance Sheets of Prashanti Co. Ltd. as on 31st December, 2013 and 2014 are as follows :

| Liabilities | 2013 Rs. | 2014 Rs. | Assets | 2013 Rs. | 2014 Rs. |
|---|---|---|---|---|---|
| Equity share capital | 10,00,000 | 14,00,000 | Land & Building | 1,60,000 | 2,40,000 |
| Profit & Loss A/c | 2,00,000 | 3,20,000 | Plant & Machinery | 10,00,000 | 16,00,000 |
| General Reserve | 1,00,000 | 1,40,000 | Stock | 2,00,000 | 1,50,000 |
| Sundry Creditors | 3,06,000 | 3,80,000 | Debtors | 3,00,000 | 3,20,000 |
| Bills payable | 80,000 | 1,00,000 | Cash | 40,000 | 40,000 |
| Outstanding expenses | 14,000 | 10,000 | | | |
| | 17,00,000 | 23,50,000 | | 17,00,000 | 23,50,000 |

Prepare Funds flow statement from the Balance Sheets and the also by using the following information :

(a) Rs. 1,00,000 depreciation has been charged on plant and machinery during 2014.

(b) A part of the machinery was sold for Rs. 16,000 during the year 2014. It had a cost of Rs. 24,000 and depreciation of Rs. 14,000 had been provided on it.

(c) Rs. 40,000 was transferred to General Reserve as part of profit.

(d) Dividend paid to shareholders was Rs. 80,000.

ಪ್ರಶಾಂತಿ ಕಂಪನಿ ನಿಯಮಿತ ಇವರ 31ನೇ ಡಿಸೆಂಬರ 2013 ಹಾಗೂ 2014 ಆಢಾವೆ ಪತ್ರಿಕೆಗಳು ಕೆಳಗಿನಂತಿವೆ :

| ಹೊಣೆಗಾರಿಕೆಗಳು | 2013 ರೂ. | 2014 ರೂ. | ಆಸ್ತಿಗಳು | 2013 ರೂ. | 2014 ರೂ. |
|---|---|---|---|---|---|
| ಸಾಮಾನ್ಯ ಶೇರು ಬಂಡವಾಳ | 10,00,000 | 14,00,000 | ಭೂಮಿ ಮತ್ತು ಕಟ್ಟಡ | 1,60,000 | 2,40,000 |
| ಲಾಭ ಮತ್ತು ಹಾನಿ ಖಾತೆ | 2,00,000 | 3,20,000 | ಸ್ಥಾವರ ಮತ್ತು ಯಂತ್ರೋಪಕರಣ | 10,00,000 | 16,00,000 |
| ಸಾಮಾನ್ಯ ಮೀಸಲು ನಿಧಿ | 1,00,000 | 1,40,000 | ಸರಕು ದಾಸ್ತಾನು | 2,00,000 | 1,50,000 |
| ಇತರೆ ಸಾಹುಕಾರರು | 3,06,000 | 3,80,000 | ಸಾಲಗಾರರು | 3,00,000 | 3,20,000 |
| ಕೊಡತಕ್ಕ ಹುಂಡಿಗಳು | 80,000 | 1,00,000 | ನಗದು | 40,000 | 40,000 |
| ಬಾಕಿಯಿರುವ ಖರ್ಚುಗಳು | 14,000 | 10,000 | | | |
| | 17,00,000 | 23,50,000 | | 17,00,000 | 23,50,000 |

ಆಢಾವೆ ಪತ್ರಿಕೆಗಳು ಹಾಗೂ ಕೆಳಗಿನ ಮಾಹಿತಿಗಳನ್ನು ಆಧರಿಸಿ ನಿಧಿ ಹರಿವು (ಚಲನ) ಪಟ್ಟಿಯನ್ನು ತಯಾರಿಸಿರಿ :

(a) ಸ್ಥಾವರ ಹಾಗೂ ಯಂತ್ರೋಪಕರಣದ ಮೇಲೆ ರೂ. 1,00,000 ಸವಕಳಿ 2014 ಸಾಲಿಗೆ ತೆಗೆದಿರಿಸಲಾಗಿದೆ.

(b) ಪ್ರಸಕ್ತ ವರ್ಷ 2014 ರಲ್ಲಿ ಯಂತ್ರೋಪಕರಣದ ಒಂದು ಭಾಗವನ್ನು ರೂ. 16,000 ಮಾರಾಟ ಮಾಡಲಾಗಿದೆ. ಅದರ ಮೂಲ ಬೆಲೆ ರೂ. 24,000 ಇರುತ್ತದೆ ಮತ್ತು ಅದರ ಮೇಲೆ ರೂ. 14,000 ಸವಕಳಿ ತೆಗೆದಿರಿಸಲಾಗಿದೆ.

(c) ಸಾಮಾನ್ಯ ನಿಧಿಗೆ ರೂ. 40,000 ವರ್ಗಾವಣೆ ಮಾಡಿರುತ್ತದೆ.

(d) ಪ್ರಸಕ್ತ ವರ್ಷದಲ್ಲಿ ರೂ. 80,000 ಲಾಭಾಂಶ ಪಾವತಿಸಲಾಗಿದೆ.

## SECTION – D / ವಿಭಾಗ – ಡ

**(1 × 15 = 15)**

Case study **(Compulsory question)** :
ಪ್ರಕರಣ ಅಧ್ಯಯನ **(ಕಡ್ಡಾಯ ಪ್ರಶ್ನೆ)** :

11. Following is the consolidated Profit and Loss Account of ABC Co. Ltd. and PQR Co. Ltd. for the year ended 31st December 2014.

(Figures are in multiples of 100)

| Particulars | ABC Co. Ltd. Rs. | PQR Co. Ltd. Rs. | Particulars | ABC Co. Ltd. Rs. | PQR Co. Ltd. Rs. |
|---|---|---|---|---|---|
| To Cost of goods sold | 2,80,000 | 1,80,000 | By Sales | 4,82,000 | 3,74,000 |
| To Administrative exp. | 36,000 | 38,000 | By Other income | 48,000 | 40,000 |
| To Selling expenses | 1,40,000 | 1,00,000 | | | |

**P.T.O.**

| Particulars | ABC Co. Ltd. Rs. | PQR Co. Ltd. Rs. | Particulars | ABC Co. Ltd. Rs. | PQR Co. Ltd. Rs. |
|---|---|---|---|---|---|
| To Other expenses | 8,000 | 10,000 | | | |
| To Loss on sale of securities | 18,000 | 16,000 | | | |
| To Income Tax | 14,000 | 16,000 | | | |
| To Net Profit | 40,000 | 50,000 | | | |
| | 5,30,000 | 4,14,000 | | 5,30,000 | 4,14,000 |

Prepare Commonsize Income Statement and answer the following questions :

(a) Which company has managed the cost efficiently?

(b) Comment on the operating efficiency of the companies.

(c) What are your comments on the profitability of the companies?

ABC ಕಂ. ನಿಯಮಿತ ಹಾಗೂ PQR ಕಂ. ನಿಯಮಿತ ಇವರು ಕ್ರೋಢೀಕರಿಸಿದ ಲಾಭ ಮತ್ತು ಹಾನಿ ಖಾತೆಗಳು 31ನೇ ಡಿಸೆಂಬರ 2014ರ ಕೊನೆಗೆ ಕೆಳಗಿನಂತಿವೆ. (ಅಂಕಿಯ ಸಂಕೇತಾಕ್ಷರಗಳು 100 ರಿಂದ ಗುಣಿಸಲ್ಪಡುತ್ತವೆ) :

| ವಿವರಗಳು | ABC ಕಂ. ನಿಯಮಿತ ರೂ. | PQR ಕಂ. ನಿಯಮಿತ ರೂ. | ವಿವರಗಳು | ABC ಕಂ. ನಿಯಮಿತ ರೂ. | PQR ಕಂ. ನಿಯಮಿತ ರೂ. |
|---|---|---|---|---|---|
| ಮಾರಾಟವಾದ ಸರಕುಗಳ ವೆಚ್ಚ | 2,80,000 | 1,80,000 | ಮಾರಾಟ ಆದಾಯ | 4,82,000 | 3,74,000 |
| ಆಡಳಿತ ಖರ್ಚುಗಳು | 36,000 | 38,000 | ಇತರೆ | 48,000 | 40,000 |
| ವಿಕ್ರಯದ ಖರ್ಚುಗಳು | 1,40,000 | 1,00,000 | | | |
| ಇತರೆ ಖರ್ಚುಗಳು | 8,000 | 10,000 | | | |
| ಹೂಡಿಕೆ ಪತ್ರಗಳ ಮಾರಾಟದ ಹಾನಿ | 18,000 | 16,000 | | | |
| ಆದಾಯ ತೆರಿಗೆ | 14,000 | 16,000 | | | |
| ನಿವ್ವಳ ಲಾಭ | 40,000 | 50,000 | | | |
| | 5,30,000 | 4,14,000 | | 5,30,000 | 4,14,000 |

ಸಾಮಾನ್ಯ ಗಾತ್ರದ ಆದಾಯ ಪಟ್ಟಿಯನ್ನು ತಯಾರಿಸಿರಿ ಮತ್ತು ಕೆಳಗಿನ ಪ್ರಶ್ನೆಗಳನ್ನು ಉತ್ತರಿಸಿರಿ :

(a) ಯಾವ ಕಂಪನಿಯು ವೆಚ್ಚ ನಿರ್ವಹಣೆಯಲ್ಲಿ ದಕ್ಷತೆ ಹೊಂದಿದೆ?

(b) ಕಂಪನಿಯ ಕಾರ್ಯ ಚಟುವಟಿಕೆ ದಕ್ಷತೆ ಬಗ್ಗೆ ನಿಮ್ಮ ಅಭಿಪ್ರಾಯ ತಿಳಿಸಿರಿ.

(c) ಕಂಪನಿಗಳು ಲಾಭ ಗಳಿಸುವ ಸಾಮರ್ಥ್ಯದ ಬಗ್ಗೆ ನಿಮ್ಮ ವಿಮರ್ಶೆ ಏನು ಇರುತ್ತದೆ?

13534/E 810

Reg. No.

# V Semester B.Com. Degree Examination, November 2015

## (Repeaters)

## Paper – I : COST ACCOUNTING – I

Time : 3 Hours] [Max. Marks : 80

***Instructions/ಸೂಚನೆಗಳು :***

*1) Attempt questions according to internal choice in each Section.*
*ಪ್ರತಿಯೊಂದು ವಿಭಾಗದಲ್ಲಿರುವ ಆಂತರಿಕ ಆಯ್ಕೆಯ ಪ್ರಕಾರ ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ.*

*2) Question No. **13** is **compulsory**.*
*ಪ್ರಶ್ನೆ ಸಂಖ್ಯೆ **13** ಕಡ್ಡಾಯವಾಗಿದೆ.*

SECTION – A/ವಿಭಾಗ – ಅ

1. Answer **any ten** of the following : **(10 × 2 = 20)**
**ಬೇಕಾದ ಹತ್ತು** ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :

(a) What do you mean by Cost Accounting?
ವೆಚ್ಚ ಲೆಕ್ಕಶಾಸ್ತ್ರ ಎಂದರೇನು?

(b) What is a Cost Unit? Give two examples.
ವೆಚ್ಚ ಘಟಕ ಎಂದರೇನು? ಎರಡು ಉದಾಹರಣೆ ಕೊಡಿರಿ.

(c) State any two advantages of Cost Accounting.
ವೆಚ್ಚ ಲೆಕ್ಕಶಾಸ್ತ್ರದ ಬೇಕಾದ ಎರಡು ಅನುಕೂಲತೆಗಳನ್ನು ಬರೆಯಿರಿ.

(d) What is Minimum stock level?
'ಕನಿಷ್ಠ ದಾಸ್ತಾನು ಮಟ್ಟ' ಎಂದರೇನು?

(e) What is Economic Order Quantity?
'ಮಿತವ್ಯಯ ಖರೀದಿ ಮೊತ್ತ' ಎಂದರೇನು?

(f) What is Time-Booking?
ಸಮಯ ವಿನಿಯೋಗ ದಾಖಲಿಸುವುದು ಎಂದರೇನು?

(g) What is the meaning of Cost Sheet?
ವೆಚ್ಚ ಪಟ್ಟಿ ಎಂದರೇನು?

(h) Name any two classifications of cost.
ಬೇಕಾದ ಎರಡು ವೆಚ್ಚದ ವಿಂಗಡನೆಗಳನ್ನು ಹೆಸರಿಸಿರಿ.

(i) What is Material Requisition Note?
ಸರಕು 'ಕೋರಿಕೆ ಪತ್ರ' ಎಂದರೇನು?

(j) What do you mean by Perpetual Inventory System?
ನಿರಂತರ ತಪಶೀಲು ಪಟ್ಟಿ ಪದ್ಧತಿ ಎಂದರೇನು?

(k) What is Overhead?
ಮೇಲು ವೆಚ್ಚ ಎಂದರೇನು?

(l) Give the meaning of Incentive Wage Plan.
ಪ್ರೋತ್ಸಾಹ ಕೂಲಿಯ ಯೋಜನೆ ಅರ್ಥವನ್ನು ನೀಡಿರಿ.

SECTION – B/ವಿಭಾಗ – ಬ

Answer **any three** of the following : **(3 × 5 = 15)**
**ಬೇಕಾದ ಮೂರು ಪ್ರಶ್ನೆಗಳಿಗೆ ಉತ್ತರಿಸಿರಿ :**

2. From the following particulars prepare cost sheet and show the sales value :

| | (Rs.) |
|---|---|
| Raw-materials as on 1.4.2014 | 1,00,000 |
| Raw-materials as on 31.3.2015 | 80,000 |
| Work in progress as on 1.4.2014 | 40,000 |
| Work in progress as on 31.3.2015 | 64,000 |
| Purchase of raw-materials | 5,00,000 |
| Direct Wages | 2,00,000 |
| Factory Overhead 80% of Direct Wages | |
| Administrative Overhead | 60,000 |
| Selling distribution Overhead | 20,000 |
| Profit 20% on Sales | |

ಕೆಳಗಿನ ವಿವರಗಳಿಂದ ವೆಚ್ಚ ಪತ್ರಿಕೆಯನ್ನು ತಯಾರಿಸಿರಿ ಹಾಗೂ ಮಾರಾಟದ ಬೆಲೆಯನ್ನು (ಮೌಲ್ಯವನ್ನು) ತೋರಿಸಿರಿ :

| | ರೂ.ಗಳಲ್ಲಿ |
|---|---|
| ಕಚ್ಚಾ ಸಾಮಗ್ರಿ 1.4.2014ಕ್ಕೆ | 1,00,000 |
| ಕಚ್ಚಾ ಸಾಮಗ್ರಿ 31.3.2015ಕ್ಕೆ | 80,000 |
| ಪ್ರಗತಿಯಲ್ಲಿ ಇರುವ ಕೆಲಸ 1.4.2014ಕ್ಕೆ | 40,000 |
| ಪ್ರಗತಿಯಲ್ಲಿ ಇರುವ ಕೆಲಸ 31.3.2015ಕ್ಕೆ | 64,000 |
| ಕಚ್ಚಾ ಸಾಮಗ್ರಿಯ ಖರೀದಿ | 5,00,000 |
| ನೇರ ಕೂಲಿ | 2,00,000 |
| ಉತ್ಪಾದನೆ ಮೇಲು ವೆಚ್ಚ ನೇರ ಕೂಲಿಯ ಪ್ರತಿಶತ 80% ರಷ್ಟು | |
| ಕಛೇರಿಯ ಮೇಲು ವೆಚ್ಚ | 60,000 |
| ಮಾರಾಟ ಹಾಗೂ ವಿತರಣೆಯ ಮೇಲು ವೆಚ್ಚ | 20,000 |
| ಮಾರಾಟದ ಬೆಲೆಯ ಮೇಲೆ 20% ಲಾಭ | |

3. From the following information calculate EOQ and Number of Orders to be placed to get annual requirements :

Annual consumption – 9000 units

Storage and carrying cost – 8% (percent) of the price

Cost of buying – Rs. 10 per order

Price per unit – Rs. 4

ಈ ಕೆಳಗೆ ಕೊಟ್ಟಿರುವ ಮಾಹಿತಿಯಿಂದ ಮಿತವ್ಯಯ ಖರೀದಿ ಪ್ರಮಾಣವನ್ನು (EOQ) ಹಾಗೂ ವಾರ್ಷಿಕ ಅವಧಿಯಲ್ಲಿ ಹಾಕಬಹುದಾದ ಖರೀದಿ ಆದೇಶಗಳನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :

ವರ್ಷದ ಅವಧಿಯಲ್ಲಿ ಉಪಯೋಗಿಸುವ ಸರಕು – 9000 ಯೂನಿಟ್‌ಗಳು

ದಾಸ್ತಾನು ಹಾಗೂ ಕೊಂಡೊಯ್ಯುವ ವೆಚ್ಚದ ಬೆಲೆಯು – 8%

ಖರೀದಿಯ ವೆಚ್ಚ ಪ್ರತಿ ಆದೇಶಕ್ಕೆ – ರೂ. 10

ಪ್ರತಿ ಯೂನಿಟ್‌ನ ಬೆಲೆ – ರೂ. 4

4. Calculate the earnings under Taylor's differential piece rate system from the given information :

Rate per hour – Rs. 6

Standard Time per unit – 10 minutes

Differential price rates to be applied :

75% of piece rate below standard

125% of piece rate at or above standard

In 8 hours of a day the actual output of the workers are :

X = 45 units

Y = 55 units

ಈ ಕೆಳಗಿನ ಮಾಹಿತಿಯಿಂದ ಟೇಲರ್ ಅವರ ವ್ಯತ್ಯಾಸದ ತುಂಡು ದರ ಪದ್ಧತಿಯಲ್ಲಿ X ಹಾಗೂ Y ಕೆಲಸಗಾರರ ಸಂಪಾದನೆಯನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :

ಸಾಮಾನ್ಯ ಪ್ರತಿ ಗಂಟೆಗೆ – ರೂ. 6

ಪ್ರತಿ ಯೂನಿಟ್‌ನ ಪ್ರಮಾಣಿತ ವೇಳೆ – 10 ನಿಮಿಷಗಳು

ಆಕರಿಸಬೇಕಾದ ವ್ಯತ್ಯಾಸ ದರ : ಪ್ರಮಾಣಕ್ಕಿಂತ ಕಡಿಮೆಯಾದರೆ ತುಂಡು ದರದ 75%

ಪ್ರಮಾಣಿತದಷ್ಟು ಅಥವಾ ಹೆಚ್ಚಾದಲ್ಲಿ ತುಂಡು ದರದ 125%

8 ಗಂಟೆಗಳ ದಿವಸದಲ್ಲಿ ಕೆಲಸಗಾರರ ಉತ್ಪಾದನೆ :

X = 45 ಯೂನಿಟ್‌ಗಳು

Y = 55 ಯೂನಿಟ್‌ಗಳು

5. Calculate Machine hour rate for machine 'A' from the following data for the month of March 2015 :

Cost the machine – Rs. 22,000

Estimated scrap value – Rs. 2,000

Estimated working life – 10,000 hours

Total hours worked during the month – 160 hours

Fixed charges allocated to the machine for the month – Rs. 240

Power used by the machine 'A' 4 units per hour at a cost of 10 paise per hour.

२४ - २०१५



'A' ಯಂತ್ರಕ್ಕೆ ಸಂಬಂಧಿಸಿದಂತೆ ಈ ಕೆಳಗಿನ ಮಾಹಿತಿಯಿಂದ ಮಾರ್ಚ್ 2015ರ ತಿಂಗಳಿಗಾಗಿ ಯಂತ್ರ ಗಂಟೆ ದರವನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ :

ಯಂತ್ರದ ಮೂಲ ಬೆಲೆ – ರೂ. 22,000

ಅಂದಾಜು ಗುಜರಿ (Scrap) ಮೌಲ್ಯ ಜೀವಿತಾವಧಿಯ – ರೂ. 2,000

ಅಂದಾಜು ಕೆಲಸದ ಗಂಟೆಗಳು – 10,000

ತಿಂಗಳಿನ ಅವಧಿಯಲ್ಲಿ ಕೆಲಸ ಮಾಡಿದ ಗಂಟೆಗಳು – 160

ಈ ಯಂತ್ರಕ್ಕೆ ತಿಂಗಳಿಗಾಗಿ ಆಕರಿಸಿದ ಸ್ಥಿರ ವೆಚ್ಚಗಳು – ರೂ. 240

ಯಂತ್ರವು ಒಂದು ಗಂಟೆ ಕಾರ್ಯ ಮಾಡಲು 4 ಯೂನಿಟ್ ವಿದ್ಯುತ್ ಶಕ್ತಿಯನ್ನು ಬಳಸುತ್ತದೆ.

ಒಂದು ವಿದ್ಯುತ್ ಶಕ್ತಿಯ ದರ – ರೂ. 0.10 ಇರುವುದು.

6. Explain in brief any five differences between Cost Accounting and Financial Accounting.
ವೆಚ್ಚ ನಿರ್ಣಯಶಾಸ್ತ್ರ ಹಾಗೂ ಹಣಕಾಸಿನ ವೆಚ್ಚಶಾಸ್ತ್ರಗಳಲ್ಲಿನ ಯಾವುದಾದರೂ ಐದು ವ್ಯತ್ಯಾಸಗಳನ್ನು ಸಂಕ್ಷಿಪ್ತವಾಗಿ ವಿವರಿಸಿರಿ.

7. From the following information relating to the consumption of chemical 'X' in an organisation :

Maximum usage – 320 litres per week

Minimum usage – 160 litres per week

Re-ordering quantity – 200 liters

Re-ordering period – 5 to 7 weeks

Emergency delivery period – 1 week

From the above information calculate the various stock levels.

ಒಂದು ಸಂಸ್ಥೆಯಲ್ಲಿ 'X' ವಸ್ತುವಿನ ಉಪಯೋಗಕ್ಕೆ ಸಂಬಂಧಿಸಿದಂತೆ ಮಾಹಿತಿ ಕೆಳಗಿನಂತಿದೆ :

ಗರಿಷ್ಠ ಬಳಕೆ – 320 ಲೀಟರ್ ಪ್ರತಿ ವಾರಕ್ಕೆ

ಕನಿಷ್ಠ ಬಳಕೆ – 160 ಲೀಟರ್ ಪ್ರತಿ ವಾರಕ್ಕೆ

ಮರು ಆದೇಶದ ಗಾತ್ರ – 200 ಲೀಟರ್

ಮರು ಆದೇಶದ ಅವಧಿ – 5 ವಾರಗಳಿಂದ 7 ವಾರಗಳು

ತುರ್ತು ಬಟವಡೆ ಅವಧಿ – 1 ವಾರ

ಮೇಲಿನ ಮಾಹಿತಿಯಿಂದ ವಿವಿಧ ದಾಸ್ತಾನು ಮಟ್ಟಗಳನ್ನು ಕಂಡು ಹಿಡಿಯಿರಿ.



P.T.O.

SECTION – C/ವಿಭಾಗ – ಕ

Answer **any three** of the following (**13** is **compulsory**) : **(3 × 15 = 45)**
ಈ ಕೆಳಗಿನ **ಬೇಕಾದ ಮೂರಕ್ಕೆ** ಉತ್ತರಿಸಿರಿ (**13 ಕಡ್ಡಾಯ** ಪ್ರಶ್ನೆ) :

8. The particulars of Receipts and Payments of material 'Z' in a Factory in January-2015 are as under :

| Date January | Particulars |
|---|---|
| 1 | Opening balance 3,000 kgs @ the rate of Rs. 10 per kg |
| 2 | Issued 600 kgs on M.R. No. – 15 |
| 5 | Purchased 800 kgs @ the rate of Rs. 15 per kg vide – G.R. No. – 9 |
| 9 | Issued 300 kgs on M.R. No. – 21 |
| 10 | Purchased 300 kgs @ the rate of Rs. 12 per kg vide – G.R. No. – 13 |
| 11 | Issued 300 kgs on M.R. No. – 27 |
| 12 | Returned to stores 25 kgs issued on M.R. No. – 15 |
| 13 | Issued 400 kgs on M.R. No. – 32 |
| 16 | Purchased 500 kgs @ the rate of Rs. 13 per kg vide – G.R. No. – 17 |
| 16 | Stores Audit Note No. 5 shortage of 10 kgs |
| 18 | Issued 350 kgs on M.R. No. – 37 |
| 20 | Returned to vendors 50 kgs from G.R. No. – 17 |
| 25 | Purchased 250 kgs @ the rate of Rs. 10 per kg vide – G.R. No. – 35 |
| 30 | Issued 200 kgs on M.R. No. – 42 |

Write Stores Ledger Account under LIFO Pricing method.

ಜನವರಿ 2015ರ ಅವಧಿಗೆ ಒಂದು ಕಾರ್ಖಾನೆಯಲ್ಲಿ ಕಚ್ಚಾ ಸಾಮಗ್ರಿ 'Z'ದ ಸ್ವೀಕರಣೆ ಹಾಗೂ ಬಿಡುಗಡೆಯು ಕೆಳಗಿನಂತಿದೆ :

| ತಾರೀಖು | ವಿವರಣೆ |
|---|---|
| ಜನವರಿ – 1 | ಆರಂಭದ ಶಿಲ್ಕು – 3000 ಕಿಲೋ ಗ್ರಾಂ ರೂ. 10 ಪ್ರತಿ ಕೆ.ಜಿ.ಗೆ |
| ಜನವರಿ – 2 | ಬಿಡುಗಡೆ – 600 ಕಿಲೋ ಗ್ರಾಂ MR ನಂ. 15 |
| ಜನವರಿ – 5 | ಖರೀದಿ – 800 ಕಿಲೋ ಗ್ರಾಂ ರೂ. 15 ಪ್ರತಿ ಕಿಲೋ ಗ್ರಾಂಗೆ GR ನಂ. – 9 |
| ಜನವರಿ – 9 | ಬಿಡುಗಡೆ – 300 ಕಿಲೋ ಗ್ರಾಂ MR ನಂ. 21 |
| ಜನವರಿ – 10 | ಖರೀದಿ – 300 ಕಿಲೋ ಗ್ರಾಂ ರೂ. 12 ಪ್ರತಿ ಕಿಲೋ ಗ್ರಾಂಗೆ GR ನಂ. – 13 |

| ತಾರೀಖು | ವಿವರಣೆ |
|---|---|
| ಜನವರಿ – 11 | ಬಿಡುಗಡೆ – 300 ಕಿಲೋ ಗ್ರಾಂ MR ನಂ. 27 |
| ಜನವರಿ – 12 | MR ನಂ. 15ರ ಬಿಡುಗಡೆ ಮಾಡಿದ ಸರಕಿನಲ್ಲಿ ಉಗ್ರಾಣಕ್ಕೆ ಹಿಂದಿರುಗಿಸಿದ್ದು 25 ಕಿಲೋ ಗ್ರಾಂ |
| ಜನವರಿ – 13 | ಬಿಡುಗಡೆ – 400 ಕಿಲೋ ಗ್ರಾಂ MR ನಂ. 32 |
| ಜನವರಿ – 16 | ಖರೀದಿ – 500 ಕಿಲೋ ಗ್ರಾಂ ರೂ. 13 ಪ್ರತಿ ಕಿಲೋ ಗ್ರಾಂಗೆ GR ನಂ. – 17 |
| ಜನವರಿ – 16 | ಉಗ್ರಾಣದ ಲೆಕ್ಕ ಸಂಶೋಧಕ ಪಟ್ಟಿ ನಂ. 5ರ ಪ್ರಕಾರ ಕೊರತೆ 10 ಕಿಲೋ ಗ್ರಾಂ |
| ಜನವರಿ – 18 | ಬಿಡುಗಡೆ – 350 ಕಿಲೋ ಗ್ರಾಂ MR ನಂ. 37 |
| ಜನವರಿ – 20 | ಮಾರಾಟಗಾರನಿಗೆ ಹಿಂದಿರುಗಿಸಿದ್ದು 50 ಕಿಲೋ ಗ್ರಾಂ GR ನಂ. – 17ರ ಪ್ರಕಾರ ಖರೀದಿ |
| ಜನವರಿ – 25 | ಖರೀದಿ –250 ಕಿಲೋ ಗ್ರಾಂ ರೂ. 10 ಪ್ರತಿ ಕಿಲೋ ಗ್ರಾಂಗೆ GR ನಂ. – 35 |
| ಜನವರಿ – 30 | ಬಿಡುಗಡೆ – 200 ಕಿಲೋ ಗ್ರಾಂ MR ನಂ. 42 |

LIFO ಬೆಲೆಯ ಪದ್ಧತಿ ಪ್ರಕಾರ ಸರಕು ಖಾತೆಯನ್ನು ಬರೆಯಿರಿ.

9. Calculate the earnings of Mr. 'P' from the following under :

(a) Halsey Plan

(b) Rowan Plan

(c) Straight Piece Rate

(d) Taylor's differential piece rate

Number of working hours per week – 48

Normal wages per hour – Rs. 9

Standard Time per piece – 20 minutes

Actual output for the week – 156 pieces

Differential piece rates applied :
80% of piece rate for below standard
120% of piece rate at or above standard

ಕೆಳಗಿನ ವಿವರಗಳಿಂದ ಶ್ರೀ 'P' ಇವರ ವಾರದ ಗಳಿಕೆಯನ್ನು ಕೆಳಗೆ ತಿಳಿಸಿದ ಪದ್ಧತಿಯಲ್ಲಿ ಕಂಡು ಹಿಡಿಯಿರಿ :

(a) ಹಾಲ್ಸೆಯವರ ಯೋಜನೆ

(b) ರೋವನ್‌ರವರ ಯೋಜನೆ

(c) ನೇರ ಉತ್ಪಾದನಾ ದರ

(d) ಟೇಲರ್‌ನ ವ್ಯತ್ಯಾಸಾತ್ಮಕ ಉತ್ಪಾದನಾ ದರ

ವಾರದ ಕೆಲಸದ ಗಂಟೆಗಳು – 48

ಪ್ರಮಾಣೀಕೃತ ಕೂಲಿ – ರೂ. 9 ಪ್ರತಿ ಗಂಟೆಗೆ

ಪ್ರಮಾಣೀಕೃತ ಸಮಯ – 20 ನಿಮಿಷ ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ

ವಾರದ ನೈಜ ಉತ್ಪಾದನೆ – 156 ಯೂನಿಟ್‌ಗಳು

ವ್ಯತ್ಯಾಸಾತ್ಮಕ ಉತ್ಪಾದನಾ ಕೂಲಿ ದರಗಳು :

ಪ್ರಮಾಣೀಕೃತ ಉತ್ಪಾದನೆಗಿಂತ ಕಡಿಮೆ – ಶೇಕಡ 80% ಕೂಲಿ ದರ

ಪ್ರಮಾಣೀಕೃತ ಉತ್ಪಾದನೆ ಅಥವಾ ಹೆಚ್ಚು – ಶೇಕಡ 120% ಕೂಲಿ ದರ

10. The following particulars relate to a company manufacturing operations :

| Production Department Overheads | Service Department Overheads |
|---|---|
| X – Rs. 800 | P – Rs. 234 |
| Y – Rs. 700 | Q – Rs. 300 |
| Z – Rs. 500 | |

The overheads of Service Departments are charged out on percentage basis as under :

| | Production Departments | | | Service Departments | |
|---|---|---|---|---|---|
| | X | Y | Z | P | Q |
| P | 20% | 40% | 30% | – | 10% |
| Q | 40% | 20% | 20% | 20% | – |

Re-apportion the service departments overheads to production departments by using "Simultaneous Equation Method".

ಈ ಕೆಳಗಿನ ವಿವರಗಳು ಒಂದು ಕಂಪನಿಯ ಉತ್ಪಾದನೆ ಚಟುವಟಿಕೆಗಳಿಗೆ ಸಂಬಂಧಿಸಿದೆ :

| ಉತ್ಪಾದನೆ ವಿಭಾಗಗಳ ಮೇಲ್ವೆಚ್ಚಗಳು | ಸೇವಾ ವಿಭಾಗಗಳ ಮೇಲ್ವೆಚ್ಚಗಳು |
|---|---|
| X – ರೂ. 800 | P – ರೂ. 234 |
| Y – ರೂ. 700 | Q – ರೂ. 300 |
| Z – ರೂ. 500 | |

ಸೇವಾ ವಿಭಾಗಗಳ ಮೇಲ್ವೆಚ್ಚಗಳನ್ನು ಶೇಕಡಾವಾರು ಈ ಕೆಳಗಿನಂತೆ ಆಕರಿಸಲಾಗುತ್ತದೆ :

| | ಉತ್ಪಾದನಾ ವಿಭಾಗಗಳು | | | ಸೇವಾ ವಿಭಾಗಗಳು | |
|---|---|---|---|---|---|
| | X | Y | Z | P | Q |
| P – | 20% | 40% | 30% | _ | 10% |
| Q – | 40% | 20% | 20% | 20% | – |

ಏಕ ಕಾಲಿಕ ಸಮೀಕರಣ ಪದ್ಧತಿ ಅನುಸರಿಸಿ ಸೇವಾ ವಿಭಾಗಗಳ ಮೇಲ್ವೆಚ್ಚಗಳನ್ನು ಉತ್ಪಾದನಾ ವಿಭಾಗಗಳಿಗೆ ಮರು ಹಂಚಿಕೆ ಮಾಡಿರಿ.

11. From the following information, calculate earnings of Ravi, Raju, Ramu and Rambo under time wages and piece rate system :

Rate per hour – Rs. 20

Piece rate is 20% of above time rate

Expected output 48 units per hour

Actual production in a day of 9 hours

Ravi – 342, Raju – 350, Ramu – 450 and Ramb – 460 units.

ಕೆಳಗೆ ಕಾಣಿಸಿದ ಮಾಹಿತಿಯಿಂದ ರವಿ, ರಾಜು, ರಾಮು ಮತ್ತು ರ್‍ಯಾಂಬೊ ಇವರ ಗಳಿಕೆಯನ್ನು ವೇಳೆಯ ಕೂಲಿ ದರ ಹಾಗೂ ತುಂಡು ದರದ ಪದ್ಧತಿಗಳಲ್ಲಿ ಕಂಡು ಹಿಡಿಯಿರಿ :

ಪ್ರತಿ ಘಂಟೆಗೆ ದರ : ರೂ. 20

ತುಂಡು ದರ : ವೇಳೆಯ ದರದ ಶೇಕಡಾ 20 ರಷ್ಟು ಹೆಚ್ಚಿಗೆ

ಅಂದಾಜಿಸಿದ ಉತ್ಪಾದಿಸಿದ ಸರಕುಗಳು : ಪ್ರತಿ ಘಂಟೆಗೆ 48 ಯೂನಿಟ್ಟುಗಳು

ನಿಜವಾದ ಉತ್ಪಾದನೆ 9 ಘಂಟೆಯ ಒಂದು ದಿನಕ್ಕೆ

ರವಿ – 342, ರಾಜು – 350, ರಾಮು – 450 ಮತ್ತು ರ್‍ಯಾಂಬೊ – 460 ಯುನಿಟ್ಟುಗಳು

12. Define Cost Accounting. Explain its advantages and limitations.
ವೆಚ್ಚ ಲೆಕ್ಕಶಾಸ್ತ್ರದ ವ್ಯಾಖ್ಯೆ ನೀಡಿರಿ. ಅದರ ಅನುಕೂಲತೆಗಳನ್ನು ಮತ್ತು ಅನಾನುಕೂಲತೆಗಳನ್ನು ವಿವರಿಸಿರಿ.

13. Company has received 3 quotations for a material as follows :

(a) Rate Rs. 240 per unit with assurance to supply any quantity.

(b) Rate Rs. 220 per unit plus Rs. 1,50,000 fixed charges for any quantity.

(c) Rate Rs. 225 per unit with a condition that :

(i) For order below 60,000 units extra charges of Rs. 25 per unit.

(ii) For order of 60,000 or above units no extra charges.

Advise the purchasing department about the selection of quotations if the ordering quantity is 50,000 units.

Show the necessary calculations.

ಒಂದು ಕಂಪನಿಯು ಈ ಕೆಳಗಿನ ಮೂರು ಕೊಟೇಶನ್ (Quotations)ಗಳನ್ನು ಪಡೆದಿದೆ :

(a) ದರ ರೂ. 240 ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ, ಎಷ್ಟೇ ಗಾತ್ರದ ಸರಕು ಪೂರೈಸುವ ಭರವಸೆಯೊಂದಿಗೆ

(b) ದರ ರೂ. 220 ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ – ಸಂಗಡ ರೂ. 1,50,000 ಸ್ಥಿರ ವೆಚ್ಚವನ್ನು ಯಾವುದೇ ಗಾತ್ರದ ಪೂರೈಕೆ ಕೊಡಿಸಬೇಕು.

(c) ದರ ರೂ. 225 ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ – ಆದರೆ ಶರತ್ತು ಏನೆಂದರೆ –

   (i) 60,000 ಯೂನಿಟ್‌ಗಳಿಗಿಂತ ಕಡಿಮೆ ಆದೇಶಗಳಿಗೆ ರೂ. 25 ರಂತೆ ಪ್ರತಿ ಯೂನಿಟ್‌ಗೆ ಹೆಚ್ಚುವರಿಯಾಗಿ ನೀಡಬೇಕು.

   (ii) 60,000 ಯೂನಿಟ್‌ಗಳು ಅಥವಾ ಹೆಚ್ಚಿನ ಆದೇಶಗಳಿಗೆ ಹೆಚ್ಚಿನ ಆಕರಣೆ ಇರುವದಿಲ್ಲ.

ಆದೇಶ ಗಾತ್ರವು 50,000 ಯೂನಿಟ್‌ಗಳು ಇದ್ದಾಗ ಯಾವ ಪೂರೈಕೆದಾರನ ಕೊಟೇಶನ್ (Quotations) ಆಯ್ಕೆ ಮಾಡಬೇಕೆಂದು ಖರೀದಿ ವಿಭಾಗಕ್ಕೆ ಸಲಹೆಯನ್ನು ನೀಡುವಿರಿ. ಅವಶ್ಯಕ ಲೆಕ್ಕಾಚಾರವನ್ನು ತೋರಿಸಿರಿ.

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Friday

Unit -2

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# Banker And Customer

Meaning ->

Banker is applicable individual person who perform's banking business.

Customer ->

A customer of bank means a person who has and account with the bank.

Defination ->

According to John paget to constitute a customer they must be sum recognisable courses or habit of dealing in the nature of regural banking business according to this view a person can be called a customer.

If he statifies to conditions

1. customer must have an account with the bank.
2. There must be sum recogisenable course or habit of dealing between a customer and banker.

Relationship between Banker and Customer

The relationship between the banker and customer starts as soon as customer opens and account in the bank by depositing sum money the relationship between banker and customer is like two sides of point such relationship between banker and customer may be divided in two parts

1. General Relationship
2. Special Relationship

1. General Relationship

The general relationship between banker and customer further divided into two types.

a. Primary relationship

b. Subsidiary relationship

a. Primary relationship

The primary relationship between a banker and customer begins as soon as customers open an account with the bank by depositing sum money. Such relationship is also known as contractual relationship between banker and customer. Contractual relationship means agreement between two parties. Such relationship will be continued and remain valid till the customer operate the account.

Primary or fundamental relationship between banker and customer which is that of debitor and creditor or borrower and lender.

Debitor and Creditor

When the banker receives the money from the customer in the form of deposit so it is property of the banker. The banker can use such money in any way he likes.

When the banker accepts deposit from the customer in that case banker is not a agent the banker is not trustee of customer money.

When the customer has taken loan

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from bank in such case banker becomes creditor & customers becomes the debitor.

Features of Debitor and creditor

Following are the special features of debitor and creditor.

1. Banker is a dignified Debtor

Banker receives money from the customer in the form of deposits so banker is called a dignified debtor.

Banker is dignified debtor for the following reasons.

* Generally barrower goes to creditor for borrowing but in case of banker as a barrower he never goes to the customer or a creditor on the other hand customer creditor himself goes to banker for giving the money in the form of deposits.
* generally amount barrowed is called debt. But in case of banker the banker barrows money from the customer it is nothing but a debt which is called as dignified deposit.
* Generally debitors is bound to repay the dept to his creditor. But in case of banker he is required to repay the deposits to the customers only when demand is made.
* Normally debitors goes to the creditor to make the important repayment of his debt but in case of banker the customer himself goes to the bank for receiving his dept.

2) Customer is only General creditor
customer having some deposit in the bank is a creditor of the bank he's only a general creditor and not a secured creditor of the bank this is because he do not get any charges on any of the assets of the bank by keeping a deposit in the bank. But a banker can be called as secured creditor when he grant a loan on advances to his customers against some security.

3) demand is necessary for repayment
Bank accepts deposits from the customers such deposits are repayable their must be a demand from customer for repayment of deposit due from banker to the customer.

4) Demand should be made at proper place.
The customer should make the demand from repayment of his deposit at the proper place that is at the same branch of the bank when his account exist it. Bank will not make repayment other than the branch bank where the account his maintain.

5) Demand should be made on business day & during business hours
According to banking regulation act payments are made within the working hours and working day for repayment of deposits demand should be maid by the customer on working day and within the working hours.

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c) Demand should be in proper form

The customers demand for the repayment of his deposits should be made in proper form this means that the demand for the repayment should be made in return or by providing printed cheque. A banker will not refund of return the customers deposit on the basis of customers oral or telephone orders.

b. Subsidiary Relationship

It been banker and customer Basically banker is a debitor of his customers in respect of the amount deposited with him by the customer but in certain circumstance banker act as Bailee, trustee and agent for his customers.

The subsidiary relationship between banker and customer are of 3 types

1. Bailee and Bailor ->

Bailee is a person to whom some goods are entrustured by the bailor for some specific purpose and who is required to return the same goods to the bailor after the purpose is over when banker receives valuables and securities for safe custody he becomes bailee and the customers a bailor. The person who gives the goods to other is called bailor and the person who receive the goods is called bailee. As a bailee there are some duties and liabilities to the bailor.

i) As a bailee banker is required to keep the safe custody of articles very carefully with him with reasonable care.

2. Bailee is required to return the valuables and securities

to the customer whenever demand is made.

3. In case if a banker fails to take reasonable care in the preservation of safe custody articles and the customer suffer a loss as a result so the banker becomes liable to compensate the customers losses.

2. Trustee and Beneficiary :-

Trustee is one who holds property for the benefit of somebody else who his called beneficiary. the benefit occurs not to the trustee but to the beneficiary. it is the duty of the trustee to operate the trust property for the benefit of beneficiary.

But in certain circumstances a banker act as truestee of the customer these circumstances are as follow:

* When the customer deposits a some of money for a specific purpose then the banker becomes trustee to the extent of that amount.
* Banker Becomes trustee inrespect of the valuable and securities deposited with him for safe custody.

3. Principle and Agent

Basically the bank act as agent on behalf of its customers. an agent is one who is employed to do an act for another or to represent another in dealing with third party.

A banker act as agent of his customer in several circumstances.

* Buyer or seller of securities
* collecting cheque, bills, dividend & interest etc.

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* Pay bills, insurance premium rent etc. on behalf of his customers as an agent.

2. Special Relationship between banker and customer

Special relationship between banker and customer refers to the mutual duty or obligation and rights of the banker and customers.

Following are special relationship between banker and customer.

1. Obligation to honour the cheque

Banker receives the deposits from the customer those deposits are repayable as and when demanded by the customers the demand should be in proper form and at proper time that is customer should present the cheque to the banker for the repayment of his deposits when the customer present the cheque it is a duty or obligation of the banker to honour the cheque and pay the money.

In India it is a statutory duty of the banker to honour the cheque as it is imposed by section 31 of the Indian negotiable instrument act of 1881.

According to Section 31 of Indian negotiable act 1881 there are some conditions to honour the cheque.

1. Availability of sufficent funds
2. Appricability of funds
3. Banker duty required to pay
4. Presentation of cheque within a reasonable time
5. No legal Bar

1. Availability of Sufficient funds in customers Account
There must be sufficient fund available in customers account then only the banker will honour the cheque if the balance of the customers account is insufficient the banker can dishonour the cheque issue by the customers the banker also cannot make a pas payment of the cheque to the extent of the balance available in order to determine the sufficient funds in the customers account only the funds available in the particular account on which the cheque is drawn should be taken into account the funds available in another account of the customer should not be considred for payment.

2. Applicability of funds
The customers funds in the hands of the banker that is the credit balance to his account must be properly applicable to the payment of his cheque presented.

3. Banker duly required to pay.
Banker is required to pay the amount to the customer that is the banker will honour customers cheque if it is correct in all respect. Such as date, amount in words, and number signature which is duly signed by drawer.

4. Presentation of cheque within the reasonable time
Cheque should be presentated within reasonable time for payment in India cheque should be presented within 3 months from the date of issue so it is a reasonable time to present a cheque.

5. No legal Bar

There must no be any legal bar preventing the payment of the customers cheque if there is legal bar then the banker can dishonour is customers cheque.

Ex- If a garnish order as been issue by the court attaching the funds in a particular account cheque drawn against such account must be dishonoured.

Dishonour of Cheque

Bank can dishonour the customer cheque for some reasons like insufficient balance stale cheque post dated cheque, on receipt of garnish order.

Wrongfull dishonour of customer cheque

Dishonour of customer cheque by bank without giving the proper reason is non as wrongfull dishonour of cheque.

2. obligation to maintain the secrecy of customer account

It is a implied duty of the banker to maintain the secrecy of the customer account banker should not disclose the information of customers account to any outsiders such as how much is deposited withdrawn how many cheques are collected and how many cheque's are issued loans etc. if the banker disclose the information of customers account to any outsiders it may affects the customers reputation or goodwill.

## * Circumstances *

Following are the circumstances through which banker's can disclose the details of customer's account

1. Where there is an express consent of the customer.
2. Where there is an implied concent of the customer
3. Where disclosure is made under compulsion of low
4. Disclosure in public interest
5. Where Banker's own interest request
6. Where credit information required by other Bank

1. Where there is an express consent of the customer

Express considred consists when the customer as instructed is banker in return to give some or all the details of his account to authorised people such as lawyer, manager, secretary etc In such case banker is justifie in disclosing information to such authorised person.

2. Where there is an implied consent of customer

customer make you the name of the banker as a reference or where the customer take loan from the banker on the basis of guarantee giving in such case it is implied that the customer has authorised the banker for furnishing necessary information to the third party on his request.

3. Where disclosure is made under compulsion of low

The banker can disclose the customers account to public authorities on the basis of low of the country.

a. court order under the banker books evidance act 1891
b. Under the income tax act 1961
c. companies act 1956
d. Under the RBI Act 1934
e. Banking Regulaction Act 1949
f. Disclosure to police

a. Court order under the banker books evidance act 1891
A court may order a banker to disclose information relating to a customers account. It is a duty of banker to provide the required information relating to customers account in court.

b. Under the income tax act 1961
As per section 131 and 133 income tax offices a power to get the information of the customers account for the purpose of accissment.

c. Under the companies act 1956
As per the section 235 and 237 the central government appoint an insepcter to investigate affairs of any joint stock company in such case it is duty of banker to provide the account information.

d. Under the RBI Act 1934
As per this act it is a duty of every bank to furnish credit information to reserve bank.

e. Under Banking Regulaction Act 1949
As per this act every bank a

Date

to submit the annual return of all accounts which have not been operated for the last 10 years.

f. Disclosure to police

Under criminal procedure code a banker is bound to disclose the information to any police or CID officers the particular of transactions relating to any customer for the purpose of investigation.

4. Disclosure in Public Interest

When a banker comes to know from his customers account that the customer is dealing with any enemy country in such case it is a duty of banker to disclose the information of the customers account.

5. Where Bankers own Interest request

Banker can disclose the information of customers account to the outsiders for the protection of his interest.

Ex- When a banker want's to take legal action against is customer for the recovery of the loan granted to him so banker is permitted to disclose the details of his customers account. To his lawyer and the court.

6) When credit information required by other Bankers

Any information required by fellow bankers in such case bank can exchange the credit information about their customers account the information must be done in strict confidence such as the information should be given in general statement but not in figures.

The bankers must be extremely carefull while answering the enquires if the report is so favourable banker may be charge with misrepresentation on the quanracy if it is too unstatisfactory and action might be brought by the customer.

* Consequences of Improper Disclosure *

The following are the effects of improper disclosure of information.

1. Liability to the customer
2. Liability of the banker to the third party

1. Liability to the customer

If the banker give information about the customers account without the permission of the customer in such case banker is liable to pay compensation to the customer.

2. Liability of the banker to the third party

Generally banker is not liable to pay the compensation to the third party because there is no any contract between banker and customer but if the information given by the banker to the outsider is false in such case Banker is liable to the third party for the payment of compensation.

3. Banker Right to lien →

Lien → lien is the right to retain property belong to another untill that due is repaid by the debitor.

lien are the classified into 3 types.

* Particular lien
* General lien
* Negative lien

* Particular lien

It is a right of creditor to retain particular property untill the debt is repaid. This kind of lien is enjoyed by 'craft's man. For ex -> Goldsmith, tailor, etc.

* General lien

It is a right of creditor to retain a any of the debitors property with him untill he receives full debt. It is enjoyed by bankers, factors etc.

* Negative lien

Bankers do not get right to retain any assets of the barrower the barrower gives a declaration to the banker that the assets maintained thair are free from any charges.

4. Right to set off

The right to set off is legal right which helps banker to combining to accounts in the same name of the customer and to adjust debit balance in accounts with the credit balance in other account. It is also called as right to off combining accounts.

Conditions for right to set off.

1) Debt due right to set off can be excersied by the banker Subjected to the following conditions:

1) Debt due
2) Debt must be certain
3) Account in the same name & same capacity
4) Notice must be given to the customer

* Debt due
Banker can excersied right to set off on debt which becomes due and not in respect of contingent debt.

* Debt must be certain
It means amount on which the right to set off is excersied must be clearly stated in the notice.

* Account in the same name and same capacity
The debt are due between the same parties and in the same right the same right means that the capacity of account holder in all accounts involved in the same and that is the credit balance held by him in one account is the same capacity in which the debit balance stands in another account.
So if the account are held by the same person in different capacity in such case banker cannot exercised his right to set of.

* Notice must be given to the customer
Banker should take an agreement form

the customer authorising him to combine the account without notice in the absence of such an agreement reasonable notice must be given to the customer.

$ Garnishee order

It is an order issued by the court to the banker to stop the payment of money belongs to particular customers account generally court issues such order on the request of creditor when the debitor fails to pay the money to creditor and creditor comes to know that debitor has an account in the bank in such case creditor may apply to the court to issue a garnishee order on the debitors bank when this order is receive banker will immediately suspend the operation of the customers account and banker cannot may any payments.

There are three parties of garnishee order

1. Judgement creditor
2. Judgement debitor
3. Garnishee

* Judgement creditor → The creditor whose request an order is issued is called judgement creditor

* Judgement debitor → The debitor whose account is suspended is called judgement debitor

* Garnishee → The banker to whom the order is given is known as Garnishee

Garnishee order is issued by the court in two parts
1. order Nisi
2. order Absolute

1. order Nisi ->
It is the first order issued by the court to the banker this order want's warn's the banker not to pay the funds belonging particular customer untill further orders are issued.

2. Order Absolute ->
It is a final order issued by the court to the banker when the banker fails to appear before the court and do not rise any objection in the court in such case bank will issue the final order.

6. Banker's Right of Appropriation of payment
If the customer as two or more current accounts in the bank an if he deposit is funds into his account it the duty of the customer to inform the bank to which account the money should be deposited if the customer fails to inform then the no question arises before the bank to which account the money should be credited.
In India rules regarding appropriation of payments are contained in section 59 & 61 of the contract act 1872.

1. Appropriation by the debitor
According to section 59

when the money is deposited into the bank by the debitor it is the duty of the debitor to inform the banker that money is applie to particular debt So banker should apply the money received according to the direction of the customer.

2) Appropriation by the creditor

According to section 61 when money is paid by the customer to the banker but debitor dose not make any specific appropriation in such case bank can appropriat and apply the payment even to the time barred debt.

If the customer donot make any specific appropriation for the payment made by him at the time of making the payment banker should appropriate the payment to any debt but banker must informed the customer how he has appropriated the payment.

7) Banker Right to charge Interest

Banker has right to charge interest on loans and advances given to the customer has per custom and rules customer has to pay the interest once in a year quater (3 months) if the customer fails to pay the interest quarterly then the interest is added to the principle, and for the next peariod interest is charged on total principle.

8) Banker Right to charg Incindental Charge

Banker as right to charge commission or incidentals charges for services rendre

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to customer.

Nowadays bank charge incidental charges for the current account because the operation of current account is expensive.

a) Law of limitation

According to the Indian limitation act 1963 debt becomes time barred if it is not repaid within the 3 years from date of contract it means the creditor cannot take any legal action against the debitor for the recovery of amount due from the debitor after the expiry of 3 years from the date of contract.

## 1.3 COMPONENTS OF BUSINESS ENVIRONMENT

Environment of a business can be basically classified into internal and external. Internal factors of environment are conditions existing within the organization. External factors of environment are conditions prevailing outside the business organizations. Following diagram gives clear picture of business environment and its various constituents.

**BUSINESS ENVIRONMENT**

- **(A) Internal**
  1. Organization Structure and Management
  2. Value System
  3. Mission and Objectives
  4. Resource
     - A. Human
     - B. Financial
     - C. Physical
  5. Marketing Factor
  6. R&D Facilities
- **(B) External**
  - **(I) Micro**
    1. Suppliers
    2. Customers
    3. Employees
    4. Competitors
    5. Marketing Intermedieries
    6. Publics
    7. Other Entities
  - **(II) Macro**
    - **a) Economic**
      1. Internal
      2. International
    - **b) Non-Economic**
      1. Political
      2. Legal
      3. Socio-cultural
      4. Demographic
      5. Technological
      6. Natural
      7. Geographical
      8. Sectorial

The above table gives clear picture of classification of environment into internal and external. External environment is further classified into economic and non-economic environment.

## A) INTERNAL ENVIRONMENT

Internal environment refers to factors or conditions that are existing within the business organization. It is the environment, which is designed by the organization to suit its convenience. It is with in the limits and control of each organization. Philip Kotler defines it as "all those factors in the enterprise that affect the enterprise from within itself." Internal environment can be strength or weakness of a organization. Business has to plan and design its internal environment to enhance its strength and minimize its weakness. Internal environment should be designed keeping the restrictions of external environment.

Internal environment has following components :

**1) Organization Structure and Management :**

Size of the organization plays a vital role in internal environment. Bigger the size of organization, more organized and systematic should be its structure. The flow of authority from top of bottom i.e. from managers to the workers at lower level and flow of responsibility from bottom to top must be clearly defined. Organization must be divided into departments, each department must be clearly defined with its objectives and functions. The overall picture of organization should be clear. There should be free flow of communication to exchange orders, ideas, doubts, plans etc. that will increase strength of organization.

Management of organization may consist of board of directors, managers and their subordinates. They are the people who plan and execute the business policies. Strength of management lives in the people who are qualified, professional and competent, Managerial people must have necessary support from the shareholders and board of directors to implement plans and programmes of business.

**2) Value System :**

Organisations should be based on values like fair practices, honesty, transparency responsibility towards society, customers, employees, nature etc. This will increase credibility and goodwill of the organization. Ventures like

Tata, Infosys, Wipro, Reliance etc. have good name and prosperous business due to their value system. Values of business must uphold culture and tradition of society and they must be ethical. Such firms will be accepted by the society and they will get necessary support from customers, government, employees and society.

### 3) Vision, Mission and Objectives :

Business of a firm is defined in terms of vision, mission and objectives. Vision is the broad horizon it states larger goals, ambitions, desires etc., which the organization would like to achieve over a period of time. Eg. These can be the market leader, expand globally etc.

Visions are expressed in mission. A mission statement expresses how the vision is going to be realized. Mission expresses the methods and procedure as to how the vision is going to be realized.

Objectives are more specific, the vision and mission statements are defined in objectives. Objectives state nature of business, business targets like profits, sales, market share etc., which the firm wants to achieve.

Vision, mission and objectives define the organization's goals and functions. These should be defined after evaluating the conditions of external environment. Vision, mission statement and objectives must be realistic, reachable and progressive. This is possible when the organization determines them based on constraints of external environment. Objectives and mission of the firm must be in tune with market conditions, customers expections and within the regulations of government then only firm can achieve desired success. Environment in which the business functions and conditions of the busines are dynamic and ever changing. Firm must define its objectives in tune with changing environment that gives directions in which firm has to move. for example, Vision, Mission objectives of Reliance Industries Ltd., are as follows.

**Mission :**

Through sustainable measures, create value for the nation, enhance quality of life across the entire socio-economic spectrum and help spearhead India as a global leader in the domains where we operate.

**Vision :**

- Create value for all stakeholders
- Grow through innovation
- Lead in good governance practices
- Use sustainability to drive product development and enhance operational efficiencies
- Ensure energy security of the nation
- Foster rural prosperity

**Values :**

Our growth and success are based on the ten core values of Care, Citizenship, Fairness, Honesty, Integrity, Purposefulness, Respect, Responsibility, Safety and Trust

**4) Resources :**

**A) Human Resources :** Real strength of any organization is its human capital or employees. Success of any organization depends on contributions made by its workers. Committed, skillful, quality workforce is the asset of any organization. Firm must have HRM (Human Resource Management) policies that develop dynamic people. Scientific selection, regular training adequate incentives will help to have dynamic man power. Change in business conditions, change in technology can be easily tackled if the firm has support of qualified and dedicated staff. Workers should be trained, motivated and developed as strength of organization to face conditions of external environment.

**B) Financial Resources :** Finance is the life blood of any organization. Firms must have adequate capital of its own. On the strength of its networth (equity + reserves) it can borrow in the form of debentures and loans. Adequate capital helps the firm to install latest technology, develop competitive marketing strategy, acquire skilled manpower etc. Firm must have proper financial planning that helps to make adequate profits. It must help to maximize wealth of equity holders in the form of better dividend and increased market price. Adequate reserves should be built, which can help for growth, expansion and innovation. Capital whether own or borrowed must be economically used without allowing for wastage. Efficient financial planning will increase profitability and performance of organization.

**C) Physical Resources :** Quality of the product and productivity of the business depends on infrastructure and technology. Firms that install modern plant and machinery, use quality raw material, design ergonomic working conditions provide adequate facilities like canteen, craeche, air conditioners etc., that can help for increase in quality and speed of work. It also results in decrease in cost due to reduced wastage. This will make the organization and workers competitive. Firms which forsee changes that are taking place in technology and other facilities and install them in their factory can become competitive. Business must continuously monitor the change taking place in external environment and incorporate such change in business. This will make the organization dynamic and competitive.

**5) Marketing Factors :** Purpose of any business is to satisfy customers by creating goods desired by people. Profitability of the business is also dependent on that.

Goods and services must reach the hands of customers then only, customers wants are satisfied and firm's profits are realized. Business must have effective marketing strategy to develop goods desired by people and deliver those goods through intermediaries and distribution channel. Business must have independent marketing department managed by competent people. Product designing, advertising, distribution channel etc. should be determined by marketing department depending on what the people accept. Firms must practise CRM (Customer Relationship Management) to regularly update knowledge of the customer i.e. his reqirements of products and services. Every competitor has his own marketing strategy. Firms have to create their own unique marketing strategy to counter the strategy of competitor.

**6) Research and Development (R&D) :** R & D Activities help an organization to continuously innovate itself. New products, marketing strategies can be developed through R&D. People always expect new things and desires of people always keep on changing. Utility theory also says that as we go on consuming more and more of a thing, the satisfaction level of it goes on decreasing. Satisafction level can be increased by innovating new things. Continuous R&D helps to innovate new products and services desired by the market. Firms which spend on R&D can be in tune with external environment, their products and policies will be in tune with expection of market. Such ventures will always make profit by offering new products and they will be dynamic.

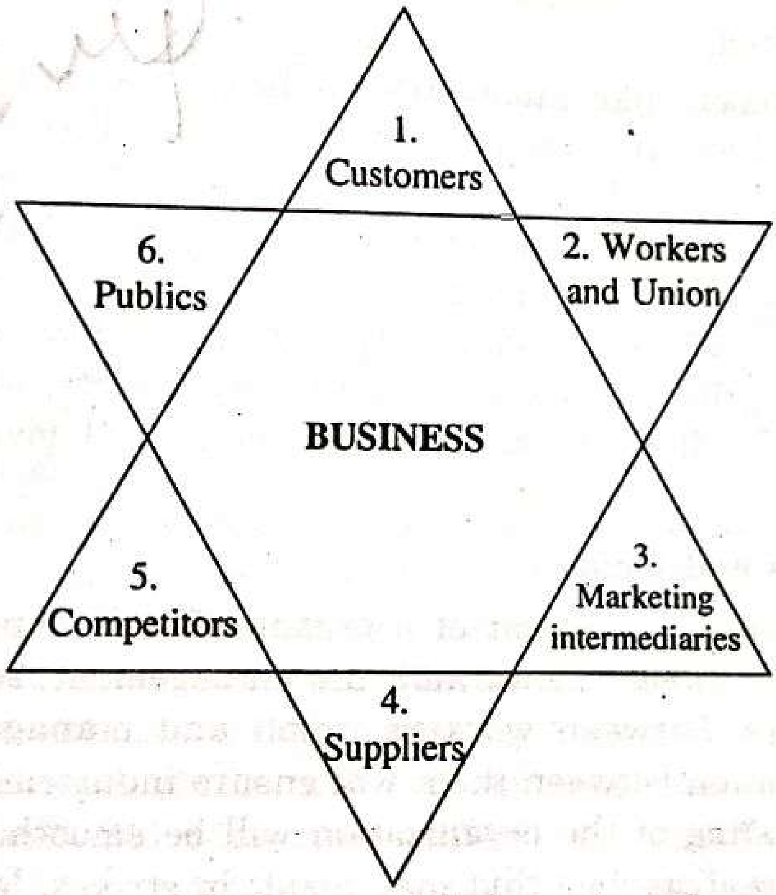
## B) External Environment

Environment which is outside the business unit and that is not in control of a firm is classified as external environment. External environment is given, firm cannot alter the conditions of external environment. External environment provides opportunities of business and threats firm has to face. It sets the limitation or restriction within which a firm has to operate.

Business environment is normally referred to factors of external environment. These factors are further classified into I) Micro II) Macro.

## I) Micro Environment :

Environment factors, which are specific and particular to each firm are micro environment factors. They are not universally applicable to all firms, their conditions will vary from one firm to the other. Following diagram shows the picture of micro environment.




Customers, workers and their trade union, marketing intermediaries, suppliers, competitors and publics are 'Important players' in micro environment. Phillip Kotler defines, "the micro environment consists of the actors in the companies immediate environment that affect the performance of the company. These include, suppliers, marketing, intermediaries, competitors, customers and the publics." The role and importance of these players is as follows :

### 1) Customers :

Business is also defined as creation of customers. Firms which can create and sustain the customers are successful organizations. Profitability of the firm is dependent on the customer's support and loyalty. Firm must have confidence of the customers, which is possible when a firm manufactures the product that are to the expectation of customers. Every firm normally undertakes 'market research' to know the likes and dislikes of product, its price, method of marketing etc. By scanning the market environment, firm can understand the requirement of the customers and accordingly products can be manufactured and marketed.

Customers like innovative products, new marketing methods that are convenient to them. Firms that experiment these things can achieve the desired success. Customer is described as king or god, organisations that understands this and make necessary arrangement to satisfy him will be the winner. Quality, reliability, service, reasonable price etc. are some of the key aspects, which a firm should adhere to win the confidence and loyalty of customers.

### 2) Workers and Union :

Business environment of a organization is dependent on relation between the workers and management, between the workers, between workers union and management. Cordial relation between them will ensure industrial peace and functioning of the organization will be smooth. If the relations are disturbed that may result in strikes, lockouts go slow attitude etc., which will hamper production and profitability of the firm. Frequent labour problem will also affect the image of firm.

Developing countries like India and China have labour laws that are in favour of workers and union. 'The hire and fire policy', which is adopted in western economies cannot be exactly adopted in country like India. Workers unions are powerful and they have strong bargaining power. If the workers do not co-operate with the management, if they go on strike, bunks etc. it will affect the performance of business. Under such conditions business should cultivate good relation with workers and their union. Forming of workers union, quality circles etc. can help to build good relation between workers and management.

Business which studies the environment of workers and their union and understands their problems can be successful by framing its labour policies so as to maintain friendly relationship. Cordial relationship with workers will help for the progress of organization.

## 3) Marketing Intermediaries :

Marketing intermediaries are the important link between the business and its customers. Products are manufactured for consumption and it is middlemen who take the product to the doors of customers. Phillip Kotler defines the intermediaries as "firms that aid the company in promoting, selling and distributing its goods to final buyer." Success in marketing depends on selection of intermediaries. Business must study the network of distributors and select those intermediaries who can push the product into the market aggressively. Business must also consider the incentives that are to be offered to these intermediaries i.e. commission, gifts etc. The cost of distribution must be compared with profits expected. Performance of the distributors must be continuously monitored. Business must continuously update the list of distributors, retain those distributors who are helpful in marketing its products, select new channel and distributors who can increase the marketing strength of business. Firm must develop alternative channel of distribution and distributors who can increase the marketing strength of business. Success in selecting right kind of channel of distribution and network of distributors helps a firm to achieve targeted sales. Firms success in marketing depends on scanning the environment of intermediaries.

4) Suppliers :

Raw materials and supplies are major element in cost of production. Major portion of working capital is locked up in current assets. There should be economic and effective use of supplies. Excess investment in raw materials may lead to wastage of raw materials and increase in the interest burden. Lower investments may affect production process due to shortage of raw materials. Firm must maintain adequate stock of raw material that ensures smooth production process and also minimizes cost of inventory. This depends on firms relationship with suppliers of raw materials. Business must have good relationship and regular contract with supplier. It must study the environment of suppliers and select those suppliers who can assure regular and timely supply of quality raw material. Business must continuously monitor the list of suppliers. It must update the list of suppliers who are supportive to the policies of the firm.

5) Competitors :

Competition is part and parcel of every business. In competition it is said 'survival of the fittest'. Every competitor has his own strategy to win over the customer. Business must scan the environment of competition to know the strategies adopted by each competitor. This will help the business to form counter strategy to popularize its product and business.

Competition is both an opportunity and a threat. Firms which study the environment of competition and prepare their business to face the competition will reap the benefits of competition. On the other hand firms which fail to evaluate the environment of competition will not be in a position to prepare effective strategy to face competition.

6) Publics :

Publics are the different group of people who take interest in the happenings of society, they protect the common interest of people and society.

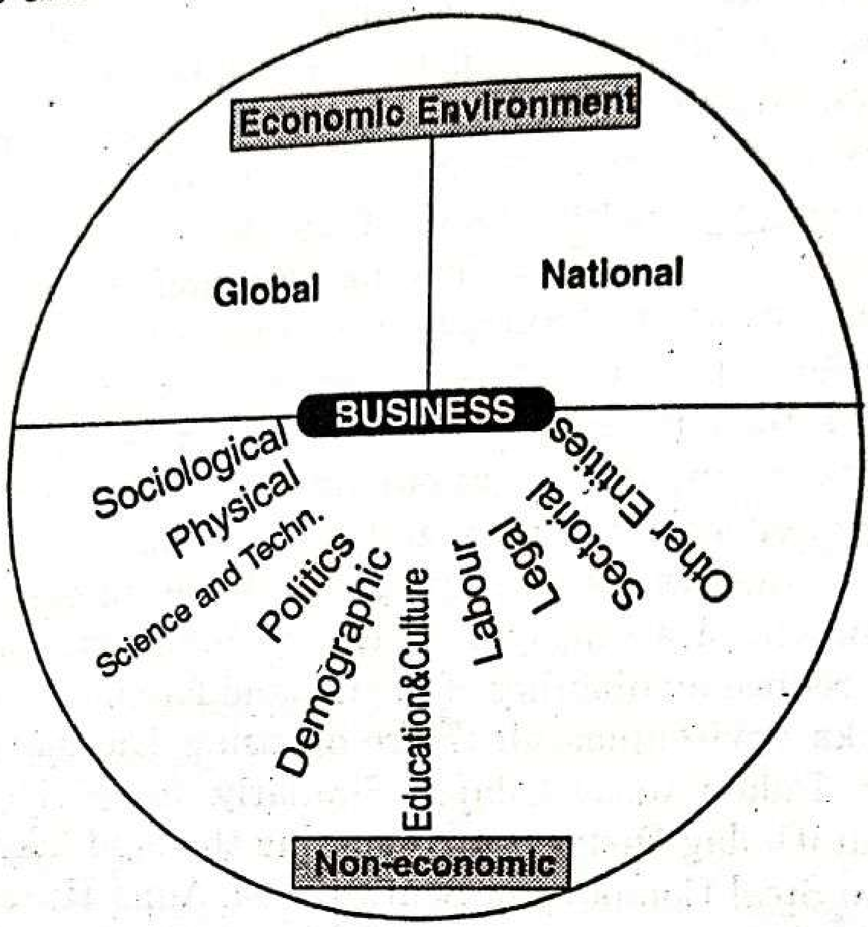
Phillip Kottler defines public as "any group that has an actual or potential interest in or impact on an organizations ability to achieve such interest" Publics watch programmes and practices of business. They will ensure that it is not functioning against the interest of nature, customers, employees etc. If the act of business is harming the interest of society, they oppose such moves. They create mass movement against acts of business by writing in newspapers, news in TVs, holding Dharana, Satyagrahas. They create awareness amongst the public regarding issues which affect public interest. Medha Patkar of NBA (Narmada Bachao Andolan) is creating mass movement against increasing the height of dam on Narmada river and its adverse effects on ecology and people. Similarly in West Bengal residents of Nandi gram have opposed the establishment of a unit of Tata car manufacturing as it is going to reduce availability of fertile land for cultivation. In Karnataka environmentalists are opposing Thermal power plant at Tadadi (near Udapi). Similarly, the residents of Halligudi (Gadag District) are opposing the land taker over for Posco Steel Company of South Korea. Anna Hazare has created a mass movement against corruption in public places.

Business to be successful must study the role of publics in society. It must ensure that the business practices do not harm the interest of society. Studying the environment of publics helps a business to decide its objectives, business practices and policies in tune with demands of society.

## II) Macro Environment :

Macro factors of environment are the larger issues which affect every business. These external factors of environment are universally applicable to every firm. Their conditions affect every firm in the same way, Phillip Kotler defines macro environment as "larger societal forces that affect all the actors in the companies micro environment – namely the demographic, economic, natural, technical, political and cultural forces."

Macro environment is classified into economic and non-economic environment. Following diagram gives clear picture of Macro Environment.




As the above diagram shows, macro environment is classified into Economic and Non-economic environment.

It is difficult to segregate economic and non-economic environment. "Economic environment of business exercises a strong influence on the non-economic environment of business as the non-economic environment can influence the economic environment. The economic environment is thus both exogenous and endogenous. It is determined as well as determining."

## A) Economic Environment

Business is an economic activity, economic factors have strong influence on the business. Economic variables like demand, supply, price, market, income, cost of production, competition determine the functioning of business.

Economic environment is further classified into :

I) International Environment
II) National Environment.

## International or Global Economic Environment :

'Vasudhaiv Kutumb' i.e. entire globe has become one family it is no more a bipolar world. We are living in a unipolar world. LPG process has taken place in all countries and trade blocks. Business restrictions between the countries have been relaxed to a great extent. This has created competition between firms and products worldwide.

- Eg. Indian firms have to compete with western MNCs and MNC's have to create market in traditional Indian and Chinese market.

Liberalization and Industrial policies in India after 1990's have opened the gate of Indian economy to foreign and multinational companies. Similarly Indian firms have opportunity to establish their firms in foreign countries.

Firms have to be competitive not only regionally or nationally, they have to attain international standard. People in every corner of the world have the opportunity to enjoy the best goods and services produced by company of any country.

One can drive Germany's Benz Car, Wear trendy dress and perfume of Italy and France, tie the Swiss watch, eat the pizzas of McDonalds etc. In such environment only those businesses houses can survive that deliver goods of international standard.

Business to be successful has to watch international environment. The development in the field of technology, arrival of new products and services, adoption of marketing strategy and management style etc. are to be carefully observed. Firms which watch happenings in the world economy and make effective adaptations to their business can survive. Firms which fail to read the happenings in world economy may have to suffer loss. Eg. After liberalization many of Indian firms received serious shock as they could not compete with the products of MNCs. The emergence Honda Motor cycle, Maruti Car replacing

Rajdoot Motor cycle, Ambassador car of India are small examples to this.

Recession in U.S., Europe and Japan have made the business firms of these countries to enter India and China. These countries offer big market for the products of foreign MNCs. Education and income levels of people in India is increasing. These have created big opportunities of business. MNC's which have made study of these changes are benefited and Indian firms which have failed to notice this change in the environment are failures.

LPG process has also created new business opportunities like BPO's, KPO's, Medical, Tourism, Hotel etc. Business people who have noticed these changes are benefited by starting their own business in the relevant field.

Business does not function in a closed door. It must be promoted to face any kind of cultural and economic shock. Eg. Crude oil price is crude example of this. Increase in its price will affect the entire economy of the world. Firms must be aware of this and adjust their business policies to such change.

## National Economic Environment :

Economic environment of a country has significant influence on business environment. Economic environment depends on economic system adopted by a country. There are three kinds of economic system.

1) Capitalist
2) Communist
3) Mixed Economy.

### 1) Capitalist :

This form of economy is more liberated, Government intervention in business is minimum. Restrictions and regulations are minimum, policies of Government are liberal towards encouraging business activities. Market determines the business conditions. Demand and supply factors, price mechanism etc. are determined by the market. Under this kind of environment business has better opportunities for growth. USA, Europe are best examples of capitalist economy. That's the reason why giant MNCs like IBM, Microsoft, Ford Motors, GM, Wall Mart, P & G etc. have grown.

**2) Communist :**

It is opposite to capitalist pattern, business is highly regulated and controlled by government. Factors of production are owned and controlled by government, role of market is minimum. China is the best example of communist regime, earlier USSR and East European Countries like Poland, Bulgeria, Czechoslovia etc. were under communist regime, but that system collapsed. Under this form of government, business opportunities are less. Individual's have no freedom of choice, they can not own property to an unlimited extent. These restrictions have led to collapse USSR, and East Germany. Freedom and Liberty of Capitalism has attracted the people of these countries to break the communist regime they are also marching towards capitalism gradually.

**3) Mixed Economy :**

It is a mix of capitalism and communism, business under this system is classified under three categories.

a. Business which is under complete ownership of government. i.e. PSEs like Indian Railways, Electricity Boards, Public Sector Banks etc.

b. Business which is managed by Government and also by private sector. Eg. Road Transport, Communication etc. In Karnataka, we have KSRTC and also private transports. In Telecommunication we have Department of Post, BSNL etc. and also various private mobile service operators like Airtel, Vodafone etc.

c. Business that are in complete control of private sector only. Eg. Trading, Hotels, Industries etc.

India is the best example of mixed economy. Economic policies in India are also gradually moving towards capitalism. Liberation process and industrial policies after as per the provisions of industrial policy.

## B) Non-economic Environment

It is difficult to distinguish business environment into economic and non-economic. Non-economic variables or factors have economic content similarly economic variables have social content in them. Both factors are interdependent not independent. Eg.

a. Political factors are dependent on economic and economic policies. Economic policies are influenced by political philosophies. The UPA-II Government led by the Congress at the national level has SP, BSP, NCP, DMK, TMC etc as its main supoorters. The economic policies of UPA government is influenced by economic ideologies of these parties.

b. Demography and economic factors are interrelated i.e. increase in population leads to increase in demand for goods and services.

Like wise every factor of business environment is interlinked, they cannot happen in isolation. For the clarity in study of business environment it is classified into economic and non-economic factors. The important non-economic factors that have impact on business are as follows :

### 1) Sociological Factors :

It is said man is a social animal, similarly it can be said business is a social organization created by man to serve the society. Keith Davis says "Business is a social institution performing a social mission and having a broad influence in the way people live and work together."

Object of business is to make profits, profit making depends on customers and support from society. Survival and success of business depends on support of all these category of people. These people have certain expectation from business i.e. workers expect fair wages and facilities, customers expect better quality and reasonable price, society expects business to be legal and ethical. Business, which fulfills these social responsibilities can achieve the desired success.

Business must study the prevailing environment in the society. Such environment changes from place to place and time to time. The environment in western society is different from the environment in Indian society. Western society is materialistic, people spend more on comforts and luxuries, they believe in consumption and satisfaction. On the other hand societies in India are conservative, they believe in saving not lavish spending on comforts and luxuries. Indian society is also gradually moving towards western culture.

Study of sociological environment helps the business to know the likes and dislikes of its people. It can gauge expectations of society. This helps the business to determine its objectives, plan its programmes and activities, which are according to the conditions of society. It helps to produce those goods that are liked by society.

**2) Physical / Ecological Factors :**

Survival of business is dependent on physical resources it receives from nature, like raw-materials, water, air etc. Business is the villan who creates ecological imbalance by exploiting the nature for its business purpose. Extraction of raw materials, cutting of the forest, release of industrial waste, air pollution etc. are creating ecological imbalance. These have resulted into natural calamities like flood, earthquake, drought, Tsunamis etc. People are becoming more aware of these aspects worldwide. The organizations like 'Green Peace' in Europe, Chipko and NBA (Narmada Bachao Andalon) in India and People like Medha Patkar, Sundarlal Bahugna Anna Hazare are creating mass movement to protect the nature and preserve ecology from the onslaught of business.

Bhopal Gas tragedy, (by union carbide) Chernobil disaster in Russia and Fukushima in Japan are the grim reminders of how industrial pollution can play havoc on mankind. Government of Karnataka is thinking of setting up a power plant at Tadadi (near Udupi), which is strongly opposed by the natives.

Business must be aware of ecological disorder it may create and opposition it may have to face from the society. Business has the responsibility to ensure that the ecological damage is minimum. It has to gain confidence of the people by undertaking measures to control and minimize pollution.

Eg. Installation of waste treatment plant, noise control equipment, and undertaking social forestry etc., can minimize pollution.

Business must study ecological environment and it must select those business activities that have minimum pollution effect. It must ensure that environmental damage is minimum. Business which takes care of nature and its environment protection will have good image.

**3) Demographic Factors :**

Demographic factors of business environment pertain to population. Size and trend of population, growth rate, population mix, age, composition, caste, family size, life style, mobility, culture, standard of living, education level. These aspects have direct effect on business as size and composition of population determines the demand.

Demographic factors in India are favorable to business, that's why there is increase in business activities and number of MNC's are entering Indian market. Increase in population in India is accompanied with increase in income levels and education. Number of middle class family are increasing, culture of the people is fast changing, young population is increasing, all these factors help for growth in business.

Business which studies demographic environment can take advantage of this trend. It can create goods and services that are liked by present generation of people. Evaluation of democratic environment helps the business to select and expand those business activities that are liked by the people.

**4) Political Factors :**

Business activities in a country are determined by political system. Political organization of a country is defined by its constitution. Constitution divides the system into legislature, executive and judiciary and also the press. They are described as four pillars of constitution. The legislature i.e. parliament at the centre and Vidhansabha (state legislature) at the state level makes policies and laws as per which people have to conduct the activities. Legislatures (MP's or MLA's are peoples representatives who are elected by the people) who belong to different political parties and each political party has its ideologies. Political party with highest majority forms the Government, the elected Government frames economic and other policies to conduct business. These policies will be implemented by executive (IAS/KAS) interpreted by judiciary and commented by the press.

Business environment in the society is strongly influenced by the political parties and system. Parties like Congress and BJP are centrists and parties like CPI, CPI (M) etc. DMK, JD(S) are leftist. Their attitude towards business is reflected in business policies they frame.

Eg. The elected Government of 2009 Parliament belongs to UPA-II (Congress, TMC, DMK, NCP and others). Congress party believes in liberalization, privatization and more freedom to business. The TMC and DMK are opposed to these ideas. They are in favour of subsidy, controls, protecting the interest of labour, importance to PSUs etc. Due to coalition Government, economic policies of UPA are compromise between congress and its allies like TMC, DMK, NCP etc. Political environment in states like Karnataka, Gujarat, Maharashtra, Andhra Pradesh have been conducive to business activities as the dominant political parties in these states are Congress, JD(S) or BJP that have pro business attitude. On the other hand states like West Bengal, Bihar, UP have not achieved expected business growth as political environment in these states does not have favourable attitude towards business. Unstable political environment will not help for stable economic policies.

Stability of the government is another important factor that influences the Government activities. Frequent change of Government may have its effect on business prospects in a particular state.

Business to be profitable must watch political development. It must study the manifestos, philosophies, election promises and policies of dominant political parties. This will help the business to frame policies and objectives to match principles of dominant political parties.

**5) Science and Technology :**

Development in the field of science and technology will increase competitiveness of business. It helps to innovate new product, new method of production and distribution that reduces cost, increase in speed, and provide more comfort and convenience.

Growth of business in a country depends on development in the field of science and technology. Indian business has not grown to the desired level due to poor investment in R&D. Business in Europe, Japan and USA have achieved tremendous growth due to application of science and technology to business.

Opening up of Indian economy and emergence of MNC's in India have changed business scenario in India. It has created competition between technologically sound MNC's and Indian firms which are using out dated technology. This has forced Indian business houses to adopt latest technology and be competitive.

Business has to study the development in the field of science and technology not only in the domestic market but also in foreign market. It has to spend on R&D activities and install the latest technology in the firm. Scanning the environment of science and technology and applying such development in the business can make the firm competitive.

**6) Education :**

Education level in the society and nature of education also determines business environment. Development of business needs skilled and trained manpower. Society which creates institutions which offer higher and technical education provide conducive environment for industrial growth. Eg. : The recent growth and concentration of IT and BT industry, the BPO's and KPO's in Karnataka is mainly due to numerous engineering and medical education institution which are providing qualified manpower. Firms like Infosys, WIPRO, Microsoft, Dell etc. have made Bangalore as their important base. Similarly big hospitals like Wockhardt, Mallya, Manipal etc. and five star hotels of big name are establishing their business in Karnataka due to availability of qualified and competent manpower.

Businesses which were earlier restricted to the USA and European countries like BPO's and Call Centers are coming to India mainly due to availability of skilled man power at lesser wage burden. Business firms can exploit such opportunities to their advantage, if they study education environment.

**7) Legal Environment :**

Activities of business are regulated by various laws. Provisions of these laws dictate the method, procedure and formalities to be completed before commencing any business. Eg. Indian Companies Act 1956 has prescribed four stages in the formation of a company i.e. a) promotion b) incorporation c) capital subscription and d) business commencement. During each of this stage a company has to fulfill many conditions.

Similarly there are other laws like MRTP Act, FEMA, Industrial Disputes Act, Consumer Protection Act, Minimum Wage Act etc. These acts ensure that business is carried to protect and safeguard interest of investors, employees, customers and society.

Growth of business depends on the extent of regulators and restrictions laid down by these acts. Before 1990, India was described as 'License Raj', it was described as permits and quotas kingdom due to numerous restrictions on business. After liberalization the provisions of various laws have been diluted, rules and procedures of managing the business have been simplified. This has led for growth in business activities.

Business must study the legal environment prevailing in the society. This will help to setup legally viable business and follow the procedures that are prescribed by laws. Firms which study the legal environment will not face any legal tangle or difficulty.

**8) Cultural Environment :**

Every society has its own unique culture. Culture can be defined as living styles and habits of people. Rituals customs, traditions, festivals, family life, dress code, food habits etc. are all part of cultural system. Based on this, culture of west is more open and liberal, their family values are different. On the other hand culture of India can be described as more traditional, there are many restrictions laid down by the society.

Eg. Vegetarian and Non-vegetarian ; People belonging to upper caste are not supposed to eat non-vegetarian food. Similarly dress codes are rigid. Wearing of tight jeans, mini skirts etc. are not appreciated.

Cultural practices will have strong influence on the business. The products manufactured by a firm, its method of advertising etc. must match the cultural norms determined by the society. Firms, which divert their operations from cultural norms, may not be accepted by common man.

When KFC (Kentucky Fried Chiken) and Macdonalds opened their retail outlets in Bangalore, they received lot of opposition from people. Similarly lot of controversy is going on regarding permission to dance bars in big cities. Certain category of people are opposed to IPL, auction of cricket players, dance girls in cricket matches and the kind of culture they are spreading. Business must be aware of cultural aspects of society. It must study the cultural environment in the society and follow those business practices that do not affect feeling and sentiments of people.

**9) Sectoral Environment :**

Firms which are doing similar business belong to a particular sector or Industry. Eg. Transport Industry, Banking Industry, Textile Industry, Steel Industry etc.

Experiences of all firms in an industry will be same. They come under one set of laws, one regulation, use similar plant and machinery and raw material and have common market. Inspite of such similarities results of individual firms will differ. Some firms may make more profit, some less and others loss. The variation in result may be due to the level and accuracy with which they study the prevailing business environment. Firms that adjust to the market conditions apply better technology and plan their business strategy to match the prevailing conditions of environment. Such firms become market leader and others follow them. Market leaders are those who read the conditions of environment and take those conditions as opportunities to the advantage of business. Eg. Advantage of their business Reliance company promoted by Late Dhirubhai Ambani has always taken advantage of prevailing business environment. Reliance petroleum, the textile units, communication, natural gas etc. went on expanding and growing by exploiting prevailing conditions of business in the sector to its advantage.

Firms, which want to perform better and have better result, must evaluate business environment prevailing in their industry. It must study strategies adopted by leading firm in their manufacturing and marketing operations. The experiences of successful firms may be made use by less successful firms to have good business results.

## 10) Labour Environment :

Labour is the important factor for production of any kind of business. Success of business depends on availability of manpower, particularly skilled man power. It also depends on the co-operation of workers and their union.

States like Bihar, UP, Orissa, West Bengal inspite of having rich natural resources have not developed to the desired extent due to non-availability of skilled manpower and hurdles created by litigant trade unions.

Trade unions with strong bargaining power and indisciplined workforce will be a hurdle to the business prospects Recent growth in business activities in South India is mainly because of educated labour force and docile unions.

Before setting up of venture, business must search for availability of manpower and nature of unionism. It also helps to design its labour policies and HRM (Human Resource Management) practices to be adopted. Business which is free from labour problems can grown and it can also have better image in the industry.

## MAJOR PLAYERS IN ENVIRON...

5-unit

WTO.

## World Trade Organization (WTO): Objectives and Functions

The **World Trade Organization (WTO)** is an intergovernmental organization that regulates international trade. The WTO officially commenced on 1 January 1995 under the Marrakesh Agreement, signed by 123 nations on 15 April 1994, replacing the General Agreement on Tariffs and Trade (GATT), which commenced in 1948.The WTO deals with regulation of trade between participating countries by providing a framework for negotiating trade agreements and a dispute resolution process aimed at enforcing participants' adherence to WTO agreements, which are signed by representatives of member governments and ratified by their parliaments. Most of the issues that the WTO focuses on derive from previous trade negotiations, especially from the Uruguay Round (1986–1994).

The WTO is attempting to complete negotiations on the Doha Development Round, which was launched in 2001 with an explicit focus on developing countries. As of June 2012, the future of the Doha Round remained uncertain: the work programme lists 21 subjects in which the original deadline of 1 January 2005 was missed, and the round is still incomplete.[8] The conflict between free trade on industrial goods and services but retention of protectionism on farm subsidies to domestic agricultural sector (requested by developed countries) and the substantiation of fair trade on agricultural products (requested by developing countries) remain the major obstacles. This impasse has made it impossible to launch new WTO negotiations beyond the Doha Development Round. As a result, there have been an increasing number of bilateral free trade agreements between governments.As of July 2012, there were various negotiation groups in the WTO system for the current agricultural trade negotiation which is in the condition of stalemate.

The WTO's current Director-General is Roberto Azevêdo,[11][12] who leads a staff of over 600 people in Geneva, Switzerland. A trade facilitation agreement, part of the Bali Package of decisions, was agreed by all members on 7 December 2013, the first comprehensive agreement in the organization's history.

The Uruguay round of GATT (1986-93) gave birth to World Trade Organization. The members of GATT singed on an agreement of Uruguay round in April 1994 in Morocco for establishing a new organization named WTO.

It was officially constituted on January 1, 1995 which took the place of GATT as an effective formal, organization. GATT was an informal organization which regulated world trade since 1948.

Contrary to the temporary nature of GATT, WTO is a permanent organization which has been established on the basis of an international treaty approved by participating countries. It achieved the international status like IMF and IBRD, but it is not an agency of the United Nations Organization (UNO).

**Structure:**

The WTO has nearly 153 members accounting for over 97% of world trade. Around 30 others are negotiating membership. Decisions are made by the entire membership. This is typically by consensus.

A majority vote is also possible but it has never been used in the WTO and was extremely rare under the WTO's predecessor, GATT. The WTO's agreements have been ratified in all members' parliaments.

The WTO's top level decision-making body is the Ministerial Conferences which meets at least once in every two years. Below this is the General Council (normally ambassadors and heads of delegation in Geneva, but sometimes officials sent from members' capitals) which meets several times a year in the Geneva headquarters. The General Council also meets as the Trade Policy Review Body and the Disputes Settlement Body. General council delegate responsibility to 3 other major

At the next level, the Goods Council, Services Council and Intellectual Property (TRIPs) Council report to the General Council. Numerous specialized committees, working groups and working parties deal with the individual agreements and other areas such as, the environment, development, membership applications and regional trade agreements.

**Objectives:**

The important objectives of WTO are:

1. To improve the standard of living of people in the member countries.

2. To ensure full employment and broad increase in effective demand.

3. To enlarge production and trade of goods.

4. To increase the trade of services.

5. To implement new world trade regime.

5. To ensure optimum utilization of world resources.

6. To protect the environment.

7. To accept the concept of sustainable development.

**Functions:**

The main functions of WTO are discussed below:

1. To implement rules and provisions related to trade policy review mechanism.

2. To provide a platform to member countries to decide future strategies related to trade and tariff.

3. To provide facilities for implementation, administration and operation of multilateral and bilateral agreements of the world trade.

4. To administer the rules and processes related to dispute settlement.

5. To ensure the optimum use of world resources.

6. To assist international organizations such as, IMF and IBRD for establishing coherence in Universal Economic Policy determination.

**WTO Agreements:**

The WTO's rule and the agreements are the result of negotiations between the members. The current sets were the outcome to the 1986-93 Uruguay Round negotiations which included a major revision of the original General Agreement on Tariffs and Trade (GATI).

GATT is now the WTO's principal rule-book for trade in goods. The Uruguay Round also created new rules for dealing with trade in services, relevant aspects of intellectual property, dispute settlement and trade policy reviews.

The complete set runs to some 30,000 pages consisting of about 30 agreements and separate commitments (called schedules) made by individual members in specific areas such as, lower customs duty rates and services market-opening.

Through these agreements, WTO members operate a non-discriminatory trading system that spells out their rights and their obligations. Each country receives guarantees that its exports will be treated fairly and consistently in other countries' markets. Each country promises to do the same for imports into its own market. The system also gives developing countries some flexibility in implementing their commitments.

**(a) Goods:**

It all began with trade in goods. From 1947 to 1994, GATT was the forum for negotiating lower customs duty rates and other trade barriers; the text of the General Agreement spelt out important, rules, particularly non-discriminations since 1995, the updated GATT has become the WTO s umbrella agreement for trade in goods.

It has annexes dealing with specific sectors such as, agriculture and textiles and with specific issues such as, state trading, product standards, subsidies and action taken against dumping.

**(b) Services:**

Banks, insurance firms, telecommunication companies, tour operators, hotel chains and transport companies looking to do business abroad can now enjoy the same principles of free and fair that originally only applied to trade in goods.

These principles appear in the new General Agreement on Trade in Services (GATS). WTO members have also made individual commitments under GATS stating which of their services sectors, they are willing to open for foreign competition and how open those markets are.

**(c) Intellectual Property:**

The WTO's intellectual property agreement amounts to rules for trade and investment in ideas and creativity. The rules state how copyrights, patents, trademarks, geographical names used to identify products, industrial designs, integrated circuit layout designs and undisclosed information such as trade secrets "intellectual property" should be protected when trade is involved.

**(d) Dispute Settlement:**

The WTO's procedure for resolving trade quarrels under the Dispute Settlement Understanding is vital for enforcing the rules and therefore, for ensuring that trade flows smoothly.

Countries bring disputes to the WTO if they think their rights under the agreements are being infringed. Judgments by specially appointed independent experts are based on interpretations of the agreements and individual countries' commitments.

The system encourages countries to settle their differences through consultation. Failing that, they can follow a carefully mapped out, stage-by-stage procedure that includes the possibility of the ruling by a panel of experts and the chance to appeal the ruling on legal grounds.

Confidence in the system is bourne out by the number of cases brought to the WTO, around 300 cases in eight years compared to the 300 disputes dealt with during the entire life of GATT (1947-94).

**(e) Policy Review:**

The Trade Policy Review Mechanism's purpose is to improve transparency, to create a greater understanding of the policies that countries are adopting and to assess their impact. Many members also see the reviews as constructive feedback on their policies.

All WTO members must undergo periodic scrutiny, each review containing reports by the country concerned and the WTO Secretariat.

Chapter - I

Bank & Banker.

B : Business
A : Accepting Deposits
N : Negotiation.
K : Keeping the confidential of customers A/c's

Meaning :-

A Bank is an Institution which deals in Money & credit. It borrows money from those who have Surplus money & lends the same to those who are in need of money.

Bank is a B. tank. of money.

Deffinitions :-

According to Sayers" Defines bank as an a Institution whose debts are widely accepted in settlement of other peoples debts to each others".

According to Sheldeen "The function of receiving money from is customers & repaying It buy honoring their cheques as an when required is the function above all other functions which distinguish a banking business from an another kind of business.

* Origion of Bank :-

The word 'Bank' is derived from Italian word 'Banca' and in latin word 'Bancus' which means the a branch & desk Some persons have Started the word Bank is derived from the Jerman word bunk which means a Joint Stock fund are a common fund collected from The large number of members of the public for the purpose of financing the Industry, commerce & agriculture.

★ Development of Banking In India :-

Banking In India is Started by two British managing agency houses Namely ferguson & company & Alexander & Company.

During 19th Century a commercial Banks were established In over country.

| | Year of establishment | | Name of the Banks |
|---|---|---|---|
| (01) | 1830 | → | Bank of Hindustan (It was under the Alexander management. It is the first joint stock Bank in ~~over~~ our country.) |
| (02) | 1839 | → | Bank of Bengal |
| (03) | 1840 | → | Bank of Bombay |
| (04) | 1843 | → | Bank of Madras |
| (05) | 1869 | → | our commercial Bank |
| (06) | 1895 | → | punjab national Bank. |
| (07) | 1906 | → | Bank of India |
| (08) | 1908 | → | Bank of Baroda |
| (09) | 1911 | → | Central Bank of India |
| (10) | 1913 | → | people Bank of India. |
| (11) | 1921 | → | Imperial Bank of India |
| (12) | 19~~35~~43 | → | RBI - Reserve Bank of India |

★ Meaning of Banker :-

Banker is applicable to Individu who performs the Banking business.

★ Banking Regulation Act 1949

Indian Banking Regulation act 1949 defines Banking as follows

Section 1 (b) of this act defines the term "Banking as accepting money for the purpose of lending ~~or~~ Investment of deposits of Money from the public repayable on demand & withdrawable by cheque draft.

* Section-5 (1) (c) - Define the term - defines "Banking company as any company which transacted the business of the banking In India.

* Section-6 - According this Section the Banking company has to undertake various Subsidiary Services which Includes collection of cheques, draft, Bills, etc acceptence of valuable for Safe custoday, collection of Interest & dividend for Investment etc.

* Section-7 - According to this Section It is compulsory every Banking company to use a path of Its name along with any of the words " Bank, Banker, Banking company" & State Bank no company Shall carry on business of Banking In India, unless, It use as a part of Its name at least one of such word."

* Section-8 :- According this Section Banking com. Engage directly or In directly In trading activities. undertakes, trading risk,

* Features of Banking :- Characteristics :-

(1) Dealing in money :- The Bank accepts deposit from the public and the same amount is given in the form of loans to needed people in this Sense banker is dealer in the money.

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(02) Dealing with credit :- Generally Banks create credit that is creation of additional money for lending. So creation of credit is unique feature of Banking.

(03) Commercial in nature :- The aim of performing all the banking of all loans the profit in this sense Bank is regarded as a commercial Institution.

(04) Nature of agent :- In addition to the basic function modern Banker possesses a the character of an agent because of modern Banks performs its various agent services (5) Deposits must be withdrawable deposits :- are withdrawable on demand. Banks accepts deposits from the public & this deposits are wrote by by cheque or drafts that is Bank issue pay cheque

★ Modern Banking :-

★ Meaning :- Modern Banking is nothing But Speed in Service, by using all new technology. under Modern banking the technology is like E-Banking, ATM, Tely Banking are used for providing functions.

★ Difference Between traditional & Modern Banking

| point | Traditional Banking | Modern Banking |
|---|---|---|
| 01) Service | This Banks provides Services such as Saving A/C, current A/c, loans & Advances Bills, deposits etc. | This Banks provides Electronic Banking Services in the form of ATM, credit card, Debit card & Internet Banking |

| | | |
|---|---|---|
| (02) Transaction | Customer has to go to the Branch of the Bank In person to perform the Basic Banking Functions. | Customers can undertake the basic Banking Functions or transaction. by sitting at home to PC |
| (03) Customer Service | Tradition Banking is more restricted | Modern Banking is less restricted. |
| (04) Cost of Service | Traditional Banking is Costly. | Modern Banking is cheaper. |
| (05) Art or Science | Traditional Banking is Just an a Art | Modern Banking is an a Science Science |
| (06) No confine Branch of Bank | It is essential To doing Banking Business. | It is not essential |

* Classification of Banks :- Into 8 types. base of their function

01) Commercial Bank.
02) Industrial Bank.
03) Central Bank
04) Agriculture Bank.
05) Exchange Bank
06) Indegenious Bank
07) Co-operative Bank
08) Regional Rural Bank.

01) Commercial Bank :- Commercial Banks are the Joint Stock Companies which deals in Money & credit a commercial Bank, May be defined as financial Institution which accepts deposits from the public & withdrawable by cheque & use that Money for lending purpose. Commercial Banks usually gives Short-term loans & ~~advantages~~. Advances

To reason here they have started giving Medium term loans & they provide working capital to Industry & trade.

**Ex:-** ICICI Bank.
State Bank.
Axis Bank
HDFC Bank

(02) Industrial Banks :- These are the Banks Established for the purpose of providing long term finance to Industries. they obtain their fund through share capital, Debentures, long-term deposits from the public. **Ex:-** IDBI Bank
(Industrial Development Bank of India)
IFC - Industrial financial corporation

(03) central Bank :- In every country there is one Bank called central Bank, which is the highest Monetary & Banking authority in the country. It is the apex (stock) Financial Institution In the Banking & Financial System of the country. It controls regulates & supervise the activity of all other banks in the country. It has Monopoly rights to issue currency notes. I mentains ~~Ext~~ Internal & External value of the Money.

**Ex:-** (1) In India RBI is the central Bank

(2) In USA FEDERAL Bank is the central Bank.

(3) In Russia GOS is the central Bank.

" Central Bank is can be defined as the strategic level of Monetary & Banking Institution primary & Engaging in controlling in regulating the Banking Monetary System of the country for the progress of the economy

(04) Agriculture Bank :- Agriculturist required both short term & long-term loans. Short-term loans are provided by Co-operative Banks. they can not provide long term loans because their resources are limited. for providing long-term loan to agriculturist for development agriculture Banks are Establish.

Ex:- ① Bank Mortgage. (this land mortgage are special Institution. they provide long-term loans to agriculturist. they provide long-term loans the period up to 20 years.

Ex: ① In USA Federal land Bank
② land Mortgage Bank
③ In England agricultural Mortgage corporation

(05) Exchange Banks :- co-operative Bank :- Co-operative means idea of living together & working together.

Co-operative means voluntary association on the basis of Equality and for some common purpose & promotion of their Economic Interest.

This Banks supply finance for Agriculture and it also provide short-term loans to the needed Agriculturist, co-operative Banks receive all kinds of deposite & Make them available to the members who are a need of finance.

Definition :

"Co-operative Banks can be defined as "An a Institution established on the co-operative principle & engaged in the normal Bank business & accepting deposits from the public, for the purpose of lending & repaying It on demand."

In India co-operative Banks has three tier structure
01) primary co-operative Bank (PCB's)
02) Central co-operative Banks (CCB's)
03) State co-operative Banks (SCB's)

01) primary co-operative Banks - These are the banks established at rural & urban area's to provide Short-term & Medium term loans to the needed agriculturist this Banks gets the loans from CCB's

02) CCB's - this Banks are established at District level this Banks give loans primary co-operative Banks & It also give & loans to Individual people this Banks are also called District (DCB's)

03) SCB's - this Banks live at the top entire co-operative Banks there is the State coperative Banks in each State. the basic function of State co-operative Bank is to provide the loans to Central co-operative Banks.

06) Exchange Bank :- Exchange Banks Banks which perform convertion of currencies that is convert Foreign currency to local currency local to foreign

Ex :- Canara Bank
Federal Bank
Bank of India etc.

Exchange Bank is also Specialised In financing the foreign trade & they supply the necessary foreign Exchange required for settlement transactions between trade engaged in foreign trade Ex :- EXIM Bank

(07) Indegenous Bank - These are the Banks established by private persons money under are called as Indegenous Bank. This Banks provide short-term loans for the needed people & they charged very high rate of Interest.

"According to Indian central Banking enquiry committee & Indegenous Bank is an Individual or private firm accepting deposits & dealing in lending money.

Ex: Mahajans
Multans
Sahukaras
Seths.

(08) Regional Rural Bank :- This Banks are established by Indian government In the year 1975. The primary object of regional rural Bank is to developed rural Economy. this Banks provide loans for the development of agriculture trade, commerce, & Industry, they also provide credit facility to Small Scale Industry.

★ Difference between Central Bank & Commercial Bank.

| points | Central Bank | Commercial Bank. |
| --- | --- | --- |
| (01) Status | It is the Top most Bank in the System of the country. | It is a part of Banking system. |
| (02) ownership | It is owned by government. | This Banks are privately owned Institution. |
| (03) objective | Economic Development of the country | earning profits. |

| | | |
|---|---|---|
| (04) Deals with | Banking & government | (05) General public |
| (05) How many Banks are there? | only one | many Banks |
| (06) Right to print & issue the notes | yes | No. |
| (07) profit Motive | It donot Make profit for Its owner | It Makes profit its owner. |

* Difference between Commercial Bank & Co-operative Banks.

| Points | Commercial Bank | Co-operative Banks |
|---|---|---|
| (01) provider of Credit | Trade & Industry | Agriculture |
| (02) Area of operation | large | Small. |
| (03) Objective | profit earning | Service to the pub |
| (04) function | Accepts the deposits from the public & grant loans to Individual & B/S. | The main function of Co-operative Bank to accept the Deposits from the member & public & provide loans to Agriculture & Small B/s |
| (05) Interest Rate Deposits | low Interest Rate | High Interest Rate |

Functions of commercial Banks.

Commercial Bank's are financial Institution which accept deposits from the public and Repayable as and when demanded by the customer.

In India during 19th century the first commercial bank has been established that is bank of Hindustan in the year 1830 which was under European management after that in the year 1839 Bank of Bengal in 1840 Bank of Bombe 1843 Bank of Madras.

The out commercial banks which was established in the year 1869 it was the first commercial banks which is purely under the management of India.

In India commercial banks are classified into two types.

1) Scheduled commercial Banks.
2) Non - Scheduled Commercial Banks.

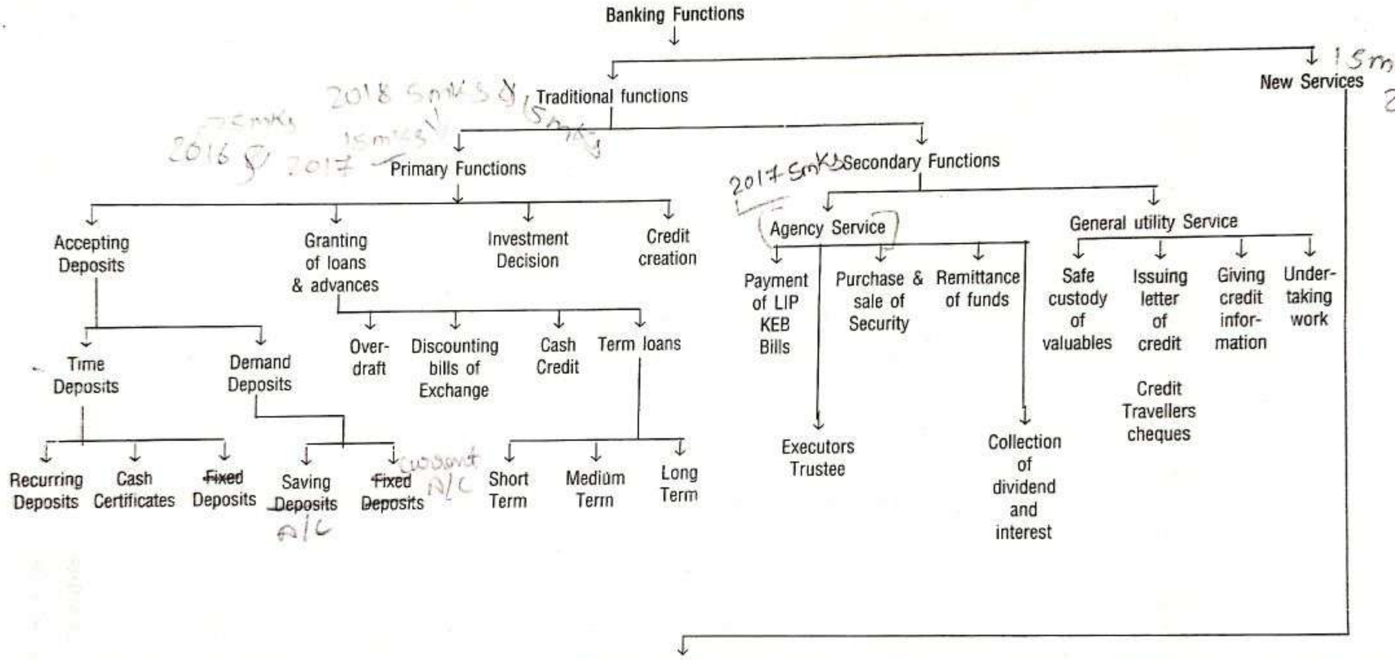
1) Scheduled Commercial Banks :- These are the banks whose paid up capital & Reserves are more than 500,000 ₹ are called Scheduled Commercial Bank.

This banks are if included in the Second Scheduled of reserve bank of India act.

2) Non - Scheduled Commercial Banks :-

These are the banks whose paid up capital & reserves are less than 500,000 ₹. These are called Non - Scheduled Commercial Bank.

These are the banks which are not included in the Second Schedule of reserve Balance of India act.




**New Services :** (1) New schemes of accepting deposits (2) Home loan A/c (3) Automatic Extension deposit scheme (4) Personal loan scheme (5) Loan participation (6) Educational loans (7) Scheme for financing small scale industries (8) Schemes for financing agriculture (9) Loan for self employment (10) Lock box and night safe service (11) Teller system (12) Bid Bonds and performance guarantees (13) Technical advice (14) Mobile Banking and extension service (15) Financing for imports and exports (16) Tax consultancy (17) Rural credit cards (18) Loan to women (19) ATM (20) Computerisation in Banks (21) Merchant Banking Services (22) On line banking (23) Net Work banking (24) Inter banking.

## Functions of Commercial Banks

In the present day, commercial banks are playing very important role and performing various functions. These functions are divided in to two classes, viz.,

(1) Traditional Services

(2) New Services

### (1) Traditional Services

Traditional Services of Banker are further classified into two classes, viz.,

1. Main or primary or principal functions.

2. Subsidiary or secondary services or functions.

The chart on page 2/2 shows the functions or commercial banker.

Main or primary services of Banker are further classified into Four classes, viz., (A) Accepting deposits and (B) Granting of loans and advances. (c) Investment (d) Credit creations

### A. Accepting Deposits :

Accepting deposits from public is the primary and main function of modern banker.

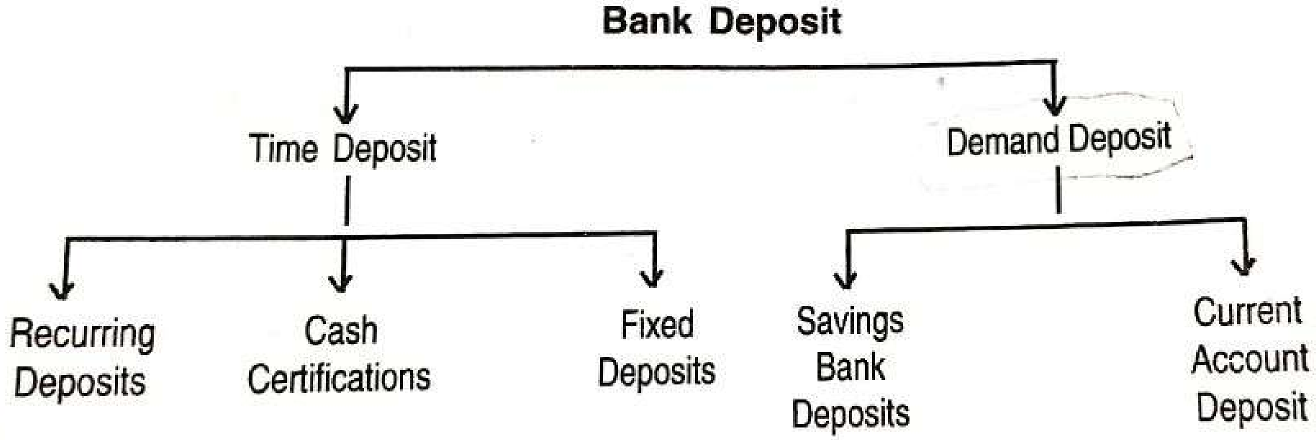
Modern banks perform various functions, among those, the main function of banks is to accept the deposits from the public. Banking is a business. Money is required to run the business. The banks collect the required amount from the public in the form of deposit with a promise that the deposits would be repaid to the customer as and when demanded. Thus banks borrow the money from the public. Banks borrow money from public by some other ways also, viz, issuing bonds, debentures, cash certificates, etc. So banks solicit the deposits from the public coming from different walks of life, and who are engaged in activities having different financial status.

When the banker accepts deposits from the customer then the relationship between banker and customer starts as a debtor (Banker) and creditor (customer). Banker is a borrower as well as a lender. It means he receives the deposits from the public and the same amount is lent to the needy persons in the form of loans and advances. Generally the banker borrows (receiving Deposits i.e. accepting deposits) at a very low rate and lends the same amount at higher rate in the form of loans and advances and earns the profit. This

**Deposits are broadly classified in to two groups**

a) Time Deposits  b) Demand Deposits

Chart shows the classification of deposits-




## (A) Time Deposits

Banks receive these deposit for a certain period it means these deposits are repayable after a certain fixed period. These deposits are not withdrawable by cheque, draft or by other means. It includes the following :

* Recurring Deposit :- is a special kind of two Deposit offered by the banks in India which helps the people with regular Income to Deposit a fixed amt. every month into their re-curring Deposits & earn Interest at the rate applicable to fixed deposit.

In Recurring deposit customer can deposit
* Some Money every month & withdraw after certain period of one year or 3 years or 5 years, & earn the Interest

* Cash certificate :- cash certificate are type of of deposits that is purchased for certain amount the account holders purchase the cash certificate for a certain ammount but needs to Make payments towards these amount only long as long as the term of the certificate.

* Fixed Deposits :- It Means Money is Deposited by the customers in the bank for a fixed period of time the period generally varies from 15 days to 25 years & above. bank pay's high rate of Interest on fixed Deposits. the rate of Interest depends upon the period for which the Money is Deposited.

II)

* Demand Deposit :- The Some of Money that is given to a bank but can not withdrawable as per the requirements of Depositer ammount that as lying in the Saving account & current A/c. are known as demand Deposit. because they can be use at any point of time. D.

Demand deposits includes
01) Saving Bank A/c
02) Current Bank A/c
03)

(01) Saving Bank A/c :- As the name suggest this type of A/c is suitable for people who have definite Income & are looking to save the Money. this type of account is more suitable to poor, middle, & salary based people they can withdraw the money as an when required withdrawals can be made by cheque, withdrawals & ATM'S, the rate of where is from bank to bank. Bank has made ristriction for withdrawals that is 25 withdrawals in quarter of a year & total withdrawals 150 in a year.

(02) current Bank A/c :-

Current A/c is a running A/c it is a continious, operation or dealing between the bank & customers it is also known as active A/c.

In current A/c customers can deposit & withdraw the ammount daily, their is no restriction withdrawals, current A/c is more suitable to business man, companies, Institution,

* Granting loans & Advances :-
01) over draft
02) cash credit
03) Discounting bills of Exchange
04) Term loans

* Granting loans & Advances:-
Bank accepts Deposit from the public out of that deposits certain amt is given to the needed people in the fields of commerce, agriculture, & trade.
following are the methods of granting loans & advances, overdraft, cash credit, discounting Bills of Exchange term loan, over

(01) over draft :- over draft means customers can withdraw more than the credit Balance Bank will give it in the form of temporary loan for 90 days & it will charge high rate of Interest.

(02) Cash credit :- Cash credit is a facility to withdraw Money from a current A/c without having credit Balance But their is limit fixed by commercial bank for borrowing of money. Bank will charge Interest on actual Amt withdrawn.

(03) Discounting Bills of Exchange :-
when customer required the Money they approaches to the bank will discount the bill & pay the amt to the customer before the due date.

(04) Term loan:- Bank will provide term loans to the customer in order to earn the profit this term loans in the form of
(01) short term loan (2) Medium term loan (3) long term loan.

(01) Short-term loan :- Short term loans are provided for less than one year.

(2) Medium term loan :- This type of loan is provided between one to five years.

(3) Long term loans : Loans are provided for the period of 5 to 10 years.

III Investments -

Investment means Bank make / Invest more fund in two type of securities.

01) government Security :-

This include both central & State government securities like treasury bills & savings etc.

02) Other Security :- This include state associated bodies like debenture of plant. electricity board. etc.

IV Credit Creation :- It is situation in which Banks make more loans to customers & business with the result that the amount of Money circulation Increase.

Secondary function

- Agency Service
  - payment of LIP & EB.
  - purchase & sales of Security
  - Remittance of funds.
  - collection of Draft, Dividend & Interest.
- General utility Service

(01) Short-term loan :- Short term loans are provided for less than one year.

(2) Medium term loan :- This type of loan is provided between one to five years.

(3) long term loans : loans are provided for the period of 6 to 10 years.

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Investment means Bank make / Invest more fund in two type of securities.

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- Agency Service
  - payment of LIP & EB.
  - purchase & sales of security
  - Remittance of funds
  - collection of Drafts, Dividend & Interest
- General utility Service

* Secondary function or Subsidiary function :-

The modern Banks performance a variety of financial & non-financial services to its customers & to the public. In addition to the primary function Bank has Introduce secondary function.

The Secondary function rendered by the Bank & in order to attract to customer & Increase the Image of the Bank. In the I of public

Secondary function are divided into ag

01) agency service 2) Jeneral Utility Service

01) Agency Service - Bank act as Agent on behalf of its customer is non as Agency service. this service are rendered by the banks to create public confidance & good will.

following are Agency service rendered by the banks

01) payment of LIP & EB (Life Insurance premium (Electricity) & Bill)

02) Purchase & sale of Security.

03) Remittance of funds

04) Collection of Draft / Dividend & Interest.

01) payment of LIP & EB :-

Bank make payments on behalf of customers. Such as life Insurance premium, electricity Bill, tend water tax, etc. when the payment is made by the bank immediately the ammount is debited from customers A/c

Jenerally this service is made by the Bank. when it obtains standing order is written for the customer, & Bank will never render the service. when the order is cancelled by the customer for this agency

Service Bank will charge Certain amount as a Service Charge

(2) purchase of Sales & Security :-

Bank buy & Sell Stock exchange Securities on Behalf of customer this Service is rendered by the bank when itt gets Instruction is written to buy & Sell the Securities on his behalf.

when the customer give to Instruction to the bank to buy & Sell Security the Bank Immediately assign the work. For this agency Service Bank will charge Certain amount as Service charge. which is known as commission.

(03) Remittance of funds :-

Money Send from One place to another place or one country to another country on behalf of customers. this remittance of fund Can be made by drafts. & cheque.

(04) Collection of Draft Dividend & Interest :-

Bank undertakes to collect the money on be half of customer.

Bank collects Drafts, Bills of Exchange, Dividend, & Interest. for rendering such Services Bank will collect Service charge.

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## (B) Miscellaneous or General Utility Services :

Modern banker is not only rendering the services to his customers but also rendering the services to the general public. These services are most useful to the general public. That is why, such services are called **General utility services** or **Miscellaneous services.**

The following are the general utility services rendered by the modern banker, viz.

(1) Safe custody of valuables.

(2) Issuing of letter of credit, travellers' cheques, gift cheques & credit cards.

(3) Giving credit information

## (1) Safe custody of valuables.

Modern banker provides safe custody services to customers. Generally safe custody rooms are well-equipped and strongly built rooms. Customer can deposit his Jewellery, Ornament, LIC policies, Debentures, Shares, Bonds and valuable articles & Documents. Safe lock rooms have watch and ward arrangements for all the twentyfour hours apart from being covered against theft and fire.

When the banker receives valuables to be deposited in safe custody, he issues safe custody receipt to the customers in detail, for the items deposited, individually. For rendering such services, banker will charge the service charges.

Generally articles for safe deposit are deposited in locker boxes or closed envelopes, while accepting sealed boxes of convenient size for safe custody, the banker should see that the box is sealed with customer's seal and contains the name and address of the depositor written thereon in bold letter in indelible ink.

Usually the safe custody articles or securities are returned to the customer himself but not to others. In case of death, insolvency or lunacy of the depositor, the articles or securities of safe custody should be delivered to his legal representative on the production of documentary evidence.

In this case, the relationship between banker and customer is that of a **bailee** and a **bailor.**

## (2) Issuing letter of credit, travellers' cheques, gift cheques, credit cards.

### Letter of Credit :

It is an important instrument in international trade i.e. importing and exporting. Letter of credit is a letter from a banker to his agent abroad or to a foreign bank, authorising the payment of specified amount to a person named in the letter. Now a days, modern banks facilitate trade and commerce by rendering valuable services to the business community. The standing reputation and goodwill earned by a bank institutions enable the bank to issue instruments known as a letter of credit in favour of traders and banks to meet the need and their customers.

The letter of credit carries a promise or an undertaking by the issuing banker requesting another party i.e. either banker trader to grant a specified amount to third party specified therein and the issuing banker finds himself to pay the money under the letter of credit.

## Types of letter of credit

There are two types of letter of credit.

(1) Travellers letter of credit (2) Commercial letter of Credit.

### (1) Travellers letter of credit

It is a letter given by one branch of a bank to another branch, situated within the country or outside the country, directing it to make payment to travellers upto the amount specified in the letter.

This letter helps persons who travel or visit any place in India or outside India. The travellers deposit a certain amount with a banker. The banker after receiving deposits, issues the advised letter of credit.

* Commercial letter of credit :-
It is a letter of credit issued by the banks in the Importer country in favor of Exporter country informing to Except or pay the bill of Exchange drawn by Exporter.

* ~~[illegible] Restrictions~~ / ~~[illegible]~~ :-

* Giving credit Information :- Now a days bank's provide credit information needs customer under this system bank collects information about other business man to their fellow bankers this system of giving credit information to customer helps the banker to give credit.

* Acting as referee :- Banks give status option about their customers to concern people in other place who may require Information on behalf of their customers who may live do have business dealing with parties about whom the status reports are required.

* New functions :-
On account of liberalisation, privatisation & globalisation the current Economic condition have increased competition in financial services provided. So today banks are providing various new services to their customers; those are.

1. New Schemes for accepting Deposits :-
Now a days various new schemes are provided for accepting deposits.
Some of new schemes for accepting Deposits are as follows.

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a) cash Certificate - There are the deposits accepted by the banks for the fixed period Interest is included by final payment customer can en-cash their certificate before maturity period.

b) Daily Savings - under this Scheme bank accept Small deposits from Small depositers every day this Scheme is More usefull to daily wage earner.

Daily Saving Scheme is also known as pigmy deposit Scheme this Scheme Introduced by Syndicate Bank.

c) Minor Savings Scheme - This Scheme is introduced in order to encourage the Saving habbits among minor childrens, under this Scheme banks accept Deposits in the name of Minor children & withdrawell facility is also availiable

d) ladies department - Some banks introduced lad department in order to attract deposits from womens in this departments all the employees are ladies. but this facility has lossed its Importance, because women banks has been Started.

e) Monthly Income Interest Scheme -

In this Scheme banks receive fixed deposit & Interest is paid Monthly Instead of yearly or half yearly under this Scheme banks accept deposit for a period of 7 years. & after Seven years, bank will pay ₹ 100. to the depositer forever.

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1) Farmers deposit Scheme :-

This Scheme is Introduced for a benifit of Farmers under this Scheme Farmers can deposit the money once or twice a year as an one they received the proceeds the Sale of crops.

2. Housing Deposit Scheme :-

This Schemes are useful for those who wish to save money for purchase or construction their own house. account holders have to deposit Money in installment for a fixed period, & on the expiry of fixed period Depositer can with draw the deposit allong with Interest.

3. personal loan Scheme :

This Scheme grant the loans to Individuals like Employee, are engaging business and having Study Income to purchase durable goods like TV, refrijetar, washing Machine, etc. there are clean loans while granting this loans the bank moves its line on the consumer durable goods brought by the consumer out of personal loan & reserve the right to Sice the goods if the borrower fail of re-payment.

04) Schemes for Financing Small Scale Industries :-

Under this Scheme bank provides Short term or long term loans to Small Scale Industries for their Expansion, innovation & Modernisation of their Industries. Similarly bank grant loans technicians or entrepreneur to establish Small Scale Industrial Units.

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(05) Educational loan :-
Now a days banks are also providing educational loan to students for their higher studies. Such loans are re-payable by the beneficiaries by way of Installment in future.

(06) Schemes for financing agriculture :- Under this Scheme bank have introduced various Scheme to help agriculture. Just like Short term loans Medium term loans, long-term loans,
- * Bank provide Short-term loans for seasonal agricultural activities
* Midium term loans are provided for purchase of agricultural equipments & pump sets etc.
* long-term loans are provided for purchase of lands, & tractors. & other assets.

(07) loan's for Self Employement :- Now a day's Banks are providing loans to graduate & other self Employement for purchase of car, Machines, Xerox machine, Auto, etc. to get Employee

(08) lock-Box - A Box is kept at the branch of the banks in which customers can put their cheque & other remitance after the Bank office hours. the Bank collects them next day en-pass the entries

* **Mobile Banking :-**

under this system banking transactions are carried down through mobile phones which is connected with concern bank which has provided this facility. The bank under this system provides free Mobile banking to its customers who contact the bank by mobile phone. Mobile banking work through series of text messages the customer will be given a specified SMS for that transaction who wants to do. He will receives the response in the form of text message on his mobile for within the few minutes.

(02) **loans to woman :-** Now a day's banks are providing finance to woman to make their self reliant for ex: Banks provide loans to woman for purchase of materials of Tailoring, Embroidery etc. Banks also provide loans to woman for home productions like pickles, papad, etc.

(03) **A.T.M =** According to A.T.M. System customers can get the money any time that is customers can withdraw through A.T.M cards from his banks. Deposit money into his bank account & check the balance.

The service of A.T.M. is available for 24 hours, for all the 365 days.

**Types of ATM.**

01) On Site ATM.

02) Off Site ATM.

(01) **On Site ATM :-** ATM installed within the bank premises is called on site ATM. it is used by specified Branch customers

(03) Off Site ATM :- These are the ATM's installed by a bank at a Important places like railway station, Bus stand, Airports, petrol Banks & opposite to School & college. this kind of ATM's are used by all kind of customer.

(04) Online Banking :- online Banking refers to use today's computer technology to avoide the time consumption, paper based aspects of traditional Banking, in order to manage our finance more quickly & effectively.
On line Banking is also known as Internet Banking.

(05) E-Banking :- E-Banking means electronic Banking. it means all the transactions are carried through electronic medias. Computer or Internet.

facilities of E-Banking.

01) Account can be open in any branch of Bank through out the world.
02) Shifting of amounts through Internet
03) withdrawal of Money
04) Deposit of Money.

(06) Tele Banking :- Tele Banking means with a ~~keep of~~ , is a System where a customer can dial the given tele Banking tele Banking number through the land line or Mobile phones. for many where The customer can Exis is amount by following the user friendly Menu.
Following are the facilities of tele Banking:

01) Balance Enquiry.
02) Enquiry of term - deposit
03) cheque Books request
04) Transfer of fund.

(07) Credit Card : Credit Card is a Small plastic Card issued by banks. which allow the credit card holder to purchase goods or Services on credit.

(08) Smart Cards :- is a plastic card which contains tiny computer chip. called as ~~Smart Cards.~~ Microprocessor which contains information.

BRM 1 - Unit. 

M.com III sem.

# Introduction

Meaning of Research

Research is a Process of Systematic enquiry or Investigation into a Specific problem or issue that leads to new Improved Knowledge.

Meaning of Business Research.

Business Research is a process of acquiring detail information of all the areas of business and use such information in Maximizing the Sales & profit of the business.

Definition.

Business Research is defined as "the Systematic and objective process of gathering, recording, analysing data For making business decision."

Charecteristics Of Research.

1. A Systematic approach is followed in research. Rules and procedures are an integral part of research that set the objective of a research process. Researches need to practice ethics and code of conduct while making observations or drawing conclusions.

2. Research is based on logical reasoning and involves both inductive and deductive methods

3. The data or knowledge that is derived is in real time, actual observations in the natural settings.

4. There is an in-depth analysis of all the data collected from research so that there are no anomalies associated with it.

5. Research creates a path for generating new questions. More research opportunity can be generated from existing research.

6. Research is analytical in nature. It makes use of all the available data so that there is no ambiguity in inference.

7. Accuracy is one of the important character of research, the information that is obtained while conducting the research should be accurate and true to its nature.
For Example, research conducted in a controlled environment like a laboratory. Here accuracy is measured of instruments used, calibrations, and the final result of the Experiment.

# Types of Research.

Following are the types of research methods.

1. Basic Research:-

Basic Research is mostly conducted to enhance knowledge. It covers fundamental aspects of research. The main motivation of this research is knowledge expansion. It is a non-commercial research and doesn't facilitate in creating or inventing anything. For example, an experiment is a good example of basic research.

2. Applied Research:-

Applied research focuses on analyzing and solving real-life problems. This type of research refers to the study that helps to solve practical problems using scientific methods. This research plays an important role in solving issues that impact the overall well-being of humans.

3. Problem oriented research:-

As the name suggests, problem-oriented research is conducted to understand the exact nature of the problem to find out relevant solutions. The term "problem" refers to having issues or two thoughts while making any decisions.

4. Problem Solving Research :-

This type of research is conducted by companies to understand and resolve their own problems. The problem-solving research uses applied research to find solutions to the existing problems.

5. Qualitative Research :-

Qualitative research is a process that is about inequiry, that helps in-depth understanding of the problems or issues in their nature settings. This is a non-statistical research method.

Qualitative research is heavily dependent on the experience of the researchers and the questions used to probe the sample. The sample size is usually restricted to 6-10 people in a sample. Open ended questions are asked in a manner that one question leads to another. The purpose of asking open ended questions is to gather as much information as possible from the sample.

Research Methodology :-

The process used to collect information and data for the purpose of making business decisions. The methodology may include publication research, interviews, surveys and other research techniques, and could include both present and historical information.

# Scope of Business Research.

1. product Research
2. Pricing Research
3. Sale or Market Research
4. Advertising Research
5. corporate Responsibilities Research
6. Business Economic & corporate Research.

## > Major Steps in Research
or
## Basic Steps in Formulating a Research Problem.

What is the most essential part of your research Project ? It is obviously the formulating of a research problem. In other words, Selecting your research topic. This is because of the quality and relevancy of your research work completely depends on it. The process in formulating a research problem requires a Series of Steps. Basically, there are 7 basic Steps in formulating a research problem.

1. Identify the Broad Study Area.
2. Dissect the Broad Study area into Subareas
3. Mark-up your Interest.
4. Study Research questions
5. Set out Objectives
6. Assess your objectives
7. Check Back

1. **Identify the Broad Study Area:-**

This is a great idea to thinking about the subject area of your interest. You should identify the field in which you would like to work a long time after your academic study or graduation. It will help you tremendously to get an interesting research topic.

2. **Dissect the Broad Study Area into Subareas:-**

In this stage, you need to dissect and specify your research broad study area into some subareas. You could consult with your supervisor on this regard. Write down subareas. For Example:- If you select unemployment as your broad study area, then dissect it into unemployment & social stability, unemployment & crime, unemployment & individual frustration etc.

3. **Mark-up your Interest:-**

It is almost impossible to study all subareas. That's why you must identify your area of interest. You should select issues in which you are possi . about. Your interest must be the most important determinant of your research study. Once you selected your research study of interest, you should delete other subareas in which you do not feel interested.

4. Study Research Questions :-

In this step, you would point out your research Questions under the area of interest as you decided in the previous stage. If you select unemployment as your study area, your questions might be "how unemployment impact on individual social status?" "How it affects social stability?" "How it creates frustration on individuals?" Define what research problem or question you are going to study?

5. Set out objectives :-

Formulate your research main objectives and Sub-objectives. Research objectives necessarily come from research questions. If you do study "Impact of unemployment on individual social status" as your research problem or research question. Then, set out what would you like to explore to address. For Example :- your main objective might be to Examine the unemployment status in a particular society or state.

6. Assess your objectives :-

Now, you should evaluate your objectives to make sure the possibility of attaining them through your research study. Assess your objectives in terms of time, budget, resources and technical expertise at your hand. You should also assess your -

P.T.O.

- research questions in light of reality.
Determine what outcome will bring your study.
Then, go to the next step.

7. Check Back :-

In this final step, you should review all the things what you have done till now in the purpose of your research study. Ask your self about your enthusiasm. Do you have enough resources to step up? If you are quite satisfied, then you forward to undertake your research work.

Nature of Research.

1. Business Research is an Economic resource.
2. Business Research is a Systematic authority.
3. Business Research is an activity of Business management.
4. Business Research is a team effort.
5. Business Research is an art and Science as a Profession.
6. Business Research is interdisciplinary System.

# * Types of Business Research:-

Variety of research activities would be helpful to categories the types of Business Research can be classified on the Basis of either technique or function, Experiment Survey and observational Studies are just few common research techniques classified business research on the basis of purpose or functions allows to us understand how the nature of the problem influenced the choice of research Method. the nature of the problem to determine whether research is.

1. Exploratory.
2. Descriptive
3. Casual.

## 1. Exploratory Research:-

Exploratory research is an important part of any marketing or business strategy. Its focus is on the discovery of ideas and insights as opposed to collecting statistically accurate data. That is why exploratory research is best suited as the beginning of your total research plan. It is most commonly used for further defining company issues, areas for potential growth, alternative courses of action, and prioritizing areas that require statistical research.

② Descriptive research :-
It is most widly used research design as indicates by this is dissertation and research report of institutions its common means of obtaining information includes use of questionnaire personal interview schedules and observation either participatory or not.

③ Causal Research :-
The investigation into an issue or to pic that looks at the effect of one thing or variable or others.
Ex :- Causal research might be used in business environment to qualify the effect that which change to its present operations will have further production level to asset in the business planning process.

Other types of Research method.

1) Generally classified
a) Quantitative research
b) Qualitative research.

2) According to nature of Study
a) Discriptive.
b) Analytical.

3) According to purpose.
a) applied Research
b) Fundamental Research.

4) According to research design.
a) Exploratory
b) Conclusive.

Research methodology.

It is the Specific procedure or techniques used to identify select product and analysis information about a topic.

Methodology is the Systematic therotical analysis of the methods applied to a field of study in otherwords all those methods which are used by researchers during the Course of Study with research problems are term as research method.

Keeping this view research method can be put into the following groups.

1) In the first group we include those methods which are concerns with collection of data already available are not sufficient to arrive at required Solution.

2) The Second group consist of those Statistical techniques, are used for establishing relationship between the data and the unknowns.

3) The third group consist of those methods which are used to evaluate the accuracy of the result of them.

Hypothesis :->

A hypothesis is a statement that may be tested and proven to be either true or false. The hypothesis maintains the statement is true. however falsifiability the ability to be disproven is also essential to testing hypothesis, in accordance with the Scientific Method

Types of Hypothesis.

1] Null Hypothesis
2] Alternative Hypothesis.

1] Null Hypothesis :-

Null Hypothesis that their is no significant difference between specified population any observe difference being due to sampling or experimental Error.

It is denoted by Ho

2] Alternative Hypothesis

It is hypothesis that same observation are influenced by some non random causes it is denoted by $H_1$ & Ha.

* Sources of Hypothesis.

1] Theory
2] observation
3] Analogies
4] Institutional & Personal Expiriment.
5] finding of i.e. Studies
6] State of Knowledge.
7] Cultural

# Research Design:-

## Definition :-

Research design is defined as a framework of methods and techniques chosen by a researcher to combine various components of research in a reasonably logical manner so that the research problem is efficiently handled.

## Characteristics of Research design or features.

There are four key characteristics of Research design.

**Neutrality :-**

The results projected in research design should be free from bias and neutral. Understand opinions about the final evaluated score and conclusion from multiple individuals and consider those who agree with the derived results.

**Reliability :-**

If a research is conducted on a regular basis the researcher involved expects similar results to be calculated every time. Research design should indicate how the research questions can be formed to ensure the standard of obtained results and this can happen only when the research design is reliable.

**Validity :**

There are multiple measuring tools available for research design but valid measuring tools are those which help a researcher in gauging results according to the objective of research and nothing else. The questionnaire developed from this research design will be then valid.

Generalization:

The outcome of research design should be applicable to a population and not just a restricted sample. Generalization is one of the key characteristics of research design.

Types of Research Design:

A researcher must have a clear understanding of the various types of research design to select which type of research design to implement for a study. Research design can be broadly classified into quantitative & qualitative research design

Qualitative Research Design

Qualitative research is implemented in cases where a relationship between collected data and observation is established on the basis of mathematical calculations. Theories related to a naturally existing phenomenon can be proved or disproved using mathematical calculations. Researchers rely on qualitative research design where they are expected to conclude "why" a particular theory exists along with "what" respondents have to say about it.

Quantitave Research Design

Quantitative research is implemented in cases where it is important for a researcher to have statisti conclusions to collect actionable insights. Numbers provide a better perspective to make important business decisions. Quantitative research design is important for the growth of any organization because any conclusion drawn on the basis of numbers and analysis will only prove to be effective for the business.

applicab

Further, research design can be divided into five types. 

1. Descriptive research Design :-

In a discriptive research design, a researcher is Solely interested in describing the Situation or case under his/her research study. It is a theory-based research design which is created by gather, analyze and presents collected data. By implementing an in-depth research design such as this, a researcher can provide insights into the why and how of research.

2. Experimental Research Design :-

Experimental Research Design is used to establish a relationship between the cause & effect of a Situation. It is causal research design where the effect caused by the independent variable on the dependent variable is observed. For example, the effect of an independent variable such as price on a dependent variable such as customer satisfaction or brand loyalty is monitored. It is a highly practical research design method as it contributes towards solving a problem at hand. The independent variables are manipulated to monitor the change it has on the dependent variable. It is often used in social Sciences to observe human behavior by analyzing two groups - affect of one group on the other.

3. Correlational Research Design :-

Correlational research is a non-Experimental research design technique which helps researchers to establish a relationship between two closely connected variable. Two different groups are required to conduct this research design method. There is no assumption while evaluating a relationship between two different variables & Statistical analysis techniques are used to calculate the relationship between them.

Correlation between two variables is concluded using a correlation co-efficient, whose value ranges between -1 and +1. If the correlation coefficient is towards +1, it indicates a positive relationship between the variables and -1 indicates a negative relationship between the two variables.

4. Diagnostic Research Design:-

In the diagnostic research design, a researcher is inclined towards evaluating the root cause of a specific topic. Elements that contribute towards a troublesome situation are evaluated in this research design method.

There are three parts of diagnostic research design:

- Inception of the issue
- Diagnosis of the issue
- Solution for the issue.

5. Explanatory Research Design:-

In Explanatory research design, the researcher's ideas & thoughts are key as it is primarily dependent on their personal inclination about a particular topic. Explanation about unexplored aspects of a subject is provided along with details about what, how & why related to the research questions.

Planning of Research.

research plan is the main part of a grant application and describes a principal Investigator's proposed research. The research plan gives a principal investigator the opportunity to discuss proposed research, stating its importance & how it will be conducted.

Steps of planning process.

Step 1 - Locating & Defining Issues or problems.

This Step focuses on uncovering the nature and boundaries of a situation or question related to marketing strategy or implementation. In defining the issues or problems, the researcher should take into account the purpose of the study, the relevant background information, what information is needed, and how it will be used in decision making.

Step 2 - Designing the research project.

This Step is focused on created a research plan or overall approach on how you are going to solve the issue or problem identified. A research plan or approach is a framework or blueprint for conducting the marketing research project. It details the procedures necessary for obtaining the required information, and its purpose is to design a study that will test the hypotheses of interest, determine possible answers to the research questions, and provide the information needed for decision making.

Step3 - Collecting data.

This step revolved around obtaining the information that you will need to solve the issue or problem identified. Data collection involves a field force or staff that operates either in the field, as in the case of personal interviewing (in-home, mall intercept, or computer-assisted personal interviewing), from an office by telephone (telephone or computer-assisted telephone interviewing), or through mail (Traditional mail and mail panel surveys with prerecruited households).

Step 4 - Interpreting Research Data.

Interpreting research data: This step is focused on examining the data & coming up with a conclusion that solves the problem.

Step 5 - Report Research findings.

The final step is to report the research findings to those who need the data to make decisions. The findings should be presented in a comprehensible format so that they can be readily used in the decision making process. In addition, an oral presentation should be made to management using tables, figures & graphs to enhance clarity and impact.

Social Science Research.

Social Science Research is a systematic method of exploring, analyzing & conceptualizing human life in order to extend, correct or verify knowledge of human behaviour and social life.

Social research seeks to find explanations to unexplained phenomena, to clarify the doubtful and correct the misconceived fact of social life.

It involves the application of scientific method for understanding and analyzing of social life in order to correct & verify the existing knowledge as a system.

## Functions of Social Science Research.

a) Discovery of facts and their Interpretation :-

Social research provides answer to questions of what, when, how and why of man, social life & institutions. Discover of facts and their inter relationship help us to discard distortions and contribute to our understanding of social reality.

b) Diagnosis of problems and their analysis :-

Our society has innumerable problems such as poverty, unemployment, economic inequality, social tension etc,. The nature and dimensions of such problems have to be diagnosed and analyzed. An analysis of problems lead to an identification of appropriate remedial actions.

c) Systematization of knowledge :-

The facts discovered through research are systematized and the body of knowledge is developed. It contributes to the growth of theory building.

d) Control over Social phenomena :-

Research in social science provides first hand information about the nature of social institutions. This knowledge helps us to control over the social phenomena.

(2)

e) Prediction :-

Social research aims at finding an order among Social fact and their casual relations. This affords a Sound basis for prediction in several cases.

f) Development planning :-

Systematic research can give us the required data base for planning and ~~designing~~ developmental. Schemes & programmes.

g) Social welfare :-

Social research can identify the causes of Social evils and problems. It can thus help in taking appropria-te remedial actions. It also provides guideline for Social welfare.

Ethical Issues in Business Research.

① Honesty and Integrity :-

This means that you need to report your research honestly, and that this applies to your methods (what you did), your data, your results and whether you have previously published any of it. You should not make up any data, including Extrapolating unreasona-bly from some of your results, or do anything which could be construed as trying to mislead anyone. It is better to undersell than over-Exaggerate your findings. When working with others, you should always keep to any agreements and act sincerely.

(2) Objectivity :-
You should aim to avoid bias in any aspect of your research, including design, data analysis, interpretation, and peer review. For example, you should never recommend as a peer reviewer, someone you know, or who you have worked with, and you should try to ensure that no groups are inadvertently excluded from your research. This also means that you need to disclose any personal or financial interests that may affect your research.

(3) Carefulness :-
Take care in carrying out your research to avoid careless mistakes. You should also review your work carefully and critically to ensure that your results are credible. It is also important to keep full records of your research. If you are asked to act as a peer reviewer, you should take the time to do the job effectively and fully.

(4) Openness :-
You should always be prepared to share your data and results, along with any new tools that you have developed, when you publish your findings, as this helps to further knowledge and advance science. You should also be open to criticism and new ideas.

(5) Confidentiality :-
You should respect anything that has been provided in confidence. You should also follow guidelines on protection of sensitive information such as patient records.

⑥ Respect for Intellectual property:-

You should never plagiarise, or copy, other pe work and try to pass it off as your own. You should always ask for permission before using other people's tools or methods, unpublished data or results. Not doing so is plagiarism. Obviously, you need to respect copyrights and patents, together with other forms of intellectual property and always acknowledge contributions to your research. If in doubt, acknowledge, to avoid any risk of plagiarism.

⑦ Responsible publication:-

You should publish to advance to state of research and knowledge, and not just to advance your career. This means, in essence, that you should not publish anything that is not new, or that duplicates someone else's work.

⑧ Legality:-

You should always be aware of laws and regulations that govern your work and be sure that you conform to them.

⑨ Animal Care:-

If you are using animals in your research, you should always be sure that your experiments are both necessary and well designed. You should also show respect for the animals you are using & make sure that they are properly cared for.

Review of literature :-

"A literature review is a comprehensive summary of previous research on a topic. The literature review surveys scholarly articles, books and other sources relevant to a particular area of research".

The review should enumerate, describe, summarize, objectively evaluate and clarify this previous research. It should give a theoretical base for the research and help you (the author) determine the nature of your research. The literature review acknowledges the work of previous researchers, and in so doing, assures the reader that your work has been well conceived. It is assumed that by mentioning a previous work in the field of study, that the author has read, evaluated and assimilated that work into the work in hand.

literature review sources can be divided into three categories are as under.

1. Primary sources

2. Secondary sources.

3. Tertiary sources.

1. Primary Sources :-

It includes high level of details. And it needs little time to publish.

Examples :- Reports, Theses, Emails, conference proceedings, Company reports, unpublished manuscript Sources, Some Government publications.

2. Secondary Sources :-

It includes medium level of detail and It need medium time to publish.

Examples :- Journals, Books, Newspapers, Some Government publication.

3. Tertiary Sources :-

It includes low level of detail. and considerabl amount of time needed to publish.

Examples :-

Indexes, Database, Catalogues, Encyclopaedias, Dictionaries, Bibliographies, Citation Indexes.

## BUSINESS AND CULTURE

Culture, which is a very intriguing and complex factor is, often, a very critical component of business environment. An important problem is that several dimensions of culture are not easily explicit. A company which sets out to do business in unfamiliar cultural environment may, therefore, encounter several problems if proper home work is not done. Many multinational businessmen agree that "cultural differences are the most significant and troublesome variables encountered by the multinational company. The failure of managers to comprehend fully these disparities has led to most international business blunders."[10]

### Meaning of Culture

There are varying definitions of culture: "Culture, in its broadest definition, refers to that part of the total repertoire of human action (and its product) which is socially, as opposed to genetically, transmitted". A very popular definition is that of E.B. Tylor: "Culture of civilization is that complex whole which includes knowledge, belief, art, morals, law, custom, and other capabilities and habits acquired by man as a member of society". Kluckhohn has defined culture very simply as "the total life way of a people" As Geert Hofstede, a noted Dutch writer and academic has nicely put it, culture is the *software of the mind*—the social programming that runs the way we think, act and perceive ourselves and others. In other words, your brain is simply the hardware that runs the cultural programming. The implication is that culture is not innate. It is learned behavior and hence can be changed.[11]

On the basis of the various definitions of culture, Francis Merill formulates the concept of culture as follows."[12] Culture:

- is the characteristically human product of social interaction;
- provides socially acceptable patterns for meeting biological and social needs;
- is cumulative, for it is handed down from generation to generation in a given society;
- is meaningful to human beings because of its symbolic quality;
- is learned by each person in the course of his development in a particular society;
- is, therefore, a basic determinant of personality; and
- depends for its existence upon the continued functioning of society but is independent of any individual or group.

Culture consists of both *material* culture and *non-material* culture. Material culture involves man-made things (e.g., automobile, television, telephone, etc.) and man-made alternations in the environment. Non-material culture includes such factors as language, ideals, beliefs, values, music, etc.

## Elements of Culture

Culture includes at least three elements, namely, knowledge and beliefs, ideals and preferences.

***Knowledge and Beliefs:*** The knowledge and beliefs refer to a people's prevailing notions of reality. They include myths and metaphysical beliefs as well as scientific realities. As Rose remarks, "one of the features of culture in general that is of special sociological interest is the shared quality of a belief system. People who share a given culture tend to take a hostile attitude towards those within their midst who cannot, or will not, accept conventional definitions of fact"[13]

***Ideals:*** Ideals refer to the societal norms which define what is expected, customary, right or proper in a given situation. Norms are enforced by sanctions, *i.e.*, by rewarding the right behaviour and punishing the wrong behaviour.

Folkways and mores are important aspects of every culture. Folkways are norms of proper behaviour (like the proper way to greet a friend) that are informally enforced. But mores are norms of obligatory behaviour considered vital to the welfare of the group.

***Preferences:*** Preferences refer to society's definitions of those things in life which are attractive or unattractive as objects of desire. Preferences may differ between cultures. Interestingly enough, the judgments of the ideal or the proper do not always correspond to our judgments of the pleasant or enjoyable. An example in point is the temptations (not proper but desirable). "All the things I really like to do are immoral, illegal, or fattening," said Alexander Woollcott.[14]

"A culture tends to provide the standards of tastes in specific lines of human activity. Taste in the most liberal sense varies greatly with the food consumption preferences of different cultures. But there is also taste in clothing, housing sexual practices, and in an endless variety of possessions and activities. What is tasteful in one culture may be highly distasteful in another."[15]

Cultural characteristics are very important in the formulation of pragmatic business strategies. The cost of ignoring customs, traditions, taboos, tastes and preferences, etc, can be very high. *For example*, in Italy, a US company that set up a corn-processing plant found that its marketing efforts failed because Italians thought of corn as "pig food". The Nestle company brews a variety of instant coffee to satisfy different national tastes.

## Organisation of Culture

The term organisation of culture refers to the social structure and the integration of traits, complexes and patterns that make up the cultural system.

That cultures are organised or integrated "does not mean that every single item of each culture is neatly and precisely integrated with everything else. It means rather that it is normal for the parts to be somewhat organised, and that culture traits receive their significance and meaning out of their relation to the rest of the culture."[16]

The social structure – "the web of organised relationships among individuals and groups that defines their mutual rights and liabilities" – together with traits, complexes and patterns, reflects the organisation of a culture.

Stratification, *i.e.*, differentiation based on criteria such as age, sex, caste, occupation, education, income and so on, is an important aspect of the social structure and cultural organisation. Each stratum is assigned or supposed/expected to have a certain rank or position, role, or limitations etc., in the societal set-up. It is important to recognise such roles and ranks for effective negotiation, promotion etc.

The organisation of a culture is determined to a large extent by major social institutions. According to MacIver and Page, institutions are established forms or conditions of procedure characteristic of group activity. The group which performs these standardised actions has been termed by them an association. According to Biesanz and Biesanz, "institutions are clusters of norms organised and established for the pursuit of some need or activity of a social group, supported by the group's knowledge, beliefs and values, as well as by the meaningful aspects of material culture".

The important common institutions of modern cultures are the economic system, the political, administrative system, the educational system; religion, family, expressionistic, aesthetic and recreational institutions, etc. Such institutions have been established to meet society's common needs of a biological, sociological, psychological, economic, and political nature – the type and nature of institutions reflect the common goals, aspirations and the ways of achieving them, definition and regulation of roles, positions, inter-relationships, etc., of the individuals and sub-groups and groups and the overall organisation of the culture.

Culture traits, complexes and patterns also help us to understand the organisation of a culture. A trait is a unit of observation. It may be a unit of normative behaviour, like shaking hands or saying *namaste*; or it may be an articraft, like a culture object such as a wooden bowl. As Lumby observes, "a culture trait is the simplest acquired material or activity pattern known; and these traits are the bricks, so to speak, of which the whole culture of the society is constructed". Most traits are related to others and fit into larger meaningful wholes called trait complexes. *For example,* the various traits involved in greeting and receiving a guest form a trait complex. A complex, thus, is a system of interrelated traits that function together as a unit and a number of complexes, in turn come together to form a culture pattern. A culture pattern "is a specific and enduring system of trait complexes". The organisation of culture may, thus, be looked upon from the point of view a meaningful integration of different traits into interrelated complexes and complexes in turn into patterns. The term culture pattern is sometimes used to designate the overall organisation of the culture; but sometimes it is used to refer to the major segments of the culture, like the religious pattern of a particular ethnic group.

## BOX 8.2 : CULTURAL ENVIRONMENT AND TECHNOLOGY

*Perhaps it is because of our need for the means of survival that Japanese science tends to concentrate more on the applied than on the theoretical. We have taken many basic ideas and turned them into practical objects, in many cases products not even thought of by the originators of the basic technology. This is inevitable, of course...*

*We Japanese have always been eager to develop our own technology, absorb aspects of technology from abroad, and blend them to make suitable objects or systems...*

*The attitude in America is much more easygoing as far as raw materials are concerned than in Japan. America has so much of everything—oil, coal, copper, gold, uranium, timber—that even today Americans do not seem to take conservation seriously. I am reminded of the American expression, "There's plenty more where that came from." We (Japanese) have no such expression. Our people also seem naturally more concerned about precision. It may have something to do with the meticulousness with which we must learn to write the complicated characters of our language. But "for whatever reason, when we tell one of our Japanese employees that the measurement of a certain part must be within a tolerance of plus or minus five,* for example, *he will automatically strive to get that part as close to zero tolerance as possible. When we started our plant in the United States, we found that workers would follow instructions perfectly. But if we said make it between plus or minus five, we would get it somewhere near plus or minus five all right, but rarely as close to zero as the Japanese workers did. We discussed what to do about this, and in no time had the answer. For the U.S. specifications, we just set the tolerance at plus or minus two, and in that range the American workers consistently gave us what we needed. If we have the need and demand zero tolerance from the American workers, we can get it if we specify it. I do not for a moment discredit the foreign worker. Sometimes you have to use a different approach where people are accustomed to different approaches.*

Courtesy : Akio Morita, *Made in Japan*, (New York, New American Library, 1986)

The culture traits, complexes and patterns differ from community to community. This indicates some of the complexities involved in multicultural business.

### Cultural Adaptation

The term cultural adaptation refers to the manner in which a social system or an individual fits into the physical or social environment. The social system may be a small group, such as the family or a larger collectivity, such as an organisation, or even a total society, like a tribal society.

Adaptation is essential for survival. The type of clothing, food and dwelling, suitable for the climatic and weather conditions, are forms of adaptations. Culture adaptation can be viewed in a very wide context. We have adapted to the energy crisis caused by the oil price hikes by modifying our energy policy and intensifying oil exploration, developing, alternative source of energy and restricting oil consumption. Humanity adapts to contagious diseases by immunisation.

Adaptation is relevant at the individual level as well. An individual who joins or accepts a new religion has to adapt himself to the beliefs and ways of that religion. A worker who becomes a member of a trade union has to fit into the objectives, rules and ways of the union. A woman who lives with her husband's family would have to fit into that family culture. An Indian who settles in the U.S.A. has to adjust to the social and physical environments there. One who joins a new organisation will have to adapt to the new environment.

The message for business is that the firm and its people will have to adapt to the environment of the different markets. As MacIver and Page remark, "every difference of environment means a difference in our habits, our ways of living. On the other hand, our habits, our ways of living, in so far as they differ, create for us a different environment, a different selection within it, and a different accommodation to it. Through a process of constant selection and constant adaptation, the moving equilibrium of life is maintained".[17]

It is often necessary to know the process and nature of the cultural environment for a successful formulation of business strategies. *For example,* while introducing new ideas, techniques, products; while segmenting the market; while formulating the product and promotion mix strategies; one should consider the extent to which different categories of consumers adapt to the new things or environment and the factors favouring and disfavouring adaptations (and also the general attitude of society to the new ideas and environment and their impact on different categories of consumers).

### Cultural Shock

Environmental changes sometimes produce culture shock – a feeling of confusion, insecurity, and anxiety caused by the strangeness of the new environment. *For example,* if a youngster, born and brought up in a large city; is posted to a bank office in a remote village, he may experience a cultural shock. Similarly, a villager may experience a cultural shock when he takes up a job in a large modern company in a far away metropolitan city or foreign nation. They have, however, to adapt to the new culture in due course if they want to survive.

Executives and other employees on foreign assignments may experience culture shock in alien environment. Sometimes the organisation itself may suffer shock. Proper home work to understand the culture can help avoid the shock. This also highlights the importance of the selection of people for foreign markets.

### Cultural Transmission

A very important character of culture is its transmissive quality. The elements of culture are transmitted among the members of the culture, from one generation to the next, and to the new members admitted into the culture. Some of the aspects of a culture may be transmitted to other cultures also.

The transmissive quality of culture makes it cumulative. Every generation inherits a stock of cultural elements, many of which have been accumulated over a long period of time. As time goes on, cultures accumulate more techniques, ideas, products and skills. It is also quite obvious that certain old elements are dropped as new ideas and traits are acquired.

Many cultural behaviours are handed down by one's parents, teachers and other elders. The reference groups play an important role in handing down new traits and ideas. Some cultural behaviours are however, "handed up" to the elders. It is not uncommon to come across elders imitating or adopting some of the new traits of the youngsters who are the trend-setters. Cultural transmission is not only downward and upward; a lot of transmission takes place among contemporaries, too, e.g., styles of dress, recreational fads; reading and learning habits, political, social and economic views. These are often transmitted among contemporaries. Cultural transmission, thus, takes place horizontally as well as vertically.

Cultural transmission takes place by means of symbolic communication. A symbol is any sign, signal or word that conveys a meaning. The great importance of language in cultural transmission is quite clear. Literature, film, TV and some other electronic gadgets, social institutions, advertising and marketing techniques, and so on, play very important roles in cultural transmission.

Transmission also facilitates cultural diffusion, *i.e.,* the spread of cultural elements from one place to another. Cultural transmission and diffusion are easy in a culture with high educational levels and a well-organised communication system. An effective communication system and high educational levels facilitate socio-economic change through better cultural transmission and diffusion, for new ideas and innovations are easily and quickly transmitted, diffused, and absorbed in such a culture. In the context of the generally low literacy rates in India, the government has realised the importance of the media, such as film T V and radio in transmitting information such as better agricultural practices and techniques, market information, the concept and importance of family planning, and so on.

The nature and process of cultural transmission and diffusion in a society is important to business decision-making. *For example*, to formulate a promotional policy for a product, a service or an idea, it is important to identify the relevant elements of transmission, to evaluate the relative effectiveness of alternative communication media, to identify the reference groups and the extent of their influence, to identify the channel of influence on the reference groups, and so on.

### Cultural Conformity

Individuals in a culture tend either to conform to the cultural norms or to deviate from them. If the culture endures as it is, most people would conform to the norms. As Inkeles observes, "the social order depends on the regular and adequate fulfilment of the role obligations incurred by the incumbents of the major status-positions in a social system. It follows that the most important process in society is that which ensures that people do indeed meet their role obligation."[18]

A student who abides by the rules of his school discipline, does his home-works promptly and studies properly is conforming to his role obligations. Similarly, an employee who works properly is conforming to his role obligation. And an employee who strikes work for a reasonable cause in response to a strike call by his union is also conforming to his role as member of the union. "When an individual has incorporated within himself the knowledge and appropriate skills necessary to the fulfilment of a role, and when he accepts the value or appropriateness of the action, sociologists speak of his having 'internalised' the role and its psychological underpinnings."[19] Such internalisation helps achieve cultural conformity.

If a society is, by and large, characterised by blind conformity, it would be very difficult to market new revolutionary ideas (including products and techniques) in such a society. Special efforts may be required in such a society to change the attitudes of the people in favour of unconventional ideas. It is also important to understand the extent and nature of the snow-balling effects of initial deviations in a society.

### Cultural Lag

The cultural lag thesis put forward by William F Orgburn says that the various parts of modern culture do not change at the same rate, and that since there is a correlation and interdependence of parts, a rapid change in one part of our culture requires readjustments through other changes in various correlated parts of that culture. These readjustments are often difficult, if not impossible, to make because of a variety of factors, ranging from ignorance to active resistance. Technological changes call for adaptive changes in non-material culture, which is inherently conservative. The cultural lag thus places constraints on the scope of social change through technological development. *For example*, in some cultures social inertia and religious sentiments come in the way of population control, though a variety of techniques are available for birth control.

International business arena is replete with cases of cultural lag. It indicates that different markets may be in different levels of readiness to accept a new product or idea. To successfully market a new idea (including product, service, technique), it is necessary to identify the factors causing the lag and to overcome them by taking appropriate measures. It would be a blunder to introduce a product to a market which is not ready to adopt it.

## CULTURAL TRAITS

Cultures have some important traits. An understanding of these cultural dimensions will be helpful in business.

## RELIGION

Different peoples have their own religious convictions, beliefs, sentiments, customs, rituals, festivals etc. The cost of ignoring certain religious aspects could be very high, sometimes even fatal, in business.

When an American fast food chain was planning to enter India, one political party stated that it would oppose the marketing of beef product in the country by the multinational. In a country where cow is regarded sacred, although there were some protests against slaughter of cow, beef is consumed by a sizable population and the number of the beef consumers in India is larger than the total population of many countries. It may, therefore, look ironic that a foreign firm should encounter this kind of a situation. Pork is banned in Muslim countries. During the holy *Ramzan* period, restaurants and the like owned by Muslims remain closed during day time. Muslims would consume the meat of only those animals/birds slaughtered following the prescribed religious rituals. Many Christians do not consume non-vegetarian during the *lent* (50 days preceding Easter) during the 24 days preceding Christmas and on all Fridays. During these periods, Christians do not conduct marriages and other celebrations like baptism. Hence, the weeks following

Christmas and Easter are seasons of such celebrations. However, it is interesting to note that although according to the Bible, Christians are expected to fast on Sundays (the Sabbath day) and devote the whole day to God, and not to indulge in any worldly activities, most of them rather eat merrily and celebrate this *holy*day. A Buddhist monk from Thailand, studying in an Indian University, who was found to be a regular non-vegetarian, was asked how the disciple of the Lord who preached *ahimsa* could be a meet eater. His answer was that as a Buddhist he was expected not to kill for meat, but if meat was available in the market he could buy and consume.

Religion may also influence the attitude towards work and wealth. "In the United States, it is common to hear people talk about the Protestant work ethic, which holds that people should work hard, be industrious, and save their money. This work ethic helped to develop capitalism in the United States because of the importance it assigned to saving and to reinvestment of capital. However, Americans are not the only people who work hard. In Asian countries where Confucianism is strong, this attitude is known as the Confucian work ethic. In Japan, it is called the Shinto work ethic."[26] An Indian economist has described the growth rate the Indian economy achieved in the earlier Five Year Plans as the Hindu growth rate.

Religion may also play a role in deciding the weekly holiday, other holidays and working hours. In several countries religious festival times are great business times. People buy new clothes, exchange gifts, sped a lot on food etc. Companies doing heavy promotions, including discounts and other incentive schemes have become very common in India.

Many religious groups consider certain days of the week or certain periods auspices for launching new ventures. On the other hand certain days and periods are regarded bad. Interestingly certain days considered as auspices by some community are considered bad days by some others. Many people, particularly Hindus, do not commence any auspicious thing start out for any important matter during *rahu kala*; important matters have to be done or commenced at *shubha muhurtha*.

Many years ago a foreign bank in Chennai introduced a promotion scheme to attract new customers. The scheme ran for a month. Although there were many enthusiastic enquiries about the details, when the period of the scheme approached the end the bank management was disappointed that the number of new accounts opened was nowhere near satisfactory. It was then that the management realised that they chose the wrong period for the promotion. The scheme was, therefore, extended for some more time.

The Islamic holy book Koran prohibits payment acceptance of interest (Interest is considered *riba* or usury.) Islamic banks do not pay regular predetermined interest to depositors nor do they charge predetermined interest rates to borrowers. Rather the banks take a share of the profits (or loss) which are again shared with depositors. What makes profit sharing, unlike interest, permissible in Islam is that only the profit sharing ratio, not the rate of return itself, is predetermined. Islamic banks now handle more assets of huge value worldwide and asset growth at Islamic banking institutions has been growing fast. Even some large Western banks now have Islamic branches. Muslims in other countries, however, do banking in the normal system prevailing there.[27]

Many business decisions in India and in several other countries are based on astrological advices. These include the decisions regarding the timing of the launch, location of the enterprise, name of the firm, brand name, business portfolio and so on. The only forecasting technique some people depend upon is astrological.

The customs of marriage, naming ceremony of the child, festivals etc, wary significantly between religions. These have implications for many types of business like textiles, jewellery, catering, consumer durables etc.

The influence of religion on politics is on the increase in many parts of the world. And politics often plays an important role in shaping economic policies and business regulation and promotion. In a number of countries religion and government are inseparably united.

## ETHNODOMINATION

In many countries one or other industry or trade is dominated by certain ethnic groups. This is particularly true of trade. Ethnodomination in distribution is defined as "a situation where an ethnic group occupies a majority position in a channel of distribution with respect to the ownership and control of physical and financial resources, or through the manipulation of social environment. The control is manipulated through the familiar coercive and collusive practices such as price setting (in both product and factor markets), exclusive dealing arrangements and discrimination among customers or suppliers.[28]

It is pointed out that a common model is an ethnic group, often a "stranger or sojourner minority, selecting or happening into a trade opportunity in a particular commodity. Members of the ethnic group then master the intricacies of supply and demand in world market for that commodity while learning the mechanics of foreign trade in the host country. Then, through time they become established and widely recognised as a reliable buyer or seller of the commodity. From this solid base, dominance is then established". In many cases of ethnodomination, it would be beneficial for the international marketer to co-opt rather than compete with the dominant ethnic group."[29]

There are a number of cases of ethnodomination in India. *For example,* the automobile spare parts business is dominated by the Sikhs. There is domination of some communities in the wholesale trade in several products. In several parts of the country, there is dominance of some or other community in banking and money lending like the Chettiars in Tamil Nadu and Vysyas in Karnataka and other places.

Regional patterns of entrepreneurship, which also have some ethnic aspects can also be observed. For example Soth Canara district in Karnataka boasts several commercial banks and NBFIs like Syndicate Bank, Canara Bank, Vijaya Bank, Corporation Bank, ICDS etc. Similarly, Trichur in Kerala is home to some banks (The South Indian Bank, Catholic Syrian Bank, Lord Krishna Bank) and NBFIs. This region is very active in stock business too. Most Kerala based large banks are promoted predominantly by Christians. When there was a move to take over The Catholic Syrian Bank by an NRI, there was opposition from a section of the community (while some of the major shareholders from the same community favored the take over move). The take over of the Tamil Nadu Mercantile Bank antoganised the Nadar community and there was even political pressure to wrest the control back to the community. Most Indian banks and NBFIs have an ethnic background.

Many ethnic businesses can go international. *For example, Punjabi* restaurants, *Udupi* restaurants, *Chinese* restaurants etc. are popular in several foreign countries. Several exporters target ethnic population abroad as in the case of Indian curry powders, pickles etc.)

## LANGUAGE

Differences in the language is a very important problem area in business. Switzerland, for examples is country with three fairly distinct cultures, divided between the French, Italian and German-speaking Swiss and the regional differences are profound. In South America there are more than 40 languages. The African continent has the largest number of languages spoken. Zaire alone has more than 200 languages. Kenya has about 40 ethnic groups, each with its own language and culture. Some 750 languages, each distinct and mutually unintelligible are spoken in Papua New Guinea.

India has numerous languages and their dialects, besides the 18 officially recognised languages. Of the 1652 mother tongues listed by the Census of India, 33 are spoken by people numbering a lakh or more.

Even some of the same words of a language have different meanings or connotations in different places. The meaning of some of the English words and idioms differ between U K and USA. *For example*, in Britain the statement "the project went a bomb" would mean that it was a great success; but in USA "the project was a bomb" means it was a massive failure. In the Spanish speaking Latin America the language vocabulary varies widely *For example*, while *Tambo* means a roadside inn in several countries like Colombia, a dairy farm in some countries like Argentina, in Chile it stands for brothel. The differences between the American and British spellings are well known. Parker Pen company has also suffered similar problems of multiple meanings in Latin America where it hoped to use *bola* to describe its ball point pen. While in some country it conveys the intended meaning, ball pen, in different countries it means "lie" or "fabrication", "revolution", and in yet another it represents an obscenity.

Problems caused by languages include, inter alia, those related to brand names and other names, and marketing communication. For example Ford's third world truck brand named *Fiera* meant "ugly old woman" in Spanish. Ford again met with a road block in respect of its top of the line product *Cliente* in Mexico where cliente is a slang for a street walker. Cue seems to be a good brand name for toothpaste, but in French-speaking countries cue is a crude slang expression for derriers. Chervrolet's brand name *Nova* in Spanish means "it doesn't go". The name proposed for a new facial cream that was to be introduced in India was *Joni* which in Indian languages stands for the most innate part of female body. Reebok unveiled a new woman's sneaker named *Incubus*, not knowing that in medieval folklore Incubus was a demon who ravished women in their sleep. Reebok was forced to discontinue the brand.

The Arabic language is read from right to left and many Arabians sequence things from right to left. A multinational blundered in the Middle East when in the advertisement of its detergent it pictured soiled clothes on the left, the box of detergent in the middle and clean clothes on the right.

In the area of translation of advertisements etc. there are two important problems. The appropriate word is not there in some languages. *For example*, in many countries, the word aftertaste does not exist, necessitating the use of several words or one or more sentences to convey the meaning. The second problem is that literal translations many a time do not convey the right meaning; sometimes they convey quite different meanings. In Japanese, General Motors' "Body by Fisher" translates as "corpse by Fisher". In Japanese, again, 3M's slogan "sticks like crazy" translates as "sticks foolishly". In some languages, Pepsi-Cola's slogan "come alive" translates as "come out of the grave."

Non-verbal communications create equally, perhaps even more, difficult problems. Body language has different interpretations in different cultures. The same symbols and gestures, may mean different things in different countries and sometimes in different regions of the same country. A symbol or gesture that represents an appreciation in one society may mean quiet a different thing in another. *For example*, the thumbs up sign, with the thumb held straight up and four fingers kept folded represents approval in countries like USA, UK and Russia; it is highly offensive in Iran and regarded rude in Australia. The OK sign with the thumb and index finger forming a circle and the other three fingers held straight up means every thing is great in USA and Germany and things are just good, not excellent, in Mexico. In most of Europe and Argentina it means an absolute zero or worthless. It is regarded a vulgar gesture in countries such as Spain, Russia, Brazil and Uruguay. In Japan it symbolizes money, mostly coins. In Tunisia it conveys a threat of bodily harm. A backward victory symbol is an insulting gesture in Europe. Shaking head left to right while conveys *no* in the United States and most of the world, it means *yes* in some countries like

Bulgaria, Saudi Arabia and Malaysia. A pat on the shoulder is offensive in Thailand while it connotes encouragement / sympathy in several countries.[30]

## CULTURE AND ORGANISATIONAL BEHAVIOR

The cultural impact on international management is reflected by several basic beliefs and behaviors. Given below are some specific examples where the culture of a society can directly affect management approaches and organisational behavior, highlighted by Hodgetts and Luthans.[31]

- ***Centralized vs. Decentralized Decision Making:*** In some societies, all important organizational decisions are made by top managers. In others, these decisions are diffused throughout the enterprise, and middle and lower-level managers actively participate in, and make, key decisions.
- ***Safety vs. Risk:*** In some societies, organizational decision makers are risk-aversive and have great difficulty with conditions of uncertainty. In others, risk-taking is encouraged, and decision making under uncertainty is common.
- ***Individual vs. Group Reward:*** In some countries, personnel who do outstanding work are given individual rewards in the form of bonuses and commissions. In others, cultural norms require group rewards, and individual rewards are frowned on.
- ***Informal vs. Formal Procedure:*** In some societies, much is accomplished through informal means. In others, formal procedures are set forth and followed rigidly.
- ***High vs. Low Organizational Loyalty:*** In some societies, people identify very strongly with their organization or employer. In others, people identify with their occupational group, such as engineer or mechanic.
- ***Cooperation vs. Competition:*** Some societies encourage cooperation between their people. Others encourage competition between their people.
- ***Short-term vs. Long-term Horizon:*** Some nations focus most heavily on short-term horizons, such as short-range goals of profit and efficiency. Others are more interested in long-range goals, such as market share and technological development.
- ***Stability vs. Innovation:*** The culture of some countries encourages stability and resistance to change. The culture of others puts high value on innovation and change.

These cultural differences influence the way that international management should be conducted.

## OTHER SOCIAL/CULTURAL FACTORS

As indicated in the preceding sections, social or cultural environment encompassing the religious aspects; language; customs, traditions and beliefs; tastes and preferences; social stratification; social institutions; buying and consumption habits etc. are all very import factors for business. One of the most important reasons for the failure of a number of companies in foreign markets is their failure to understand the cultural environment of these markets and to suitably formulate their business strategies. This section takes a look at certain social factors relevant to business, some of which would have overlapping with those described in the preceding sections.

### Consumer Preferences, Habits and Beliefs

What is liked by people of one culture may not be liked by those of some other culture. Significant differences in the tastes and preferences may exist even within the same country, particularly when the country is very vast, populous and multi-cultural, like India. *For example,* An important socio-political environment confronting the business is the growth of consumerism and the legislative measures to protect the consumers. Consumer movement had its conspicuous beginning and development in the United States. There has been a growth of consumer awareness in most countries leading to growth of consumerism and growing demand for consumer protection.

Though consumerism is not well developed in India, there are several consumer organizations in India like the Consumer Guidance Society of India (CGSI), Mumbai, and the Consumer Education and Research Centre (CERC), Ahmedabad, which are doing commendable work. Some of these organisations are very active in conducting product testing and exposing substandard quality and adulteration. The demand for regulatory measures or effective implementation of available measures for consumer protection have been substantiated by the results of such tests. ISI certification of food colours, demanded by the C.G.S.I., is now mandatory. Consumer organisations also play an important role in redressing consumer grievances.

Since 1993, the Consumer Education and Research Society (CERS), sponsored by the CERC, has been performing a very bold and commendable task of comparative testing of consumer goods at their in-house laboratory, and since 1998 its publication *INSIGHT – The Consumer Magazine* has been carrying our test results far and near. The media have been publishing the test findings. These test results enable the consumers to evaluate and compare product of different companies. The manufacturers can, therefore, no longer be complacent.

Consumer movement is growing, *albeit* slowly, in India. It may gather momentum from the growing consumer awareness and the growing feeling that the consumer is ruthlessly exploited and taken for a ride. Many products fail to satisfy the quality requirements and many sellers do not favourably respond to the genuine grievances of consumers.

Many products tests conducted by some consumer organisations have brought to light alarming facts regarding product quality and safety and, they have, therefore, been very vehemently demanding governmental action ensuring quality standards. Consumers have been increasingly taking resort to redressal measures. In short, the business can no more take the consumer for granted.

This does not, however, mean that consumerism is necessarily a problem for the business. Consumerism is, in fact, regarded as an opportunity by consumer oriented businessmen, as described later in this chapter.

## CONSUMER RIGHTS

Consumers in the advanced countries, obviously, are much more conscious of their rights than in countries like India. In 1962 President John F. Kennedy, and in 1965 President Johnson emphasised the consumer rights and gave an impetus to consumerism in the U.S.A. and other countries.

Important consumer rights include:

1. Right against exploitation by unfair trade practices.
2. Right to protection of health and safety from the goods and services the consumers buy or are offered free.
3. Right to be informed of the quality and performance standards, ingredients of the product, operational requirements, freshness of the product, possible adverse side effects and other relevant facts concerning the product or service.
4. Right to be heard if there is any grievance or suggestions.

5. Right to get the genuine grievances redressed.

6. Right to choose the best from a variety of offers.

7. Right to a physical environment that will protect and enhance the quality of life.

The Consumer Protection Act, 1986, has listed the consumer rights it seeks to protect in India. Details of the Act are given at the end of the Chapter.

**BOX 10.1 : RESPONSIBILITIES OF CONSUMERS**

*Rights do not exist independently. Like a coin which has obverse and reverse, consumer rights have the other side, consumer responsibilities. They are :*

1. *The consumer should not make vague or general complaints, but have a specific complaint, with supporting information and proof such as a bill.*
2. *The consumer should try to understand the view-point of the seller before making a complaint.*
3. *In some situations, consumers have to cooperate with the sellers. For instance, in observing a queue, or in a situation of coin shortage using the coupons issued by the bus transport system.*
4. *Consumers, in asserting their rights, should not inconvenience or hurt other sections of the public. For instance,* Rasta Roko, Satyagraha, Dhama, Bandh *etc., are ways of expressing one's anger and generating enthusiasm for a cause among general public, but they result in disrupting normal life for others. This is not justified.*
5. *The consumers should, as a rule, complain against a system and not attack individuals who are incumbents of posts. In most cases, the system or practices call for a change, not individuals. If "'A" is removed or transferred from a post, "B" will occupy it, and may be equally helpless in the particular situation to serve consumers!*

Courtesy : M.R. Pai, *Guidelines to Consumers – Rights & Responsibilities, 1987*
(Sponsored in public interest by the Rotary Club of Bombay)

## EXPLOITATION OF CONSUMERS

Consumers are, however, by and large, practically denied most of these rights. They are exploited by a large number of restrictive and unfair trade practices. A situation has developed in which the public have become victims of false claims for products blatantly advertised. Behavioural science is extensively applied to marketing to ruthlessly, exploit the consumers by stimulating the weak points and soft corners of their mind. Misleading, false or deceptive advertisements are quite common. Many a time the advertisements deliberately give only half truths so as to give a different impression than is the actual fact. Thus, advertisements may, be misleading because things that should be said have not been said, or, because advertisements are composed or purposefully presented in such a way as to mislead. The situation is such that misrepresentations about the quality of a product or the potency of a drug or medicine can be projected without much risk.

As the High-Powered Expert Committee on Companies and MRTP Acts, popularly known as the Sachar Committee, points out, fictitious bargains are another common form of deception. Many devices are used to lure buyers into believing that they are getting something for nothing or at a nominal value for their money. Prices may be advertised as generally reduced and cut when in reality the goods may be sold at sellers' regular prices.

Thus, apart from the monopolistic and restrictive trade practices that have the effect of restricting competition and increasing the market imperfections to the common detriment, consumer exploitation through unfair trade practices that mislead or dupe the customers has become widespread. And it is this situation that has largely led to the growth of consumerism.

## CONSUMERISM

Philip Kotler defines Consumerism as "a social movement seeking to augment the rights and powers of the buyers in relation to sellers".[1] Boyd and Allen state that "although often abused as a term, consumerism may be best defined as the dedication of those activities of both public and private organisations which are designed to protect individuals from practices that impinge upon their rights as consumers."[2]

In his speech delivered at the 44th Annual General Meeting of *Hindustan Lever Ltd.* in 1977, the Chairman, Mr. T. Thomas, rightly pointed out: "While the producer has the power or the right to design the product, distribute, advertise and price it, the consumer has only the power of not buying it. One may argue that the producer runs the greater risk in spite of having several rights because the veto power remains with the consumer. However, the consumer often feels that while he has the power of veto, he is not always fully equipped to exercise that power in his best interests. This situation may be the effect of lack of information, too much indigestible information or even misinformation from one or several competing producers. This problem facing the consumer has led to 'consumerism'. It is worthwhile to note that consumerism, like several other social movements, e.g., independence movement Civil Rights movement, etc., has been the result of a social conflict and cannot, therefore, be wished away. It will be with us till the conflict facing the consumer is resolved".

*Consumerism, interpreted as a collective endeavour of the consumers to protect their interests, is a manifestation of the failure of the business, including that of the public sector, and the government to guarantee and ensure the legitimate rights of the consumers.*

## CONSUMER PROTECTION

For effective consumer protection, a practical response on the part of three parties, the business, the government and the consumers, is essential.

*Firstly*, the business, comprising the producers and all the elements of the distribution channels, has to pay due regard to consumer rights. The producer has an inescapable responsibility to ensure efficiency in production and the quality of output. He should also resist the temptation to charge exorbitant prices in a seller's market. Many a time, the imperfections on the supply side, like hoarding and blackmarketing, mercilessly gouge the consumer. Hence, a socially responsible producer should see to it that whatever is produced reaches the ultimate consumer in time and at reasonable prices.

As T. Thomas observes, "Restraint is best exercised voluntarily than through legislation which will otherwise become inevitable. Advertising agencies and marketing management have a very important role to play in this respect. By overplaying the claims, they will be cutting the very branch on which they are perched."[3]

*Secondly*, the Government has to come to the rescue of the helpless consumer to prevent him from being mislead, duped, cheated and exploited. It should also take special care of the vulnerable sections.

The UN Guidelines for Consumer Protection points out that "the governmental role in consumer protection is vital and finds expression through policy-making, legislation and the development of institutional capacity for its enforcement. To provide a legal basis for enforcing basic consumer rights, every country needs to have an irreducible minimum of consumer protection legislation, covering physical safety, promotion and protection of consumers' economic interests, standards for the safety and quality of goods and services, distribution facilities, redress, and education and information programmes. Governments also require the necessary machinery to enforce such legislation." The Guidelines encourage Governments to develop, strengthen or maintain a strong consumer protection policy. In so doing, each Government must set its own priorities for the protection of consumers in accordance with its economic and social circumstances and the needs of its population.

The UN Guidelines also calls upon the Government to establish distribution facilities for essential consumer goods and services. It is suggested that Governments should, where appropriate, consider: *(a)* Adopting or maintaining policies to ensure the efficient distribution of goods and services to consumers; where appropriate, specific policies should be considered to ensure the distribution of essential goods and services where this distribution is endangered, as could be the case particularly in rural areas. Such policies could include assistance for the creation of adequate storage and retail facilities in rural centres, incentives for consumer self-help and better control of the conditions under which essential goods and services are provided in rural areas; and *(b)* encouraging the establishment of consumer cooperatives and related trading activities, as well as information about them especially in rural areas.

Further, according to the Guidelines, Governments should establish or maintain legal and/or administrative measures to enable consumers or, as appropriate, relevant organizations to obtain redress through formal or informal procedures that are expeditious, fair, inexpensive and accessible. Such procedures should take particular account of the needs of low-income consumers. Governments should also encourage all enterprises to resolve consumer disputes in a fair, expeditious and informal manner, and to establish voluntary mechanisms, including advisory services and informal complaints procedures, which can provide assistance to consumers. Information on available redress and other dispute-resolving procedures should be made available to consumers.

The motive of private gain tempts business to maximise income by socially undesirable trade practices; and this calls for government intervention. Statutory action to protect the interests of consumers has become quite common. In the United Kingdom, for instance, the Trade Description Act, 1968, prohibits the use of misleading descriptions of goods of services or misleading representation of price reductions. In a number of countries, pro-consumer legislation contains provisions that enable an affected party to seek remedy for compensation for the loss or damage suffered by it at the hands of a person who has indulged in prohibited practices. This is true of the Sherman Act and the Clayton Act of the U.S.A.; the Federal Act of Switzerland, the Act Against Restraint of Competition of Spain, the Act concerning Prohibition of Private Monopoly and Maintenance of Fair Trade of Japan, the Trade Practices Act of Australia, the Combines Investigation Act of Canada, etc.

In some countries, statutory bodies are empowered to require the advertiser to substantiate the claims made in the advertisements. For instance, the Federal Trade Commission (FTC) of the United States can seek *affirmative disclosures.* That is, if information in an advertisement is considered insufficient by the FTC, the Commission may require a company to disclose in its advertising some of the deficiencies or limitations of its product or service so that the consumer can judge its negative, as well as, positive attributes. The FTC can also require the advertisers to submit on demand by the Commission data to back-up advertising claims for a product's safety, performance, quality or price comparability. The intent of this *substantiation* is to help consumers make more reasoned choices by having information made available to them. Members of many industry groups, including automobiles, appliances, soaps and detergents, television sets, dentistries, hearing aids, and all over-the-counter drugs have been ordered to provide the Commission with documentation in support of their designated advertising claims. *Corrective advertising* requirements have increasingly been a part of many FTC consent orders. Corrective advertising doctrines are based upon the idea that inaccurate information has a ready been communicated by advertisers, and that corrective advertising is needed to eliminate the lingering effects of such information.[4]

*Thirdly,* consumers should accept consumerism as a means of asserting and enjoying their rights. Consumerism should succeed in making the business and the government more responsive to the rights of the consumers.

Peter Drucker has remarked that "consumerism is the shame of the total marketing concept,"[5] implying that the concept is not widely implemented. Consumerism reflects not only the failure of the business to widely implement the marketing concept but also the need to give the business policies a social orientation so as to enhance long-run social welfare. As Philip Kotler observes, "consumerism is a clarion call for a *revised marketing concept.*" [6] Hence, the original marketing concept has to be broadened to include the societal marketing concept.

"The societal marketing concept calls for customer orientation backed by integrated marketing aimed at generating customer satisfaction and long-run consumer welfare, as the key to attaining long-run profitable volume".[7]

As Kotler points out, "the addition of long-run consumer welfare asks the businessman to include social and ecological considerations in his product and market planning. He is asked to do it not only lo meet his social responsibilities but also because failure to do this may hurt his long-run interests as producer."[8] Thus, the message of consumerism is not a setback for marketing but rather points to the next stage in the evolution of enlightened marketing. Just as the sales concept said that sales were all important, and the original marketing concept said that consumer satisfaction was also important, the societal marketing concept has emerged to say that long-run consumer welfare is also important."[9] Hence, he feels that consumerism will be enduring, beneficial, pro-marketing and ultimately profitable. "Consumerism mobilises the energies of consumers, businessmen and government leaders to seek solutions to several complex problems in a technologically advanced society. One of these is the difference between serving consumer desires efficiently and serving their long-run interests. To marketers, it says that products and marketing practices must be found which combine short-run and long-run values for the consumer. It says that a societal marketing is an advance over the original marketing concept and a basis for earning increased consumer goodwill and profits. The enlightened marketer attempts to satisfy the consumer *and* hence his total well-being on the theory that what is good in the long-run for consumer is good for business." [10]

## PLIGHT OF THE INDIAN CONSUMER

An examination of the important problems facing the Indian consumer would make clear the need for more effective government intervention and consumer movement to safeguard consumer rights.

The following factors make the plight of the Indian consumer miserable.

1. Short supply of many goods and services, especially of essential items, is a very serious problem afflicting the Indian consumer. The demand-supply imbalance has produced all the associated evils of profiteering, hoarding and blackmarketing, corruption, nepotism, irresponsiveness and arrogance towards consumers. Although the situation has improved as a result of the increase in competition due to liberalization, it is still far from satisfactory.

2. The Indian consumer has also been the victim of lack of *effective* or *workable competition*. "Competition among sellers, even though imperfect, may be regarded as effective or workable if it offers buyers real alternatives sufficient to enable them, by shifting their purchases from one seller to another, substantially to influence quality, service, and price. Competition, to be effective, need not involve the standardization of commodities; it does, however, require the ready substitution of one product for another; it may manifest itself in differences in quality and service as well as in price. Effective competition depends also upon the general availability of essential information; buyers cannot influence the behaviour of sellers unless alternatives are known. It requires the presence in the market of several sellers, each of them possessing the capacity to survive and grow, and the preservation of conditions which keep alive the threat of potential competition from others.... The test of effectiveness and workability in competition among sellers is thus to be found in the availability of buyers of genuine alternatives in policy among their sources of supply." [11]

   (The above two points should not be confused as one and the same. Short supply refers to quantitative insufficiency whereas lack of effective competition refers to dearth of enough alternatives.)

3. Many products with which consumers in advanced countries are quite familiar are still new to a very large segment of the Indian consumers. The unfamiliarity of the consumers with product features makes the sale of substandard, inferior or even defective products easier in India than in advanced countries.

4. Due to low literacy levels and unsatisfactory information flows, the Indian consumers, by and large, are not conscious of all their rights. This encourages irresponsible and unscrupulous business attitudes and tactics.

5. It has been said that the legal process in India is comparatively time-consuming and cumbersome. This discourages the consumers from seeking the redressal of their grievance by means of the judicial process.

6. Consumerism in India is not well organised and developed.

7. Though the public sector had been developed and expanded to serve the public interest by providing effective competition to the private sector, increasing production, improving distribution, etc., it failed to produce benefits that were commensurate with the investment. It is an irony that though consumer welfare is an avowed objective of the public sector, in certain areas the poor performance of the public sector monopolies has made the

plight of the consumer more miserable. Some of them have even been charged with unfair trade practices.

8. Though there are a number of laws to safeguard the interests of consumers, they are not effectively implemented and enforced to achieve the objectives.

The above factors call for effective State intervention and consumerism to ensure the rights of consumers.

# Capital Of A Company

* **Share Capital :—**

A Share Capital is the money a Company raises by issuing Common or preferred Stocks.

* **Meaning of Share :—**

A Company's capital is divided into small equal units of a finite number. Each unit is known as a Share.

In Simple terms, a Share is a percentage of ownership in a Company or a financial asset.

Investors who hold Shares of any Company are known as Shareholders.

* **Kinds of Shares or Types of Shares :—**

There are mainly two Types of Shares. They are

1. Equity Shares
2. Preference Shares

**1 Equity Shares :—**

Equity shares also known as ordinary Shares or Common Shares, represent the owner's Capital in a Company. The holders of these Shares are the real owners of the Company.

They have a control over the working of the company. Equity Share holders are paid dividend after paying it to the preference Shareholders. The Rate of dividend on these Shares depends upon the profits of the Company. They may be paid a higher rate of dividend or they may not get anything. These Shareholders take more risk as compared to preference Shareholders.

## * Characteristics of Equity Shares :—

The following are the most significant features of equity shares

1. **Maturity :—**

Equity Shares provide permanent Capital to the Company & cannot be redeemed during the life time of the company. Equity Shareholders can demand refund of their Capital only at the time of liquidation of a Company.

2. **Claims / Right to Income :—**

Equity Shareholders have a residual claim on the Income of a Company. They have a claim on income left after paying dividend to preference shareholders. The rate of dividend on these shares is not fixed. it depends upon the earnings available after paying dividends on preference Shareholders. In many Cases they may not get anything if profits are insufficient, or may get even a higher rate of dividend. That is why, Equity Shares are also known as Variable Income Security.

3. **Claim on Assets :—**

Equity Shareholders have a residual claim on ownership of company's Assets. In the event of liquidation of a Company the assets are utilised first to meet the claims of Creditors & preference Shareholders but everything left thereafter belongs to the equity shareholders. Thus, equity Shares provide a cushion to absorb losses on liquidation & may, usually, remain unpaid.

4. **Right to Control or Voting Rights :—**

Equity Shareholders are the real owners of the Company. They have Voting rights in the meeting of the Company & have a control over the working of the company. The control in case of a Company rests with the Board of directors who is elected by the equity Shareholders. Directors are appointed in the Annual General Meeting by majority Votes.

5 Pre emptive Right :—

The pre-emptive right protects equity shareholders by ensuring that management cannot issue additional shares to persons of their choice in order to strengthen their control over the company. It also protects them from dilution of their financial interest in the company.

6 Limited Liability :—

Another distinct feature of equity shares is limited liability. Thus, although equity shareholders are the real owners of the company, their liability is limited to the value of share they have purchased. If a shareholder has already fully paid the share price, he cannot be held liable further for any losses of the company even at the time of liquidation. This enables the equity shareholders to enjoy the ownership of a firm without risking unlimited liability as is the case in sole-proprietorship or partnership firms.

* Advantages of Equity Shares

1 Equity Shares do not create any obligation to pay a fixed rate of dividend

2 Equity Shares can be issued without creating any charge over the assets of the company

3 It is a permanent source of capital & the company has not to repay it except under liquidation

4 Equity shareholders are the real owners of the company who have the voting rights

5 In case of profits, equity shareholders are the real gainers by way of increased dividends & appreciation in the value of shares.

* **Disadvantages of Equity Shares :—**

1. If only equity share are issued, the company cannot take the advantage of trading on equity
2. As equity capital cannot be redeemed there is a danger of over capitalisation
3. Equity shareholders can put obstacles in management by manipulation & organising themselves.
4. During prosperous periods higher dividends have to be paid leading to increase in the value of shares in the market & speculation
5. Investors who desire to invest in safe securities with a fixed income have no attraction for such shares.

2. **Preference Shares :—**

As the name suggests these shares have certain preferences as compared to other types of shares. These shares are given two preferences. There is a preference for payment of dividend. Whenever the company has distributable profits the dividend is first paid on preference share capital. Other shareholders are paid dividend only out of the remaining profits if any. The second preference for these shares is the repayment of capital at the time of liquidation of company.

A fixed rate of dividend is paid on preference share capital. Preference shareholders do not have voting rights, so they have no say in the management of the company.

* **Types of Preference Shares :—**

Preference shares are of the following types

1. **Cumulative Preference Shares :—**

These shares have a right to claim dividend for those years also for which there are no profits. Whenever there are divisible profits, cummulative preference shares are paid dividend for all the previous years in which dividend could not be declared.

2 **Non - Cumulative Preference Shares :—**

The holders of these shares have no claim for the arrears of dividend. They are paid a dividend if there are sufficient profits. They cannot claim arrears of dividend in subsequent years.

3 **Redeemable Preference Shares :—**

Normally, the capital of a company is repaid only at the time of liquidation. Neither the company can return the share capital nor can the shareholders demand its repayment. The company, however can issue redeemable preference shares if Articles of Association allow such an issue. The company has right to return redeemable preference share capital after a certain period. The Companies Act has provided certain restrictions on the return of this capital. The shares to be redeemed should be fully paid up. The company should redeem these shares either out of profits or out of fresh issue of capital. The object of these restrictions is that the resources of the company are not depleted.

4 **Irredeemable Preference Shares :—**

Those shares which cannot be redeemed unless the company is liquidated are known as Irredeemable preference shares.

5. **Participating Preference Shares :—**

The holders of these shares participate in the surplus profits of the company. They are firstly paid a fixed rate of dividend & then a reasonable rate of dividend is paid on equity shares. If some profits remain after paying both these dividends, then preference shareholders participate in the surplus profits. The mode for dividing surplus profits between preference & equity shareholders is given in the Articles of Association.

6 Non Participating Preference Shares :—

These shares on which only a fixed rate of dividend is paid are known as non-participating preference shares. These shares do not carry the additional right of sharing of profits of the company.

7 Convertible Preference Shares :—

The holders of these shares may be given a right to convert their holdings into equity shares after a specific period. These are called convertible preference shares. The right of conversion must be authorised by the Articles of Association.

8 Non-convertible Preference shares :—

The shares which cannot be converted into equity shares are known as non-convertible preference shares.

* Features of Preference Shares :—

The following are the most significant features of preference shares

1 Maturity :—

Generally, preference shares resemble equity shares in respect of maturity. These are perpetual (irredeemable) & the company is not required to repay the amount during its life time. It is only at the time of liquidation that a company has to repay the preference shareholder after meeting the claim of creditors but before paying back the equity shareholders.

2 Claims on Income :—

A fixed rate of dividend is payable on preference Shares. preference shareholders have prior Claims on income (dividend) over equity shareholders whenever the company has distributable profits the dividend is first paid on preference Share Capital only after payment of stipulated dividend on prefered Stock, the company can pay any dividend to other (equity) Shareholders.

3 Claims on Assets :—

Preference Shares have a preference in the repayment of Capital at the time of liquidation of a company Their claims on assets are Superior to those of equity Shareholder In the event of winding up of the company their claims is to be Settled first before making any payment to the equity Shareholders But as they are not real owners of the company the preference shareholder usually do not have any right in the surplus assets of the company

4 Control :—

Ordinarily, preference Shareholders do not have any voting rights so they do not have any Say in the management or Control of the company. However under Section 87 of the Companies Act 1956 preference Shareholders can vote on a resolution which directly affects the rights to be attached to their preference Shares. They can also vote on every kind of resolutions placed before the meeting of the company if the dividend due on their shares or any part there of has remained unpaid.

5 Hybrid Form of Security :—

Preference Share Capital in the real sense represents a hybrid form of Security as it includes some features of

equity & other of debt financing. It resembles equity in the sense that 1) Payment of dividend is not obligatory
2) Preference dividend is payable only out of distributable profits
3) It is not deductible as an expenses while determining tax liability of the company.

At the same time it has certain characteristics of debt financing such as 1) It carries a fixed rate of dividend
2) It entitles to a right to its holder prior to equity shareholders & 3) It does not provide a right to vote.

* **Advantages or Merits of preference Shares**

Preference shares provide a number of advantages both to the company as well as investors or shareholders.

a Company's point of view. The company has the following advantages by issuing the preference shares

1 There is no legal obligation to pay dividend on preference shares. Preference dividend is payable only out of distributable profits at the discretion of the management. Hence a company does not face a financial burden or legal action if it does not pay dividend.

2 Preference shares provide a long-term capital for the company

3 There is no liability of the company to redeem preference shares during the life time of the company.

4 Redeemable preference shares have the added advantage of repayment of capital whenever there are surplus funds with the company.

5 As a fixed rate of dividend is payable on preference shares these enable a company to adopt trading on equity

6 As preference share capital is generally regarded as part of company's net worth it enhances the credit worthiness of a firm

7 Preference shares do not carry voting rights under normal circumstances & hence there is dilution of control.

8 As no Specific assets are pledged against preferred Stock the mortagageable assets of the company are Conserved.

B) Investor's. Shareholders Point of View : Investors in preference Shares enjoy the following advantages

1 It earns a fixed rate of dividend
2 It is a Superior Security over equity Shares
3 It provides preferential rights in regard to payment of dividends & repayment of Capital at the time of liquidation of the company
4 Preference Shares although carry no Voting rights but the holders of Such shares can vote on matters directly affecting their rights as well as on all resolutions if the dividend due on their Shares is remaining unpaid.

* Disadvantages of Preference Shares :—

Inspite of many advantages, preference shares suffer from many short comings.

a) Company's Point of View :—

1) It is an expensive Source of finance as compared to debt because generally the investor's expect a higher rate of dividend on preference shares as compared to the rate of interest on debentures.

2) Cumulative preference Shares become a permanent burden so far as the payment of dividend is concerned.

3) Although there is no legal obligation of a company to pay dividend on preference shares but frequent delays or non payment adversely affect the Creditworthiness of the firm

4) Preference share dividend is not a deductible expenses while calculating tax while interest is a deductible Expenses Thus there is a tax disadvantage to the company

5) In some cases preference shares carry even the voting right & hence the control & management of the company may be diluted.

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B) Share holders Point of View :-

1) As the preference shareholders ordinarily do not have any Voting rights they remain at the mercy of the management for the payment of dividend & redemption of their capital.

2) The rate of dividend on preference shares is usually lower as compared to the equity shares

3) Preference shareholders do not have any charge on the assets of the company while debentures, usually provide a charge on all the assets of the company

4) The market prices of preference shares fluctuate much more than that of debentures.

## PREFERENCE SHARES COMPARED WITH EQUITY SHARES

| S. No | Preference capital | Equity share capital |
|---|---|---|
| 1 | Preference shares are entitled to a fixed rate of dividend. | The rate of dividend on equity shares depends upon the amount of profit available and the funds requirements of the company for future expansion etc. |
| 2 | Dividend on the preference shares is paid in preference to the equity shares. | The dividend on equity shares is paid only after the preference dividend has been paid. |
| 3 | In case of winding up, preference share holder get preference over equity share holders with regard to the payment of capital. | In case of winding up, equity share holder get payment of capital after the payment of capital to preference shareholders. |
| 4 | Dividend on preference share may be cumulative. | The dividend on equity shares is paid only after the preference dividend has been paid and it is not cumulative. |
| 5 | The voting rights of preference shareholders are restricted. A preference shareholder can vote only when his special rights as a preference shareholder are being varied, or on any resolution for the winding up of the company or for the repayment or reduction of its equity or preference share capital or their dividend has not been paid for a period of two | An equity shareholder can vote on all matters affecting the company. |

| | | |
|---|---|---|
| | years or more [section 47(2)]. | |
| 6 | No bonus shares/right shares are issued to preference share holders. | A company may issue rights shares or bonus shares to the company's existing equity shareholders. |
| 7 | A company can issue preference shares which are redeemable within 20 years except in certain circumstances. | Equity shares cannot be redeemed except under a scheme involving reduction of capital or buy back of its own shares. |
| 8 | Voting right of a preference shareholders on a poll shall be in proportion to his share in the paid-up preference share capital of the company. | Voting right of an equity shareholders on a poll shall be in proportion to his share in the paid-up equity share capital of the company. |

* **Debentures OR Bonds :—**

A company may raise long-term finance through public borrowings These loans are raised by the issue of debentures.

A debenture is an acknowledgement of a debt.

According to Thomas Evelyn "A debenture is a document under the Company's Seal which provides for the payment of a principal Sum and interest thereon at regular intervals, which is usualy Secured by a fixed or floating charge on the company's property or undertaking & which acknowledges a loan to the company's property.

* **Types of Debentures :—**

1 **Simple, Naked or Unsecured debentures :—**

These debentures are not given any Security on assets They have no priority as compared to other Creditors They are treated alongwith unsecured Creditors at the time of winding up of the company. So they are just unsecured Creditors.

2 **Secured or Mortgaged Debentures :—**

These debentures are given security on assets of the company. In case of default in the payment of interest or principal amount, debentureholders can sell the assets in order to satisfy their claims.

3 **Bearer Debentures :—**

These debentures are easily transferable. They are just like negotiable instruments. The debentures are handed over to the purchaser without any registration deed. Anybody purchasing them with a consideration & in good faith becomes the lawful owner of the debentures.

4 **Registered Debentures :—**

As compared to bearer debentures which are transferred by mere delivery, registered debentures require a procedure to be followed for their transfer.

5 **Redeemable Debentures :—**

These debentures are to be redeemed on the expiry of a certain period. The interest on the debenture is paid periodically but the principal amount is returned after a fixed ~~as~~ period. The time for redeeming the debenture is fixed at the time of their issue.

6. **Irredeemable debentures :—**

Such debentures are not redeemable during the life time of the company. They are payable either on the winding up of the company or at the time of any default on the part of the company.

7 **Convertible Debentures :—**

Sometimes convertible debentures are issued by a company & the debentureholders are given an option to exchange the debentures into equity shares after the lapse of a specified period.

8 Zero Interest : Bonds / Debentures :—

It is an instrument recently introduced in India by some companies It is usualy a convertible debenture which yields no interest The company does not pay any interest on such debentures But the investor in a zero interest bond is compensated for the loss of interest through conversion of such bond into equity shares at a specified future date.

9 Zero Coupon Bonds :—

Another instrument which has recently become popular in India is the zero coupon bond (ZCB) Zero Coupon bond does not carry any interest but it is sold by the company at a deep discount from its eventual maturity value.

10 First Debentures and Second Debentures :—

The debentures which have to be paid back first or who have preference over other debentures in payment of interest are called first debentures & the debentures who rank after these are known as second debentures.

11 Guaranteed Debentures :—

These are debentures or bonds on which the payment of interest & principal of is guaranteed by third parties generally banks & Government etc.

12 Collateral Debentures :—

A company may issue debentures in favour of a lender of money generally the banks & financial institutions as a colla-teral i.e subsidiary or secondary security for a loan raised by it.

13 Other Innovative Debt Instruments :—

In the fast changing capital market scenario the corporate sector has devised many other innovative debt instruments for raising funds from the market.

## * Features of Debentures or Bonds

The Salient Characterstics of debentures are as follows.

1 **Maturity :—**

Although debentures provide long term funds to a company they mature after a Specific period Generally the debentures are to be repaid at a definite time as stipulated in the issue. The company must pay back the principal amount on these debentures on the given date otherwise the debentureholders may force winding up of the Company as Creditors.

2 **Claims on Income :—**

A fixed rate of interest is payable on debentures unlike shares a company has a legal obligation to pay the interest on due dates irrespective of its level of earnings Even if a company makes no earnings or incurs loss, it is under an obligation to pay interest to its shareholders

3 **Claims on Assets :—**

Even in respect of claim on assets, debentureholders have priority of claim on assets of the Company They have to be paid first before making any payment to the preference or equity shareholders in the event of liquidation of the company

4 **Control :—**

Since debentureholders are Creditors of the Company & not its owners they do not have any control over the management of the Company. They do not have any Voting rights to elect the directors of the company or on any other matters.

5 **Call Feature :—**

Issue of debentures Sometimes provides a call feature which entitles the company to redeem its debentures at a certain price before the maturity date since the call feature provides advantages to the company at the expenses of its debentureholders the call price is usually more than the issue price

* **Advantages of Debentures :—**

I. **Advantages to the Company**

1) Debentures provide long term funds to a company.
2) The rate of interest payable on debentures is usually lower than the rate of dividend paid on shares
3) The Interest on debentures is a tax-deductible expense & hence the effective cost of debentures (debt-capital) is lower as compared to ownership securities where dividend is ~~lower~~ as not a tax deductible expenses
4) Debt financing does not result into dilution of control because debentureholders do not have any voting rights
5) A company can trade on equity by mixing debentures in its capital structure & thereby increase its earnings per share
6) Many companies prefer issue of debentures because of the fixed rate of interest attached to them irrespective of the changes in prices
7) Debentures provide flexibility in the capital structure of a company as the same can be redeemed as & when the company has surplus funds & desires to do so
8) Even during depression when stock market sentiment is very low a company may be able to raise funds through issue of debentures or bonds because of certainity of income & low risks to investors

II) **Advantages to Investors :—**

1) Debentures provide a fixed regular & stable source of income.
2) It is comparatively a safer investment because debentureholders have either a specific or a floating charge on all the assets of the company & enjoy the status of a superior creditor in the event of liquidation of the company

3) Many investors prefer debentures because of a definite maturity period.

4) A debenture is usually more liquid investment & an investor can sell or mortgage his instrument to obtain loans from financial institutions.

5) The interest of debentureholders is protected by various provisions of the debenture trust deed & the guidelines issued by the Securities & Exchange Board of India in this regard.

* Disadvantages of Debentures

I) From the point of View of Company

1) The fixed interest charges & repayment of principal amount on maturity are legal obligations of the company.

2) Charge on the assets of the company and other protective measures provide to investors by the issue of debentures usually restrict a company from using this source of finance.

3) The use of debt financing usually increases the risk perception of investors in the firm.

4) Cost of raising finance through debentures is also high because of high stamp duty.

II) From the Point of View of Investors

1) Debentures do not carry any voting rights & hence its holders do not have any controlling power over the management of the company.

2) Debentureholders are merely creditors & not the owners of the company.

3) Interest on debentures is fully taxable while shareholders may avoid tax by way of stock dividend (bonus dividend) in place of cash dividend.

4) The prices of debentures in the market fluctuate with the changes in the interest rates.

5) Uncertainty about redemption also restricts certain investors from investing in such securities.

## * Types of Issue of Shares :—

When Companies are on the look to raise money for their business operations they use various means for the Same They are

### 1) Initial Public Offering (IPO) :—

An IPO is referred to a flotation which an issuer or a company proposes to the public in the form of ordinary stocks or shares They are generally offered by new & medium sized firms looking for funds to grow. However it can be done by big privately-owned firms seeking to transform themselves into an openly traded firm.

### 2) Follow On Public Offering (FPO) :—

A follow on offering is an issuance of stocks subsequent to the company's initial public offering. A follow-on offering can be either of two types (or a mixture of both) dilutive & non-dilutive. A follow on offering is preceded by release of prospectus similar to IPO

For Ex :- Google's Initial public offering (IPO) included both a primary offering and a secondary offering.

### 3) Right Issue :—

In a right Issue shares or convertible Securities are offered to the existing shareholders are offered to the existing shareholders at a concessional rate on a stipulated date fixed by the company itself. The main aim of issuing right shares is to raise additional funds by offering shares to the existing equity shareholders in the proportion of their holdings rather than making a fresh Issue.

### 4) Composite Issue :—

A Composite Issue is one in which an already listed company offers shares on the public-cum-rights basis & makes concurrent allotment of the shares

5) Bonus Issue :—

As the name itself suggests it is the free additional shares distributed to the current shareholders in the proportion of the fully paid-up equity shares held by them on a particular date. The issue of these shares is made out of the Company's free reserves or Securities premium account.

6) Private placement :—

If a company offers shares to a selected group of investors which can be mutual funds, banks, insurance companies, pension funds & so forth to raise capital is called private placement.

## * Book Building Process

### Book Building :—

The method of offering shares by providing a price range so that the investors can decide on the right price is called as Book Building.

Book Building is actually a price discovery method in which the investors bid for the shares of the company during IPO/FPO. They are given a price range in which the investors have to bid for the shares. In this method the company doesn't fix up a particular price for the shares but instead gives a price range. E.g. ₹ 80-100.

The lowest price (₹ 80) is known as floor Price.
The highest Price (₹ 100) is known as Cap price.
The price at which the shares are allotted is known as cut off price.

## Book Building Process :-

[Company plans an IPO via the Book Build Route]

↓

Appoints a merchant banker as book runner

↓

[Issues a draft prospectus (containing all mandatory comp. any disclosures other than Price)]

↓

Draft Prospectus filed Simultaneously with concerned authority (SEBI)

↓

[Book runner appoints Syndicates members & registered intermediaries to gather Subscription]

↓

price discovery begins through the bidding process

↓

[At close of bidding, book runner & company decide upon the allocation & allotments]

The entire process begins with the Selection of the lead manager an investment banker whose job is to bring the issue to the public. Both the lead manager & the issuing Company fix the price range & the issue size. Next Syndicate members are hired to obtain bids from the investors. Normally the issue is kept open for 5 days. once the offer period is over, the lead manager & issuing company fix the price at which the shares are sold to the investors. If the issue price is less than the Cap price, the investors who bid at the cap price will get a refund & those who bid at the floor price will end up paying the additional money.

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For Ex : If the cut off in the above example is fixed at ₹ 90 those who bid at ₹ 80 will have to pay ₹ 10 per share & those who bid at ₹ 100 will end up getting the refund of ₹ 10 per share. Once each investor pays the actual issue price the shares are allotted.

# 3. CHEQUES

Date 31 8 2019

Meaning

Cheque it is an order to bank to pay stated sum charge of money from the drawers account.

Defination

According to section 6 of the Indian negotiable instrument act 1881 a cheque is a bill of exchange drawn on a specified banker and not expressed to be payable otherwise then on demand.

pairies in a Cheque

* Drawer -> The person who's draws a cheque is called Drawer.

* Drawee -> The banker on whom it is drawn is called drawee.

* Payee -> The person who receives the money of the cheque is called payee.

Types or classification of cheque

1) Bearer Cheque
2) order Cheque
3) open Cheque
4) Stale Cheque
5) post dated Cheque
6) Self Cheque
7) yourself cheque
8) cross cheque
9) Ante date Cheque

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1) Bearer cheque

When the words "or Bearer" appearing on the face of the cheque are not cancelled the cheque is called Bearer cheque The bearer cheque is payable to the person specifide their or to any other else who present it to the bank for payment.

Bearer cheques are risky Because if the cheque is lossed the finder of the cheque can collect payment from the bank.

2) Order cheque

When the word Bearer appears on the face of a cheque is cancelled and when its place the word "or order" is written on the face of the cheque the cheque is called order cheque. Such a cheque is payable to the person's specifide their in as the payee.

3. Open Cheque

Open cheque are those that are paid across the counter. There is a great risk involved in case of open cheque because the open cheque may be stollen or lost and the finder can get encashed

4. Stale Cheque

If a cheque is presented for payment after 3 months from the date of the cheque is called stale cheque. A stale cheque is not honoured by the bank.

5. Post dated cheque

If a cheque bears a date which is

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yet to come than it is non as post dated cheque a post dated cheque cannot be honoured earlier than the date on the cheque.

6. Self Cheque

Self cheque is when you right on you self where you will act as both drawer and payee

7. Yourself Cheque

A Cheque which is used to send money through RTGS or NEFT (Real time Gross Settlement (National electronic Financial Tr ) in a bank can mentioned it as yourself in the name Box is called yourself cheque

8. Cross Cheque

Cross cheque means drawing to parlel lines on the face of the cheque with or without additional words like, and company's, a/c payee, not negotiable. A crossed cheque cannot be encashed at the cash counter of a bank but it can only we Credited to payee's account.

9. Ante dated Cheque

If a cheque bears a date earlier than the date on which it is presented to the bank is called ante dated cheque.

10. Blank Cheque

It is a cheque which as no date, no name and no amount written but already signed is called blank cheque or A cheque on which the drawer puts his signature and leaves other column blank is called blank cheque

Short

# Crossing of Cheque

Drawing to paler line across the face of the cheque with or without the words and company, or any abbreviations thereof.

Crossing is direction from drawer to payee banker to pay the amount of the cheque through the bank and not directly to the person presenting the cheque at the counter the purpose of crossing of cheque is to minimise risk of loss or stolen.

The object of crossing of cheque is that the amount of cheque is paid to customers bank account.

## Methods of crossing

There are mainly two methods of crossing those are

1. General crossing
2. Special crossing

### 1. General Crossing

According to Section 123 of nego-tiable instrument act 1881 defines where a cheque bears crossing across its space in addition to the words and company or and abbreviations there of between two parallel line.

The main object of general crossing is that the amount of cheque should be paid to a banker and it may be any banker the lines may be drawn on the face of the cheque at any place usually at the left hand top corner of the cheque.

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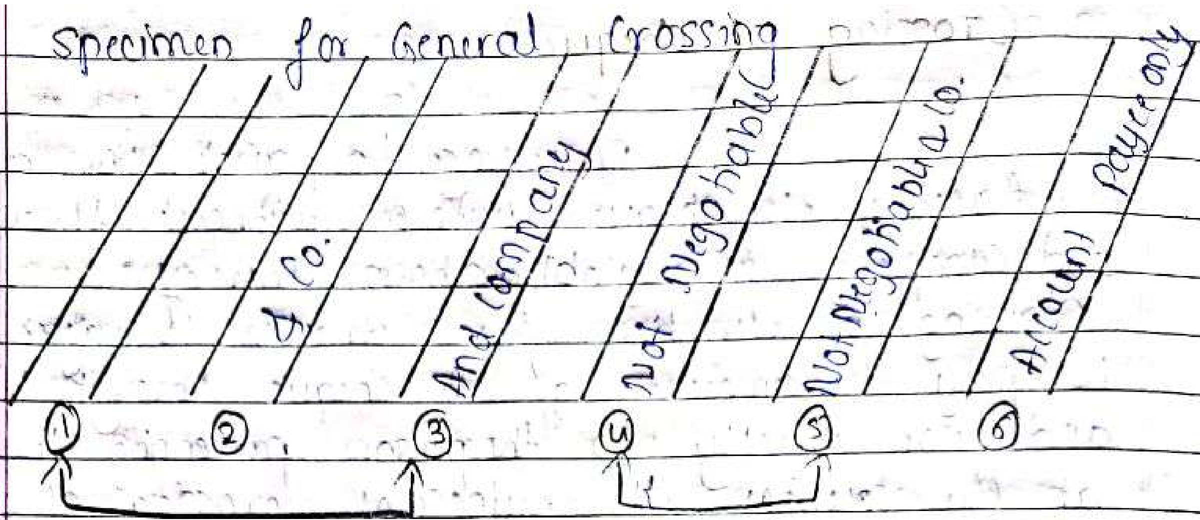
Specimen for General Crossing




Among the above given speciman number 1, 2 & 3 identifies that the banker can pay the amount to the customer and it may be any bank so if the cheque bears the above speciman it can be presented to banker through account holder it means cheque first of all credited to account holder afterwards account holder withdraw the amount & pay to consider customer.

Speciman number 4 and 5 indicates that cheque should not be further transfered.

Speciman number 6 indicates that account payee means that the payment should be made through account only that is account holder suppose the customer dose not have any account first of all he should open the account and the cheque should be credited to the account afterwords the customer can withdraw the money.

2) Special Crossing

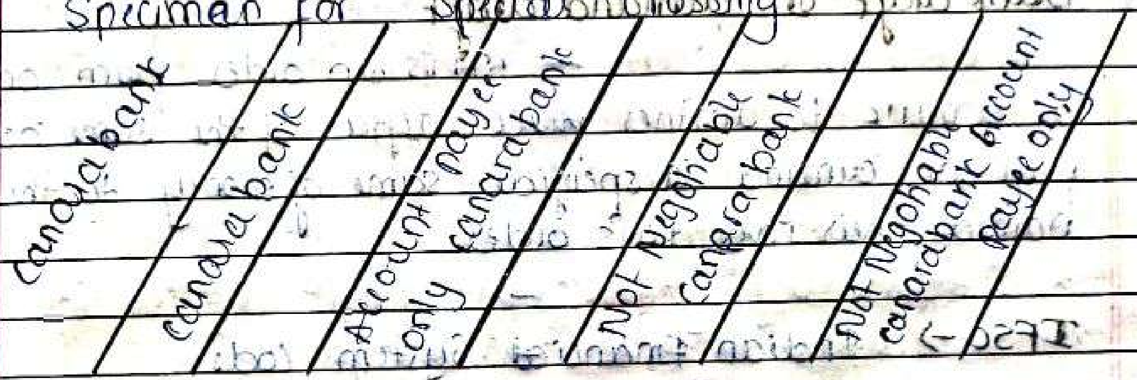
According to section 124 of negotiable instrument act 1881 defines where a cheque bears crossing across its space face in addition to the name of the bank either with or without

Even then drawing to parallel line on the face of the cheque is in practice generally name of the bank is maintained in between two parallel line.

the words not negotiable and account payee.

Special crossing means maintaining the name of the banker on the face of the cheque. the main object of special crossing is that the cheque should be paid to a particular banker. drawing to parallel line is not necessary in the case of special crossing.

Speciman for Special Crossing




marking of cheque

when bank certifies a cheque by writing on it words like approved or good on the cheque any bank may also put it's seal on it such cheques are also called marked cheque or certified cheque

material Alteration

Any change made in the cheque is non as material Alteration. when the changes are made in the cheque such cheque becomes invalid they may be valid by obtaining confirmation by the drawer and bank will honour the cheque

MICR → magnetic Ink Character Recognition

MICR technology under which there is a machinized cheque and draft processing processing system using magnetic Ink character recognized format

MICR is a digit code which helps to identify a particular bank branch every bank branch is having unique MICR code.

The code the contains the information first 3 numbers indicates city code, next 3 number indicates Bank code and last 3 number indicates branch code.

* Bank draft or demand draft

- It is an order from one branch of a bank to another place office of the same bank to pay one demand a specific same of money to the person named their in or to is order.

SYNB0005554 Bank Branch

ICIC0001470

IFSC → Indian Financial System code

IFSC is a 11 digit Alpha numerical code that is used to identify the bank branch you may find IFSC code on your bank passbook or on the Cheque book.

| | General crossing | special crossing |
|---|---|---|
| line | Two parallel line required for general cross | not required |
| Name of banker | The name of collecting banker is not necessary | The name of collecting bank is necessary |
| conversion | general crossing conversion to special crossing | Special crossing cannot be converted into general crossing |
| payment | It can be paid to any other banker | It should be paid by payment

# International Trade Theories.

Page No.
Date

## I Absolute Cost - Advantage Theory

This Theory is developed by Adam Smith in 1776. If a country is favourable with its climate quality of land and natural resources etc. Than it will be able to produce a goods at low cost this means the country is enjoying absolute cost advantage.

Adam Smith explains this theory with two country two commodity & one factor.

ex: If country A produces 25 barrels of wine or 10 bales of cloth per unit of labour per day and if country B produce 10 barrels of wine or 15 bales of cloths per unit of labour per day Than the country A has absolute advantage in production of wine while country B has an absolute advantage in producing cloth. This means that country A is specialised in producing of wine and shall export wine to country B.

Country B is specialised in production of cloth and it can export it to country A.

However, the theory of absolute cost advantage fails to explain about the occurance of trade between two countries if one particular country produces both the goods with a absolute cost advantage

## II. Comparative Cost Advantage

The most basic concept in the whole of international trade theory is a principle of comparative advantage which is introduced by David Ricardo in 1817 It remains a measure influence on much international trade policy

& it is important to understand the modern global economy.

The principle of compariative cost advantage state that a country should specialise in producing & exporting those products in which it has comporative cost advantage compared with other countries should import those goods in which it has comporative cost advantage.

It is quite common that some countries have the advantage of producing some goods @ a lower cost compare to other countries this is due to availability of cheap labour skilled labour, cheap & quality raw material advanced technology etc. availability of this factors help to produce & sell the products @ lower cost similarly other countries have disadvantage in producing other goods.

Ex. Japan has advantage in producing electronics @ low cost were as India has advantage in producing textiles.

Con * Conclusion :

1 B's. b/w 2 countries is profitable when a country produces 1 goods @ lower cost & the other country produces another goods @ lower cost

2 B's b/w 2 countries is also profitable when a country produces more than one product efficiently but when it produce one of these products comporatively @ a great efficiency then the other product.

3 Both the nations can engage in international B's when 1 country specialise in the production in which it has great efficiency in the product

The only cost of

Assumptions

1. only Elemenation of cost
2. Production is a subject to the law of constant return.
3. There are no trade barriers
4. Trade is free from cost of transportation

3. Heckscher Ohlin Theory:

In early 1900 an International Trade Theory called Factor proportion Theory introduced by two Swidish economist Eli heckscher and Bertil Ohlin This Theory is also called as Heckscher's Ohlin Theory or HO Theory.

The Heckscher Ohlin theory states that the country should produce and Export the goods that requires resources that are available in large amount and import the goods that requires resources in short supply.

This Theory differs from the theory of comparative advantage and absolute advantage this theory focus on the productivity of the production process for a particular goods on the contrary the Heckscher's Ohlin Theory states that a country should specialise production and exporting using a factors that are available in large quantity and the cheapest not produce as earlier theory stated the goods it produces most efficiently

Heckscher's Ohlin Theory is preferred to Riccardo Theory by many economist because it makes few simplifying assumptions.

Wassily leontief published a study were he tested the Validity of Heckechers Ohlin Theory The Study Shows that US was more abudant in Capital comp-ared to other countories Therefore US could Export Capital intensive goods and Import labour intensive goods so he found that the U.S was less capital intensive than Import.

4. Product Life Cycle Theory

~~Ramo~~ Raymond Veomon developed a International product life Cycle in 1960. The International product life Cycle Theory States that a company will begin to export its product and later take on forign direct investment as the product moves through its life cycle inventually a countory's Export becomes its import although the model is developed around the U.S It can be generalised and applied to any of developed and innovative markets. Ex: I phone.

5 Mercantilism

According to wide 2000 The trade theory States that Nations should accumulate financial wealth in the form of gold by encouraging Export and discouraging Imports This called Mercantile.

According to this theory Other measures of countories well begins such as living Standard or human development are irrelevant.

Ex: Spices of Indian Products

(6) International Trade Theory

International business takes different approaches from the ricardian and heckscher ohlin model on why countries engaged in international trade both recardo & Heckscher assume costant returns to scale where them if all factors of productions are double than output than output will be also double. but a firm or a Industry may have increasing returns to scale or economic of scale in y That when all factors of productions are double output more than doubles which will necessiate a bigger market and thus foreign firms to engage in international trade where there is large market the new trade noted that the bigger the size of a ferm or industry the more the efficiency of its operations in that the cost per unit of output falls as a firm or Industry increases output the increases in output must however be met with an increase in the market size if it has to be sustainable.

New Trade Theory of International business says why country is engaged in international trade is apposite to the assumption made in the recardian & heckscher model that there is a perfect competition in the market in that all income from production is paid to owners of factors of production and there is no excess or existance of monopoly profit.

There are two types of economic of scale were considered in explaining that a countryes is engaged in international trade because of economies of scale the first one is internal economy in which average cost of individual firms will fall as they produce more output and become larger

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Date

and the Second one is the external economy in which average cost of the industry in a country will reduce as it produces more output and grows larger

(6) International Trade Theory

International business takes different approaches from the recardian and heckscher ohlin model on why countries engaged in international trade both recardo & Heckshear assume costant return to scale where them if all factors of productions are double than output than output will be also double. but a firm or a Industry may have increasing returns to scale or economic of scale in y That when all factors of productions are double output more than doubles which will necessiate a bigger market and thus foreign firms to engage in international trade where there is large market the new trade noted that the bigger the size of a firm or industry the more the efficiency of its operations in that the cost per unit of output falls as a firm or Industry increases output the increases in output must however be met with an increase in the market size if it has to be sustainable

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Unit - V

## International Business Environment.

Prof. Laxman. M. Hirekodi.

### Latest EXIM Policy:

'India's foreign trade policy' also known as export & import policy (EXIM) in general, aims at developing export potential, improving export performance, encouraging foreign trade and creating favorable balance of payments position.

- Commerce ministry released Foreign Trade policy [EXIM policy] 2015-2020.
- New foreign trade policy: $ 900 billion exports by 2020. [58 Lakh Crore ₹].
- The target to be doubled India's exports in goods & services over the next five years. [from $465 billion to $ 900 billion) and upping the Indian share of the world exports pie from the current 2% to 3.5% over the same period.
- Exports through 'Make in India' underlined in Foreign Trade policy by commerce Minister Nirmala Sitharaman. (Now she is finance minister)


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The Govt of India notifies the foreign trade policy for the period of five years (2015 To 2020] under the section 5 of the Foreign Trade Act 1992. The Current EXIM policy covers the period 2015 To 2020. The ~~FTP~~ FTP is updated every year on the 31 of March and modifications, improvements and new schemes become effective from 1st April of every year.

All types of changes or modifications related to the EXIM policy is normally announced by the union Minister of Commerce and Industry who co-ordinates with the Ministry of Finance, The Directorate General of Foreign Trade and Network of DGFT Regional offices.

Some Highlights of the present Foreign Trade policy (EXIM policy) 2015-2020. are as follows:

- India to be made a significant participant in world trade by 2020.
- EXIM policy 2015-20 introduced two new schemes
  (1) Merchandise [illegible] Export from India scheme (MEIS).
  (2) Service Exports from India scheme (SEIS).
  (These two schemes promote exports from India like commercial products & services).

- ~~New~~ New EXIM policies reduced obligations by 25% and ~~boo~~ boosted domastic production.
- New EXIM policies provided Incentives for SEZs. ~~als~~ also.
- E-commerce of handicrafts([illegible]), handloome([illegible]), booke etc are eligible for benifits of MEIS.
- Agricultural and village industry products to be supported accross the globe at rates of 3% to 5% under MEIS. Higher level of supports to be provided to processed and packaged agricultural and food items under MEIS.
- Industrial products to be supported in major markets at rates ranging from 2% to 3%.
- Manufacturers who are also status holder will be enabled to self certify their manufactured goods as originating from India. Tax and duty on Indian manufacturers ~~has~~ have been reduced to boost Make in India vision.
- Inter-ministeral consultations to be held online for issue of various licences.
- No need to repeatedly submit physical copies of documents which are already available at Exporter & Importer proofile.
- Validity period of SCOMET export authorisation extended from present 12 months to 24 months.

SCOMET = Export of Special chemicals, organisms, materials, Equipments and Technologies.


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- Calicut Airport, Kerala and Arakonam ICDs (Inland Container Depots), Tamilnadu notified as registered ports for Import & Export.
- Vishakhapatnam and Bhimavaram added as Towns of Export Excellence.
- Certificate from independent charted engineer for EPCG authorisation is no longer required.

EPCG = Export promoting Capital Goods.

## "An Overview of International Economic Institution and Their working".

The global economy is administered, financed and regulated by three institutions - The world Trade organization (WTO), The International Monetary Fund (IMF), and The International Bank for Reconstruction and Development. (IBRD).

### 1 WTO:

* The world Trade organization is an intergovernmental organization that regulates international trade.

* The WTO officially commenced on 1st January 1995. under the 'Marrakesh Agreement', signed by 123 nations by replacing the 'General Agreement on Tarriffs and Trade (GATT) which was commenced in 1948.


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* WTO is the largest international economic organization in the world.

* The WTO deals with regulation of trade in goods, services and intellectual property between participating countries by providing a frame work for negotiating trade agreements & a dispute resolution process aimed at enforcing participants adherence to WTO agreements.

* Headquarters: 1) Centre William Rappard.
  2) Geneva.
  3) Switzerland.

* Region served: world wide.
* Membership: 164 ~~6~~ member Countries
* Official language: English, French, Spanish.
* Director General: Roberto Azevedo.

* Staff: 640.
* website: www. wto. org

[signature]


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* WTO oversees the implementation, administration & operation of the covered agreements.

* WTO provides a forum for negotiations & settling disputes.

* The WTO shall administer Trade policy Review mechanism.

* WTO has been achieving greater coherence in global economic policy making.

* WTO has been cooperating with IMF & IBRD.

* Organizational structure:

1) Council for Trade in Goods.

2) Council for Trade-Related Aspects of Intellectual property Rights (TRIPS) [patents, copyrights, Design, Trademark, Trade secrets].

3) Council for Trade in Services.

4) Trade Negotiations Committee.


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* Principles of WTO:

1. Non-Discrimination.
2. ~~Bi~~ Binding of enforceable commitment.
3. Reciprocity [Exchange of things for mutual benefits).
4. Transparency.
5. safety valves.

* Decision making:

The WTO describes itself as "a rules based member driven organization - all decisions are made by the member governments and the rules are the outcome of negotiations among members". The WTO agreement foresees votes where consensus (সর্বসম্মত) can not be reached, but the practice of consensus dominates the process of decision making.

* Examples of some Trade related Agreements:

1. Agreement on Agriculture (AOA)
2. The General Agreement on Trade in Services.


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③ The agreement on Trade Related Aspects of Intellectual property Rights,

④ The agreement on Technical Barriers to trade [The object ~~of~~ ensures that technical negotiations of standard testing, certification procedures, Removing all kinds of obstacles to trade).

* India was one of the 76 Governments that become members of the WTO on the first day of the formation of the WTO.

* India is expected to play a leader's role for the developing countries in WTO. Recently, India is experiencing the problems from the ~~WTO~~ WTO due to dumping. Indian agriculture sector is affected ~~badly~~ Negatively compared to Industrial sector by dumping.

IMF:

International Monetary Fund is an international Organization headquartered in Washington, D.C. 189 Countries working to foster global monetary cooperation secure financial stability, facilitate international trade, promote high employment and sustainable economic growth, & reduce poverty around the world.


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* IMF is formed in 1945 at the Bretton woods Conference. At initial stage only 29 Countries were members.

* IFM plays a central role in the management of balance of payments. and avoids international financial Crises.

* Countries contribute funds to a pool through a quota system from which countries experiencing balance of payments problems can borrow money.

* offical Language → English.
* M.D.: christine Lagarde
* parent org: UNO.
* staff : 2700.
* web: imf.org

Functions: according to IMF itself, it works to foster global growth and economic stability by providing policy, advices and financing the members by working with all the nations to help them to achieve macroeconomic stability & reduce poverity


Scanned by CamScanner

* IMF oversee the fixed exchange rate arrangements between Countries. Thus helping governments to manage their exchange rates and allowing these governments to prioritise economic growth.

* IMF provides short term capital to aid the balance of payments. This assistance was meant to prevent the spread of international economic crises.

* IMF also recomends what type of government policy would ensure economic recovery.

WORLD BANK:

* Formation: July 1945.
* Type: Monetary International Financial org.
* Legal status: Treaty.
* Headquarters: Washington, D.C. US.
* Membership: 189 Countries.
* Key people: Jim Yong Kim (president).
* Website: www.worldbank.org.


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The world bank is an international financial institution that provides loans to countries of the world for capital projects. It comprises two institutions ~~to~~: The International Bank for Reconstruction and Development (IBRD), and The International Development Association (IDA). The world Bank is a component of the world Bank Group. The world Bank stated goals is the reduction of poverty which its Articles of Association defined as Commitments to the promotion of foreign investment and international trade and to facilitation of capital investment.

World Bank Groups:

(a) The world Bank is different ~~from~~ from the world bank group, an extended family of five international organizations:

1. IBRD.
2. IDA.
3. IFC [International finance Corporation]
4. MIGA [Multilateral Investment Guarantee Agency]
5. ICSID [International Centre for settlement of Investment Disputes].


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# Objectives and Functions of World Bank:

* To provide long-run capital to member countries for economic development.
* To inspire long-run capital investment among member countries.
* To provide guarantee for loans granted to small & large units and other projects of member countries.
* To ensure the implementation of development projects.
* providing various technical services to the member countries. (For this purpose the bank has established " The Economic Development Institute" and staff is in Washington.
* Determining the quantities of loans, interest rates and terms & conditions etc.
* providing loans to private investors belonging to member countries on its own guarantee, but for this loan private investors have to seek prior permission from those countries where this amount will be collected.


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# LMS MANAGEMENT SYSTEM

K. L. E. Society's
G. I. Bagewadi Arts, Science and Commerce College,
Nipani - 591237
Accredited at 'A' level by NAAC with CGPA 3.35
Affiliated to Rani Channamma University, Belagavi, Karnataka, India
Website : www.klegibnpn.edu.in ☎ (08338) 220116 E-mail : klegib_npn@yahoo.co.in

## LMS management system, e-recourses

| S. No. | Name of the Teacher | Tools and techniques (Zoom/Google Meet/Webex/ Teamlink, etc) | Subject Videos Recorded by Faculty and uploaded in YouTube / Google Drive (Provide the link) (Videos Recorded by Faculty only) | Google Classroom Code |
|---|---|---|---|---|
| 1. | Dr. C V Koppad | 12 (ZOOM) | **https://youtu.be/7Efl5NPNjw8** | otedoeb,<br>xjrqytq |
| 2. | Dr. A. S. Jaganure | (YouTube) | **https://youtu.be/-dxg3qvlHy4** | --- |
| 3. | Dr. S. B. Solabannavar | **(YouTube)** | https://youtu.be/nIuV_5FXzCs | --------- |
| | | | https://youtu.be/aOOpqt--DkM | --------- |
| | | | https://youtu.be/dnl5x6BoC_4 | --------- |
| 4. | Dr R. G. Kharabe | YOU TUBE | https://youtu.be/BtDysrHxQ_A | --- |
| | | | https://youtu.be/s5j7Ge0FuOk | --- |
| | | | (https://youtu.be/BgpzSTsFTsI | --- |
| | | | https://youtu.be/bRRBDapzv0c | --- |
| | | | https://youtu.be/JMsL_nw113s | --- |
| | | | https://youtu.be/BtDysrHxQ_A | --- |
| | | | https://youtu.be/s5j7Ge0FuOk | --- |
| | | | https://youtu.be/BgpzSTsFTsI | --- |
| 5. | Prof. Deepa Koppal | | https://youtu.be/kMm81bqTwdA | |

| | | | | |
|---|---|---|---|---|
| 6. | Dr.B.M.Hiremath | 02 (SKYPE) | https://youtu.be/prEmSVP6Mjg https://join.skype.com/UceKdh5Gx6v9 https://youtu.be/2i_M6vO11Xk | otedoeb, xjrqytq |
| 7. | Prof. Sangeeta Sansuddi | 10 (SKYPE) | https://join.skype.com/WTOo9yd6nxVh | otedoeb, xjrqytq, 2t7qw6n, v6iqohj |
| 8. | Prof. Abhijeet Tavakari | 06 (SKYPE) | https://join.skype.com/XgFYh15navOD | o2fwzhq, otedoeb, xjrqytq |
| 9. | Prof. Priyanka Kamate | 15 (ZOOM) | Meeting id: 9998156827 Password: 0NvRMk | o2fwzhq, otedoeb, 2t7qw6n, v6iqohj |
| 10. | Prof. S.A.Deshpande | 13 (ZOOM) | https://www.youtube.com/watch?v=gnwXcL0IDaU, https://www.youtube.com/watch?v=Gxr6IAzFJJk https://www.youtube.com/watch?v=S3BEJai4MjQ https://www.youtube.com/watch?v=AG_wK_Lb8RI meeting id : 7466647541 | o2fwzhq, otedoeb, 2t7qw6n, v6iqohj |
| 11. | Prof. Shruti Mirje | 08 (ZOOM) | https://youtu.be/sejOF5LOwwY, Meeting id: 7662581947 | o2fwzhq, otedoeb, 2t7qw6n, v6iqohj |

| | | | | |
|---|---|---|---|---|
| 12. | Prof. Snehal Hirekude | 05 (ZOOM) | https://youtu.be/28qKDRowR_Y | o2fwzhq, |
| 13. | Prof. Pallavi Ankali | 15 (ZOOM) | Meeting id: 8063927855 | 2t7qw6n,<br>v6iqohj |
| 14. | Dr. B.S. Kamble | **YouTube** | https://youtu.be/cJ2pVBOH0Oc | --------- |
| 15. | Dr. B.S. Kamble | **YouTube.** | https://youtu.be/dRdBqk_kO6M | --------- |
| 16. | Shri .M.S.VANAKI | **YouTube** | https://youtu.be/KzmLaRk51CE | --------- |
| 17. | Shri .M.S.VANAKI | **YouTube** | https://youtu.be/QAgVAvaIcGw | --------- |
| 18. | Shri .M.S.VANAKI | **YouTube** | https://youtu.be/QAgVAvaIcGw | --------- |
| **19.** | **Dr S.M.Raymane** | Zoom -01 | https://us04web.zoom.us/j/<br>713113189?pwd=bwznqkhHdERT<br>TWmYtB6ZHFoanLWUT09 | **7131131089**<br>**Password**<br>**:4KAEc9G** |
| **20.** | **Dr S.M.Raymane** | Zoom -01 | https://us04web.zoom.us/j/ | 7131131089<br>Password:4KA<br>Ec9G |
| 21. | Prof.V.B.Dharwad | 07<br>(Zoom App) | https://us04web.zoom.us/j/7892118217<br>9?pwd=N2pKMUNvUWZzZWtTd29T<br>V3lQN0NIZz09 | Meeting ID:<br>789 2118 2179 |
| 22. | Dr.(Smt).G.A.Chougala | 05<br>(Zoom App) | https://us04web.zoom.us/j/5719859409<br>?pwd=dHVrU0VvNmc2K0hMbkEzbW<br>RHejBnZz09 | Meeting<br>ID:<br>571 985 9409 |
| 23. | Shri.B.M.Janagouda | 06<br>(Zoom App) | https://us04web.zoom.us/j/3294681519<br>?pwd=a2txNkdJd1dVVmh6MFg3aFVp<br>YkdwUT09 | Meeting ID:<br>329 468 1519 |
| 24. | Shri.Kumara Talawar | 04<br>(Zoom App) | https://us04web.zoom.us/j/8444472980<br>?pwd=NkVBRmliZURyTUY2MCtyN<br>EU2YWFVZz09 | Meeting ID:<br>844 447 2980 |
| 25. | Prof. Vinayak Sutar | 08 (Zoom) | ------------ | 3914705447 |

| | | | | |
|---|---|---|---|---|
| 26. | Prof. S.S.Kothiwale | 01 (Zoom App) | https://us04web.zoom.us/j/6471051353 ?Pwd=TG1DaU1mUk5SRONkWkJHO HJ5Zy9zdz09 | Meeting ID: 6471051353 |
| 27. | Prof . S.S.Ghorpade | 01 (Zoom App) | https://us04web.zoom.us/j/6471051353 ?Pwd=TG1DaU1mUk5SRONkWkJHO HJ5Zy9zdz09 | Meeting ID: 6471051353 |
| 28. | Prof . M.S.Holimath | 01 (Zoom App) | https://us04web.zoom.us/j/6471051353 ?Pwd=TG1DaU1mUk5SRONkWkJHO HJ5Zy9zdz09 | Meeting ID: 6471051353 |
| 29. | Prof. S.S.Kothiwale | 01 (Zoom App) | https://us04web.zoom.us/j/6471051353 ?Pwd=TG1DaU1mUk5SRONkWkJHO HJ5Zy9zdz09 | Meeting ID: 6471051353 |
| 30. | Prof . S.S.Ghorpade | 01 (Zoom App) | https://us04web.zoom.us/j/6471051353 ?Pwd=TG1DaU1mUk5SRONkWkJHO HJ5Zy9zdz09 | Meeting ID: 6471051353 |
| 31. | Smt. S. B. Patil | - | Google class | Mngzttb |
| | | | Google class | Kokggm4 |
| 33. | Dr. Smt S. V. Shirgave | Zoom | https://us04web.zoom.us/j/73318806067?pwd=UGNGcmpYOVRpYVZhemltcEkzSk5XQT09 Google class | Yl7cqad |
| 34 | Miss Priya Patil | zoom | https://us04web.zoom.us/j/2415997107?pwd=MGtMQ1ZrUTEyUEI3VkVmSEh1Y0FYZz09 | |
| 34. | Smt. S. S. Sunnal | | Googleclass | J36ccbf |
| | Smt. S. B. Patil | - | Google class | Mngzttb |
| | | | Google class | Kokggm4 |

| | | | | |
|---|---|---|---|---|
| 36. | Miss Priya Patil | One on zoom | https://us04web.zoom.us/j/2415997107 ?pwd=MGtMQ1ZrUTEyUEI3VkVmS Eh1Y0FYZz09 | |
| | Smt. S. S. Sunnal | | Googleclass | J36ccbf |

## Department Of Mathematics

| UG | | | | |
|---|---|---|---|---|
| **Name of the Teacher** | **No. of online classes engaged (zoom/Google meet/Webex/Teamlink, etc.** | | **Subject videos recorded by faculty and uploaded in you tube / Google drive(provide the link) (Videos Recorded by faculty only)** | **Google class room code** |
| 37. Dr. M. M. Shankrikopp | B.Sc. II sem. | 36 Zoom and small videos | **Unit IV : Cone**<br>https://www.youtube.com/watch?v=H5faRd57rtY<br>https://www.youtube.com/watch?v=EH7inJeGGOo<br>https://www.youtube.com/watch?v=-Fh4k4_snlY<br>https://www.youtube.com/watch?v=t6bTa34FXnU<br>https://www.youtube.com/watch?v=N2Fe4U9mpNc<br>https://www.youtube.com/watch?v=updEvI2gs48<br>https://www.youtube.com/watch?v=-tu1lqT_tUw<br>https://www.youtube.com/watch?v=fm6qlvYaAZE<br>https://www.youtube.com/watch?v=wMpKZGHt5kw<br>https://www.youtube.com/watch?v=N54s7SFuF7Y<br>https://www.youtube.com/watch?v=64ofjLHCYCo<br>https://www.youtube.com/watch?v=v_XrfgRha_U<br>https://www.youtube.com/watch?v=64ofjLHCYCo<br>https://www.youtube.com/watch?v=smnRqdI7mHY<br>**Unit V: Cylinder**<br>https://www.youtube.com/watch?v=UXfGkde0lFA<br>https://www.youtube.com/watch?v=U32XLFw1Yag<br>https://www.youtube.com/watch?v=MGTSqPmBT_I<br>https://www.youtube.com/watch?v=zjhI8mCGAqc<br>https://www.youtube.com/watch?v=qPo3CquXTWs<br>https://www.youtube.com/watch?v=huqLRrWU2Gg<br>https://www.youtube.com/watch?v=5A5SwCkNEtc<br>https://www.youtube.com/watch?v=krlIhHgR0Wo<br>https://www.youtube.com/watch?v=EVFGb0WEPqk<br>https://www.youtube.com/watch?v=OPK9K2A7BvQ<br>https://www.youtube.com/watch?v=HA_QWMexY3k<br>https://www.youtube.com/watch?v=p00B1HdsKrg<br>https://www.youtube.com/watch?v=cEyqGg1vpk8 | Zoom : Personal meeting ID 664-125-6874 |

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|---|---|---|---|---|
| | | | **Unit III: Envelopes**<br><br>https://www.youtube.com/watch?v=WGB4LChS08I<br>https://www.youtube.com/watch?v=3ywV9z_NwDI | |
| | B.Sc. IV sem | Google forms | **Online test conducted**<br>**Paper I: 5.5.2020**<br>https://docs.google.com/forms/d/e/1FAIpQLSdd4JcY4PsMch0sCv9pRWkaLy6wn3oChD59u0rFtFyzBaQSiA/viewform?usp=sf_link | - |
| | B.Sc. VI sem. | youtube | **Unit IV of paper II**<br>https://youtu.be/nwR1ayS3pl<br>https://youtu.be/iV_9UfeWjPE<br>https://youtu.be/iV_9UfeWjPE<br>https://youtu.be/WZHiaHZ_M | |
| 38. Miss G. L. Karguppi. | B.Sc. II sem | 4 | **Unit III: Asymptotes**<br><br>https://www.youtube.com/watch?v=h065bwLh2N8<br>https://www.youtube.com/watch?v=XbYGtg1YgfI<br>https://www.youtube.com/watch?v=1vzU5HQCto4<br>https://www.youtube.com/watch?v=eQr3GzvqTsw | |
| | B.Sc. IV Sem. | 6 | **Unit IV and V: Linear Differential Equations**<br><br>https://www.youtube.com/watch?v=6_7ce_SDrJA<br>https://www.youtube.com/watch?v=r6ZQJaeVBP4<br>https://www.youtube.com/watch?v=4rWZ6i3l2rA<br>https://www.youtube.com/watch?v=gsHiCTJWk3E<br>https://www.youtube.com/watch?v=g-nJ9CkiJCI<br>https://www.youtube.com/watch?v=ghE2_6X5sDY | |
| | B.Sc. VI Sem. | 6 | **Unit IV and V of paper III: Laplace Transform**<br>https://www.youtube.com/watch?v=0z3yj4XN0Ck<br>https://www.youtube.com/watch?v=jwKDPW0yEOk<br>https://www.youtube.com/watch?v=bws-Uk2aUY0<br>https://www.youtube.com/watch?v=VgRkEGwbWWU<br>https://www.youtube.com/watch?v=4Hc_4kqprgo<br>https://www.youtube.com/watch?v=kJm_EdZMxtw | |

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|---|---|---|---|---|
| 39. Prof.Sammed Chougale | B.Sc II Sem. | 6 Zoom app | **B. Sc. II Sem, Paper II: Number Theory**<br><br>https://www.youtube.com/watch?v=YAFadtcTruA<br>https://www.youtube.com/watch?v=nclu1pZzo-A<br>https://www.youtube.com/watch?v=Cvmi91IMiKM<br>https://www.youtube.com/watch?v=r0EjAbaerQc<br>https://www.youtube.com/watch?v=AIDI7tyBqW8<br>https://www.youtube.com/watch?v=UqwjhVPieRI | Personal meeting ID 745 424 06796 |
| 40. Prof. J. N. Magadum | B. Sc. IV sem. | 7 videos | **Fourier series : channel link**<br>https://t.me/joinchat/AAAAAFRl1OnPMxWZyTz7hg | |
| 41.Prof. Miss Vinaya Khot | B. Sc. IV sem. | 2 videos | **Cauchy 's Integral test**<br>telegram channel | |
| 42. Smt. K. N.Samaje | M.Sc I and II | zoom | | Personal meeting ID 745 424 06796 |

| **Class** | **Paper** | **Date** | **Google form Link** | **Time** |
|---|---|---|---|---|
| B.Sc II sem. | I | 11.5.2020 | https://docs.google.com/forms/d/e/1FAIpQLSdBmWTuXOXhBq8-dPY7BqmrDfA-vJ2tkKEh293GEg2S-xX0QQ/viewform?usp=sf_link | 11.00 am to 12.00 |
| | II | 4.5.2020 | https://docs.google.com/forms/d/e/1FAIpQLSf02REpTJpMhjZ4M42XBwt8CR48_qmMlzacs7TynvtRQi23RQ/viewform?usp=sf_link | 3.00 pm to 4.00 pm |
| B. Sc. IV Sem. | I | 5.5.2020 | https://docs.google.com/forms/d/e/1FAIpQLSdd4JcY4PsMch0sCv9pRWkaLy6wn3oChD59u0rFtFyzBaQSiA/viewform?usp=sf_link | 11.00 am to 12.00 |
| | II | 12.5.2020 | https://docs.google.com/forms/d/e/1FAIpQLSds3ii9b8QGQ-JNjo10IAwOP_vRYI31Ei7YkoeO9zb4UpvRlQ/viewform?usp=sf_link | 11.00 am to 12.00 |
| B.Sc.VI sem | I | 12.05.2020 | https://docs.google.com/forms/d/e/1FAIpQLScr-nE3DSaqeSOQNl9LHee6F4Rdq8jNhnJYSe02pvjHaflY7g/viewform?usp=sf_link | 11.00 am to 12.00 |
| | II | 04.05.2020 | https://docs.google.com/forms/d/e/1FAIpQLSc6D7tkUQ_WV_IiJbn7faNBNnx2NKlr2Uv70RK2oT2PWtC1mA/viewform?usp=sf_link | 11.00 am to 12.00 |
| | III | 13.05.2020 | https://docs.google.com/forms/d/e/1FAIpQLSf1aw2tuPySB5ojlYqG4Zp1gUJrwjvRkFKMv-H-5b42D3voOw/viewform?usp=sf_link | 11.00 am to 12.00 |
| Feedback on online teaching | | 20.05.2020 | https://docs.google.com/forms/d/e/1FAIpQLSf1aw2tuPySB5ojlYqG4Zp1gUJrwjvRkFKMv-H-5b42D3voOw/viewform?usp=sf_link | - |

**PG**

| Name of the Teacher | No. of online classes engaged (zoom/Google meet/Webex/Teamlink etc. | | Subject videos recorded by faculty and uploaded in you tube / Google drive(provide the link) (Videos Recorded by faculty only) | Google class room code |
|---|---|---|---|---|
| Prof. Sammed Chougale | M.Sc. II Sem. | 17 Zoom | Classes engaged on zoom meeting not uploaded in you tube Zoom PMI: 845 424 0679 | pad3hno |
| | M.Sc IV sem. | 15 Zoom | | 7rvacme |
| Prof. Jinendra Magadum | M.Sc. II Sem. | 26 videos | Partial Differential equation: channel link https://t.me/joinchat/AAAAAFBMCpA9z8My2cFLKw | pad3hno |
| | M.Sc IV sem. | 9 videos | Advanced Numerical Analysis: channel link https://t.me/joinchat/AAAAAE8BNY0LEWSZP8Fycw | 7rvacme |
| Miss Vinaya Khot | M.Sc. II Sem. | 5 Videos | https://youtu.be/M3uVGP7vnqw Remaining three not uploaded in you tube because of NET problem Sent to students through telegram channel | pad3hno |
| | M.Sc IV sem. | 6 Videos | Sent to students through telegram channel | 7rvacme |
| Prof. Karuna Samaje | M.Sc. II Sem. | Zoom 25 | Classes engaged on zoom meeting not uploaded in you tube Zoom PMI:840-407-0490 | pad3hno |
| | M.Sc IV sem. | Zoom 10 | | 7rvacme |
| Prof.(Miss) Nikita Jadhav | M.Sc. II Sem. | 4 Videos | Telegram channel https://t.me/joinchat/AAAAAEbXI5a7JAhTH6gPMw remaining 3 are not uploaded in you tube but sent to students through whatsapp. | 36jhs6q |
| | M.Sc IV sem. | 8 Videos | Videos sent through telegram channel not uploaded in you tube | c4v3cjn |
| Internal Test | M.Sc. II and M.Sc. IV sem. | | Internal Test was conducted from 12.05.2020 to 17.05.2020 In Google class room. | |

## Study Material Links:

### 1) Shri V.B.Dharwad

1. **https://drive.google.com/file/d/1ddX2XsgDSfYkBYzz_aWh86qBQGhPbs8d/view?usp=sharing**
2. **https://drive.google.com/file/d/17pR-JMnX6II36cz5cX5FtMEMWcm4uQuw/view?usp=sharing**

3. https://drive.google.com/file/d/1QYEQ2juYDkCdKNfXI9nIahoARCaqr7kK/view?usp=sharing
4. https://drive.google.com/file/d/1CEdPsPWogJmS-lvx4ZvxWiT2ZL7JAwWS/view?usp=sharing
5. https://drive.google.com/file/d/1iOuZhBYKxCLSZ-FApfyAkzH7La7zTzuU/view?usp=sharing
6. https://drive.google.com/file/d/1_uK_c3jBWVu18vmuNBXa7rwT6sv4kiBk/view?usp=sharing
7. https://drive.google.com/file/d/1DbSISI_orfhp4oNPlqTwvR17xsVTyqhr/view?usp=sharing

## 2) Dr.(Smt) G.A.Chougala

1. https://drive.google.com/file/d/1LXZN_RyFw_Kju15LNcD_F7zCq04LF-n6/view?usp=drivesdk
2. https://drive.google.com/file/d/1LYXOlgUHF9m_GcEPxQsg_4hM4mWEmaUE/view?usp=drivesdk
3. https://drive.google.com/file/d/1L_AdlILRb-R2nHTJnwJTQmfauvX6UdJK/view?usp=drivesdk
4. https://drive.google.com/file/d/1LgQ-EK58TaA3EMjB7JV8on7WI3mocI1s/view?usp=drivesdk

## 3) Dr. B.M.Janagouda

1. https://drive.google.com/file/d/1SLoX4VOPHbKII0Luca4ftlUERA6Su4Ar/view?usp=sharing
2. https://drive.google.com/file/d/1S1F-Z1883OdofJ0iRxm1c30QOEJ1Pnfo/view?usp=sharing
3. https://drive.google.com/file/d/1SAFfTc5w9bSkB-og2qMMzX7VKy-_FB5T/view?usp=sharing
4. https://drive.google.com/file/d/1SUK7ssWrQUL6KUmt_PdNSAReLa-Dnb5l/view?usp=sharing
5. https://drive.google.com/file/d/1Skf6qodiO-v8qrinvqUZLyGRb_RvGRnN/view?usp=sharing
6. https://drive.google.com/file/d/1Rtnr1IkIOkiBd5N9zy_hH5scRGyENVI3/view?usp=sharing
7. https://drive.google.com/file/d/1Swewz40dmPLBFGGN5grjqDfDqx7csDq_/view?usp=sharing
8. https://drive.google.com/file/d/1RZUYy_iFkpKXdN_ojhRS02VAJtcoAbht/view?usp=sharing
9. https://drive.google.com/file/d/1Ru8qMYbWs2EDiKMcmFlZj4_q05oRcb5V/view?usp=sharing

10. https://drive.google.com/file/d/1STWMZ7k7_9I0IpUI5NoyzEPFe_kyFkNG/view?usp=sharing
11. https://drive.google.com/file/d/1SqkRGG8QxWNkIQwQfx3vRw7hg0nJTZnR/view?usp=sharing
12. https://drive.google.com/file/d/1T4JK6Z52UHN1HhVt9XYsMspoHJJssfdO/view?usp=sharing
13. https://drive.google.com/file/d/1S-Ma3Al92eu3umUQgUWD6zwf3ctIKYTh/view?usp=sharing
14. https://drive.google.com/file/d/1-Hw1Jdu2OpsPW6cLiV9YbFRuXARdfu8y/view?usp=sharing
15. https://drive.google.com/file/d/1SyuKwWTC4OG9GLRrRG0uXBe27L9OentH/view?usp=sharing
16. https://drive.google.com/file/d/1TLc8CcpuQjjLiM4odgxVCBLB1036DSl9/view?usp=sharing
17. https://drive.google.com/file/d/1TJ917Fk7Wh2v_aLzkRR6g9bs-lyrRhsp/view?usp=sharing
18. https://drive.google.com/file/d/1TPoj6r9HoTgcDgTxeECK5406q3_JJT0M/view?usp=sharing
19. https://drive.google.com/file/d/1TgOJL68yKZN4cT8Bo2Z0jFU08FDB3pGP/view?usp=sharing
20. https://drive.google.com/file/d/1TQCNfkMxfsxDaB12jUpt2HFLM90hdWeD/view?usp=sharing
21. https://drive.google.com/file/d/1TDjdd2W1cY0atGcaywBvxe3PqOn7UNQA/view?usp=sharing
22. https://drive.google.com/file/d/1TvxYkBwuD0sByangcdgGRZ5ytTw6AyKu/view?usp=sharing